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A MULTIPLET TABLE FOR NEUTRAL HELIUM (^{4}He I) WITH TRANSITION RATES

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ABSTRACT

This paper combines the precise determination of the energy levels of ^{4}He I from calculations and experiments with theoretical transition probabilities to present multiplet tables and finding lists for the fine structure of the helium atom. The tabulated transition rates and oscillator strengths include corrections for singlet–triplet mixing and spin-orbit coupling, but not the higher order relativistic terms nor the finite nuclear mass, although the latter are tabulated for future use. The results are consistent with laboratory lifetimes and oscillator strengths, but very few measurements are accurate enough to be stringent tests. An Appendix discusses the corrections for finite nuclear mass.

Subject headings: atomic data — ISM: atoms — stars: atmospheres

Online material: machine-readable tables

1. INTRODUCTION

Since helium is the second most abundant element in the universe, it is important to have an accurate knowledge of the wavelengths and oscillator strengths of its spectral lines to interpret observations. The present paper provides these basic data with improved accuracies over earlier compilations for the abundant isotope ^{4}He of the neutral atom.

Although the three-body helium atom cannot be calculated exactly, detailed results in a series of papers by Drake & Martin (1998), Drake & Goldman (1999), and Morton et al. (2006) predict extremely accurate level separations from the ionization limit. In the last paper the authors combined these theoretical *ionization energies* with precise laboratory measurements of a few low-lying transitions to derive energy levels for ^{4}He I with relative errors of 0.2 MHz $= 6.7 \times 10^{-6}$ cm^{-1} or less for all but seven of the lowest levels. They also calculated isotope shifts and hyperfine shifts to produce equally precise levels for the rare ^{3}He isotope. The present paper combines these results with calculations of transition probabilities to generate multiplet tables for ^{4}He I.

This paper is restricted to the permitted and semiforbidden electric-dipole transitions of ^{4}He I with $n \leq 10$ and $l \leq 7$. Bauman et al. (2005) have considered transitions involving higher levels of neutral helium as well as its photoionization and recombination. Other interesting features of the He I spectrum include magnetic-dipole lines calculated by Drake (1971), magnetic quadrupole lines by Drake (1969), Baklanov & Denisov (1997), and Lach & Pachucki (2001), electric-quadrupole transitions by Cann & Thakkar (2002), two-photon decays by Derevianko & Johnson (1997), and doubly excited states by Wu et al. (2003).

In this paper we will follow the usual practice with helium by omitting the $1s$, l, and parity labels on the spectroscopic terms, so that for example 3 $^3P_2$ represents $1s3p$ $^3P^o_2$.

2. ENERGY LEVELS, WAVENUMBERS, AND WAVELENGTHS

Most of the ^{4}He energy levels used here originate from calculations that adopted values for the Rydberg R_∞, the fine-structure constant α, and the mass ratio M/m_e of the nucleus to the electron from the CODATA 2002 revision of the fundamental constants (Mohr & Taylor 2005). An additional parameter was the nuclear charge radius $r_c(^4\mathrm{He}) = 1.673(1)$ fm from Borie & Rinker (1978).

Through connections with the theoretical ionization energies of 3 $^3D_{3,2,1}$ and 3 $^1D_2$, laboratory measurements of ^{4}He provided the levels 2 $^3S_1$, 2 $^1S_0$, 2 $^3P_{2,1,0}$, 2 $^1P_1$, 3 $^1S_0$, 3 $^3P_{2,1,0}$, 3 $^1P_1$, 4 $^3S_1$, and 5 $^3S_1$ as well as the ionization potential (IP) of 5,945,204,290(33) MHz $=$ 198,310.6690(11) cm^{-1} from the 1 $^1S_0$ ground state. The 0.0011 cm^{-1} uncertainty in the IP affects the absolute values of all levels, and produces an error of 4×10^{-6} Å in the extreme ultraviolet resonance lines at wavelengths less than 600 Å, but cancels out in all other differences tabulated here. Table 1 lists the errors for the seven largest cases and the corresponding errors for wavelengths shortward of 10000 Å for typical transitions. The error for all other levels is 0.2 MHz $= 6.7 \times 10^{-6}$ cm^{-1} or less.

Wavenumbers ν_{ul} in cm^{-1} and wavelengths λ_{ul} in Å are calculated from the respective upper and lower energy levels E_u and E_l in cm^{-1} according to

$$\nu_{ul} = E_u - E_l, \quad \lambda_{ul} = 10^8/\nu_{ul} \tag{1}$$

and mean values $\bar{\nu}_{ul}$ and $\bar{\lambda}_{ul}$ for a multiplet from the mean energies

$$\bar{E}_u = \frac{1}{g_{Mu}} \sum_u g_u E_u \quad \text{and} \quad \bar{E}_l = \frac{1}{g_{Ml}} \sum_l g_l E_l, \tag{2}$$

where the statistical weights are $g = 2J + 1$ and

$$g_{Mu} = \sum_u g_u \quad \text{and} \quad g_{Ml} = \sum_l g_l. \tag{3}$$

3. TRANSITION PROBABILITIES

The theoretical transition probabilities and oscillator strengths adopted here follow from the formulation described by Drake (2006) using simplified wave functions appropriate for infinite

TABLE 1
^{4}He I Levels with Largest Uncertainties

Level	Energy E (cm^{-1})	E Error (cm^{-1})	λ Error (Å)
1 $^1S_0$	198310.6690	0.0011	0.000004
3 $^1S_0$	184864.829321	0.000033	0.000030
3 $^3S_1$	183236.791701	0.000067	0.000033
4 $^1S_0$	190940.226355	0.000020	0.000005
4 $^3S_1$	190298.113260	0.000023	0.000005
5 $^1S_0$	193663.512095	0.000010	0.000002
5 $^3S_1$	193346.991344	0.000010	0.000002
All other levels		<0.000007	<0.000007

nuclear mass, but now with the explicit inclusion of the relativistic effects of singlet–triplet mixing and spin-orbit coupling as perturbations. However, these calculations do not include higher order relativistic terms nor the corrections for the finite nuclear mass, namely, the decreased Rydberg constant also known as mass scaling, the radiation of the nucleus moving about the center of mass, and the mass-polarization term resulting from the transformation of the Hamiltonian to coordinates centered on the nucleus. These last effects are discussed in the Appendix.

Thus, for a radian frequency $\omega_{ul} = 2\pi c/\lambda_{ul}$, the transition probability or transition rate is

$$A_{ul} = \frac{4\alpha}{3c^2}\omega_{ul}^3 |\langle\psi_l|\boldsymbol{r}_1 + \boldsymbol{r}_2|\psi_u\rangle|^2, \quad (4)$$

where α is the fine-structure constant, $\boldsymbol{r}_1 + \boldsymbol{r}_2$ is the dipole operator, and ψ_u and ψ_l are the wave functions corresponding to the above approximations. The reciprocal lifetime of a state u is the sum of the decays to all lower states,

$$1/\tau_u = \sum_l A_{ul}. \quad (5)$$

The absorption oscillator strength f_{lu} for infinite nuclear mass is

$$f_{lu} = \frac{2m_e}{3\hbar}\omega_{ul}|\langle\psi_l|\boldsymbol{r}_1 + \boldsymbol{r}_2|\psi_u\rangle|^2 = \frac{m_e}{8\pi^2\alpha\hbar}\lambda_{ul}^2\frac{g_u}{g_l}A_{ul}, \quad (6)$$

For internal consistency we have used the calculated nonrelativistic mean multiplet values of wavelengths and frequencies for infinite nuclear mass quoted in Tables 2 and 3 for these derivations of A_{ul} and f_{lu} rather than the true values known from Morton et al. (2006). The quantity ν/ν_∞ in these tables indicates that the maximum error is 0.37% and typically it is much smaller.

Multiplet values A_{Mul} and f_{Mlu} are useful for comparisons with earlier calculations, which usually assumed LS coupling, ignored singlet–triplet mixing, and quoted only multiplet averages. Following Wiese et al. (1966), the triplet means are determined by summing over the allowed triplet–triplet transitions,

$$(\bar{\lambda})^3 g_{Mu} A_{Mul} = \sum_u g_u \sum_l \lambda^3 A_{ul} \quad (7)$$

and

$$\bar{\lambda} g_{Ml} f_{Mlu} = \sum_l g_l \sum_u \lambda f_{lu}, \quad (8)$$

where $\bar{\lambda}_{ul} = 10^8/(\bar{E}_u - \bar{E}_l)$ for $\bar{\lambda}_{ul}$ in Å and $\bar{E}$ in cm^{-1} from equation (2). The singlet means are simply the singlet–singlet values. Note that many authors of theoretical papers condense their tabulations by quoting emission f-values f_{ul}, sometimes with a minus sign, when S lies above P or P above D.

4. TABLE FORMAT

Tables 2 and 3 present the results for the singlet–singlet and triplet–triplet multiplets of ^{4}He I while Tables 4, 5, 6, and 7 give the results for individual lines including the intersystem singlet–triplet transitions and Tables 8 and 9 provide finding lists. For both upper and lower levels, the computations ranged over $n = 1$ to 10, $S = 0$ and 1, $l = L = 1$–7, and all permitted J. From these combinations we have limited the tabulated output to transition wavenumbers >1 cm^{-1} or wavelengths $<10^8$ Å.

4.1. *Tables 2 and 3*

Tables 2 and 3 present various parameters pertaining to whole multiplets that are useful in assessing the uncertainties and relating these results to earlier papers, most of which quoted only multiplet values. Note that these are means of the singlet–singlet and triplet–triplet transitions separately.

N.—Sequential multiplet number.

Lower–upper.—Level designations $n^{2S+1}L$.

g_{Mu}, g_{Ml}.—Total statistical weights for the upper and lower terms from equation (3).

$\bar{\lambda}$ (Å).—Mean multiplet wavelength in vacuum or, if $\lambda >$ 10000 Å, $\bar{\nu}$ (cm^{-1}) mean wavenumber, the true values from equations (1) and (2).

$\bar{\lambda}_\infty$(Å) *or* $\bar{\nu}_\infty$ (cm^{-1}).—Mean multiple nonrelativistic wavelength or wavenumber for infinite nuclear mass corresponding to the energy used in the calculation of A and f.

TABLE 2
^{4}He Singlet–Singlet Transitions

N	Lower–Upper	g_{Ml}–g_{Mu}	Wavelength (Å) Wavenumber (cm^{-1}) λ or ν	Infinite Nuclear Mass λ_∞ or ν_∞	Ratio ν/ν_∞	η	τ (ns)	f_{Mlu}	f_{Mul}
1	1 ^1S–2 ^1P	1–3	584.334357	584.234477 A	0.999829071	−0.283	5.5528E−01	2.7616E−01	9.2053E−02
2	1 ^1S–3 ^1P	1–3	537.029918	536.937713 A	0.999828305	0.211	1.7243E+00	7.3435E−02	2.4478E−02
3	1 ^1S–4 ^1P	1–3	522.213086	522.123498 A	0.999828446	0.417	3.9640E+00	2.9863E−02	9.9543E−03
4	1 ^1S–5 ^1P	1–3	515.616842	515.528467 A	0.999828603	0.526	7.6249E+00	1.5040E−02	5.0132E−03
5	1 ^1S–6 ^1P	1–3	512.098563	512.010850 A	0.999828719	0.593	1.3059E+01	8.6277E−03	2.8759E−03

Table 2 is available in its entirety in the electronic edition of the *Astrophysical Journal Supplement*. A portion is shown here for guidance regarding its form and content.

TABLE 3

^{4}He Triplet–Triplet Multiplets

N	Lower–Upper	$g_{Ml}-g_{Mu}$	Wavelength (Å) Wavenumber (cm^{-1}) λ or ν	Infinite Nuclear Mass λ_∞ or ν_∞	Ratio ν/ν_∞	η	τ (ns)	f_{Mlu}	f_{Mul}
296..................	2 ^3S–2 ^3P	3–9	9230.935878	9232.241142 cm^{-1}	0.999843050	−1.187	9.7886E+01	5.3907E−01	1.7969E−01
297..................	2 ^3S–3 ^3P	3–9	3889.744806	3889.374083 A	0.999903143	7.555	9.4805E+01	6.4461E−02	2.1487E−02
298..................	2 ^3S–4 ^3P	3–9	3188.665402	3188.366131 A	0.999905627	5.888	1.3852E+02	2.5769E−02	8.5896E−03
299..................	2 ^3S–5 ^3P	3–9	2945.964405	2945.687014 A	0.999905600	5.334	2.1929E+02	1.2491E−02	4.1635E−03
300..................	2 ^3S–6 ^3P	3–9	2829.913164	2829.645530 A	0.999905294	5.065	3.3833E+02	6.9823E−03	2.3274E−03

Table 3 is available in its entirety in the electronic edition of the *Astrophysical Journal Supplement*. A portion is shown here for guidance regarding its form and content.

TABLE 4

Multiplet Table for ^{4}He Singlet–Singlet Transitions

N	Lower–Upper	J–J	Air Wavelength (Å)	Vacuum Wavelength (Å) or Wavenumber (cm^{-1})	E_l (cm^{-1})	E_u (cm^{-1})	A_{ul} (s^{-1})	Sum A_{ul} (s^{-1})	f_{lu}	f_{lu}/f_{LS}
1..................	1 ^1S–2 ^1P	0–1		584.334357	0.0	171134.896946	1.7989E+09	1.8009E+09	2.7616E−01	1.0000
2..................	1 ^1S–3 ^1P	0–1		537.029918	0.0	186209.364940	5.6634E+08	5.7996E+08	7.3435E−02	1.0000
3..................	1 ^1S–4 ^1P	0–1		522.213086	0.0	191492.711909	2.4356E+08	2.5227E+08	2.9863E−02	1.0000
4..................	1 ^1S–5 ^1P	0–1		515.616842	0.0	193942.462294	1.2582E+08	1.3115E+08	1.5040E−02	1.0000
5..................	1 ^1S–6 ^1P	0–1		512.098563	0.0	195274.908466	7.3174E+07	7.6575E+07	8.6277E−03	1.0000
6..................	1 1S–7 ^1P	0–1		509.998293	0.0	196079.087570	4.6224E+07	4.8499E+07	5.4055E−03	1.0000
7..................	1 1S–8 ^1P	0–1		508.643376	0.0	196601.400247	3.1031E+07	3.2619E+07	3.6095E−03	1.0000
8..................	1 1S–9 ^1P	0–1		507.718095	0.0	196959.692816	2.1826E+07	2.2974E+07	2.5296E−03	1.0000
9..................	1 1S–10 ^1P	0–1		507.058021	0.0	197216.089562	1.5929E+07	1.6784E+07	1.8413E−03	1.0000

Table 4 is available in its entirety in the electronic edition of the *Astrophysical Journal Supplement*. A portion is shown here for guidance regarding its form and content.

TABLE 5

Multiplet Table for ^{4}He Triplet–Triplet Transitions

N	Lower–Upper	J–J	Air Wavelength (Å)	Vacuum Wavelength (Å) or Wavenumber (cm^{-1})	E_l (cm^{-1})	E_u (cm^{-1})	A_{ul} (s^{-1})	Sum A_{ul} (s^{-1})	f_{lu}	f_{lu}/f_{LS}
296............	2 ^3S–2 ^3P	Mean	10830.1711	10833.137758	159855.974330	169086.910208	1.0216E+07	1.0216E+07	5.3907E−01	1.0000
		1–2	10830.3398	10833.306444	159855.974330	169086.766473	1.0216E+07	1.0216E+07	2.9948E−01	1.0000
		1–1	10830.2501	10833.216751	159855.974330	169086.842898	1.0216E+07	1.0216E+07	1.7969E−01	1.0000
		1–0	10829.0911	10832.057472	159855.974330	169087.830813	1.0216E+07	1.0216E+07	5.9897E−02	1.0000
296............	2 ^3S–2 ^3P	Mean		9230.935878 cm−1	159855.974330	169086.910208	1.0216E+07	1.0216E+07	5.3907E−01	1.0000
		1–2		9230.792143 cm−1	159855.974330	169086.766473	1.0216E+07	1.0216E+07	2.9948E−01	1.0000
		1–1		9230.868568 cm−1	159855.974330	169086.842898	1.0216E+07	1.0216E+07	1.7969E−01	1.0000
		1–0		9231.856483 cm−1	159855.974330	169087.830813	1.0216E+07	1.0216E+07	5.9897E−02	1.0000

Table 5 is available in its entirety in the electronic edition of the *Astrophysical Journal Supplement*. A portion is shown here for guidance regarding its form and content.

TABLE 6

MULTIPLET TABLE FOR ^{4}He SINGLET–TRIPLET TRANSITIONS

N	Lower–Upper	J–J	Air Wavelength (Å)	Vacuum Wavelength (Å) or Wavenumber (cm^{-1})	E_l (cm^{-1})	E_u (cm^{-1})	A_{ul} (s^{-1})	Sum A_{ul} (s^{-1})	f_{lu}
557.............	1 ^1S–2 ^3P	0–1		591.412071	0.0	169086.842898	1.7758E+02	1.0216E+07	2.7935E−08
558.............	2 ^1S–2 ^3P	0–1		2809.402757 cm−1	166277.440141	169086.842898	2.9656E−02	1.0216E+07	1.6899E−08
559.............	2 ^1P–3 ^3D	1–2	6679.6768	6681.521139	171134.896946	186101.548689	1.5101E+04	7.0719E+07	1.6845E−04
560.............	2 ^1P–4 ^3D	1–2	4922.4093	4923.783479	171134.896946	191444.482131	2.4751E+03	3.1192E+07	1.4993E−05
561.............	2 ^1P–5 ^3D	1–2	4388.1483	4389.381125	171134.896946	193917.151929	8.7627E+02	1.6411E+07	4.2184E−06
562.............	2 ^1P–6 ^3D	1–2	4143.8791	4145.047688	171134.896946	195260.071736	4.2072E+02	9.6697E+06	1.8062E−06
563.............	3 ^1S–3 ^3P	0–1		699.754574 cm−1	184864.829321	185564.583895	2.3266E−03	1.0548E+07	2.1370E−08
564.............	3 ^1P–4 ^3D	1–2		5235.117191 cm−1	186209.364940	191444.482131	8.9436E+02	3.1192E+07	8.1539E−05

Table 6 is available in its entirety in the electronic edition of the *Astrophysical Journal Supplement*. A portion is shown here for guidance regarding its form and content.

TABLE 7

MULTIPLET TABLE FOR ^{4}He TRIPLET–SINGLET TRANSITIONS

N	Lower–Upper	J–J	Air Wavelength (Å)	Vacuum Wavelength (Å) or Wavenumber (cm^{-1})	E_l (cm^{-1})	E_u (cm^{-1})	A_{ul} (s^{-1})	Sum A_{ul} (s^{-1})	f_{lu}
742.............	2 ^3S–2 ^1P	1–1	8863.6613	8866.095052	159855.974330	171134.896946	1.4423E+00	1.8009E+09	1.7000E−08
743.............	2 ^3P–3 ^1D	1–2	5874.4603	5876.088412	169086.842898	186104.966689	1.2324E+04	6.3721E+07	1.0632E−04
		2–2	5874.4339	5876.062023	169086.766473	186104.966689	4.3097E+03	6.3721E+07	2.2307E−05
744.............	2 ^3P–4 ^1D	1–2	4471.0947	4472.349352	169086.842898	191446.455741	2.2565E+03	2.6983E+07	1.1276E−05
		2–2	4471.0794	4472.334065	169086.766473	191446.455741	7.9822E+02	2.6983E+07	2.3933E−06
745.............	2 ^3P–5 ^1D	1–2	4026.0138	4027.151538	169086.842898	193918.289901	8.3444E+02	1.3929E+07	3.3809E−06
746.............	3 ^3S–3 ^1P	1–1		2972.573239 cm−1	183236.791701	186209.364940	1.4612E−01	5.7996E+08	2.4796E−08
747.............	3 ^3P–3 ^1D	1–2		540.382794 cm−1	185564.583895	186104.966689	2.3169E+00	6.3721E+07	1.9854E−05
		2–2		540.404769 cm−1	185564.561920	186104.966689	8.0194E−01	6.3721E+07	4.1232E−06

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TABLE 8

FINDING LIST FOR ^{4}He WAVELENGTHS <10000 Å

Wavelength (Å)	N	Lower–Upper	g_{Ml}–g_{Mu}	A_{Mul} (s^{-1})	f_{Mlu}	log gf
507.058................................	9	1 ^1S–10 ^1P	1–3	1.593E+07	1.841E−03	−2.735
507.718................................	8	1 ^1S–9 ^1P	1–3	2.183E+07	2.530E−03	−2.597
508.643................................	7	1 ^1S–8 ^1P	1–3	3.103E+07	3.610E−03	−2.443
509.998................................	6	1 ^1S–7 ^1P	1–3	4.622E+07	5.405E−03	−2.267
512.099................................	5	1 ^1S–6 ^1P	1–3	7.317E+07	8.628E−03	−2.064
515.617................................	4	1 ^1S–5 ^1P	1–3	1.258E+08	1.504E−02	−1.823
522.213................................	3	1 ^1S–4 ^1P	1–3	2.436E+08	2.986E−02	−1.525
537.030................................	2	1 ^1S–3 ^1P	1–3	5.663E+08	7.344E−02	−1.134
584.334................................	1	1 ^1S–2 ^1P	1–3	1.799E+09	2.762E−01	−0.559

Table 8 is available in its entirety in the electronic edition of the *Astrophysical Journal Supplement*. A portion is shown here for guidance regarding its form and content.

TABLE 9
FINDING LIST FOR ^{4}He WAVENUMBERS <10,000 cm^{-1}

Wavelength (Å)	N	Lower–Upper	$g_{Ml}-g_{Mu}$	A_{Mul} (s^{-1})	f_{Mlu}	log gf
9974.121	64	3 ^1D–7 ^1P	5–3	3.942E+04	3.563E−04	−2.749
9969.634	757	3 ^3D–7 ^1F	12–7	2.873E+05	2.527E−03	−1.518
9969.621	351	3 ^3D–7 ^3F	15–21	1.123E+06	2.370E−02	−0.449
9966.214	65	3 ^1D–7 ^1F	5–7	9.289E+05	1.962E−02	−1.008
9966.209	573	3 ^1D–7 ^3F	5–7	2.866E+05	6.057E−03	−1.519
9925.762	350	3 ^3D–7 ^3P	15–9	8.443E+04	7.706E−04	−1.937
9860.763	50	3 ^1P–7 ^1D	3–5	1.125E+06	2.889E−02	−1.062
9860.308	567	3 ^1P–7 ^3D	3–5	9.100E+01	2.339E−06	−5.154

Table 9 is available in its entirety in the electronic edition of the *Astrophysical Journal Supplement*. A portion is shown here for guidance regarding its form and content.

$\bar{\nu}/\bar{\nu}_\infty$.—The ratio of mean transition energies for true and infinite-nuclear mass.

η.—The coefficient of y in the factor $(1+\eta y)$ necessary to correct f to finite nuclear mass as described in the Appendix.

$\bar{\tau}_{ul}$ (s^{-1}).—Mean lifetime of the term's upper levels from equation (5) with $\Sigma_l A_{ul}$ averaged over all u.

f_{Mlu}.—The multiplet absorption oscillator strength for infinite nuclear mass.

f_{Mul}.—The multiplet emission oscillator strength for infinite nuclear mass $=g_{Ml}f_{Mlu}/g_{Mu}$.

As noted in the Appendix, the $(1+\eta y)$ correction for finite nuclear mass should not be applied until the higher order relativistic corrections are known.

4.2. *Tables 4, 5, 6, and 7*

These tables list the data for the individual transitions under the following headings.

N.—Sequential multiplet number.

Lower-upper.—Level designations $n^{2S+1}L$.

J–J.—Lower and upper *J*-values J_l and J_u.

Wavelength (Å) λ.—"Air" for $\lambda > 2000$ Å calculated following Peck & Reeder (1972) and "Vacuum" calculated from the energy levels of Morton et al. (2006) according to equation (1). If the vacuum wavelength exceeds 10000 Å, the two numbers are replaced by the more useful vacuum wavenumber followed by cm^{-1}. The important 2 ^3S–2 ^3P transition at 10833 Å or 9231 cm^{-1} is tabulated both ways.

E_l, E_u (cm^{-1}).—Lower and upper level energies from Paper I.

A_{ul} (s^{-1}).—The spontaneous transition rate including singlet–triplet mixing and spin-orbit coupling.

Sum A_{ul} (s^{-1}).—The sum of all A_{ul} to lower levels, which is the reciprocal of the lifetime of the upper level. The numbers are nearly identical for all three *J*-values of each upper triplet term.

f_{lu}.—The absorption oscillator strength or *f*-value including singlet–triplet mixing and spin-orbit coupling.

$f_{lu}/f_{\rm LS}$.—The ratio of the above *f*-value to one for pure LS-coupled states with no singlet–triplet mixing.

In Table 5 the first line for each multiplet gives the mean value calculated according to equations (2), (3), (7), and (8) followed by individual lines in order of decreasing wavelength or increasing wavenumber. The listed wavelengths and energy levels should be reliable to the quoted number of figures, except for transitions involving the seven $^1S_0$ and $^3S_1$ levels in Table 1. However, for *A* and *f*, the neglect of some relativity corrections and the finite nuclear mass probably causes errors of a few parts in 10^3 for most transitions and somewhat more for the n ^1D-n ^1P lines noted in the Appendix. Nevertheless, we have quoted five figures for comparison with previous calculations.

The ratio in the final column of Tables 4 and 5 shows the effects of including singlet–triplet mixing and spin-orbit coupling. Among the triplet–triplet multiplets, all those involving *F* and higher angular momentum states have some *f*-values of some lines changed significantly, while for the singlets, only the *D–F* lines are severely affected and the *F–G* ones by about 10%. In most cases the transfer of oscillator strength to the intersystem lines reduces the ratio for both the triplet–triplet and singlet–singlet lines below unity, although a few cases such as 5 $^3G_4$–6 $^3H_5$ gain a little from the corresponding 5 $^1G_4$–6 $^1H_5$.

At the beginning of Table 6 we have added the very weak electric-dipole transition 1 $^1S_0$–2 $^3P_1$at 591.412 Å with $A=$ 177.58 s^{-1}calculated by Lach & Pachucki (2001), compared with 178.7 s^{-1} by Johnson et al. (1995) and 176.4 s^{-1} by Drake (1979). Both the later papers included the negative-energy even-parity *P* states in the continuum omitted in the earlier paper.

4.3. *Tables 8 and 9*

Table 8 provides a finding list for transitions shortward of 10000 Å ordered by increasing wavelength and Table 9 for wavelengths longward of 10000 Å $=$ 10,000 cm^{-1} ordered by decreasing wavenumber. Since all wavelengths in a multiplet are close to each other, we have tabulated only mean multiplet values. The column headings are as follows:

λ_M (Å).—Mean multiplet wavelength in vacuum or, if $\lambda >$ 10000 Å, ν_M (cm^{-1}) mean wavenumber, the true values from equation (1).

N.—Sequential multiplet number.

Lower-upper.—Level designations $n^{2S+1}L$.

g_{Ml}, g_{Mu}.—Total statistical weights for the lower and upper terms from equation (3).

A_{Mul} (s^{-1}).—The multiplet spontaneous transition rate including singlet–triplet mixing and spin-orbit coupling.

f_{Mlu}.—The multiplet absorption oscillator strength corresponding to A_{Mul}.

$\log g_{Ml}f_{lu} = \log g_{Mu}f_{Mul}$.—The logarithm of the weighted multiplet absorption or emission oscillator strength.

5. COMPARISONS WITH EARLIER CALCULATIONS

There is a long history of increasing sophistication in the calculation of oscillator strengths for neutral helium including configuration interaction in a central field by Green et al. (1966), a variational method with Hylleraas wave functions by Schiff et al. (1971), an extension of these results with double Hylleraas-type

TABLE 10

THEORETICAL RESULTS FOR $3\,^3P$–$4\,^3D$

f_{lu}	Source
0.47757	This paper
0.44763	Chen (1994a, 1994b)
0.47760[a]	Cann & Thakkar (1992)
0.47663	Theodosiou (1987)
0.4476	Kono & Hattori (1984)
0.4766	Green et al. (1966) length
0.4790	Green et al. (1966) velocity

[a] Chen (1994b) miscopied this as 0.44760.

basis functions by Kono & Hattori (1984), the Coulomb approximation with a realistic central field by Theodosiou (1987), close coupling by Fernley et al. (1987), explicitly correlated wave functions by Cann & Thakkar (1992), and *B*-spline basis functions by Chen (1994a, 1994b). None of these considered singlet–triplet mixing nor spin-orbit coupling as we do, and only Theodosiou included transitions involving ^1F, ^1G, ^3F, and ^3G terms.

With the help of the useful summaries provided by Chen, we have compared our results with the earlier calculations of *S*, *P*, and *D* transitions and found good agreement, particularly with Kono & Hattori and Chen, where the match usually was better than one part in a thousand. A surprising exception is the transition $3\,^3P$–$4\,^3D$ listed in Table 10, where both of these papers are about 6% lower than the other five determinations, possibly the result of misprints. The *D*–*F* and *F*–*G* calculations of Theodosiou agree with our LS *f*-values, but not with our perturbed results in Tables 4 and 5, as expected, because of the singlet–triplet and spin-orbit effects.

6. COMPARISONS WITH EXPERIMENTS

For many years the theoretical oscillator strengths and lifetimes of the strongest transitions in He I have been sufficiently accurate to be used as checks on experimental apparatus and

TABLE 11

COMPARISON WITH EXPERIMENTAL LIFETIMES

Upper Singlet Level	Primary Decay	This Paper (ns)	GLSD03[a] (ns)	ZSBLPHL03[b] (ns)	ES91[c] (ns)	CKB84[d] (ns)	LMP83[e] (ns)	KH79[f] (ns)	VPS87[g] AVPS81[g] (ns)	ACLMM76[h] (ns)
$3\,^1S$	7281 Å	54.65	54(1)		56.3(20)			55.9(7)		
$4\,^1S$	5048 Å	88.00	89(5)		88.7(30)					
$5\,^1S$	4438 Å	146.6			149(5)					
$6\,^1S$	4169 Å	232.9			235(8)		230(7)			
$7\,^1S$	4024 Å	351.6					360(18)			
$8\,^1S$	3936 Å	507.9					513(30)			
$9\,^1S$	3878 Å	707.0					625(40)			
$2\,^1P$	584 Å	0.5553		0.560(14)						
$3\,^1P$	537 Å	1.7243		1.71(4)						1.7225(46)
$4\,^1P$	522 Å	3.964		3.96(8)						
$5\,^1P$	516 Å	7.625		7.59(15)		7.4(4)				
$6\,^1P$	512 Å	13.06		13.0(3)		14.0(3)				
$7\,^1P$	510 Å	20.62		20.4(4)		21.6(12)	22.7(34)			
$8\,^1P$	509 Å	30.66		30.5(9)			31.7(40)			
$9\,^1P$	508 Å	43.53		43.3(16)			43.6(31)			
$10\,^1P$	507 Å	59.58		59.8(15)			54.9(38)			
$3\,^1D$	6678 Å	15.69							15.3(3)	
$4\,^1D$	4922 Å	37.06						31.3(4)		
$4\,^1F$	5347 cm^{-1}	72.29							74(2)	
$5\,^1F$	7816 cm^{-1}	139.8							133(5)	

Upper Triplet Level	Primary Decay	This Paper (ns)	GLSD03[a] (ns)	VMRS95[i] (ns)	ES91[c] (ns)	SEK87[j] (ns)	KP78[k] (ns)	KH79[f] (ns)	VPS78[g] (ns)	LS65[l] (ns)
$3\,^3S$	7065 Å	35.90	38(1)	35.94(20)				35.7(6)		
$4\,^3S$	4713 Å	62.37	61(3)							
$2\,^3P$	10830 Å	97.89								105(5)
$3\,^3P$	3889 Å	94.80	105(9)		$105(^{+5}_{-10})$	96.4(82)	97.6(45)	104(8)		
$4\,^3P$	3188 Å	138.5			164(7)					
$5\,^3P$	2495 Å	219.3			245(15)					
$3\,^3D$	5876 Å	14.14		14.12(6)					14.2(6)	

[a] Gans et al. (2003).
[b] Zitnik et al. (2003).
[c] Erman & Sundström (1991).
[d] Charnay et al. (1984).
[e] Larsson et al. (1983).
[f] Kono & Hattori (1979).
[g] von Oppen et al. (1978); Aynacioglu et al. (1981).
[h] Astner et al. (1976).
[i] Volz et al. (1995).
[j] Silim et al. (1987).
[k] Kramer & Pipken (1978).
[l] Lifsitz & Sands (1965).

TABLE 12

Comparison with Experimental Oscillator Strengths for Resonance Transitions

Transition Lower–Upper	λ (Å)	This Paper	ZFWZZX97[a]	GR95[b]	LMZHWS95[c]	CCB91[d]	TWA89[e]
$1\ ^1S_0$–$2\ ^1P_1$	584	0.2761	0.276(16)	0.2700(76)	0.269(15)	0.280(7)	0.273(8)
$1\ ^1S_0$–$3\ ^1P_1$	537	0.07344	0.0739(44)	0.0737(23)		0.0741(7)	0.071(3)
$1\ ^1S_0$–$4\ ^1P_1$	522	0.02986	0.0304(18)			0.0303(7)	
$1\ ^1S_0$–$5\ ^1P_1$	516	0.01504	0.0154(9)			0.0152(3)	
$1\ ^1S_0$–$6\ ^1P_1$	512	0.008628	0.00930(56)			0.00892(50)	
$1\ ^1S_0$–$7\ ^1P_1$	510	0.005406				0.0587(30)	

Transition Lower–Upper	λ (Å)	This Paper	AKMFY82[f]	WV77[g]	BTWV75[h]	DV71[i]	BL71[j]
$1\ ^1S_0$–$2\ ^1P_1$	584	0.2761	0.270(14)	0.262(18)			0.275(7)
$1\ ^1S_0$–$3\ ^1P_1$	537	0.07344	0.078(7)		0.073	0.076(4)	0.073(5)
$1\ ^1S_0$–$4\ ^1P_1$	522	0.02986	0.030(5)			0.029(2)	
$1\ ^1S_0$–$5\ ^1P_1$	516	0.01504	0.016(2)				
$1\ ^1S_0$–$6\ ^1P_1$	512	0.008628	0.0094(16)				

[a] Zhong et al. (1997).
[b] Gibson & Risley (1995).
[c] Larsson et al. (1995).
[d] Chan et al. (1991).
[e] Tsurubuchi et al. (1989).
[f] Alexandrov et al. (1982).
[g] Westerveld & Van Eck (1977).
[h] Backx et al. (1975) normalized to $f(\lambda584) = 0.276$.
[i] De Jongh & Van Eck (1971) normalized to $f(\lambda584) = 0.276$.
[j] Burger & Lurio (1971).

procedures. Thus, it is useful to compare our results with the available measurements.

Theodosiou (1984) compiled a comprehensive list of laboratory lifetimes. There is general agreement with his calculations and ours, but considerable scatter among the measurements of individual decays, possibly caused by cascades from higher levels. In Table 11 we have quoted some more recent measurements along with the important ones of Larsson et al. (1983) and Astner et al. (1976) from the earlier list as well as a few omitted by Theodosiou. In this table the only serious discrepancy is the Kono-Hattori (1979) lifetime for $4\ ^1D$, which is too short by 14 σ. The average of all 19 lifetimes for this level listed by Theodosiou is 37.2 ± 4.2 ns, consistent with our prediction. There is excellent agreement with the exceptionally accurate measurements of $3\ ^3S$ and $3\ ^3D$ by Volz et al. (1995) and $3\ ^1P$ by Astner et al. (1976). Of course, a lifetime tests the rates of only the very strongest transitions that contribute to the decay.

With one exception, Tables 12 and 13 show similar good agreement between our numbers and experimental f-values including the weak intersystem transition $2\ ^3P$–$3\ ^1D$ measured by Fujimoto et al. (1986). The $2\ ^3P$–$3\ ^3D$ measurement by Dubreuil & Catherinot (1980) deviates from the calculation by 7 σ, but the excellent agreement of the $3\ ^3D$ lifetimes in Table 6 supports the theoretical value. Chan et al. (1991) listed eight additional measurements for $1\ ^1S$–$2\ ^1P$ prior to 1970. They are consistent with our calculations, although some have large errors.

TABLE 13

Comparison with Experimental Oscillator Strengths for Nonresonance Transitions

Transition Lower–Upper	λ (Å)	$1/\lambda$ (cm^{-1})	This Paper	FHOST86[a]	KKT83[b]	DC80[c]	SVP80[d]
$2\ ^1S_0$–$2\ ^1P_1$	4857	cm^{-1}	0.3764		0.362(19)		
$2\ ^1S_0$–$3\ ^1P_1$	5016	Å	0.1514			0.147(11)	
$2\ ^1P_1$–$3\ ^1S_0$	7281	Å	0.04849			0.048(3)	
$2\ ^1P_1$–$3\ ^1D_2$	6678	Å	0.7100			0.70(4)	
$3\ ^1D_2$–$3\ ^1P_1$	104	cm^{-1}	0.01268				0.0119(3)
$2\ ^3S_1$–$3\ ^3P$	3889	Å	0.06446			0.0671(20)	
$2\ ^3P$–$3\ ^3S_1$	7065	Å	0.06951			0.0696(20)	
$2\ ^3P$–$3\ ^3D$	5876	Å	0.6102			0.566(6)	
$2\ ^3P_1$–$3\ ^1D_2$	5874	Å	10.632×10^{-5}				
$2\ ^3P_2$–$3\ ^1D_2$	5874		2.231×10^{-5}				
Net–$3\ ^1D_2$	5874		4.783×10^{-5}	$4.75(62) \times 10^{-5}$			

[a] Fujimoto et al. (1986).
[b] Kostenko et al. (1983).
[c] Dubreuil & Catherinot (1980).
[d] Szostak et al. (1980).

7. FUTURE INVESTIGATIONS

A few more experimental lifetimes and *f*-values of high accuracy, as well as measurements of additional intersystem transitions, would provide useful tests of these calculations. On the theoretical side, the higher order relativistic corrections should be the next step. They are expected to enter at the 0.2% level except for cases of accidental degeneracy where the percentage change could be larger. The comparable corrections for the finite nuclear mass already are available through the η terms in Tables 2 and 3 discussed in the Appendix. Other effects such as the finite nuclear size are negligible at this stage. The calculation of *A*- and *f*-values for the rare ^{3}He isotope will depend on slightly different wave functions and energies and must include the individual hyperfine components.

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APPENDIX

CORRECTIONS FOR FINITE NUCLEAR MASS

The small corrections due to finite nuclear mass are not normally included in discussions of radiative decay rates in atoms, but they become important if accuracies better than a few parts in 10^4 [i.e., of order μ/M, where $\mu = m_e M/(m_e + M)$ is the reduced electron mass] are required. The relevant theory was first discussed by Fried & Martin (1963) and extended by Yan & Drake (1995) and Drake (2006). Here we wish to amplify the last reference to cover ions as well as neutral atoms by including terms involving the motion of the center of mass (c.m.) in the radiation field in addition to the motion of the charged nucleus relative to the center of mass.

The nonrelativistic Hamiltonian H_a for an atom or ion with nuclear mass M and charge Ze at postion $\boldsymbol{R}_N$ and N electrons of mass m_e and charge $-e$ at positions $\boldsymbol{R}_i$ is

$$H_a = \frac{1}{2M}\boldsymbol{P}_N^2 + \frac{1}{2m_e}\sum_{i=1}^{N}\boldsymbol{P}_i^2 - \sum_{i=1}^{N}\frac{Ze^2}{|\boldsymbol{R}_i - \boldsymbol{R}_N|} + \frac{1}{2}\sum_{j}^{N}\frac{e^2}{|\boldsymbol{R}_j - \boldsymbol{R}_i|}, \tag{A1}$$

where $\boldsymbol{P}_i = -i\hbar\partial/\partial\boldsymbol{R}_i = -i\hbar\nabla$ and the center of mass is at

$$\boldsymbol{R}_c = \frac{M\boldsymbol{R}_N + m_e\sum_{i=1}^{N}\boldsymbol{R}_i}{M + Nm_e}. \tag{A2}$$

The Schrödinger equation

$$H_a|u\rangle = E_u|u\rangle \tag{A3}$$

determines the energy levels E_u and the eigenvectors $|u\rangle$.

Transforming to coordinates $\boldsymbol{r}_i = \boldsymbol{R}_i - \boldsymbol{R}_N$ and $\boldsymbol{R}_c$ gives

$$\begin{aligned} H_a = {} & \frac{1}{2\mu}\sum_{i=1}^{N}\boldsymbol{p}_i^2 + \frac{1}{2M}\sum_{j}^{N}\boldsymbol{p}_i\cdot\boldsymbol{p}_j + \frac{1}{2(M + Nm_e)}\boldsymbol{P}_c^2 \\ & - Ze^2\sum_{i=1}^{N}\frac{1}{|\boldsymbol{r}_i|} + \frac{e^2}{2}\sum_{j}^{N}\frac{1}{|\boldsymbol{r}_j - \boldsymbol{r}_i|}, \end{aligned} \tag{A4}$$

where $\boldsymbol{p}_i\cdot\boldsymbol{p}_j$ is the mass-polarization operator and the term in $\boldsymbol{P}_c = -i\hbar\partial/\partial\boldsymbol{R}_c = -i\hbar\nabla$ must be included whenever there is a net charge on the atom to account for the motion of the center of mass relative to the inertial frame represented by the coordinates $\boldsymbol{R}_N$ and $\boldsymbol{R}_i$.

Again following Drake (2006) in the application of the interaction Hamiltonian, the general equation for the averaged decay rate for a single photon transition from upper state u to lower state l in the dipole approximation at distances well outside atomic dimensions is

$$A_{ul} = (4/3)\alpha\omega_{ul}\left|\langle l|\boldsymbol{Q}_p|u\rangle\right|^2. \tag{A5}$$

Here $\alpha = e^2/\hbar c$ is the fine-structure constant, ω_{ul} is the transition radian frequency, and

$$\boldsymbol{Q}_p = \frac{1}{m_e c}\sum_{i=1}^{N}\boldsymbol{P}_i - \frac{Z}{Mc}\boldsymbol{P}_N = \frac{Z_p}{m_e c}\sum_{i=1}^{N}\boldsymbol{p}_i - \frac{1}{c}\frac{(Z - N)}{(M + Nm_e)}\boldsymbol{P}_c \tag{A6}$$

is the dimensionless velocity form of the transition operator before and after the transformation to the coordinates $\boldsymbol{r}_i$ and $\boldsymbol{R}_c$. The commutator

$$[H,\ \boldsymbol{Q}_r] = \hbar\omega_{ul}\boldsymbol{Q}_p \tag{A7}$$

determines the corresponding length forms

$$\boldsymbol{Q}_r = \frac{i\omega}{c}\sum_{i=1}^{N}\boldsymbol{R}_i - \frac{i\omega}{c}Z\boldsymbol{R}_N = \frac{i\omega}{c}Z_r\sum_{i=1}^{N}\boldsymbol{r}_i - \frac{i\omega}{c}(Z-N)\boldsymbol{R}_c, \tag{A8}$$

where

$$Z_p = \frac{Zm_e + M}{M} \quad \text{and} \quad Z_r = \frac{Zm_e + M}{Nm_e + M}. \tag{A9}$$

In either coordinate system, it follows from equations (A7) and (A3) that

$$\langle l|\boldsymbol{Q}_p|u\rangle = \langle l|\boldsymbol{Q}_r|u\rangle \tag{A10}$$

and since $[\boldsymbol{p},\ \boldsymbol{r}] = -3i\hbar$,

$$[\boldsymbol{Q}_p,\ \boldsymbol{Q}_r] = \frac{3\hbar\omega}{m_e c^2}\left[Z_p Z_r N + \frac{m(Z-N)^2}{M+Nm_e}\right] = \frac{3\hbar\omega}{m_e c^2}\left(N + \frac{m_e}{M}Z^2\right). \tag{A11}$$

If we define the negative emission oscillator strength f_{ul} and the positive absorption oscillator strength $f_{lu} = -f_{ul}g_u/g_l$ in terms of the transition rate in the usual way,

$$-f_{ul} = \frac{g_l}{g_u}f_{lu} \equiv \frac{m_e c^2}{2\alpha\hbar\omega_{ul}^2}A_{ul}. \tag{A12}$$

Substituting from equation (A5) and using equation (A10) gives

$$\begin{aligned} f_{ul} &= \frac{2m_e c^2}{3\hbar\omega_{ul}}\langle l|\boldsymbol{Q}_p|u\rangle\langle l|\boldsymbol{Q}_r|u\rangle \\ &= \frac{m_e c^2}{3\hbar\omega_{ul}}\{\langle l|\boldsymbol{Q}_p|u\rangle\langle l|\boldsymbol{Q}_r|u\rangle + \langle l|\boldsymbol{Q}_r|u\rangle\langle l|\boldsymbol{Q}_p|u\rangle\} \\ &= \frac{m_e c^2}{3\hbar\omega_{ul}}\{\langle u|\boldsymbol{Q}_p|l\rangle\langle l|\boldsymbol{Q}_r|u\rangle - \langle u|\boldsymbol{Q}_r|l\rangle\langle l|\boldsymbol{Q}_p|u\rangle\}. \end{aligned} \tag{A13}$$

Summing over all states, including the continuum, and counting emissions as negative, we have

$$\begin{aligned} \sum_l f_{ul} &= \frac{m_e c^2}{3\hbar}\{\langle u|\boldsymbol{Q}_p\boldsymbol{Q}_r/\omega_{ul}|u\rangle - \langle u|\boldsymbol{Q}_r\boldsymbol{Q}_p/\omega_{ul}|u\rangle\}\langle l|l\rangle \\ &= \frac{m_e c^2}{3\hbar}\langle u|[\boldsymbol{Q}_p, \boldsymbol{Q}_r/\omega_{ul}]|u\rangle \\ &= N + Z^2 m_e/M. \end{aligned} \tag{A14}$$

The inclusion of the finite nuclear mass has added the term Z^2m_e/M to the usual Thomas-Reiche-Kuhn sum rule $\Sigma f_{ul} = N$. Thus, the sum is 2.000274 for ^{4}He I, while it is 2 for positronium (Ps) and 3 for the negative ion (Ps^-), as expected for 2 and 3 particles of the same mass. The expressions for f_{ul} in equations (A12) and (A13) differ from those proposed by Yan & Drake (1995) and Drake (2006), who included extra factors of Z_p and Z_r as well as omitting the final terms in equations (A6) and (A8) in order to retain the strict $\Sigma f_{ul} = N$, the number of electrons, for any M/m_e. However, it now seems preferable to adopt the revised definition of f_{ul} that maintains the traditional ratio to A_{ul} and gives a sum of 3 for Ps^-.

Following equation (11.4) of Drake (2006), the actual calculation of the energy levels and oscillator strengths first involves a scaling to dimensionless parameters for the length operator $\rho_i = r_i/a_\mu$, the momentum operator $-i\hbar\partial/\partial\rho_i = -i\hbar a_\mu\partial/\partial r_i$, and energy $\varepsilon = a_\mu E/e^2$, where $a_\mu = (\mu/m_e)a_0$, $a_0 = \hbar^2/m_e e$ is the Bohr radius and e is the electron charge in e.s.u. For the purpose of presenting the results for neutral helium, for which the radiation field causes no motion of the center of mass, it is instructive to separate the effects of the mass scaling from mass polarization. If the latter is neglected in H_a and its effect on the wave function, as is the case for large L, then comparing the terms in the pure length form of equation (A13) with the corresponding one for infinite nuclear mass we have $\omega = (\mu/m_e)\omega_\infty$ and $\langle \boldsymbol{r}_i\rangle = (m_e/\mu)\langle \boldsymbol{r}_i\rangle_\infty$, so the mass scaling of the oscillator strength is

$$f = \frac{\mu}{m_e}\left(\frac{m_e}{\mu}\right)^2 f_\infty = \left(1 - \frac{\mu}{M}\right)^{-1} f_\infty \approx \left(1 + \frac{\mu}{M}\right) f_\infty. \tag{A15}$$

Thus, in the absence of mass polarization, the mass scaling is $f \approx (1+y)f_\infty$, where $y = \mu/M$, which equals 1.370746×10^{-4} for ^{4}He I. In Tables 2 and 3, the influence of mass polarization appears as the factor η different from unity in $f_M = (1 + \eta y)f_\infty$.

We did not apply the $(1 + \eta y)$ factor to the f-values listed in the tables because the finite mass correction for $n_l \neq n_u$ usually has the opposite sign to that for the omitted relativity terms. See, for example, the 2 ^2P lifetime calculations for ^{7}Li I in Table IX of Yan et al. (1998). Thus, we prefer to ignore both corrections rather than include just one. However, we have tabulated η in Tables 2 and 3 to show which transitions could be affected significantly and to have the numbers available when the remaining relativistic corrections become known. Since these are expected to change f-values by about 0.2%, η must exceed 14.6 to have a similar effect. Only the transitions $n\ ^1P$–$n\ ^1D$ $(3 \leq n \leq 10)$ with $32.0 \geq \eta \geq 30.6$ significantly exceed this limit, while the multiplets $n\ ^3S$–$(n+1)\ ^3P$ $(5 \leq n \leq 9)$ with $12.2 \leq \eta \leq 15.4$ are comparable. Otherwise, the higher order relativistic corrections probably dominate.

REFERENCES

Alexandrov, Y. M., Kozlov, M. G., Makhov, V. N., Fedorchuk, R. V., & Yakimenko, M. N. 1982, Opt. Spectrosc. (USSR), 52, 129
Astner, G., Curtis, L. J., Lijeby, L., Mannervk, S., & Martinson, I. 1976, Z. Phys. A, 279, 1
Aynacioglu, A. S., von Oppen, G., Perschmann, W.-D., & Szostak, D. 1981, Z. Phys. A, 303, 97
Backx, C., Tol, R. R., Wright, G. R., & Van der Weil, M. J. 1975, J. Phys. B, 8, 2050
Baklanov, E. V., & Denisov, A. V. 1997, Quantum Electronics (Moscow), 27, 463
Bauman, R. P., Porter, R. L., Ferland, G. J., & MacAdam, K. B. 2005, ApJ, 628, 541
Borie, E., & Rinker, G. A. 1978, Phys. Rev. A, 18, 324
Burger, J. M., & Lurio, A. 1971, Phys. Rev. A, 3, 64
Cann, N. M., & Thakkar, A. J. 1992, Phys. Rev. A, 46, 5397
———. 2002, J. Phys. B, 35, 421
Chan, W. F., Cooper, G., & Brion, C. E. 1991, Phys. Rev. A, 44, 186
Charnay, D. J., King, G. C., & Buckman, S. J. 1984, J. Phys. B, 17, 3173
Chen, M.-K. 1994a, J. Phys. B, 27, 865
———. 1994b, J. Phys. B, 27, 4847
De Jongh, J. P., & Van Eck, J. 1971, Physica, 51, 104
Derevianko, A., & Johnson, W. R. 1997, Phys. Rev. A, 56, 1288
Drake, G. W. F. 1969, ApJ, 158, 1199
———. 1971, Phys. Rev. A, 3, 908
———. 1979, Phys. Rev. A, 19, 1387
———. 2006, in Springer Handbook of Atomic, Molecular and Optical Physics, ed. G. F. W. Drake (New York: Springer)
Drake, G. W. F., & Goldman, S. P. 1999, Canadian J. Phys., 77, 835
Drake, G. W. F., & Martin, W. C. 1998, Canadian J. Phys., 76, 679
Dubreuil, B., & Catherinot, A. 1980, Phys. Rev. A, 21, 188
Erman, P., & Sundström, G. 1991, Phys. Rev. A, 43, 5790
Fernley, J. A., Taylor, K. T., & Seaton, M. J. 1987, J. Phys. B, 20, 6457
Fried, Z., & Martin, A. D. 1963, Nuov. Cimento, 29, 574
Fujimoto, T., Hirabayashi, A., Okuda, S., Shimizu, K., & Takuma, H. 1986, J. Phys. B, 19, 571
Gans, T., Lin, C. C., Shulz-von der Gathen, V., & Dobele, H. F. 2003, Phys. Rev. A, 67, 012707
Gibson, N. D., & Risley, J. S. 1995, Phys. Rev. A, 52, 4451
Green, L. C., Johnson, N. C., & Kolchin, E. K. 1966, ApJ, 144, 369
Johnson, W. R., Plante, D. R., & Sapirstein, J. 1995, Adv. At. Mol. Opt. Phys., 35, 255
Kono, A., & Hattori, S. 1979, J. Opt. Soc. Am., 69, 253
———. 1984, Phys. Rev. A, 29, 2981
Kostenko, V. A., Kasyanenko, S. V., & Tolmachev, Y. A. 1983, Opt. Spectrosc. (USSR), 54, 440
Kramer, P. B., & Pipken, F. M. 1978, Phys. Rev. A, 18, 212
Lach, G., & Pachucki, K. 2001, Phys. Rev. A, 64, 042510
Larsson, J., & Melvel, E., Zerne, R., L'Hullier, A., Wahlström, C.-G., & Svanberg, S. 1995, J. Phys. B, 28, L53
Larsson, M., Mannfors, B., & Pendleton, W. R. 1983, Phys. Rev. A, 28, 3371
Lifsitz, J., & Sands, R. H. 1965, Bull. Am. Phys. Soc., 10, 1214
Mohr, P. J., & Taylor, B. N. 2005, Rev. Mod. Phys., 77, 1
Morton, D. C., Wu, Q., & Drake, G. W. F. 2006, Canadian J. Phys., 84, 83
Peck, E. R., & Reeder, K. 1972, J. Opt. Soc. Am., 62, 958
Schiff, B., Pekeris, C. L., & Accad, Y. 1971, Phys. Rev. A, 4, 885
Silim, H. A., El-Farrash, A. H., & Kleinpoppen, H. 1987, Z. Phys. D, 5, 61
Szostak, D., von Oppen, G., & Perschmann, W.-D. 1980, Phys. Lett. A, 76, 376
Tsurubuchi, S., Watanabe, K., & Arikawa, J. 1989, J. Phys. B, 22, 2969
Theodosiou, C. 1984, Phys. Rev. A, 30, 2910
———. 1987, At. Data Nucl. Data Tables, 36, 97
Volz, U., Marger, D., Roth, H., & Schmoranzer, H. 1995, J. Phys. B, 28, 579
von Oppen, G., Perschmann, W.-D., & Szostak, D. 1981, Z. Phys. A, 286, 243
Westerveld, W. B., & van Eck, J. 1977, J. Quant. Spectrosc. Radiat. Transfer, 17, 131
Wiese, W. L., Smith, M. W., & Glennon, B. M. 1966, Atomic Transition Probabilities Vol. 4: Hydrogen through Neon (Washington: NBS)
Wu, X.-L., Gou, B.-C., & Wang, F. 2003, Canadian J. Phys., 81, 1419
Yan, Z.-C., & Drake, G. W. F. 1995, Phys. Rev. A, 52, R4316
Yan, Z.-C., Tambasco, M., & Drake, G. W. F. 1998, Phys. Rev. A, 57, 1652
Zhong, Z. P., Feng, R. F., Wu, S. L., Zhu, L. F., Zhang, X. J., & Xu, K. Z. 1997, J. Phys. B, 30, 5305
Zitnik, M., Stanik, A., Bucar, K., Lambourne, J. G., Penent, F., Hall, R. I., & Lablanquie, P. 2003, J. Phys. B, 36, 4175