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Many-body localization and quantum thermalization

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It is the common wisdom that time evolution of a many-body system leads to thermalization and washes away quantum correlations. But one class of system—referred to as many-body localized—defy this expectation.

The main thrust of quantum many-body physics in the past 50 years or so has been the effort to classify the zero-temperature quantum phases of matter and describe their low-energy excitations. This remarkably successful endeavour has led to the explanation of striking macroscopic quantum phenomena, such as superconductivity and the quantum Hall effects, and to the theoretical prediction of topological insulators.

Recent advances in realizing synthetic many-body systems that are very well isolated from the environment motivated the community to start looking beyond near-equilibrium behaviour. In a common experimental scheme with ultracold atoms, one can prepare a simple initial state and let it evolve under the unitary time evolution generated by the many-body Hamiltonian under well-controlled conditions. Such experiments also offer new ways to probe the state of the system with unprecedented detail. These developments raise the natural question: can the classification of quantum matter be extended to describe states that emerge in the course of quantum dynamics?

The conventional wisdom had long been that generic quantum systems ultimately approach a state of thermal equilibrium. In this process, quantum correlations encoded in the initial state are scrambled as the local degrees of freedom become ever more entangled across the system. The only surviving structures are the fluctuations of conserved densities, whose slow relaxation modes are described by classical hydrodynamics. This picture explains why the dynamics of macroscopic systems normally appears classical even if they are fundamentally governed by quantum mechanics.

The headline from the past few years, however, is that the classical fate of a closed many-body system is not inevitable. There is at least one class of system that fails to thermalize and can retain retrievable quantum correlations to arbitrarily long times through the phenomenon of many-body localization (MBL). In this Perspective, I survey in rather broad strokes how our understanding of thermalization and MBL has developed in the past few years, and how this facilitates the effort of classifying emergent quantum phenomena away from thermal equilibrium. I will close with a discussion of the many intriguing open questions concerning MBL, which call for further research. For more comprehensive reviews on the topic I refer readers to refs [1–3].

The study of MBL has a long history. Already in his first famous paper on localization[4], Anderson stated that his motivation for studying the problem was to understand the dynamics of a many-body system. Anderson postulated that a system of interacting spins, not coupled to an external heat bath and subject to sufficiently strong disorder, may also fail to achieve thermal equilibrium. In his remarks, Anderson recognized two unique aspects of the problem. First, the crucial importance of having a closed system—coupling to an external bath, such as phonons in a solid, would lead to delocalization and thermalization. Second, in order to classify a system as non-thermalizing we must consider its dynamics at non-vanishing energy densities. It is not enough to understand the ground state and low-energy excitations.

A non-interacting system in which the single-particle states are Anderson-localized certainly cannot thermalize. But it is not obvious a priori that it will not thermalize if we add even very weak interactions. A system localized to a given many-body configuration could potentially delocalize by attempting a resonant transition—that is, a transition with a matrix element larger than the energy detuning—to any one of the enormous number of multi-particle configurations (exponentially large in the system size). On the other hand, when the interaction is turned off, only single-particle transitions are possible, which reduces the number of opportunities for resonance to be only linear in the system size. Nonetheless, refs [5,6] argued convincingly that the localized phase is stable. These works computed the decay of a single particle into a shower of particle–hole excitations, each of which decays again in the same manner, by including the leading processes at each level within a self-consistent Born approximation.

Basko et al.[5] further argued that a system that is localized at low temperature (or energy density) can become delocalized above a critical temperature due to the increased phase space for transitions. The critical energy density at which the transition occurs is called a many-body mobility edge.

Ten years later, John Imbrie provided a mathematical proof of the stability of an MBL state in certain one-dimensional spin chains[7]. While this proof has placed the existence of an MBL phase on firm ground, it also inspired general arguments, which called some of the early results into question. For example, De Roek et al.[8] argued that an MBL phase with many-body mobility edges may be unstable. Similar arguments also suggest that the MBL phase cannot exist in two or three dimensions[9]. The validity of these arguments is still debated in the community. However, there is no doubt that one-dimensional systems with bounded spectra, such as a spin chain or lattice fermions, can be in a state in which all many-body states are localized (Fig. 1). Such a system can undergo a disorder-tuned MBL transition even at infinite temperature.

The hallmark of MBL, as seen already in early numerical simulations[10,11], is that certain observables do not relax to their thermal-equilibrium values even after long time evolution. More recently, similar signatures of ergodicity-breaking have also been observed experimentally in systems of ultracold atoms[12] and of trapped ions[13]. Both systems can achieve extremely good isolation from the environment. Figure 2 shows the main result of ref. [12]: a density modulation imprinted on the initial state fails to relax to a vanishing value, which would be expected in the equilibrium state. While true MBL cannot be seen in solids because of the inevitable coupling to a bath of delocalized phonons, possible precursors of a MBL transition at a finite temperature were observed in transport experiments with indium oxide films[14].

Once the existence of MBL had been established, the focus turned to gaining a better understanding of the dynamical phenomena, which characterize MBL as a new distinct phase of matter. A first indication that the dynamics in the MBL state

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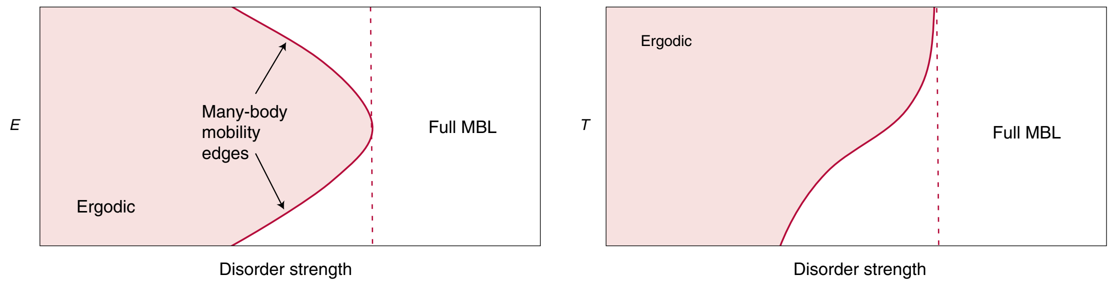

Fig. 1 | Schematic phase diagram of a model with a bounded many-body spectrum (for example, lattice fermions or spins). a, The phase diagram in the space of energy density versus disorder strength. It is currently debated if the many-body mobility edges mark true phase transitions or only sharp crossovers. In the latter case the actual phase transition occurs on the dashed line into the full MBL phase. **b**, The phase diagram in the space of temperature versus disorder strength.

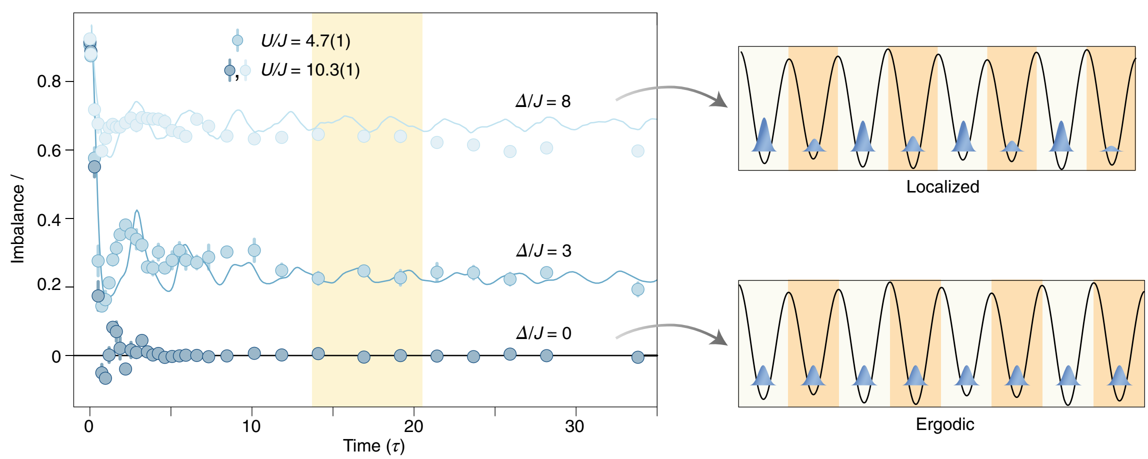

Fig. 2 | Experimental observation of MBL in an ultracold atomic system. The plot shows relaxation of a density wave imposed as an initial state of interacting fermions in a one-dimensional optical lattice with quasi-periodic potential. Beyond a critical strength of the quasi-periodic lattice detuning Δ, the density wave fails to relax to its vanishing thermal equilibrium value. Adapted from ref. [12], AAAS.

obey some universal structure, distinct from the special non-interacting case, came from studies of entanglement entropy[15,16]. The entanglement entropy of a subsystem A can be viewed as a measure of the information lost from that part of the system as it becomes entangled with the rest of the system, thereby making this information non-local.

In thermalizing systems, the entanglement entropy of an artificially prepared state with low entanglement typically grows linearly in time until it reaches the value expected for a system with a given energy density, in which the entanglement entropy follows the thermodynamic volume law. In a localized system, on the other hand, one might expect that information would remain localized in the vicinity of the subsystem, leading to saturation of the entanglement entropy to a constant that is proportional to the localization length, rather than to the subsystem size. Hence, it was surprising to find that in generic MBL systems the entanglement entropy continues to grow slowly as $\log t$ for time t, saturating only when it reaches a volume law, albeit with a smaller entropy density than the expected thermal entropy. Only in a non-interacting Anderson insulator does the entanglement entropy saturate to a constant independent of the system size. Recently, evidence of this logarithmic growth of the entanglement entropy has been seen experimentally in a system of ultracold atoms[17].

Today we understand these results within a universal effective description of the MBL phase. Consider the spin chain Hamiltonian

$$H(J_\perp) = \sum_i \epsilon_i \sigma_i^z + \sum_i J^z \sigma_i^z \sigma_{i+1}^z + J_\perp \sum_i (\sigma_i^+ \sigma_{i+1}^- + \text{H.c.}) \tag{1}$$

with ϵ_i uncorrelated random variables and H.c. is the Hermitian conjugate. The case $H(0)$, without the coupling $J_\perp$, is trivially localized: all spin operators σ_i^z commute with the Hamiltonian and their expectation values cannot relax. The spectrum is then composed of product states that are mutual eigenvectors of all σ_i^z. If the system remains localized for a finite range of $J_\perp$ then there is a sense of adiabatic continuity between eigenstates of $H(0)$ and those of $H(J_\perp)$. The latter can still be labelled by the eigenvalues of a set of almost local integrals of motion τ_i^z, related to the original spins through a quasi-local unitary transformation U.

$$\tau_i^z = U^\dagger \sigma_i^z U = Z\sigma_i^z + \text{exponential tail} \tag{2}$$

Because U is quasi-local, the non-local contributions decay exponentially with their range. Crucially, τ_i^z has a non vanishing overlap Z with the original local spin operator. Based on this picture, refs [18,19] have postulated the following effective model for the MBL phase, written entirely with the local integrals of motion

$$\hat{H}_{\text{MBL}} = \sum_i \tilde{h}_i \hat{\tau}_i^z + \sum_{ij} J_{ij} \hat{\tau}_i^z \hat{\tau}_j^z + \sum_{ijk} J_{ijk} \hat{\tau}_i^z \hat{\tau}_j^z \hat{\tau}_k^z + \cdots \tag{3}$$

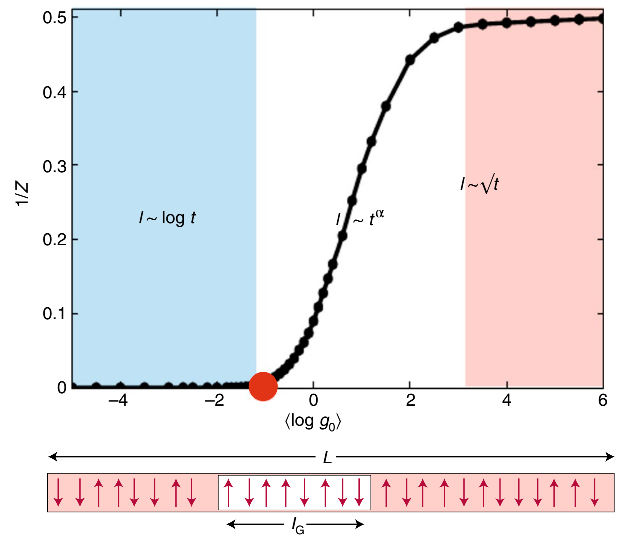

Fig. 3 | Subdiffusive transport in the thermal state near the MBL transition. The dynamical exponent z that governs the relaxation time τ of a disturbance of size l, $\tau(l) \sim l^z$, is exactly 2 in normal diffusive systems. However, the relaxation in the thermal regime near the MBL critical point is subdiffusive, with $1/z$ vanishing continuously at this point. The top panel shows the continuous change of $1/z$ as calculated in the RG scheme of ref. [33]. The bottom panel shows an insulating inclusion (Griffiths region) inside the thermal phase, which is the cause of the anomalous transport. The size of the largest Griffiths region in a system of length L is typically of length $l_G \sim \xi \log L$. This region constitutes a bottleneck for transport leading to the timescale $\tau \sim \tau_0 \exp(l_G / l_*) \sim L^{\xi/l_*}$.

that is, with all commuting terms. All the interactions J_{ij}, J_{ijk} and so on decay exponentially with their range. This Hamiltonian can be viewed as a fixed point of a renormalization group (RG) scheme, which diagonalizes the Hamiltonian through successive local unitary transformations[20,21].

Unlike conventional quantum phases, which flow to a simple fixed point at low energy, the effective Hamiltonian (3) is valid at all energy scales and defines true integrals of motion. In this sense, the MBL state is similar to integrable models. In both cases, the breaking of ergodicity is due to an extensive set of integrals of motion that cannot relax. But while any small perturbation destroys conventional integrability and leads to relaxation, the integrals of motion in an MBL state are robust to the changes in the Hamiltonian parameters within a finite range.

The effective model (3) allows the prediction of a wealth of properties of the localized phase. The most immediate one is that certain local observables do not relax to their equilibrium values because they have a finite overlap with an integral of motion τ_i^z. At the same time, the components τ_i^x and τ_i^y are not conserved. Entangling these components at long ranges due to the exponentially decaying interactions leads to the logarithmic growth of the entanglement entropy. Because of this process, one might think that the MBL state retains only the classical information encoded in τ_i^z rather than the full quantum information in a local qubit. Remarkably, however, the dephasing and entanglement of the perpendicular components can be, at least partially, undone even after arbitrary long time evolution with spin echo techniques employing only local operations[22,23]. This suggests possible uses of the MBL state as a resource for quantum information storage or processing.

The fact that quantum information persists in the dynamics is a reflection of the radically different structure of many-body eigenstates in the MBL phase compared to those of conventional systems. All the eigenstates of a fully MBL system—assuming no mobility edges—are related to product states by a quasi-local unitary transformation that takes the Hamiltonian (1) to the effective model (3). As a consequence, the entanglement entropy of a subsystem must obey an area law in any eigenstate[18,24]. This is to be contrasted with thermalizing systems, where generic eigenstates exhibit a volume law consistent with the thermodynamic entropy at the same energy density.

An important consequence of the effective model (3) is that it implies the existence of not one, but many different MBL phases, distinguished by the structure of the local integrals of motion[25]. For example, if the underlying model has a discrete symmetry, such as $\mathbb{Z}_2$, the integrals of motion could be either even or odd under this symmetry. In the latter case, the eigenstates of the system all spontaneously break the $\mathbb{Z}_2$ symmetry. This phenomenon, unique to MBL systems, has been termed localization-protected eigenstate order. The eigenstates can also realize symmetry-protected topological states, which in equilibrium could occur only in the ground state. Consequently, these states can host edge modes, which allow coherent manipulation, even if the bulk of the system is hot and incoherent[22].

A whole new arena for quantum matter is opened by the possibility of driving MBL systems with time-periodic perturbations. Anyone who has operated a microwave oven knows that driving conventional systems in this manner causes them to heat up indefinitely. By contrast, an MBL system can retain its coherence and localization properties provided that the drive frequency exceeds a certain threshold[26–28]. Moreover, the fact that the drive breaks the usual translational symmetry in time down to a discrete symmetry opens the door to realizing new states of matter that are not possible in static Hamiltonian systems. In particular, the system can exhibit persistent oscillations that spontaneously break the residual discrete time translational symmetry in presence of the drive[29,30]. Such a phase, known as a time crystal, cannot exist in thermal equilibrium[31]. However, a stable time crystal can occur and has been observed experimentally in an MBL system[32].

Having discussed the unique dynamical properties of the MBL phase, I now turn to the question how this state gives way to a thermalizing fluid when we tune a parameter, such as the disorder strength. Part of the difficulty and the interest in studying this problem is that it differs from conventional phase transitions in crucial ways. The stability of MBL rests on the vanishing probability of many-body resonances in the system. As the disorder weakens, resonances involving an increasing number of sites can form. The critical point occurs where an infinite resonant cluster proliferates in the system and destabilizes the MBL phase. Semi-phenomenological RG schemes have been developed in an attempt to describe the universal aspects of this process[33,34].

While there is no controlled theory of the critical point, different RG schemes lead to rather similar results. In particular, a fixed point is found with a correlation length ξ (localization length on the insulating side) that diverges with a critical exponent $u \approx 3.3$ in one dimension. An unusual finding is that typical observables exhibit critical behaviour only as the transition is approached from the thermal side, but not from the localized side. In particular, the transport and relaxation time τ scales exponentially with distance l as $\tau \sim e^{l/\xi_*}$, where ξ_* behaves as a localization length that remains finite at the critical point. Thus, while the critical cluster thermalizes its surroundings, it does so exponentially slowly and in a way that is controlled by a microscopic scale. It is a challenge to find observables that are sensitive to the diverging size of the critical cluster.

The critical behaviour on the thermal side is also interesting and more readily observable than that in the MBL phase. RG schemes[33,34] as well as numerical studies[35–37] find that the thermal phase close to the critical point in one dimension exhibits subdiffusive transport. Thus, a disturbance of size l decays on a time scale $\tau \sim l^z$ with dynamical exponent $z > 2$ ($z = 2$ for normal diffusion). Within the RG analysis[33,34] the subdiffusive behaviour is attributed to the effect of rare insulating regions inside the thermal fluid.

Although the probability for finding a long critical region is suppressed exponentially, such a region will also cause an exponentially long delay of the relaxation, hence it can have an effect on the average relaxation times. Such rare region effects are known in other contexts as Griffiths phenomena. Figure 3 gives a schematic depiction of a Griffiths region. The dynamical exponent z generated by the Griffiths effects is predicted to change continuously on approaching the MBL transition where it diverges together with the correlation length—that is, $z \sim \delta h^{-\nu}$ ref. [33]

The Griffiths physics underlying the critical behaviour on the thermal side relies on the rare fluctuations of the uncorrelated random potential. However, recent experiments that explored the MBL phase and phase transition with ultracold atoms employed deterministic quasi-periodic potentials. It is believed that the MBL transition is controlled by a different fixed point in this case[38,39]. However, even a rudimentary understanding of such a critical point is lacking.

Furthermore, most of the insights we currently have on the dynamics in the MBL phase and phase transition pertain exclusively to one-dimensional systems, leaving considerable uncertainty concerning the MBL phase in higher dimensions. An attempt to describe the formation of a critical cluster in higher dimensions from finite ergodic seeds[9], as done for one-dimensional systems within the RG schemes[33,34], leads to a surprising outcome. The result of ref. [9] implies that a finite-size resonant (ergodic) cluster is always sufficient to destabilize the surrounding MBL phase in $d > 1$ and thermalize the entire (infinite system) in an 'avalanche-like' process. To be sure, the analysis relies on a non-trivial assumption regarding the applicability of random matrix theory to describe each stage of the avalanche, which must be tested independently.

The ongoing efforts to describe the MBL phase transition or the instability towards thermalization in higher dimensions can be seen in the broader context of understanding the dynamics of quantum thermalization and onset of quantum chaos. Holographic theories, which provide one route to control the calculations of thermalization, correspond to the limit in which thermalization is very fast, saturating the theoretical bounds[40]. The MBL phase transition marks the opposite limit in which thermalization just emerges as an instability of a localized quantum system. Studying this transition can provide a new starting point from which to investigate these questions with the control afforded by the proximity to a dynamical critical point. Such studies could benefit from the development of numerical methods for computing the dynamics of thermalizing systems, which would provide unbiased tests of theoretical ideas. New methods using matrix product states[41] and operators[37,42] have been developed and shown to capture the thermalization dynamics and emergent transport properties of certain thermalizing quantum systems that thermalize fast. It remains to be seen if such methods can be extended to capture the critical dynamics close to the MBL transition.

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