



## **Nonlinear control applied to 4D transport aircraft guidance**

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### **Aerospace Engineering**

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Dedicated to someone special...



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A few words about the university, financial support, research advisor, dissertation readers, faculty or other professors, lab mates, other friends and family...



## Resumo

Inserir o resumo em Português aqui com o máximo de 250 palavras e acompanhado de 4 a 6 palavras-chave...

**Palavras-chave:** palavra-chave1, palavra-chave2,...





## Abstract

Due to their inherent non-linear dynamics, designing control systems for both rotary and fixed wing aircraft is a non-trivial task, where using linear control approaches often reveal to be inaccurate. Feedback linearization is an approach to non-linear control design that algebraically transforms a non-linear system onto a linear one. From the linearised system linear control techniques can be used. However this approach, as well as other state-of-the art control systems such as model predictive control, backstepping and gain scheduling require the *a priori* exact mathematical model of the system to be controlled [1]. The lack of such a model leads to errors in the calculation of the model inversion, necessary to implement feedback linearization. Indeed, while feedback linearization control shows good tracking, it has poor disturbance rejection, that ultimately lead to these inversion errors [2]. One solution to this problem that has gained momentum over the last years is the use augmentation algorithms, namely neural networks and other intelligent algorithms, to minimize and cancel this error[3], [4], [5]. This thesis therefore aims to discuss and implement the existing alternatives to augment the precision, robustness and overall performance of feedback linearization control.

**Keywords:** non-linear control, feedback linearization, neural network, flight control



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# Chapter 1

## Introduction

Insert your chapter material here...

### 1.1 Motivation

Define: usefulness of NLI vs robust control and problems of NLI

### 1.2 Topic Overview

Brief description of the solutions to be studied and how they will be used to improve control

### 1.3 Objectives

Goals: create systems able to adapt to changes of the airplane's dynamic

### 1.4 Thesis Outline

Background includes in this order: NLI theory, NLI problematic, neural network theory Implementation:  
? Results: ???



# Chapter 2

## Background

On this chapter the aircraft model used for this work will be described firstly. A theoretical model of the plane's dynamics will be discussed, and implementation detail will be provided on the chapter 3. The control strategy used will then follow, giving an overview of the feedback linearisation approach, as well as its limitations, namely its sensitivity to inversion errors and external interferences. An attempt to solve these limitation will be made, suggesting some solutions and finally describing the chosen methodology for this case.

### 2.1 Airplane Model

The work made in this thesis was built on top of the work done by H. Escamilla Nuñez and F. Mora Camino on 4D trajectory tracking [6]. The model used in this work of a six degree of freedom transport aircraft will be described in this section.

#### 2.1.1 Frames of reference

The first step before describing the dynamics of a commercial aircraft will be to define the frames of reference used to do so. The first frame of reference, on which 4D trajectories are described, corresponds to the WGS84 frame of reference. A second frame of reference corresponding to the aircraft's body frame will be used to provide its fast rotational dynamics. Lastly all aerodynamic forces will be applied in the axial directions of the wind frame. This frame is aligned to the wind speed vector relative to the airplane, given by both the angle of attack  $\alpha$  and the sideslip angle  $\beta$ . For these last two frames of reference, a rotation matrix can be defined from the wind frame to the body frame by

$$R_{WB} = \begin{bmatrix} c_\alpha c_\beta & -c_\alpha s_\beta & -s_\alpha \\ s_\beta & c_\beta & 0 \\ s_\alpha c_\beta & -s_\alpha s_\beta & c_\alpha \end{bmatrix} \quad (2.1)$$

To describe the attitude of the plane Euler angles will also be used, namely  $\phi\{-\pi, \pi\}; \theta\{-\frac{\pi}{2}, \frac{\pi}{2}\}; \psi\{-\pi, \pi\}$ .

From these angles the rotation matrix from the body to the earth's frame is given by

$$R_{BE} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \quad (2.2)$$

## 2.1.2 Fast Dynamics

The considered actuators of the aircraft that control its attitude are given by  $\delta = [\delta_{ail} \delta_{ele} \delta_{rud}]^T$ , each applying a torque along an axis of the body frame. These torques are given by

$$\begin{bmatrix} L' \\ M \\ N \end{bmatrix} = \frac{1}{2} \rho S V_a^2 \left( \begin{bmatrix} b C_l \\ \bar{c} C_m \\ b C_n \end{bmatrix} + C_\delta \delta \right) \quad (2.3)$$

where  $\bar{c}$  and  $b$  represent the wing chord length and span respectively,  $C_\delta$  and the moment coefficients  $[C_l C_m C_n]^T$  are given by

$$C_\delta = \begin{bmatrix} b C_{l\delta_{ail}} & 0 & b C_{l\delta_{rud}} \\ 0 & \bar{c} C_{m\delta_{ele}} & 0 \\ b C_{n\delta_{ail}} & 0 & b C_{n\delta_{rud}} \end{bmatrix} \quad (2.4)$$

$$\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} = \begin{bmatrix} C_{l\beta} \beta + C_{l_p} p \frac{b}{2V_a} + C_{l_r} r \frac{b}{2V_a} \\ C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} q \frac{\bar{c}}{2V_a} \\ C_{n\beta} \beta + C_{n_p} p \frac{b}{2V_a} + C_{n_r} r \frac{b}{2V_a} \end{bmatrix} \quad (2.5)$$

Where  $p, q, r$  are the body's angular rates ( $\Omega = [p \ q \ r]^T$ ) and  $V_a$  is the airspeed. The method of obtaining of the coefficients of equation 2.5 will be provided in the chapter to follow. Having defined the torques applied to the aircraft the rotational dynamics equation can now be stated as per [6],  $I$  being the aircraft's inertial matrix.

$$\dot{\Omega} = I^{-1} M_{ext} - I^{-1} \Omega \times (I \Omega) \quad (2.6a)$$

$$\dot{\Omega} = \frac{1}{2} \rho S I^{-1} V_a^2 \left( \begin{bmatrix} b C_l \\ \bar{c} C_m \\ b C_n \end{bmatrix} + C_\delta \delta \right) - I^{-1} \Omega \times (I \Omega) \quad (2.6b)$$

These two equations can be rearranged to account for the effect of the wind, allowing further on to simulate the behaviour of the airplane in the presence of wind disturbances. Let  $\vec{V}_G = [u \ v \ w]^T$  be the speed of the CG relative to the ground,  $\vec{V}$  the speed of the CG relative to the air mass and  $\vec{W}$  the

speed of the wind relative to the ground, then as per Etkin and Reid [7]

$$\vec{V}_G = \vec{V} + \vec{W} = \begin{bmatrix} V_a c_\alpha c_\beta + V_{w_x} \\ V_a s_\beta + V_{w_y} \\ V_a s_\alpha c_\beta + V_{w_z} \end{bmatrix} \quad (2.7)$$

and  $\alpha$  and  $\beta$  can be computed by

$$\alpha = \arctan\left(\frac{w}{u}\right) \quad (2.8a)$$

$$\beta = \arctan\left(\frac{v}{V_a}\right) \quad (2.8b)$$

From these three equations, differentiating 2.8a and 2.8b comes that

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} H_{11} & 1 & H_{13} \\ H_{21} & 0 & H_{23} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \quad (2.9)$$

SEE ANNEX HERE

The angular rates are also related to the Euler angles. The relationship between the euler angles and the rotation rates is also one that will prove useful when implementing the model on a Matlab simulation, and is given by

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & tg_\theta s_\phi & tg_\theta c_\phi \\ 0 & c_\phi & -s_\phi \\ 0 & \frac{s_\phi}{c_\theta} & \frac{c_\phi}{c_\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (2.10)$$

## 2.2 Guidance Dynamics

This subsection on the forces applied to aircraft, introducing a new actuation variable, the thrust force  $T$ . These forces are applied along the three axis of the wind frame, lift, drag and side force, given by

$$\begin{bmatrix} D \\ Y \\ L \end{bmatrix} = \frac{1}{2} \rho S V_a^2 \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} \quad (2.11)$$

Once again, the method used to compute these coefficients will be given in the chapter 3 in detail. These coefficients, similarly to the moment coefficient, are functions of the angle of attack, sideslip angle and airspeed, the three most relevant variables when determining aerodynamic forces and moments. Although aerodynamic forces are usually expressed on the wind frame, as the thrust is always applied along the  $x$  axis of the body frame, it is necessary to rotate the aerodynamic forces to this frame. This way the sum of the airplane's forces can be obtained.

$$\begin{bmatrix} F_{xa} \\ F_{ya} \\ F_{za} \end{bmatrix} = R_{WB} \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} \quad (2.12)$$

From Newton's Second Law comes the aircraft's acceleration

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(F_{xa} + T) - gs_\theta + rv - qw \\ \frac{1}{m}F_{ya} + gc_\theta s_\phi + pw - ru \\ \frac{1}{m}F_{za} + gc_\theta c_\phi + qu - pv \end{bmatrix} \quad (2.13)$$

An expression in the Earth frame can also be obtained

$$\begin{bmatrix} \ddot{x}_E \\ \ddot{y}_E \\ \ddot{z}_E \end{bmatrix} = \frac{1}{m} R_{BE} \begin{bmatrix} F_{xa} + T \\ F_{ya} \\ F_{za} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (2.14)$$

### 2.2.1 Actuator Dynamics

Finally, to simulate the delay response in actuation in order to have a realistic simulation, first order systems were introduced to the actuator dynamics as per [6]. For the control surfaces  $\delta_i$ , given a desired  $\delta_i^d$  comes

$$\dot{\delta}_i = \frac{1}{\xi_i}(\delta_i^d - \delta_i) \quad (2.15)$$

Similarly for the thrust

$$\dot{T} = \frac{1}{\xi_T}(T^d - T) \quad (2.16)$$

$\xi_i$  and  $\xi_T$  being time constants. As the responsiveness of the resultant thrust will be much slower than that of the control surfaces,  $\xi_T \gg \xi_i$ .

## 2.3 Feedback linearisation

Feedback linearisation, also known as dynamic inversion, is an approach based on the idea of algebraically transforming a non-linear system into a linear one, from which linear control laws can be used to control the resulting system. Unlike Jacobian linearisation, that assumes linearity of the system around an equilibrium value, feedback linearisation implements a feedback loop that cancels non-linearities of the given system [8]. Given a nonlinear system

$$\dot{x} = f(x, u) \quad (2.17)$$

one must find both a state transformation  $z = z(x)$  and an input transformation  $v = v(x, u)$  in order

for the transformed system to be a linear and time-invariant system  $\dot{z} = Az + Bv$ . This process is called Input-State linearisation.

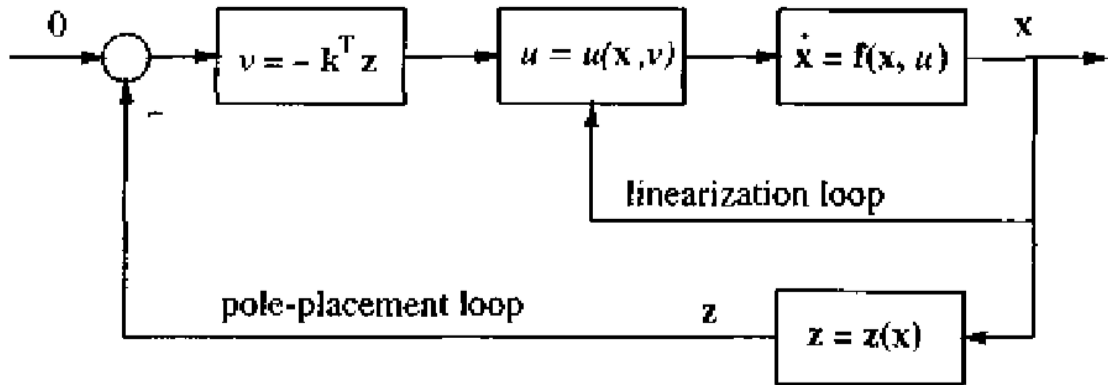


Figure 2.1: Feedback linearisation example

EXPLAIN NECESSARY MATHEMATICAL CONCEPTS HERE

### 2.3.1 SISO systems

Although an aircraft is always considered as a MIMO system, a description of this control concept will firstly be provided for a general SISO case, before generalising to a MIMO case. Nonlinear systems that can be represented as  $\dot{x} = f(x) + g(x)u$  will be discussed in this section. From [8], a definition of Input-State linearisation is to find, for a nonlinear system of relative degree  $n$ , a *diffeomorphism*  $\phi : \Omega \rightarrow R^n$  and nonlinear control law

$$u = \alpha(x) + \beta(x)v \quad (2.18)$$

such that the new state variables  $z = \phi(x)$  and *pseudo-input*  $v$  satisfy the linear time invariant system  $\dot{z} = Az + Bv$ , where the following equations are satisfied

$$\begin{cases} \dot{z}_i = z_{i+1} & \text{if } i < n-1 \\ \dot{z}_n = v & \text{if } i = n-1 \end{cases} \quad (2.19)$$

In order to find such a function and control law, one must find a state  $z_1$  such that

$$\nabla z_1 \text{ad}_f^i g = 0 \quad i = 0, \dots, n-2 \quad (2.20a)$$

$$\nabla z_1 \text{ad}_f^{n-1} g \neq 0 \quad (2.20b)$$

The remaining states are then obtained from  $z(x) = [z_1 \quad L_f z_1 \quad \dots \quad L_f^{n-1} z_1]^T$  and the input transformation from

$$\alpha(x) = -\frac{L_f^n z_1}{L_g L_f^{n-1} z_1} \quad (2.21a)$$

$$\beta(x) = \frac{1}{L_g L_f^{n-1} z_1} \quad (2.21b)$$

### 2.3.2 MIMO systems

These concepts can also be extended to MIMO systems in a similar manner, by similarly differentiating the outputs until the inputs explicitly appear. This time however, there are individual relative degrees per output. The sum of these relative degrees is called total degree  $r$  and must satisfy  $r < n$ ,  $n$  being the order of the system. Given a MIMO system

$$\dot{x} = f(x) + G(x)u \quad (2.22)$$

$$y = h(x) \quad (2.23)$$

the  $\phi$  functions are defined as  $\phi_j^i(x) = L_f^{j-1} h_i(x)$  and satisfy a condition similar to 2.19

$$\dot{\phi}_1^i(x) = \phi_2^i, \dots, \dot{\phi}_{r_i-1}^i = \phi_{r_i}^i \quad \text{and} \quad \dot{\phi}_{r_i}^i = L_f^{r_i} h_i(x) + \sum_{j=1}^m L_{g_j} L_f^{r_i-1} h_i(x) u_j \quad (2.24)$$

As this equation holds for  $1 < i < p$ ,  $p$  being the number of outputs of the system, giving an expression for the pseudo control input  $v$

$$v = \begin{bmatrix} \dot{\phi}_{r_1}^1 \\ \vdots \\ \dot{\phi}_{r_p}^p \end{bmatrix} \quad (2.25)$$

It is also interesting to note the resulting system is not only linear, but also completely decoupled, as each new pseudo input only affects one output.

## 2.4 Limitations of feedback linearisation

Although feedback control answers to many of the problems of linear controllers such as PID and LQR, it has a big downside. Indeed, being a model based controller, it requires the system model to be quite accurately known. Let  $v$  be the vector of pseudo inputs, for a second order  $\ddot{x} = f(x, \dot{x}, u)$ , if it is input-output linearisable and the exact system is known, then it comes that, from [9]

$$\ddot{x} = v \quad (2.26)$$

However, even for simpler models, a real system is difficult to accurately describe, and an approximation  $\hat{f}$  of  $f$  is usually chosen, resulting in  $v = \hat{f}(x, \dot{x}, u)$ . Therefore, defining a dynamic inversion error,  $\Delta(x, \dot{x}, u) = f(x, \dot{x}, u) - \hat{f}(x, \dot{x}, u)$ , comes that, as stated in [9]

$$\ddot{x} = v + \Delta(x, \dot{x}, u) \quad (2.27)$$



There can be many causes for a dynamic inversion error to be present, as not only modelling uncertainties can lead to larger errors, but also external interference (wind gusts) and actuator faults. The goal of this section will be to present the reader with some solutions to minimize this error.

### 2.4.1 Fuzzy Logic Systems

Fuzzy logic systems are based on the paradigm of continuous levels of truth between 0 and 1, rather than the usual discrete true or false levels of truth. Using these continuous truth values, a set of IF-ELSE rules is chosen. The goal of this type of controller is to duplicate the way a pilot would respond to a given flight situation. These rules can base their control input based on variables such as roll, angle of attack and sideslip [? ]. This control method is based on three main parts. The first one is *fuzzification*, that consists of converting the plant outputs to fuzzy logic values. *Rule-based inference*, using the previously mentioned set of rules, a fuzzified control input is computed, which then goes through the *defuzzification* part of the control algorithm. Fuzzy systems have some advantages: its linguistic interpretation of human knowledge facilitates the interpretation of results, and the knowledge base can be improved through addition of new rules. However, this method comes with some disadvantages. First of all, the set of rules that will improve the control of the aircraft is entirely dependent on expert knowledge, thus resulting in an empiric set of rules. This also means the method has no capability of generalization, as the control is only applied to specific cases, of the form "if the error is greater than a given value, apply this compensation to control input". It is also not robust to topological changes of the system [? ]. For these reasons this method will not be further studied and implemented in this work.

### 2.4.2 Neural Networks

Artificial Neural Networks (NN) are a biologically inspired simulation of brain's nervous system. They are composed of simulated neurons and synapses. The sum of the inputs of a neuron are summed and the result is fed into an activation function, which then returns a bounded value, either between -1 and 1 or 0 and 1. The inputs and outputs of a neuron are multiplied by a weight, representing the synapses between neurons. We get the following equation for one neuron

$$y(x_1, x_2, \dots, x_n) = f\left(\sum_{i=1}^n w_i x_i + b\right) \quad (2.28)$$

where  $b$  is the bias term and  $f$  a given activation function. Neurons are then set in three different types of layers: the input, the output and the hidden layers. These are connected between each other, and the hidden part can be composed of several layers. The weights can then be tuned based on experience, making it suitable for intelligent learning. To tune these weights learning algorithms are used. There are two types of learning algorithms: batch and online training. It is called training to the process of iteratively reaching the ideal set of weights that will minimise the error between the output of the neural network and that of a given unknown function.

Batch training is mostly based on a property NN is that, according to the Universal Approximation

Theorem showed in 1989 by George Cybenko, a feed forward neural network can approximate any continuous function. The network is trained by providing input and output pairs. The weights are found in order to approximate the output of the neural network to that of the original function. Once these weights are computed, the network can then be used to compute outputs from inputs that were not part of the initial knowledge of input-output pairs.

Although this learning method approach is used in many of NN applications, it cannot however be easily used to improve an existing feedback linearisation controller, as it requires an *a priori* knowledge of the modelling error for each given input.

Online training, contrary to batch training, can deal with dynamically changing environments. This training paradigm is not based on an *a priori* knowledge set, and instead relies on update laws that compute the new weights after each control iteration. It is this method that will be retained to adaptively correct the inversion error from the feedback linearisation. Training a network online, however, is less trivial when compared to batch training. An algorithm named backpropagation will be presented to provide a way of training a network online.

The backpropagation algorithm is one of the most common NN training algorithm, commonly used in batch training, used to update the weights of each layer of a network, in order to minimize a cost function. However, research has been done in order to use backpropagation techniques to create online adaptive and augmented control such as [10], [11], [12]. Indeed, it is also possible to use such algorithms to continuously update the weights of a network as the data is generated. Backpropagation can only be implemented if the activation function of the NN is differentiable.

The backpropagation algorithm can be described by a two phase cycle, Propagation and Weight update. During this last phase the gradient descent algorithm is used to optimize a cost function. As such, a cost function  $E$  must be chosen, and must be a function of the outputs of the neural network. A commonly used error function is

$$E = \frac{1}{2}(y_d - y)^2 \quad (2.29)$$

Where  $y_d$  is the desired output of the network and  $y$  is the actual output of the network. The  $\frac{1}{2}$  factor is added to be cancelled when differentiating. For a given neuron  $i$ , the sum of its inputs  $sum_i$  is given by

$$sum_i = \sum_{k=1}^n w_{ki} x_k \quad (2.30)$$

Where  $w_{ki}$  is the weight of the  $k^{th}$  input of neuron  $i$ , and  $x_k$  are the outputs of the neurons from the previous layer. Given an activation function  $f(x)$ , we define the output of neuron  $i$  as

$$x_i = f(sum_i) \quad (2.31)$$

In order to use the gradient descent algorithm, the derivative of the cost function with respect to a given weight  $w_{ij}$  must be computed. To do so, the chain rule is used as such

$$\frac{\partial E}{\partial w_{ij}} = \frac{\partial E}{\partial x_j} \frac{\partial x_j}{\partial sum_j} \frac{\partial sum_j}{\partial w_{ij}} \quad (2.32)$$

These three factors can now be easily computed. The last two factors are independent of weather  $x_j$  is a hidden or output layer and will for this reason be calculated firstly. From 2.30 and 2.31 comes that

$$\frac{\partial x_j}{\partial sum_j} = \frac{\partial f(sum_j)}{\partial sum_j} \quad (2.33)$$

For an output layer, where values may be unbounded, linear output functions may be used, for which the equation above has a trivial solution. For output layers of classification neural networks or for hidden layers in general, logistic or tangent sigmoid functions can be used instead. For the first case let  $f(x) = \frac{1}{1+e^{-x}}$ , then

$$\frac{\partial f(sum_j)}{\partial sum_j} = f(sum_j)(1 - f(sum_j)) \quad (2.34)$$

For the case of a tangent sigmoid activation function  $f(x) = \tanh(x)$  comes

$$\frac{\partial f(sum_j)}{\partial sum_j} = 1 - f^2(sum_j) \quad (2.35)$$

For the term  $\frac{\partial sum_j}{\partial w_{ij}}$ , from 2.30 this partial derivative is easily computed

$$\frac{\partial sum_j}{\partial w_{ij}} = \frac{\partial (\sum_{k=1}^n w_{kj} x_k)}{\partial w_{ij}} = x_i \quad (2.36)$$

For the output layer, the first term of equation 2.32 can easily be evaluated, as  $x_j = y$  for such a case, giving

$$\frac{\partial E}{\partial x_j} = \frac{\partial (\frac{1}{2}(y_d - y)^2)}{\partial y} = y - y_d \quad (2.37)$$

For input layers however, this derivative is less obvious to estimate. One can consider that the error function  $E$  is a function of the inputs of all neurons that take  $x_j$  as input. Let  $S$  be such a set of inputs, from the total derivative to  $x_j$  comes that

$$\frac{\partial E}{\partial x_j} = \sum_{s \in S} \left( \frac{\partial E}{\partial x_s} \frac{\partial x_s}{\partial sum_s} \frac{\partial sum_s}{\partial x_j} \right) = \sum_{s \in S} \left( \frac{\partial E}{\partial x_s} \frac{\partial f(sum_s)}{\partial sum_s} w_{js} \right) \quad (2.38)$$

Note that the term  $\frac{\partial E}{\partial x_s}$  represent the all the derivatives of the errors with respect to the outputs of the next layer of neurons. Taking into account that the last layer is easily evaluated without this method, the derivatives of all hidden neurons can be computed. Knowing the value of 2.32, an update law for the weights can now be stated from the gradient descent algorithm

$$\Delta w_{ij} = -\alpha \frac{\partial E}{\partial w_{ij}} \quad (2.39)$$

Where  $\alpha$  is a learning coefficient, and its choice is crucial to assure a fast convergence of the solution. Indeed, while a exceedingly large learning coefficient can cause this algorithm to miss the minimum error, a small learning coefficient will also increase the convergence rate. The  $-1$  factor is added in the equation above to converge to a local minima, removing it would make this algorithm maximize the cost

function. A limitation of the backstepping algorithm that must also be taken into account is that it only guarantees local minima convergence.

## Chapter 3

# Implementation

This section will go in depth on the implementation of the concepts presented in the chapter 2. The plane model to be controlled was firstly implemented and verified. The first goal after this first step was to design a controller that would allow the aircraft to follow a trajectory from several position waypoints through time. The influence of certain parameters and knowledge of the exact plane model will be studied and discussed in chapter 4. The final goal will be to improve the controller and its robustness by reducing tracking error through the use of an adaptive neural network.

### 3.1 Plane Model

The chosen commercial aircraft that will be simulated is the Boeing 737-200, an aircraft with over 30 years of service for which information of flight parameters is readily available. The simulation was made in a cruise flight environment, at 200 m/s velocity at 10000 m above the ground. The chosen inertial matrix for this aircraft is given by

$$\begin{bmatrix} 1278369.56 & 0 & -135588.17 \\ 0 & 3781267.79 & 0 \\ -135588.17 & 0 & 4877649.98 \end{bmatrix} \quad (3.1)$$

The ISA atmospheric model was used to measure the air density at any given height. The time constants

Table 3.1: Boeing 737-2 parameters

Weight $m$	52390 kg
Wing Span $b$	28.35 m
Wing Area $S$	102.0 m <sup>2</sup>
Wing mean chord $\bar{c}$	4.35 m
Length $l$	30.53 m

used for the actuators was  $\xi_{\delta_i}$  50ms and  $\xi_T = 4$ s for the engines.

### 3.1.1 Plane Dynamics

In order to simulate the aircraft's fast dynamics, its moments must firstly be computed in order to use equation 2.6. The torque caused by the actuators as modelled as equation 2.4, using

$$\begin{aligned} C_{L\delta_{ail}} &= 0.02 \quad rad^{-1} \\ C_{L\delta_{rud}} &= 0.002 \quad rad^{-1} \\ C_{M\delta_{ele}} &= -0.003 \quad rad^{-1} \\ C_{N\delta_{ail}} &= -0.002 \quad rad^{-1} \\ C_{N\delta_{rud}} &= -0.07 \quad rad^{-1} \end{aligned}$$

The remaining aerodynamic coefficients from equation 2.5 were obtained from the work of Hector Escamilla Nuñez in [6]. In this work, neural networks were used to interpolate data from the USAF Stability and Control Digital Data Compendium. A two layer feed forward network, with a hidden sigmoid activation function and a linear output activation function, was then trained using the gathered data with the Bayesian Regulation training algorithm [6]. Indeed as stated previously, neural networks can approximate any continuous function according to the Universal Approximation Theorem. This method is therefore optimal to accurately approximate the variation of these coefficients that mainly vary with the angle of attack  $\alpha$  and airspeed  $V_a$ . The only exception to this method was the drag coefficient, given for this aircraft by

$$C_D = 0.0176 + 0.0515C_L^2 \quad (3.2)$$

The sum of the moments can be computed and used to obtain the rotation rates  $p$ ,  $q$  and  $r$ . Equation 2.10 can also be used and integrated to obtain the Euler angles. These will also be necessary for frame of reference changes, namely from body to earth and vice-versa. The aerodynamic forces were computed from equation 2.11 using the outputs of the neural networks. The body forces and acceleration relative to the earth frame were then obtain from equations 2.12 and 2.13. The effects of the wind were also taken into account by adding the wind speed to the integration of the acceleration of the aircraft relative to the earth as per 2.7. At this point the values of  $\alpha$  and  $\beta$  were also obtained for their respective equations 2.8a and 2.8b.

A simplified block diagram of the plane simulator is given by figure 3.1.

## 3.2 Controller Implementation

To invert such a complex system, two layers of inversion are proposed by H. Escamilla [6], namely for the fast and slow dynamics. Directly controlling these are three of the four actuator control inputs, the control surfaces for the aileron, elevon and rudder. These will therefore be the output of the nonlinear inversion control. To invert the fast dynamics the equation 2.6 must be differentiated once, to account for both the actuator dynamics 2.15 and the effects of the wind. Doing so yields the jerk vector given by, as per [6]

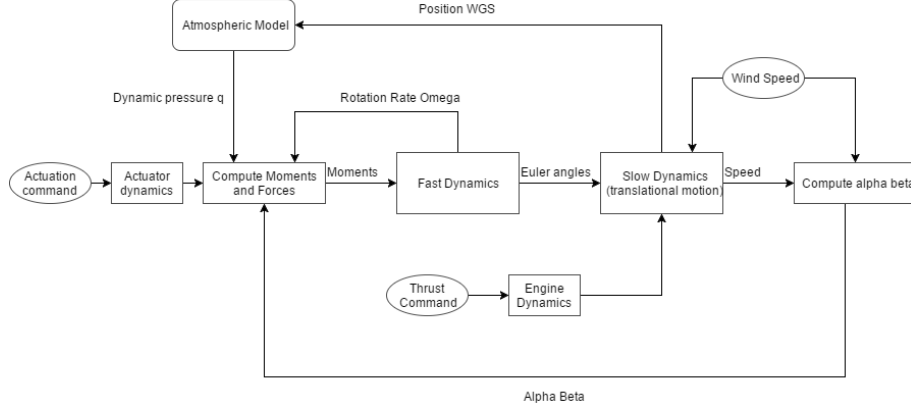


Figure 3.1: Plane dynamics simulator diagram

$$\begin{bmatrix} \ddot{p} \\ \ddot{q} \\ \ddot{r} \end{bmatrix} = \frac{1}{2} \rho S I^{-1} \left\{ V_a^2 C_\delta \xi \begin{bmatrix} \delta_{ail}^d - \delta_{ail} \\ \delta_{ele}^d - \delta_{ele} \\ \delta_{rud}^d - \delta_{rud} \end{bmatrix} + V_a^2 C_c \dot{R}_a + 2V_a \dot{V}_a \left( \begin{bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{bmatrix} + C_\delta \begin{bmatrix} \delta_{ail} \\ \delta_{ele} \\ \delta_{rud} \end{bmatrix} \right) \right\} + I^{-1} \left( \frac{1}{4} \rho S V_a C_k - I_n \right) \dot{\Omega} \quad (3.3)$$

Where

$$I = \begin{bmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{bmatrix}$$

$$I_n = \begin{bmatrix} -Eq & (C-B)r - Ep & (C-B)q \\ (A-C)r + 2Ep & 0 & (A-C)p - 2Er \\ (B-A)q & (B-A)p + Er & Eq \end{bmatrix}$$

$$\xi = \begin{bmatrix} \frac{1}{\xi_{ail}} & 0 & 0 \\ 0 & \frac{1}{\xi_{ele}} & 0 \\ 0 & 0 & \frac{1}{\xi_{rud}} \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0 & bC_{l_\beta} & -\frac{b^2}{2V_a^2}(C_{l_p}p + C_{l_r}r) \\ \bar{c}C_{m_\alpha} & 0 & -\frac{\bar{c}^2}{2V_a^2}C_{m_q}q \\ 0 & bC_{n_\beta} & -\frac{b^2}{2V_a^2}(C_{n_p}p + C_{n_r}r) \end{bmatrix}$$

$$C_k = \begin{bmatrix} b^2C_{l_p} & 0 & b^2C_{l_r} \\ 0 & \bar{c}^2C_{m_q} & 0 \\ b^2C_{n_p} & 0 & b^2C_{n_r} \end{bmatrix}$$

$$\dot{R}_a = [\dot{\alpha} \quad \dot{\beta} \quad \dot{V}_a]^T$$

To do a feedback linearisation, a pseudo input must be chosen, in this case  $\tau = \ddot{\Omega}$ . The feedback control law will therefore be, solving equation 3.3 to  $\delta^d$ ,

$$\begin{bmatrix} \delta_{ail}^d \\ \delta_{ele}^d \\ \delta_{rud}^d \end{bmatrix} = \frac{1}{V_a^2} \xi^{-1} C_\delta \left\{ \frac{2I}{\rho S} \tau - \frac{2}{\rho S} \left( \frac{1}{4} \rho S V_a C_k - I_n \right) \dot{\Omega} - 2V_a \dot{V}_a \left( \begin{bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{bmatrix} + C_\delta \delta \right) - V_a^2 C_c \dot{R}_a \right\} + \delta \quad (3.4)$$

(Check if conditions from MIMO NLI are respected)

The wind effects will appear in the terms including  $\dot{R}_a = [\dot{\alpha} \quad \dot{\beta} \quad \dot{V}_a]^T$ , as the equation defining  $\dot{V}_a$  can be obtained differentiating the norm of the speed relative to the ground. SEE ANNEX HERE

Should all of the parameters mentioned in the equations above be known and no inversion error be made, the resulting system  $\Omega = \tau$  will be both linear and decoupled, having three pseudo inputs  $\tau = [\tau_p \quad \tau_q \quad \tau_r]^T$ . As mentioned in section 2.3, the next step of the nonlinear inversion shall be to propose a linear controller for this resulting system. Taking the desired rotation rates  $\Omega^d$  as inputs comes a control law for  $\tau$ , firstly considering without neural network adaptation

$$\tau = -K_P(\Omega - \Omega^d) - K_D(\dot{\Omega} - \dot{\Omega}^d) + \ddot{\Omega} \quad (3.5)$$

Where the  $K_P$  and  $K_D$  coefficients are chosen to assure stability of KD NOT EMPIRICALLY?

From this point the next step shall be to obtain the desired values of  $\Omega$ . These were obtained using another PD controller, using Euler angle reference values to control rotation rates. Among these reference values to be followed,  $\tau_r$  was set to zero. This was made to make use of the decoupling through feedback linearisation of the plane's movement, using a roll and pitch motion to turn the aircraft. This results in the following control of rotation rate

$$p = k_P(\phi^d - \phi) + k_d(\dot{\phi}^d - \dot{\phi}) \quad (3.6a)$$

$$q = k_P(\theta^d - \theta) + k_d(\dot{\theta}^d - \dot{\theta}) \quad (3.6b)$$

$$r = 0 \quad (3.6c)$$

### 3.3 Neural Network

### 3.4 Verification and Validation

The goal in this section will be to validate the behaviour of the model in cruise conditions. Assuming cruise conditions comes that



$$T = D \quad (3.7)$$

$$W = L \quad (3.8)$$

$$L' = M = N = 0 \quad (3.9)$$

In order to verify the model described so far, the required thrust to have cruise conditions will be computed for a given plausible value of  $\alpha$ . From this point the airspeed of the aircraft can also be computed. The graph of  $C_L$  versus alpha was also obtained from its respective neural network, given by 3.2 From

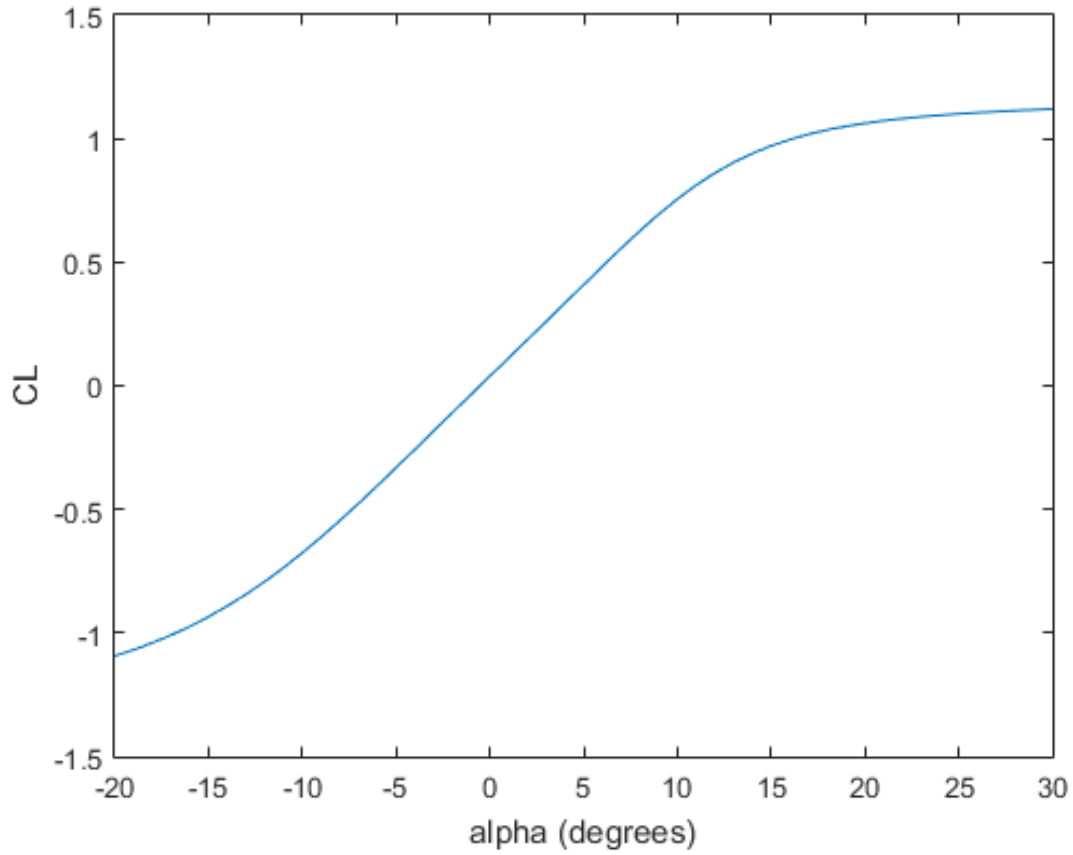


Figure 3.2:  $C_L$  versus  $\alpha$  graph from Neural Network

the cruise conditions 3.9, knowing that  $L = \frac{1}{2}\rho S V^2 C_L$ , solving for the airspeed  $V$  comes that

$$V = \sqrt{\frac{2mg}{\rho S C_L}} \quad (3.10)$$

The required thrust can also be computed from both 3.9, 3.10 and 3.2, knowing the  $C_L$  for a given angle of attack. Proposing some values of  $\alpha$ , the following results are obtained

Table 3.2: Required cruise conditions for different values of  $\alpha$

$\alpha(^{\circ})$	$C_D$	$C_L$	$V(ms^{-1})$	$T(N)$
0	0.017677131	0.0387	707.4010791	224047.3585
2	0.019379779	0.1859	322.7613368	51133.84325
4	0.023345134	0.334	240.7952785	34283.79709
6	0.029604436	0.4828	200.279994	30076.58604

# Chapter 4

## Results

Insert your chapter material here...

### 4.1 Problem Description

Description of the baseline problem...

### 4.2 Baseline Solution

Analysis of the baseline solution...

### 4.3 Enhanced Solution

Quest for the optimal solution...

#### 4.3.1 Figures

Insert your section material and possibly a few figures...

Make sure all figures presented are referenced in the text!

#### Images



Figure 4.1: Caption for figure.



(a) Airbus A320



(b) Bombardier CRJ200

Figure 4.2: Some aircrafts.

Make reference to Figures 4.1 and 4.2.

By default, the supported file types are *.png,.pdf,.jpg,.mps,.jpeg,.PNG,.PDF,.JPG,.JPEG*.

See [http://mactex-wiki.tug.org/wiki/index.php/Graphics\\_inclusion](http://mactex-wiki.tug.org/wiki/index.php/Graphics_inclusion) for adding support to other extensions.

## Drawings

Insert your subsection material and for instance a few drawings...

The schematic illustrated in Fig. 4.3 can represent some sort of algorithm.

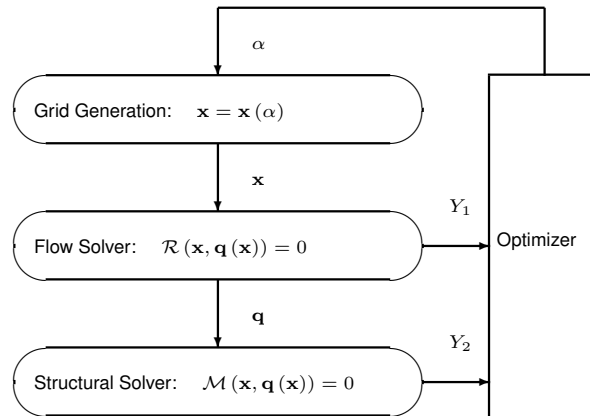


Figure 4.3: Schematic of some algorithm.

## 4.3.2 Equations

Equations can be inserted in different ways.

The simplest way is in a separate line like this

$$\frac{dq_{ijk}}{dt} + \mathcal{R}_{ijk}(\mathbf{q}) = 0. \quad (4.1)$$

If the equation is to be embedded in the text. One can do it like this  $\partial\mathcal{R}/\partial\mathbf{q} = 0$ .

It may also be split in different lines like this

$$\begin{aligned} &\text{Minimize} && Y(\alpha, \mathbf{q}(\alpha)) \\ &\text{w.r.t.} && \alpha, \\ &\text{subject to} && \mathcal{R}(\alpha, \mathbf{q}(\alpha)) = 0 \\ &&& C(\alpha, \mathbf{q}(\alpha)) = 0. \end{aligned} \tag{4.2}$$

It is also possible to use subequations. Equations 4.3a, 4.3b and 4.3c form the Navier–Stokes equations 4.3.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0, \tag{4.3a}$$

$$\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j + p \delta_{ij} - \tau_{ji}) = 0, \quad i = 1, 2, 3, \tag{4.3b}$$

$$\frac{\partial}{\partial t} (\rho E) + \frac{\partial}{\partial x_j} (\rho E u_j + p u_j - u_i \tau_{ij} + q_j) = 0. \tag{4.3c}$$

### 4.3.3 Tables

Insert your subsection material and for instance a few tables...

Make sure all tables presented are referenced in the text!

Follow some guidelines when making tables:

- Avoid vertical lines
- Avoid “boxing up” cells, usually 3 horizontal lines are enough: above, below, and after heading
- Avoid double horizontal lines
- Add enough space between rows

Model	$C_L$	$C_D$	$C_{My}$
Euler	0.083	0.021	-0.110
Navier–Stokes	0.078	0.023	-0.101

Table 4.1: Table caption.

Make reference to Table 4.1.

Tables 4.2 and 4.3 are examples of tables with merging columns:

An example with merging rows can be seen in Tab.4.4.

If the table has too many columns, it can be scaled to fit the text width, as in Tab.4.5.

	Virtual memory [MB]	
	Euler	Navier–Stokes
Wing only	1,000	2,000
Aircraft	5,000	10,000
(ratio)	5.0×	5.0×

Table 4.2: Memory usage comparison (in MB).

		$w = 2$			$w = 4$		
		$t = 0$	$t = 1$	$t = 2$	$t = 0$	$t = 1$	$t = 2$
$dir = 1$							
	$c$	0.07	0.16	0.29	0.36	0.71	3.18
	$c$	-0.86	50.04	5.93	-9.07	29.09	46.21
	$c$	14.27	-50.96	-14.27	12.22	-63.54	-381.09
$dir = 0$							
	$c$	0.03	1.24	0.21	0.35	-0.27	2.14
	$c$	-17.90	-37.11	8.85	-30.73	-9.59	-3.00
	$c$	105.55	23.11	-94.73	100.24	41.27	-25.73

Table 4.3: Another table caption.

ABC	header			
	1.1	2.2	3.3	4.4
IJK	group		0.5	0.6
			0.7	1.2

Table 4.4: Yet another table caption.

Variable	a	b	c	d	e	f	g	h	i	j
Test 1	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
Test 2	20,000	40,000	60,000	80,000	100,000	120,000	140,000	160,000	180,000	200,000

Table 4.5: Very wide table.

### 4.3.4 Mixing

If necessary, a figure and a table can be put side-by-side as in Fig.4.4



Legend		
A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Figure 4.4: Figure and table side-by-side.





## **Chapter 5**

# **Conclusions**

Insert your chapter material here...

### **5.1 Achievements**

The major achievements of the present work...

### **5.2 Future Work**

A few ideas for future work...



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# Appendix A

## Vector calculus

In case an appendix is deemed necessary, the document cannot exceed a total of 100 pages...

Some definitions and vector identities are listed in the section below.

### A.1 Vector identities

$$\nabla \times (\nabla \phi) = 0 \tag{A.1}$$

$$\nabla \cdot (\nabla \times \mathbf{u}) = 0 \tag{A.2}$$



## **Appendix B**

# **Technical Datasheets**

It is possible to add PDF files to the document, such as technical sheets of some equipment used in the work.

### **B.1 Some Datasheet**

### BENEFITS

#### Maximum Light Capture

SunPower's all-back contact cell design moves gridlines to the back of the cell, leaving the entire front surface exposed to sunlight, enabling up to 10% more sunlight capture than conventional cells.

#### Superior Temperature Performance

Due to lower temperature coefficients and lower normal cell operating temperatures, our cells generate more energy at higher temperatures compared to standard c-Si solar cells.

#### No Light-Induced Degradation

SunPower n-type solar cells don't lose 3% of their initial power once exposed to sunlight as they are not subject to light-induced degradation like conventional p-type c-Si cells.

#### Broad Spectral Response

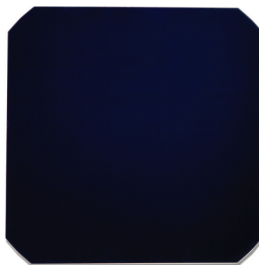
SunPower cells capture more light from the blue and infrared parts of the spectrum, enabling higher performance in overcast and low-light conditions.

#### Broad Range Of Application

SunPower cells provide reliable performance in a broad range of applications for years to come.

The SunPower™ C60 solar cell with proprietary Maxeon™ cell technology delivers today's highest efficiency and performance.

The anti-reflective coating and the reduced voltage-temperature coefficients provide outstanding energy delivery per peak power watt. Our innovative all-back contact design moves gridlines to the back of the cell, which not only generates more power, but also presents a more attractive cell design compared to conventional cells.



SunPower's High Efficiency Advantage

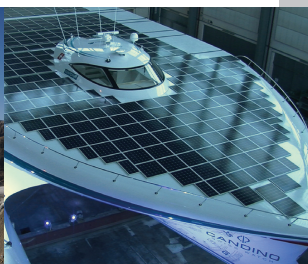
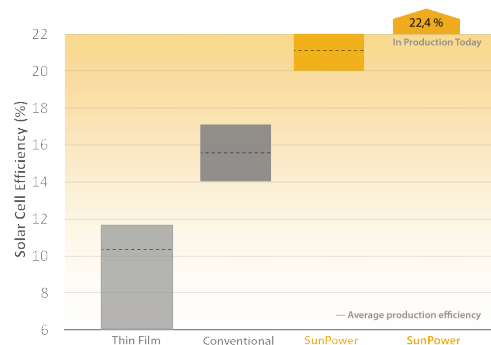


Photo courtesy of 3S Photovoltaics

### Electrical Characteristics of Typical Cell at Standard Test Conditions (STC)

STC: 1000W/m<sup>2</sup>, AM 1.5g and cell temp 25°C

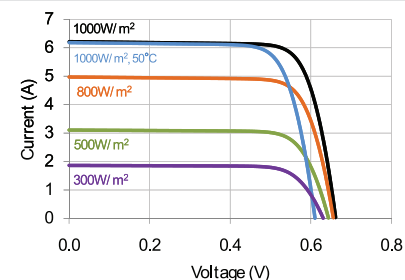
Bin	P <sub>mp</sub> (Wp)	Eff. (%)	V <sub>mp</sub> (V)	I <sub>mp</sub> (A)	V <sub>oc</sub> (V)	I <sub>sc</sub> (A)
G	3.34	21.8	0.574	5.83	0.682	6.24
H	3.38	22.1	0.577	5.87	0.684	6.26
I	3.40	22.3	0.581	5.90	0.686	6.27
J	3.42	22.5	0.582	5.93	0.687	6.28

All Electrical Characteristics parameters are nominal  
Unlaminated Cell Temperature Coefficients  
Voltage: -1.8 mV / °C Power: -0.32% / °C

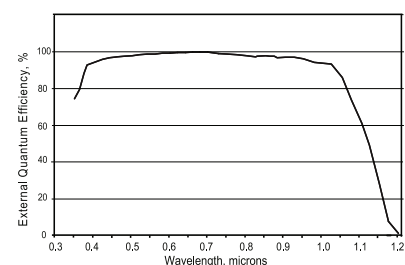
### Positive Electrical Ground

Modules and systems produced using these cells must be configured as "positive ground systems".

### TYPICAL I-V CURVE



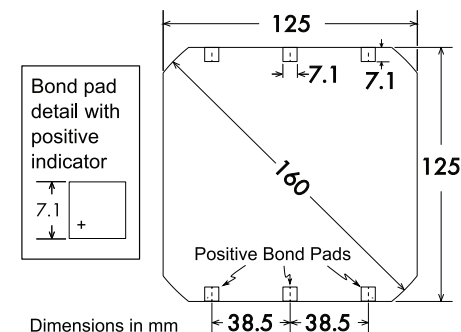
### SPECTRAL RESPONSE



### Physical Characteristics

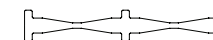
Construction:	All back contact
Dimensions:	125mm x 125mm (nominal)
Thickness:	165µm ± 40µm
Diameter:	160mm (nominal)

### Cell and Bond Pad Dimensions



Bond pad area dimensions are 7.1mm x 7.1mm  
Positive pole bond pad side has "+" indicator on leftmost and rightmost bond pads.

### Interconnect Tab and Process Recommendations



Tin plated copper interconnect. Compatible with lead free process.

### Packaging

Cells are packed in boxes of 1,200 each; grouped in shrink-wrapped stacks of 150 with interleaving. Twelve boxes are packed in a water-resistant "Master Carton" containing 14,400 cells suitable for air transport.

Interconnect tabs are packaged in boxes of 1,200 each.

### About SunPower

SunPower designs, manufactures, and delivers high-performance solar electric technology worldwide. Our high-efficiency solar cells generate up to 50 percent more power than conventional solar cells. Our high-performance solar panels, roof tiles, and trackers deliver significantly more energy than competing systems.