Bubble Sort

5 9 8 2 1

1st (n-1) comp

1.1 5 9 8 2 1

1.2 5 8 9 2 1

1.3 5 8 2 9 1

1.4 5 8 2 1 9

2nd (n-2) comp

5 8 2 1 | 9

2.1 5 8 2 1 | 9

2.2 5 2 8 1 | 9

2.3 5 2 1 8 | 9

3rd (n-3) comp

5 2 1 | 8 9

3.1 2 5 1 | 8 9

3.2 2 1 5 | 8 9

4th 2 1 | 5 8 9 (n-4) comp

4.1 1 2 | 5 8 9 → ans = 1 2 5 8 9

Total comp = (n-1) + (n-2) ... 1

$\frac{n(n-1)}{2}$

$O(N^2)$

TC depends on no of comparisons

```java
public static void bubbleSort(int[] arr) {
  //write your code here
  int n = arr.length;
  for(int i=0;i<n;i++) {
      for(int j=0;j<n-i-1;j++) {

          if(isSmaller(arr,j+1,j) == true) {
              swap(arr,j + 1,j);
          }
      }
  }
}
```

```java
// used for swapping ith and jth elements of array
public static void swap(int[] arr, int i, int j) {
  System.out.println("Swapping " + arr[i] + " and " + arr[j]);
  int temp = arr[i];
  arr[i] = arr[j];
  arr[j] = temp;
}

// return true if ith element is smaller than jth element
public static boolean isSmaller(int[] arr, int i, int j) {
  System.out.println("Comparing " + arr[i] + " and " + arr[j]);
  if (arr[i] < arr[j]) {
    return true;
  } else {
    return false;
  }
}
```

The main summary of Bubble Sort is that the heaviest element (largest) will be at its sorted position after the first iteration.

So, the largest elements keep on moving at the last of the array.

Best Case : Array Already Sorted

$O(N^2)$

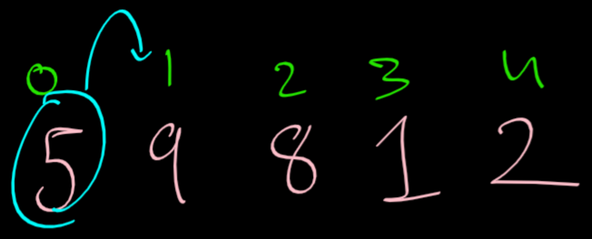
Selection Sort

TC = O(N²)

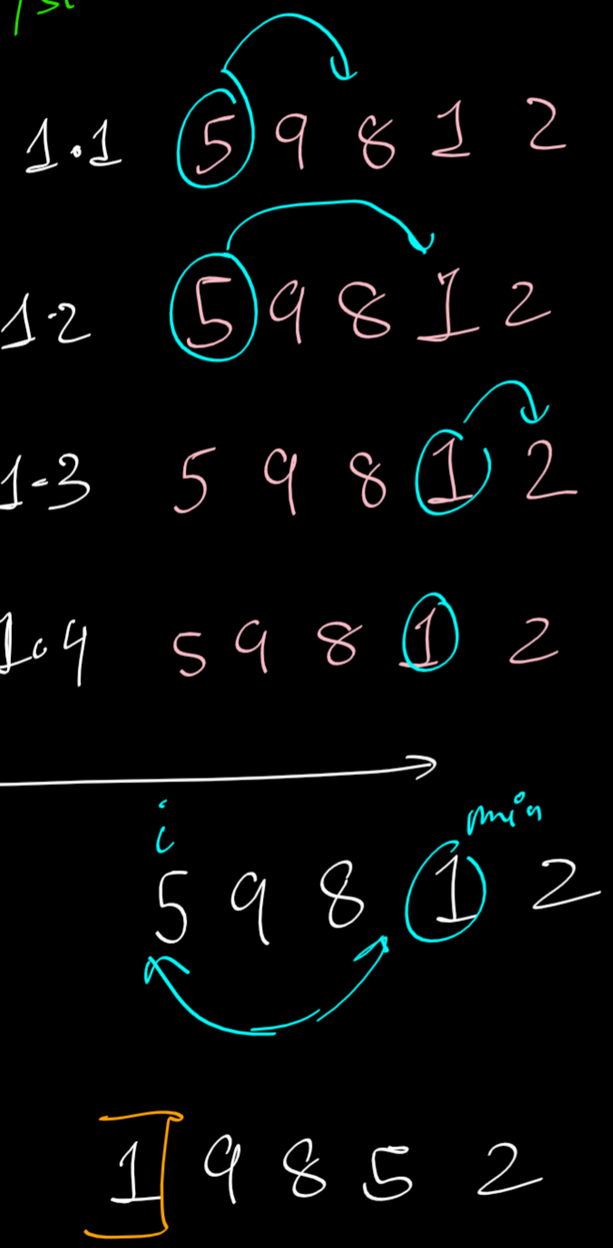
find i^{th} minimum element in every iteration.

swap the min element with i^{th} ele.

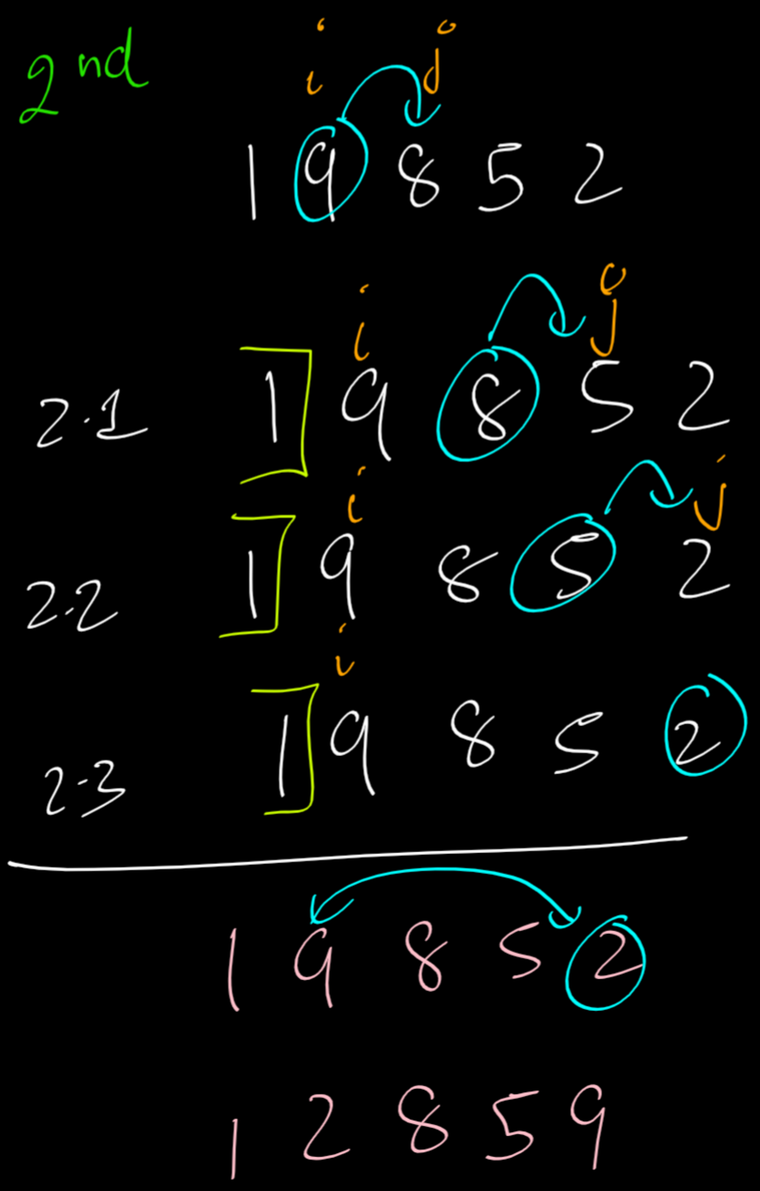
(in every sub-iteration, we are comparing ele at min with the ele at j^{th} pos)



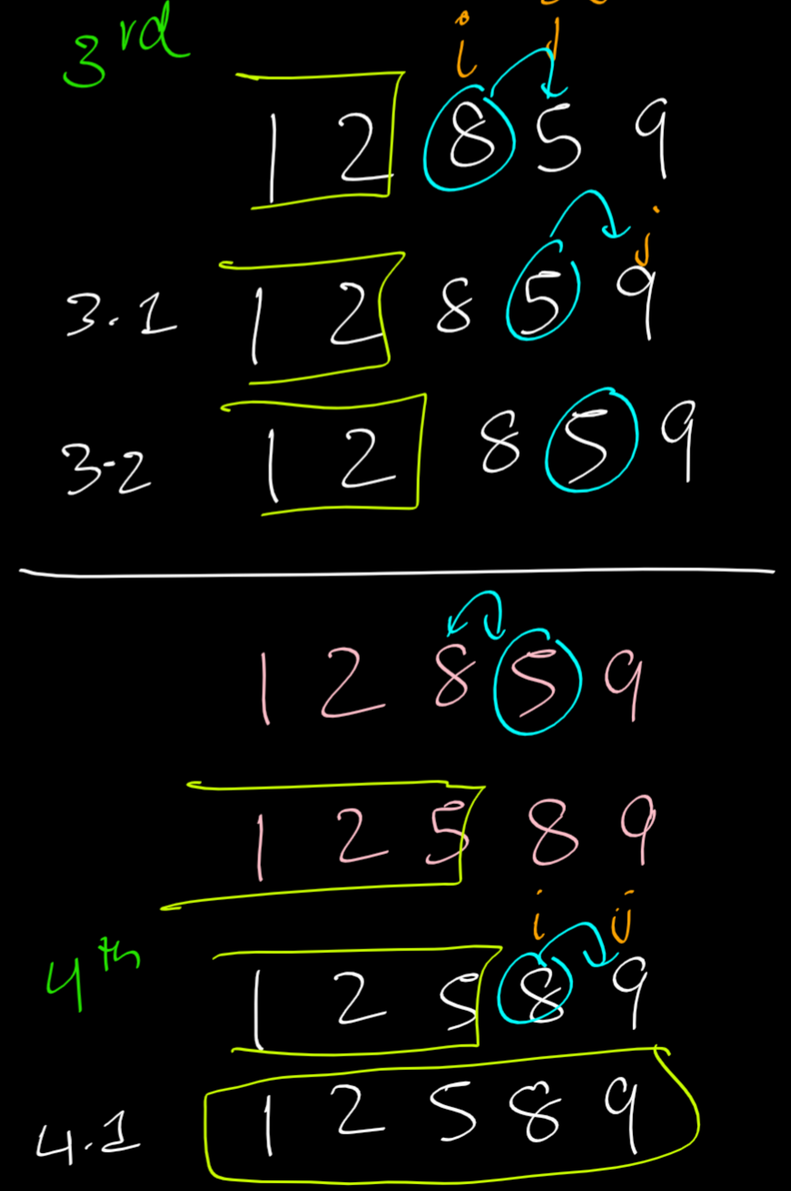
1st



2nd



3rd



Selection Sort Code

```java
public static void selectionSort(int[] arr) {
  //write your code here
  int n = arr.length;
  for(int i=0;i<n - 1;i++) {
    int min = i;
    for(int j=i+1;j<n;j++) {
      if(isSmaller(arr,j,min) == true) {
        min = j;
      }
    }
    swap(arr,i,min);
  }
}
```

If array is already sorted, then also $TC = O(N^2)$

Bubble Sort is worst than selection sort because of high no of swaps.

Data Structure Operations Cheat Sheet

Data Structure Name	Average Case Time Complexity				Worst Case Time Complexity				Space Complexity
	Accessing n^{th} element	Search	Insertion	Deletion	Accessing n^{th} element	Search	Insertion	Deletion	Worst Case
Arrays	$O(1)$	$O(n)$	$O(n)$	$O(n)$	$O(1)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Stacks	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$
Queues	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$
Binary Trees	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Binary Search Trees	$O(logn)$	$O(logn)$	$O(logn)$	$O(logn)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Balanced Binary Search Trees	$O(logn)$	$O(logn)$	$O(logn)$	$O(logn)$	$O(logn)$	$O(logn)$	$O(logn)$	$O(logn)$	$O(logn)$
Hash Tables	N/A	$O(1)$	$O(1)$	$O(1)$	N/A	$O(n)$	$O(n)$	$O(n)$	$O(n)$

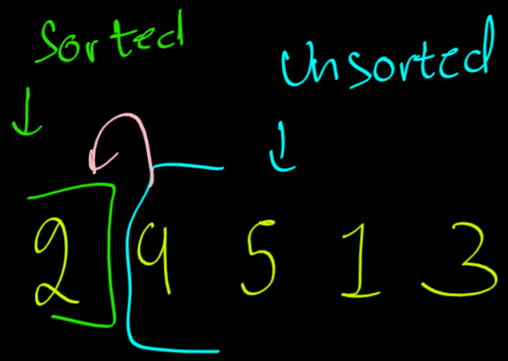
Note: For best case operations, the time complexities are $O(1)$.

Sorting Algorithms Cheat Sheet

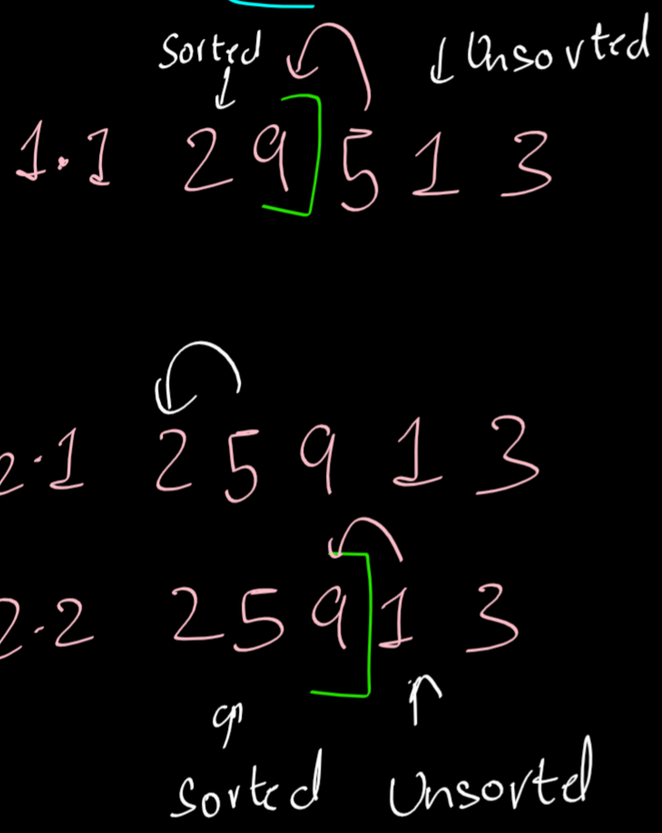
Sorting Algorithm Name	Time Complexity			Space Complexity	Is Stable?	Sorting Class Type	Remarks
	Best Case	Average Case	Worst Case	Worst Case			
Bubble Sort	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$	Yes	Comparison	Not a preferred sorting algorithm.
Insertion Sort	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$	Yes	Comparison	In the best case (already sorted), every insert requires constant time
Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$	Yes	Comparison	Even a perfectly sorted array requires scanning the entire array
Merge Sort	$O(nlogn)$	$O(nlogn)$	$O(nlogn)$	$O(n)$	Yes	Comparison	On arrays, it requires $O(n)$ space; and on linked lists, it requires constant space
Heap Sort	$O(nlogn)$	$O(nlogn)$	$O(nlogn)$	$O(1)$	No	Comparison	By using input array as storage for the heap, it is possible to achieve constant space
Quick Sort	$O(nlogn)$	$O(nlogn)$	$O(n^2)$	$O(logn)$	No	Comparison	Randomly picking a pivot value can help avoid worst case scenarios such as a perfectly sorted array.
Tree Sort	$O(nlogn)$	$O(nlogn)$	$O(n^2)$	$O(n)$	Yes	Comparison	Performing inorder traversal on the balanced binary search tree.
Counting Sort	$O(n + k)$	$O(n + k)$	$O(n + k)$	$O(k)$	Yes	Linear	Where k is the range of the non-negative key values.
Bucket Sort	$O(n + k)$	$O(n + k)$	$O(n^2)$	$O(n)$	Yes	Linear	Bucket sort is stable, if the underlying sorting algorithm is stable.
Radix Sort	$O(dn)$	$O(dn)$	$O(dn)$	$O(d + n)$	Yes	Linear	Radix sort is stable, if the underlying sorting algorithm is stable.

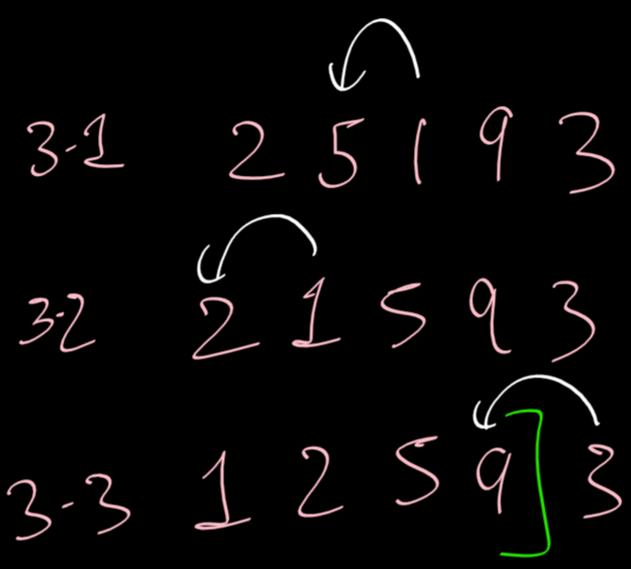
In place Algorithms

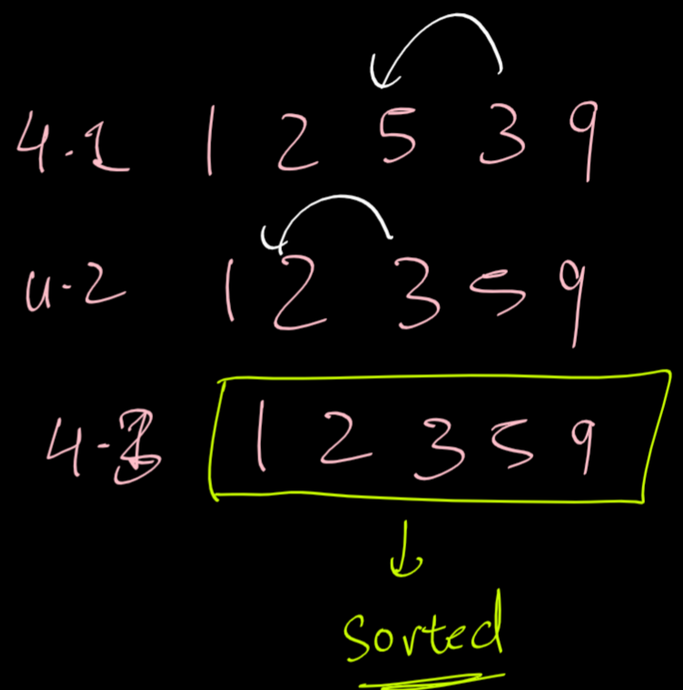
Insertion Sort



$O(N^2)$








```java
public static void insertionSort(int[] arr) {
  //write your code here
  for(int i=1;i<arr.length;i++) {
      for(int j=i;j>0;j--) {
          if(isGreater(arr,j-1,j)) {
              swap(arr,j-1,j);
          } else {
              break;
          }
      }
  }
}
```

Merge 2 Sorted Array (Two pointer technique)

(M) A: 2 3 6 7 9 10

(N) B: 1 4 5 7 10 12 13 14

(M+N) C =

0	1	2	3	4	5	6	7	8	9	10	11	12	13
1	2	3	4	5	6	7	7	9	10	10	12	13	14

Compare A[i] & B[j] and add the smaller element into C.

After one array is completed, copy all the elements of the second array as it is.

```java
public static int[] mergeTwoSortedArrays(int[] a, int[] b){
  //write your code here
    int[] c = new int[a.length + b.length];

    int i = 0, j = 0, k = 0;

    while(i < a.length && j < b.length) {

        if(a[i] <= b[j]) {
            c[k] = a[i];
            i++;
            k++;
        } else {
            c[k] = b[j];
            j++;
            k++;
        }
    }

    while(j<b.length) {
        c[k] = b[j];
        j++;
        k++;
    }

    while(i<a.length) {
        c[k] = a[i];
        i++;
        k++;
    }
    return c;
```

A.length = n

B.length = m

TC = O(n+m)

Merge Sort (Divide & Conquer)

0 1 2 3 4 5 6 7 8 9 10

{5 2 6 3 1 9 4 8 7 2 5}

faith that both get sorted using MS

{1 2 3 5 6 9} {2 4 5 7 8}

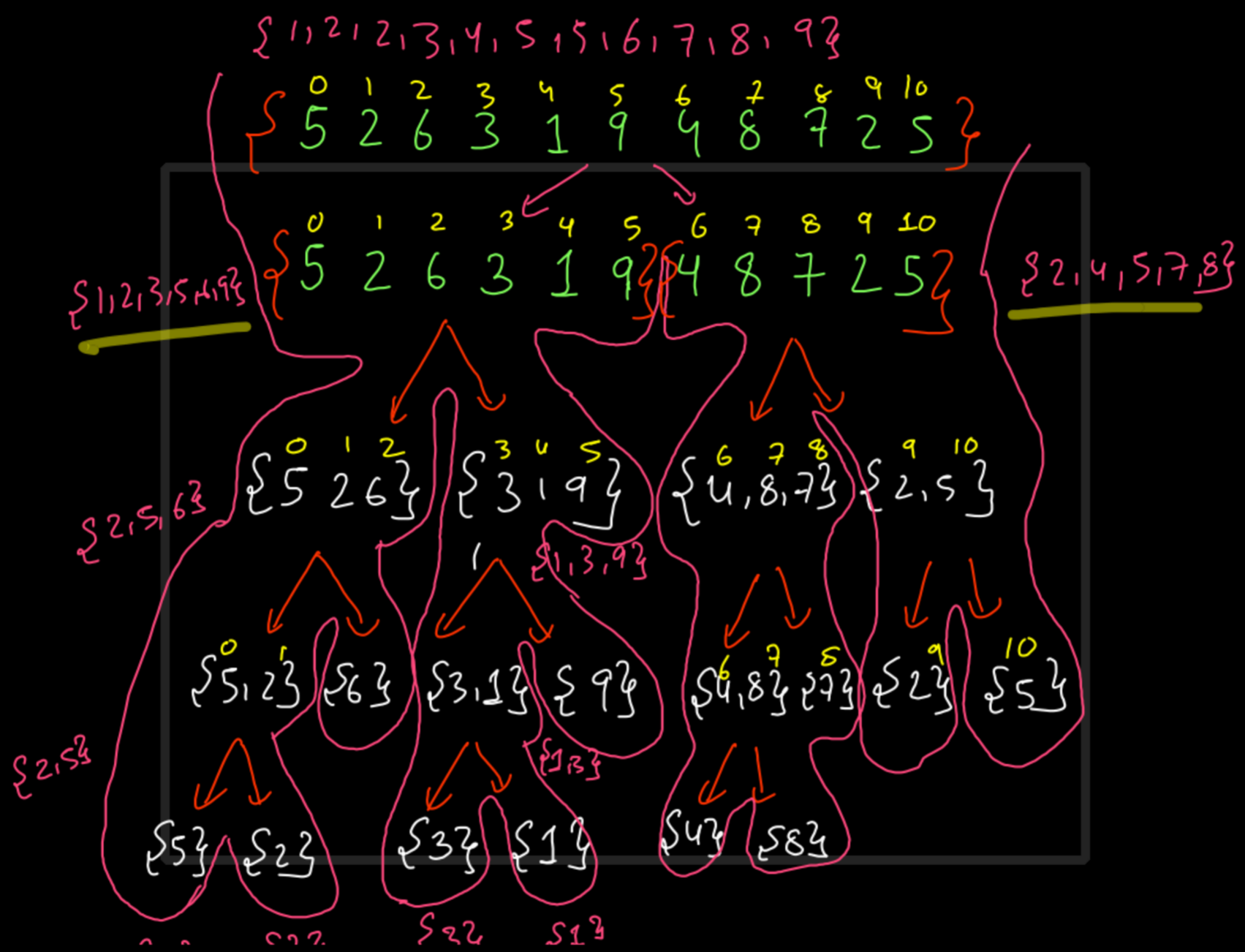
Merge them

{1 2 2 3 4 5 5 6 7 8 9}

Divide & Conquer: Divide big problem into 2 almost equally sized sub problems. (Recursion faith)

Now Conquer (Merge in this case)

Not Mandatory that conquer will be after divide only.


{1,2,2,3,4,5,5,6,7,8,9}
0 1 2 3 4 5 6 7 8 9 10
5 2 6 3 1 9 4 8 7 2 5
{1,2,3,5,4,9}
{2,4,5,7,8}
{2,5,6}
{1,3,9}
{2,5,4}
{1,3}

```java
public static int[] mergeSort(int[] arr, int lo, int hi) {
  //write your code here

  if(lo == hi) {
      int[] baseArr = new int[1];
      baseArr[0] = arr[lo];
      return baseArr;
  }

  int mid = (lo + hi)/2;

  int[] leftSortedArray = mergeSort(arr,lo,mid);
  int[] rightSortedArray = mergeSort(arr,mid+1,hi);

  int[] sortedArray = mergeTwoSortedArrays(leftSortedArray,rightSortedArray);

  return sortedArray;
}
```

Calls = 2

Height = $\log_2 N$

$$(calls)^{h} + (pre+post)*h = (2)^{\log_2 n} + (0+N)*\log_2 N \quad \text{(N: merge)}$$

$$= n + n\log_2 N$$

$$\boxed{O(\log_2 N)}$$

$$T(n) = 2T(n/2) + N$$

$$2\,T(n/2) = (2T(n/4) + N/2)\,2$$

$$2^2\,T(n/4) = (2T(n/8) + N/4)\,2^2$$

$$\vdots$$

$$2^{k-1}(T(1)) = (2T(0) + 1)\,2^{k-1}$$

$$T(n) = 2^k + N + \frac{N}{2} \times 2 + \frac{N}{2^2} \times 2^2 \ldots \text{k times}$$

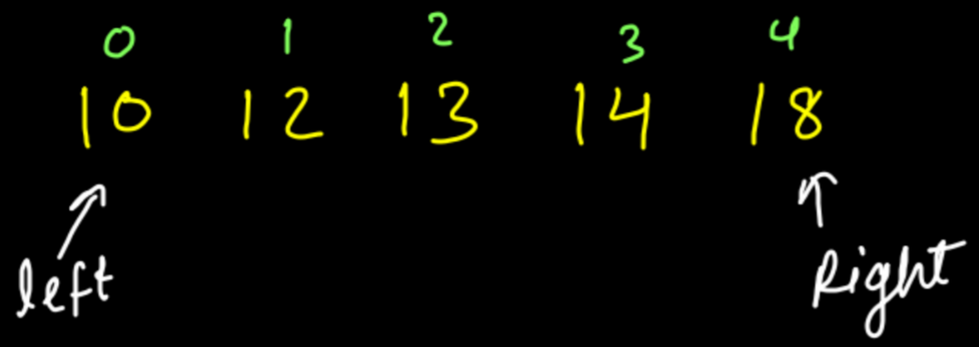
$$T(n) = 2^{\log_2 N} + N \log_2 N \;(k = \log_2 N) = O(N \log_2 N)$$

① Extra/Auxi Space: $O(N)$ {at merging}

② Input/output : $O(N)$

③ Recursion (all Stack Space: $O(\log_2 N)$
(Height of Stack/Tree)

Target Sum Pair




```
//brute force
for(int i=0;i<nums.length;i++) {
    for(int j=i+1;j<nums.length;j++) {
        if(nums[i] + nums[j] == target) {
            return new int[]{i=1,j+1};
        }
    }
}

return null;
```

2 pointer solution

```
int lo = 0;
int hi = nums.length-1;

while(lo < hi) {
    if(nums[lo] + nums[hi] < target) {
        lo++;
    } else if(nums[lo] + nums[hi] > target) {
        hi--;
    } else {
        return new int[]{lo+1,hi+1};
    }
}

return null;
```

nums[left] + nums[right] > target

right--;

bada kuch zyada he
bada reh gaya

nums[left] + num[right] < target

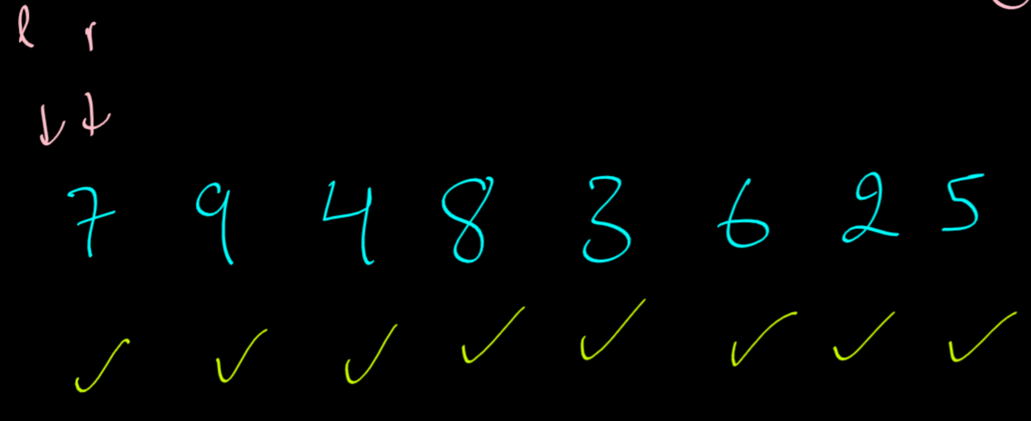
left++;

chota kuch < yada he
chota reh gaya

$T(n) = T(n-1) + O(1)$

$TC = O(N)$

Partition an Array



pivot = 5

(r to n)
unexplored

(0 to l-1)
≤ pivot

(l to r-1)
> pivot

Initial Situation of the Array

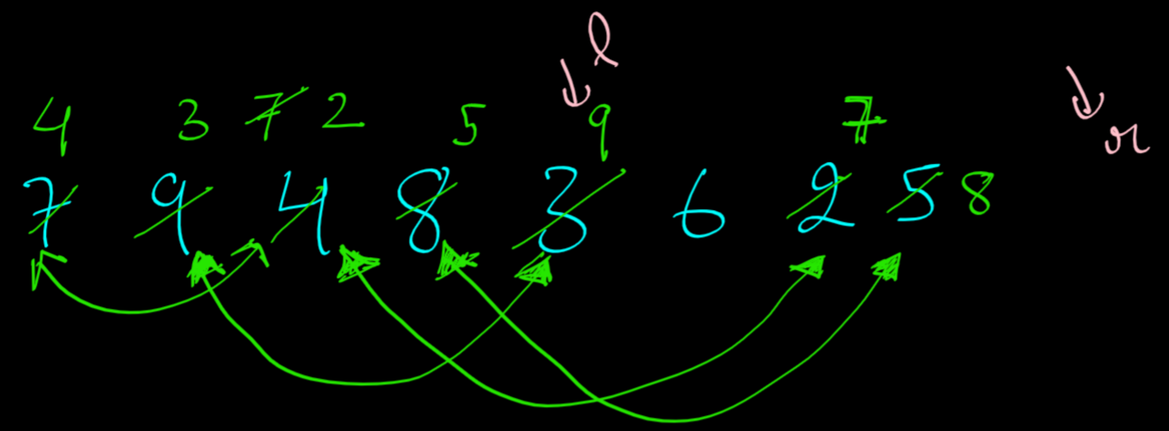
```
if (arr[right] > pivot)
    right++
else{
    swap(arr[l], arr[r]);
    left++; right++;
}
```

Left pointer → first element of greater region
Right pointer → first ele of unexplored region

relative order inside the array does NOT MATTER

≤ pivot → left > pivot → right

Procedure:



(r to n)
unexplored

(0 to l-1)
≤ pivot

✓ ✓ ✓ ✓

(l to r-1)
> pivot

✓ ✓ ✓ ✓

Partitioned Array: 4 3 2 5 9 6 7 8

```java
public static void partition(int[] arr, int pivot){
  //write your code here

  int left = 0;
  int right = 0;

  for(int i=0;i<arr.length;i++) {
    if(arr[i] > pivot) {
      right++;
    } else {
      int temp = arr[left];
      swap(arr,right,left);
      left++;right++;
    }
  }

}
```

Sort 01

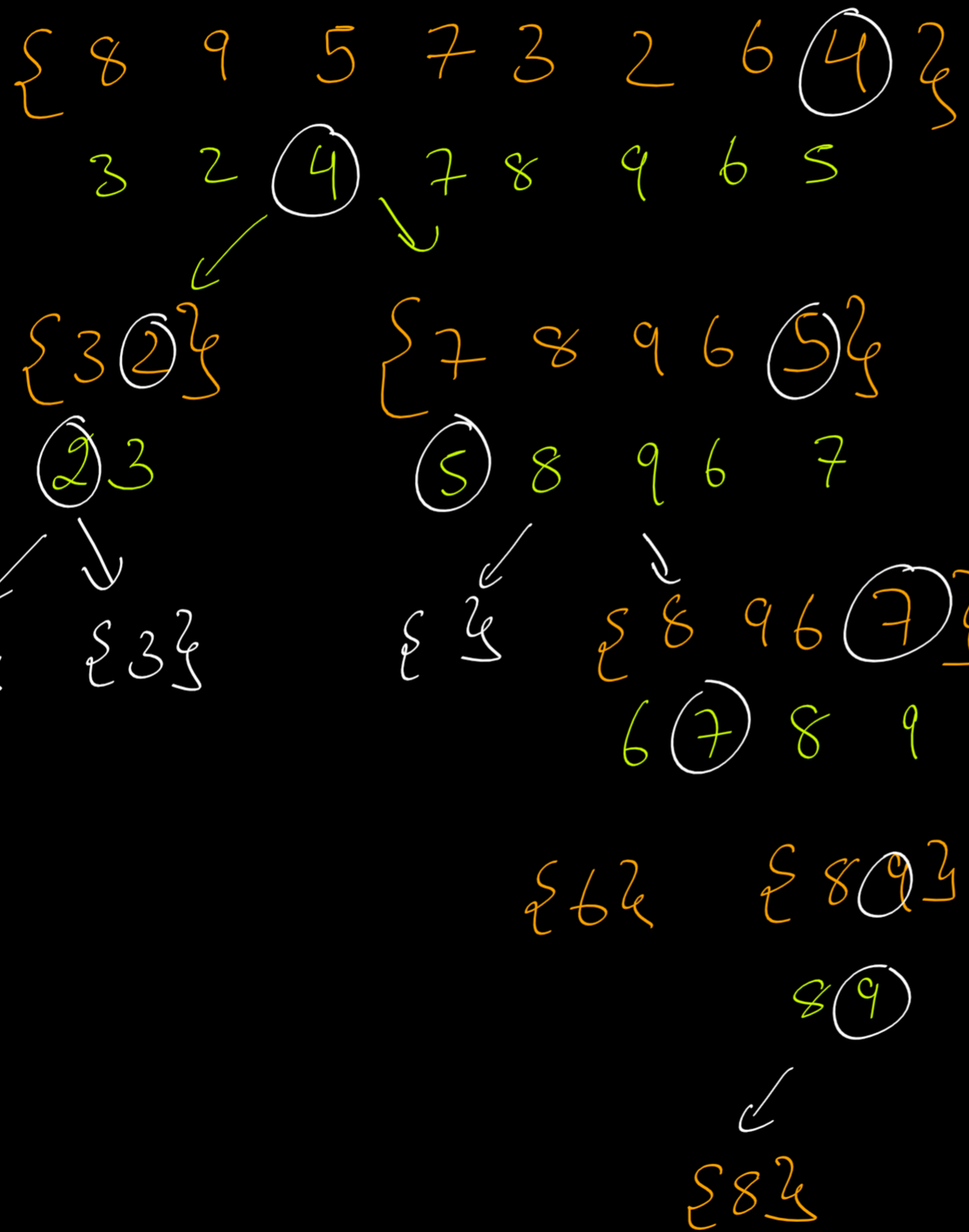
Same as the partition of array. Just make pivot = 1 or 0 and partition accordingly.

```java
public static void sort01(int[] arr){
  //write your code here
  int left = 0;
  int right = 0;

  for(int i=0;i<arr.length;i++) {
    if(arr[i] > 0) {
      right++;
    } else {
      int temp = arr[left];
      swap(arr,right,left);
      left++;right++;
    }
  }

}
```

Quick Sort



Partitions In Preorder

Subarray [l-r]

pivot = arr[right]

⇓

why?

↳ Partitioning on pivot will result in pivot element to be on its' current index (its' sorted position)

QuickSort Recursive Code

```java
public static void quickSort(int[] arr, int lo, int hi) {
  //write your code here
  if(lo > hi) {
      //no element in the array
      return;
  }

  int partitionIndex = partition(arr,arr[hi],lo,hi);

  quickSort(arr,lo,partitionIndex-1);
  quickSort(arr,partitionIndex + 1, hi);
}
```

Decrease and Conquer approach

Space

Input/output → O(N)

Extra space → O(1)

Recursion call stack → worst case O(n) (depth of Recursion tree)

Avg case O(h) = O(log n)