MSc. High Performance Computing

TCD

Quantitative Finance

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Recall: Forward Rates

Today t_1, effective date t_2, maturity date t_3
FRA payoff at t_3:

$$\Pi(t_3) = N\,\delta(t_2, t_3)\,\{K - L(t_2, t_3)\} \tag{1}$$

FRA contract value at trade time t_1

$$\Pi(t_1) = N\,\{\delta(t_2, t_3)\,K\,P(t_1, t_3) + P(t_1, t_3) - P(t_1, t_2)\}$$

Fair FRA rate K is the *forward rate* $F(t_1; t_2, t_3)$

$$K = \frac{1}{\delta(t_2, t_3)}\left(\frac{P(t_1, t_2)}{P(t_1, t_3)} - 1\right) =: F(t_1; t_2, t_3)$$

Express $\Pi(t_1)$ in terms of F:

$$\Pi(t_1) = N\,P(t_1, t_3)\,\delta(t_2, t_3)\,\{\,K - F(t_1; t_2, t_3)\,\} \tag{2}$$

Pricing recipe: Compare $\Pi(t_3)$ to $\Pi(t_1)$

- multiply the payoff by discount factor $P(t_1, t_3)$
- replace $L(t_2, t_3)$ by $F(t_1; t_2, t_3)$

FRA - Replication

Combine a long deposit with maturity t_3 and a short deposit with maturity t_2 both starting today. This portfolio represents effectively an investment over the future period $[t_2, t_3]$:

	Flow at t_1	Flow at t_2	Flow at t_3
Deposit 1	$+N$	$-N/P(t_1, t_2)$	0
Deposit 2	$-N$	0	$+N/P(t_1, t_3)$
Total	0	$-N/P(t_1, t_2)$	$+N/P(t_1, t_3)$
Multiplied	0	-1	$+P(t_1, t_2)/P(t_1, t_3)$

Difference between flows between subsequent cash flow amounts is the fair forward interest rate, i.e.

$$F(t_1; t_2, t_3)\,\delta(t_2, t_3) := \frac{P(t_1, t_2)}{P(t_1, t_3)} - 1.$$

Money Market Futures

Money market futures are the exchange traded equivalents of FRAs. However, there are differences:

- clearing house
- standardization: typically fixed expiries t_2 on 3rd Wednesday of March, June, September, December
- margining: daily settlement of the daily change in future contract value; the settlement amount builds up gradually

Value at expiry

$$\Pi(t_2; t_2, t_3) = 100 \cdot \{ 1 - L(t_2, t_3) \}$$

Approximate futures contract value at $t < t_2$:

$$\Pi(t; t_2, t_3) = 100 \cdot \{ 1 - F(t; t_2, t_3) \}$$

(to be refined later on)

Floating Rate Note

A Floating Rate Note (FRN) is a bond that pays a *floating* rate that is typically expressed as a Libor rate $L(t, T)$ plus/minus a spread s, applied to notional amount N.

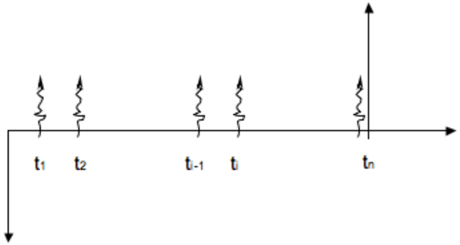


Figure: Floating Rate Note cashflow schematic

A FRN is a cash instrument that is particularly sensitive to the credit worthiness (default risk) of the bond issuer. Here we are ignoring this complication for the moment and assume that the bond is default-risk-free (to be refined later on).

Pricing an FRN

To value the FRN, we decompose into a series of fixed coupons based on spread s and a series of FRAs. Fixed payments are discounted, each FRA is treated following the FRA recipe, so that

$$\Pi(t) = N \sum_{i=1}^{n} \{ F(t; t_{i-1}, t_i) + s_i \} \; \delta(t_{i-1}, t_i) \; P(t, t_i) + N \, P(t, t_n) \tag{3}$$

The amortising flavour is given by

$$\Pi(t) = \sum_{i=1}^{n} \{ N_i \left(F(t; t_{i-1}, t_i) + s_i \right) \delta(t_{i-1}, t_i) + N_{i-1} - N_i \} \; P(t, t_i) + N_n \, P(t, t_n)$$

Constant Notional FRN

For constant notional and zero spread, the pricing formula simplifies to

$$\Pi(t) = N \sum_{i=1}^{n} F(t; t_{i-1}, t_i)\, \delta(t_{i-1}, t_i)\, P(t, t_i) + N\, P(t_n).$$

Insert the definition of the forward rate in terms of discounts,
$F(t; t_1, t_2)\delta(t_1, t_2) = P(t, t_1)/P(t, t_2) - 1$:

$$\begin{aligned}
\Pi(t) &= N \sum_{i=1}^{n} \left(P(t, t_{i-1}) - P(t, t_i)\right) + N\, P(t, t_n) \\
&= N \left\{ P(t, t_0) - P(t, t_1) + P(t, t_1) - P(t, t_2) + \cdots + P(t, t_n) \right\} \\
&= N\, P(t, t_0).
\end{aligned}$$

Consecutive terms cancel ("telescoping").
A FRN is always worth par (is fair) when we are at the start of an interest period.

FRN intra interest period

The elegant telescoping sum is a little spoiled by reality when we are (time t) within an interest period, e.g. the first one:
The amount to be paid at the next payment date is not unknown, but rather Libor, fixed at the last payment date. So we have one fixed coupon outstanding and a sum of subsequent future floating coupons. We assume $t_0 \leq t < t_1$. Libor $L(t_0, t_1)$ for payment at t_1 is fixed, the others still unknown:

$$\Pi(t) = N\, L(t_0, t_1)\, \delta(t_0, t_1)\, P(t, t_1) + N \sum_{i=2}^{n} \left(P(t, t_{i-1}) - P(t, t_i)\right) + N\, P(t, t_n) \quad (4)$$

and the telescoping yields again

$$\Pi(t) = N\, \{1 + L(t_0, t_1)\, \delta(t_0, t_1)\}\; P(t, t_1) \quad (5)$$

Compared to a coupon bond, the simple FRN (constant notional, zero spread) value is always close to the face amount N, and equal to N on fixing dates.
If we reintroduce a constant spread s into the valuation formula, (5) is extended to

$$\Pi(t) = N\, \{1 + L(t_0, t_1)\, \delta(t_0, t_1)\}\; P(t, t_1) + N\, s \sum_{i=1}^{n} \delta(t_{i-1}, t_i)\, P(t, t_i).$$

What is a Swap?

Interest rate swaps (IRS) are at their heart a very simple instrument. The idea is that one counterparty receives fixed coupons on agreed dates and pays floating coupons on possibly the same or different dates to the second counterparty. The rationale is that a swap together with a bond can turn a fixed rate bond into a FRN and vise versa.
In fact the combination of swap and bond is so ubiquitous that is called an Asset Swap and is considered an instrument in its own right. We will see more of Asset Swaps, and options on them, later.
Interest rate swaps are very widely used as both trading and hedging instruments and are considered highly liquid. The standard interbank discount curve is in fact generally built from swap quotes.

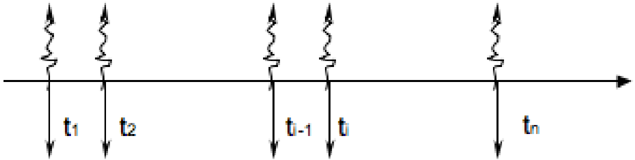


Figure: Swap cashflow schematic

Example Termsheet

Pricing

The IRS is a contract to exchange fixed interest rate payments against a cash flow based on floating rates. The floating reference rates used are typically Libor rates. For pricing, we decompose the Swap into two legs, a fixed and a floating leg so that the Swap value is

$$\Pi = \Pi_{float} - \Pi_{fixed} \tag{6}$$

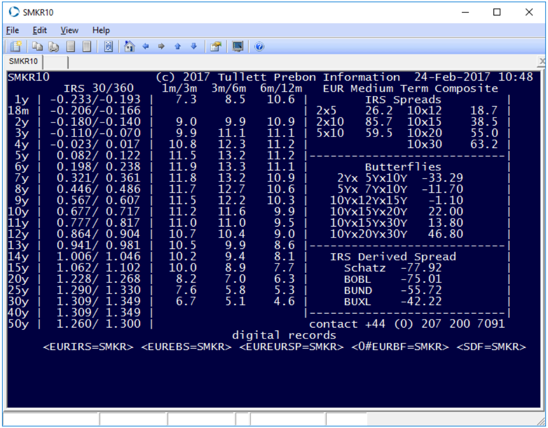
assuming we are receiving the floating payments and paying fixed.
Float and fixed leg of an IRS look much like a FRN and Fixed Coupon Bond,

$$\begin{aligned}
\Pi_{float}(t) &= \sum_{i=1}^{I} N_i \left(F(t; t_{i-1}, t_i) + s_i\right) \delta(t_{i-1}, t_i)\, P(t, t_i) \\
\Pi_{fixed}(t) &= \sum_{j=1}^{J} N_j\, k_j\, \delta(t'_{j-1}, t'_j)\, P(t, t'_j),
\end{aligned}$$

but note that

- there is no final notional redemption and no intermediate amortization payment
- the number of terms of fixed and floating legs may differ; floating payments are typically more frequent (e.g. semiannual) than fixed payments (annual)

Swap Quotes



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EUR Medium Term Composite

	IRS 30/360	1m/3m	3m/6m	6m/12m
1y	-0.233/-0.193	7.3	8.5	10.6
18m	-0.206/-0.166			
2y	-0.180/-0.140	9.0	9.9	10.9
3y	-0.110/-0.070	9.9	11.1	11.1
4y	-0.023/ 0.017	10.8	12.3	11.2
5y	0.082/ 0.122	11.5	13.2	11.2
6y	0.198/ 0.238	11.9	13.3	11.1
7y	0.321/ 0.361	11.8	13.2	10.9
8y	0.446/ 0.486	11.7	12.7	10.6
9y	0.567/ 0.607	11.5	12.2	10.3
10y	0.677/ 0.717	11.2	11.6	9.9
11y	0.777/ 0.817	11.0	11.0	9.5
12y	0.864/ 0.904	10.7	10.4	9.0
13y	0.941/ 0.981	10.5	9.9	8.6
14y	1.006/ 1.046	10.2	9.4	8.1
15y	1.062/ 1.102	10.0	8.9	7.7
20y	1.228/ 1.268	8.2	7.0	6.3
25y	1.290/ 1.330	7.6	5.8	5.3
30y	1.309/ 1.349	6.7	5.1	4.6
40y	1.309/ 1.349			
50y	1.260/ 1.300			

IRS Spreads

2x5	26.2	10x12	18.7
2x10	85.7	10x15	38.5
5x10	59.5	10x20	55.0
		10x30	63.2

Butterflies

2Yx 5Yx10Y	-33.29
5Yx 7Yx10Y	-11.70
10Yx12Yx15Y	-1.10
10Yx15Yx20Y	22.00
10Yx15Yx30Y	13.80
10Yx20Yx30Y	46.80

IRS Derived Spread

Schatz	-77.92
BOBL	-75.01
BUND	-55.72
BUXL	-42.22

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Figure: Swap quotes as of 24/02/2017

IRS Credit Risk, Collateral, ISDA

- IRS are mainly traded between financial institutions. On trade date, an IRS is typically fair, i.e. $\Pi = 0$.
- An IRS can develop a significant value during its life and pulls back to zero towards its maturity.
- This *potential future exposure* creates a credit risk: if our Swap counterpart defaults at a time when the IRS has developed a positive value Π' in our favour, then the default causes a loss for us, because re-entering into the same IRS with another counterpart will cost us Π'.
- However, IRS between banks are typically closed in combination with a *collateral agreement*:

Regularly (e.g. weekly) both parties agree the current value Π of the IRS; the party for which Π is negative pays $|\Pi|$ to a collateral account, so that in the case of its default, the current value can be settled to the counterpart. This measure removes credit risk from the transaction largely, much like in the case of financial futures where daily profit/loss is paid to a margin account. IRS collateral is normally paid in cash, but the parties may also agree to pledge securities as collateral.

In the absence of collateral agreements (e.g. when an IRS is traded between a bank and corporate customer), credit risk can be important. How this affects Swap valuation in pricing is the subject of a later chapter on credit.

Bootstrap

If we want to trade a simple instrument that is quoted in the market, e.g. a standard 10Y Swap spot start, we can simply read the fair Swap rate off the appropriate Broker screen.
However, if we want to price an instrument that was traded in the past and is still alive or not quite standard (e.g. because it is amortizing or has an unusual payment schedule), then we need to mark this slightly unusual instrument to market e.g. by decomposing it into market quoted simple instruments or interpolating adequately between those. Such decomposition and interpolation is done in two steps:

- build a discount curve from appropriately chosen instruments and then
- use this discount curve to price the instrument according to the formulas introduced in the previous sections.

Conventions

A discount curve is a collection of discount factors P for dates that should span the entire life of the instrument to price from the earliest date to maturity or beyond. The choice of source instruments depends on which kind of instrument we want to price eventually. We will use EUR denominated instruments throughout this section. Conventions are then:

- spot days 2
- calendar TARGET for fixings and payments
- day count Actual 360 for Deposit rates
- day count convention 30/360 for Swap rates
- fixed Swap payment frequency annual

Choice of Instruments

Let's assume we want to price an IRS.

- Source instruments should include Swaps to make sure that a vanilla quoted Swap is repriced correctly by the curve.
- The floating side of an IRS is a series of Deposits and their rates are fixed at Libor or Euribor. If the Swap's floating side is based on 6M-Euribor then we should make sure that we include a 6M Deposit in the instrument list, if it is based on 3M Euribor we should include a 3M Deposit (convergence of forwards to fixings)
- As we approach payment dates we will also need interim points on the discount curve. To avoid arbitrary interpolation we may include more Deposit instruments, ON, TN, SN for the very short end, 1M, 2M, etc.
- We may also consider criteria such as liquidity of the market quoted instruments, i.e. how actively they are traded and hence how relevant they are for pricing or *mark-to-market*.

Example choice of instruments:

- Deposits: ON, TN, SN, 1M, 2M, 3M, 6M, 9M, 12M
- Swaps: 2Y, 3Y, 4Y, 5Y, 7Y, 10Y, 12Y vs 6M Euribor

The Bootstrap: Deposit Segment

First step: Compute discount factor for $t_1 = 1D$ from the ON Deposit:

$$P(t_0, t_{1D}) = \frac{1}{1 + r_{ON}\,\delta(t_0, t_{1D})}.$$

Second step: TN Deposit, actually a forward instrument:

$$r_{TN}\,\delta(t_{1D}, t_{2D}) = \frac{P(t_0, t_{1D})}{P(t_0, t_{2D})} - 1 \qquad \rightarrow \qquad P(t_0, t_{2D}) = \frac{P(t_0, t_{1D})}{1 + r_{TN}\,\delta(t_{1D}, t_{2D})}.$$

Similarly up to the 12M Deposit:

$$\begin{aligned}
P(t_0, t_{3D}) &= \frac{P(t_0, t_{2D})}{1 + r_{SN}\,\delta(t_{2D}, t_{3D})} \\
P(t_0, t_{1M}) &= \frac{P(t_0, t_{2D})}{1 + r_{1M}\,\delta(t_{2D}, t_{1M})} \\
&\vdots \\
P(t_0, t_{12M}) &= \frac{P(t_0, t_{2D})}{1 + r_{12M}\,\delta(t_{2D}, t_{12M})}
\end{aligned}$$

Note: SN, 1M, 2M, etc. Deposits are spot-starting, with effective date t_{2D}.

The Swap Segment

Recall:

- An IRS can be regarded as a a Swap of a Coupon Bond and FRN with value $\Pi = \Pi_{float} - \Pi_{fixed}$.
- Quoted IRS are fair, i.e. $\Pi = 0$ or $\Pi_{float} = \Pi_{fixed}$.
- Float leg value reduces to $\Pi_{float} = N\, P(t_{2D})$

Fixed leg:

$$\Pi_{fixed} = N\, c_M \sum_{i=1}^{M} \delta(t_{i-1}, t_i)\, P(t_0, t_i) + N\, P(t_0, t_M),$$

So:

$$P(t_{2D}) = c_M \sum_{i=1}^{M} \delta(t_{i-1}, t_i)\, P(t_0, t_i) + P(t_0, t_M)$$

for a Swap with maturity t_M. Solve for $P(t_M)$:

$$P(t_0, t_M) = \frac{P(t_0, t_{2D}) - c_M \sum_{i=1}^{M-1} \delta(t_{i-1}, t_i)\, P(t_0, t_i)}{1 + c_M\, \delta(t_{M-1}, t_M)}$$

This is a recurrence relation, solve step by step for increasing t_M.

Interpolation

When pricing requires discount factors for intermediate dates that do not coincide with source instrument maturities, we need to interpolate.
Some choose to interpolate the source data, i.e. insert "fake" Deposits and Swaps with desired maturities and rates that are e.g. linearly interpolated between market quotes.
Alternatively we can interpolate the discount factors or associated zero rates $P(t,T) = \exp(-z(T)\,t)$. Choices here include

- linear, zero
- log-linear, discount (= piecewise flat forward)
- natural cubic spline, zero
- financial cubic spline, zero

Curve Data

Instrument	Term	Mid Rate
DEPOSIT	1D	1.050
DEPOSIT	2D	1.050
DEPOSIT	1W	1.375
DEPOSIT	2W	1.525
DEPOSIT	3W	1.625
DEPOSIT	1M	1.750
DEPOSIT	2M	1.950
DEPOSIT	3M	2.150
DEPOSIT	4M	2.200
DEPOSIT	5M	2.230
DEPOSIT	6M	2.270
SWAP	1Y	2.114
SWAP	18M	2.131
SWAP	2Y	2.275
SWAP	3Y	2.578
SWAP	4Y	2.841
SWAP	5Y	3.050
SWAP	6Y	3.229
SWAP	7Y	3.382
SWAP	8Y	3.512
SWAP	9Y	3.631
SWAP	10Y	3.739

Instrument	Term	Mid Rate
DEPOSIT	1D	1.050
DEPOSIT	2D	1.050
DEPOSIT	1W	1.375
DEPOSIT	2W	1.525
DEPOSIT	3W	1.625
DEPOSIT	1M	1.750
DEPOSIT	2M	1.950
DEPOSIT	3M	2.150
FRA	3x6	1.768
FRA	6x9	1.738
SWAP	1Y	1.923
SWAP	15M	1.935
SWAP	18M	1.986
SWAP	21M	2.059
SWAP	2Y	2.155
SWAP	3Y	2.489
SWAP	4Y	2.769
SWAP	5Y	2.989
SWAP	6Y	3.176
SWAP	7Y	3.335
SWAP	8Y	3.469
SWAP	9Y	3.592
SWAP	10Y	3.704

Figure: Yield curves based on 6M tenor Swaps (left) and 3M tenor Swaps (right), data asof 26/01/09, Deposits from Kliem, FRAs and Swaps from ICAP.

Zero Curves

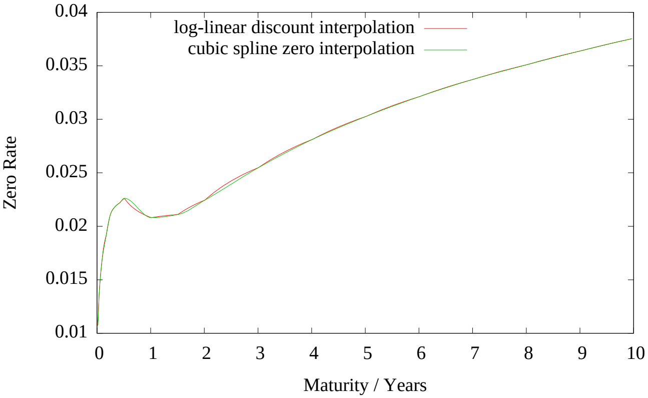

Figure: Zero curves based on data in the table above (left) with log-linear discount and cubic natural spline zero interpolation.

Forward Curves

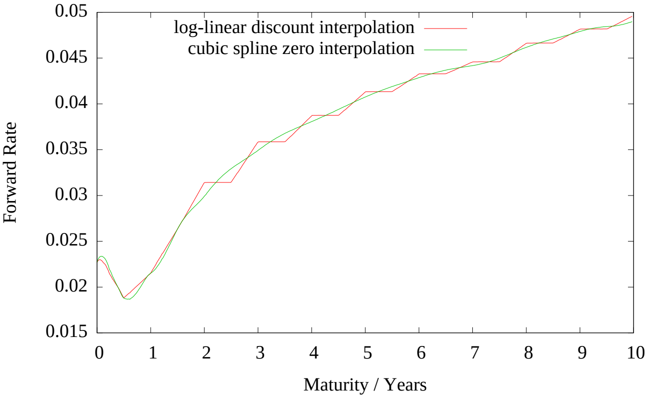

Figure: 6M forward curves based on data in the table above (left) with log-linear discount and cubic natural spline zero interpolation.

Consistency of Forward Rates

It is instructive to check whether the curve implied from deposits and swaps is capable of reproducing quoted FRA rates accurately:

Term	Quoted	Implied	Difference
3x6	1.768	2.381	+0.613
6x9	1.738	1.906	+0.168
3x9	1.894	2.148	+0.254
6x12	1.901	1.895	-0.006
12x18	2.153	2.145	-0.008
18x24	2.617	2.619	+0.002

The difference on FRAs with 3M tenor is significant whereas the FRAs with 6M tenor are repriced with less than a basis point difference. This underlines how sensitively the choice of source instruments is linked to the instrument to be valued.

More Curve Data

Instrument	Term	Mid Rate
DEPOSIT	1D	1.050
DEPOSIT	2D	1.050
DEPOSIT	1W	1.375
DEPOSIT	2W	1.525
DEPOSIT	3W	1.625
DEPOSIT	1M	1.750
SWAP	2M	1.685
SWAP	3M	1.609
SWAP	4M	1.57
SWAP	5M	1.544
SWAP	6M	1.529
SWAP	7M	1.521
SWAP	8M	1.521
SWAP	9M	1.527
SWAP	10M	1.539
SWAP	11M	1.552
SWAP	12M	1.569
SWAP	2Y	1.93
SWAP	3Y	2.32
SWAP	4Y	2.629
SWAP	5Y	2.869
SWAP	6Y	3.07
SWAP	7Y	3.239
SWAP	8Y	3.381
SWAP	9Y	3.51
SWAP	10Y	3.628

Instrument	Term	Mid Rate
DEPOSIT	1D	1.050
DEPOSIT	2D	1.050
EONIA	1W	1.26
EONIA	2W	1.267
EONIA	3W	1.275
EONIA	1M	1.28
EONIA	2M	1.217
EONIA	3M	1.172
EONIA	4M	1.148
EONIA	5M	1.135
EONIA	6M	1.129
EONIA	7M	1.127
EONIA	8M	1.134
EONIA	9M	1.145
EONIA	10M	1.162
EONIA	11M	1.178
EONIA	12M	1.195
EONIA	15M	1.258
EONIA	18M	1.347
EONIA	21M	1.447
EONIA	2Y	1.550
EONIA	3Y	1.950
EONIA	4Y	2.277
EONIA	5Y	2.530
EONIA	6Y	2.740
EONIA	7Y	2.917
EONIA	8Y	3.065
EONIA	9Y	3.199
EONIA	10Y	3.321

Figure: Yield curves based on 1M tenor Swaps (left) and Eonia Swaps (right), data asof 26/01/09, Deposits from Kliem, FRAs and Swaps from ICAP. Note that the fixed EONIA Swap rate is based of Actual 360 rather than 30/360 as the 1M, 3M and 6M tenor Swaps.

Overnight Index (EONIA) Swap

Typical float leg tenors for IRS are 3M and 6M, the standard for EUR based IRS being 6M until recently.

However, short term interest rate risk management created need for shorter tenor IRS, also before the current crisis started. With the financial crisis the demand for short tenor instruments has risen significantly, and we find brokers quote more short term instruments than before.

The Overnight Index Swap (OIS) marks the extreme short end of the tenor spectrum:

- Floating payments based on Overnight Index L_{ON} for 1D periods
- Let $t_0 < t_1 < \cdots < t_n$ denote the business days spanning a single coupon period and $L_{ON}(t_i, t_{i+1})$ the ON fixing on time t_i for period from t_i to t_{i+1}.
- No daily floating payments, but compounding and annual payments of rate

$$r \cdot \delta(t_0, t_n) := \left\{ \prod_{i=1}^{n} (1 + L_{ON}(t_{i-1}, t_i)\, \delta(t_{i-1}, t_i)) \right\} - 1.$$

Zero Curves, Various Tenors

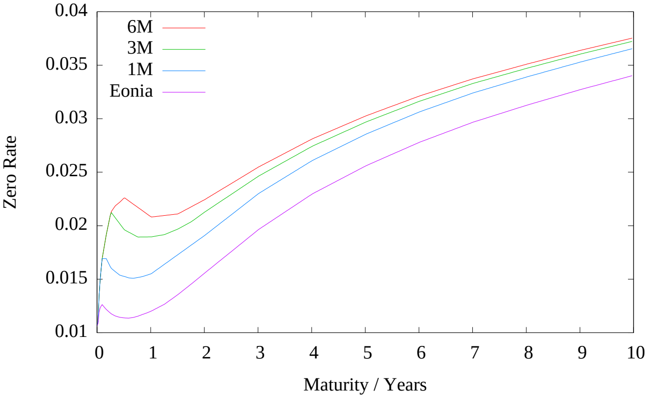

Figure: Zero curves built from 6M, 3M, 1M and Eonia Swaps, respectively.

Forward Curves, Various Tenors

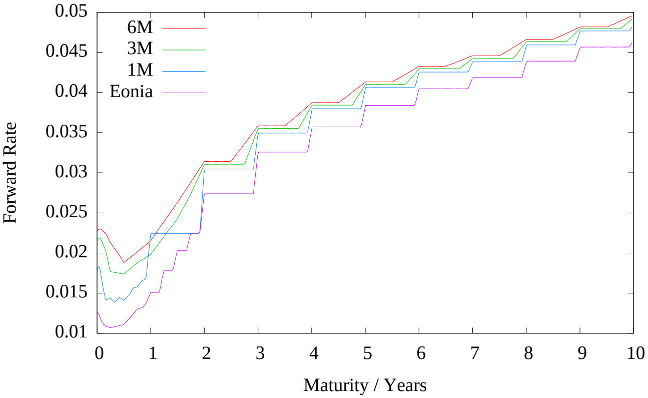

Figure: Forward curves built from 6M, 3M, 1M and Eonia Swaps, respectively, in tables 4 and 7. The forward tenors correspond, i.e. 6M, 3M, 1M, 1D. Interpolation method is log-linear discount.

Basis Swap

A Basis Swap is an OTC contract to exchange different kind of floating payments. One typical example are exchanges of Libor payments for different tenors such as 3M and 6M, e.g.:

	BS	
	Pay	Receive
Currency	EUR	EUR
Notional	1,000,000	1,000,000
Reference	Euribor-3M	Euribor-6M
Spread	+10 bp	
Frequency	quarterly	semiannual
Term	5Y	5Y

Basis Swap Quotes

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USD LIBOR Basis Swaps Composite

	L (1M) / L (3M)			L (3M) / L (6M)			L (1M) / L (6M)			L (3M) / L (12M)		
6M	1M+	20.000	22.000	3M+	21.000	23.000	1M+	42.000	44.000			
1Y	1M+	19.375	21.375	3M+	19.625	21.625	1M+	40.000	42.000	3M+	41.125	51.125
18M	1M+	18.875	20.875	3M+	18.750	20.750	1M+	38.625	40.625			
2Y	1M+	18.375	20.375	3M+	17.875	19.875	1M+	37.375	39.375	3M+	37.750	47.750
3Y	1M+	17.375	19.375	3M+	16.875	18.875	1M+	35.250	37.250	3M+	36.875	46.875
4Y	1M+	16.250	18.250	3M+	16.000	18.000	1M+	33.250	35.250	3M+	36.500	46.500
5Y	1M+	15.250	17.250	3M+	15.250	17.250	1M+	31.500	33.500	3M+	35.250	45.250
7Y	1M+	13.375	15.375	3M+	15.000	17.000	1M+	29.375	31.375	3M+	34.125	44.125
10Y	1M+	11.500	13.500	3M+	15.125	17.125	1M+	27.625	29.625	3M+	32.750	42.750
12Y	1M+	10.750	12.750	3M+	15.250	17.250	1M+	27.000	29.000	3M+	31.625	41.625
15Y	1M+	10.125	12.125	3M+	15.375	17.375	1M+	26.500	28.500	3M+	31.000	41.000
20Y	1M+	10.500	12.500	3M+	15.125	17.125	1M+	26.625	28.625	3M+	30.375	40.375
25Y	1M+	12.000	14.000	3M+	14.875	16.875	1M+	27.875	29.875	3M+	29.625	39.625
30Y	1M+	12.875	14.875	3M+	14.500	16.500	1M+	28.375	30.375	3M+	28.500	38.500

Logical Records <USDLBS=SMKR> Conventions <SMKR96AINFO>

Basis Swap Spreads

- Basis Swaps trade at spreads different from zero.
- The origin of basis swap spreads is credit risk:
- Although Swaps are typically collateralized, their variable rate fixing is based on an unsecured cash instrument (Deposit) which is exposed to credit risk, i.e. a Deposit counterpart has some probability of default within the life time of the Deposit which would wipe out the repayment. This credit risk is larger the longer the Deposit tenor. Via the Swap's float leg fixings, this implicit credit risk shows up in the swap quotes.
- Recently the credit risk differential between 3M and 6M tenors has become significant.

The basis swap spread between e.g. 6M and 3M tenor IRS can be computed from discount curves for these tenors as built above. The derivation of the "basis swap spread formula" as a function of discount curves is done as an exercise.

Consistency of Basis Swap Quotes

The following table shows a snapshot of quoted basis spreads taken at the same time as the data in the tables above, compared to "implied" spreads.

Term	Quoted Spread	Implied	Difference
1Y	+22.4	+21.3	1.1
2Y	+13.9	+12.9	1.0
3Y	+10.4	+9.3	1.1
4Y	+8.6	+7.5	1.1
5Y	+7.3	+6.2	1.1
6Y	+6.4	+5.2	1.2
7Y	+5.7	+4.4	1.3
8Y	+5.2	+3.8	1.4
9Y	+4.8	+3.5	1.3
10Y	+4.5	+3.1	1.4

Figure: Basis Swap Spreads as of 19/01/09 from ICAP, collected at the same snapshot time as data in tables above. The quotation is against 6M Euribor flat. The implied spread is computed from discount curves for 3M and 6M tenor instruments, respectively.

Asset Swap

An *asset swap (ASW)* is an interest rate swap with terms and conditions that match an underlying asset, typically a bond position, perfectly (as long as there are no defaults).

For example, if the underlying is a long position in a fixed rate bond, then the ASW's pay leg has the same maturity, payment dates and coupon rates as the bond. Cash flows match with the exception of the final redemption of the asset which is not mirrored by the swap.

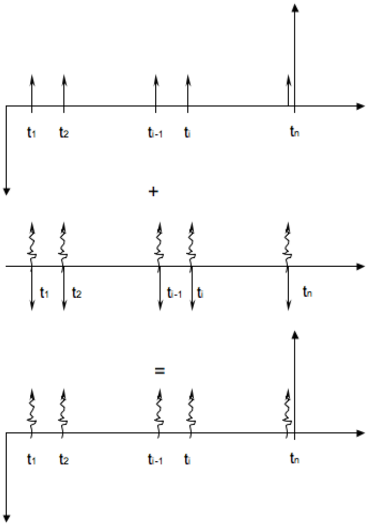
The ASW's receive leg has floating cash flows linked to Euribor or Libor plus/minus a spread, the *ASW spread*.

If the asset's payments are not simply fixed but structured in some way, this structure is mirrored by the swap pay leg accordingly.

ASW Package

Combination of underlying asset and matching ASW: Interest rate payments of the asset are offset by the payments of the ASW's corresponding leg

Net cash flows of the ASW package agree with those of a floating rate note with spread:



ASW Pricing

The ASW package value is the sum of the value of its components,

$$\Pi_{Package}(t) = \Pi_{Asset}(t) + \Pi_{ASW}(t)$$

The *fair ASW spread* is the spread that turns the package into a *par* bond, i.e.

$$\Pi_{Package}(t) = N.$$

In more detail, the values of a simple asset and ASW (constant notional, fixed asset payments) are given by

$$\begin{aligned}
\Pi_{Asset}(t) &= N\left\{c\sum_{i=1}^{I}\delta(t_{i-1},t_i)\,P_{Asset}(t_i) + P_{Asset}(T)\right\}, \qquad t_I = T \\
\Pi_{ASW}(t) &= N\left\{-c\sum_{i=1}^{I}\delta(t_{i-1},t_i)\,P_{ASW}(t_i) + P_{ASW}(t_0) - P_{ASW}(T) + s\cdot A(t)\right\}
\end{aligned}$$

where

$$A(t) = \sum_{j=1}^{J}\delta'(t_{j-1},t_j)\,P_{ASW}(t_j), \qquad t_J = T$$

is the ASW's float leg annuity.

If the discount curves for both asset and ASW are given, one can therefore back out the fair ASW spread s from $\Pi_{Package}(t) = \Pi_{Asset}(t) + \Pi_{ASW}(t) = N$. The result is:

$$\begin{aligned} s &= \frac{1}{A(t)} \left\{ c \sum_{i=1}^{I} \delta(t_{i-1}, t_i) \left[P_{ASW}(t_i) - P_{Asset}(t_i) \right] \right. \\ &\qquad \left. + P_{ASW}(T) - P_{Asset}(T) + 1 - P_{ASW}(t_0) \right\} \end{aligned} \tag{7}$$

The ASW spread is another way of expressing the difference between asset and ASW discount curves. This difference originates from credit (and also liquidity) risk: If the asset's issuer is more likely to default than a typical Libor panel bank, then the asset discount factors should be systematically smaller than the corresponding swap discount factors which leads to a positive ASW spread, and vice versa. We will come back to ASW pricing and related options in later sections.

Market Evolution 2008

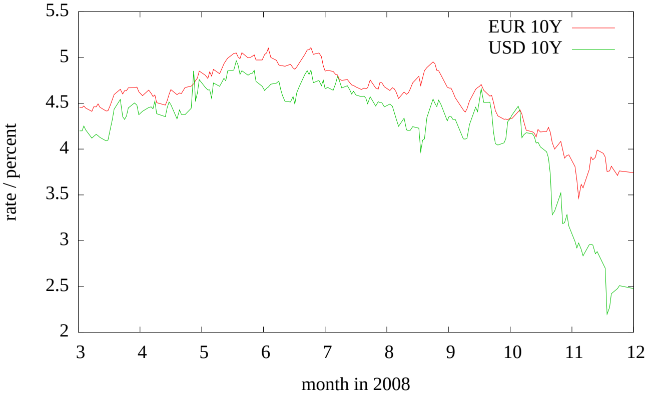


Figure: Evolution of 10Y EUR and USD swap rates in 2008.

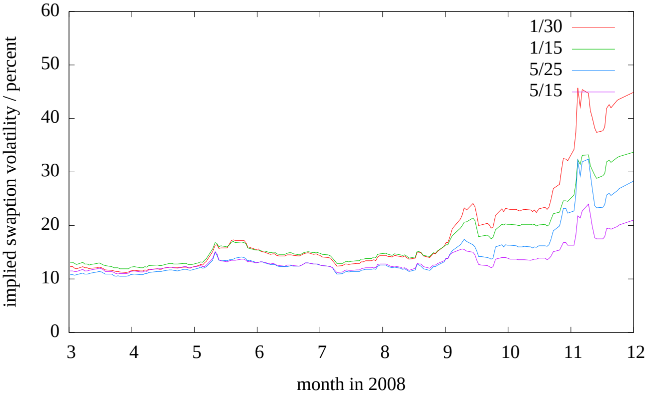


Figure: Evolution of swaption volatilities in 2008, 1/30 meaning option expiry in 1 year, underlying swap term 30 years.

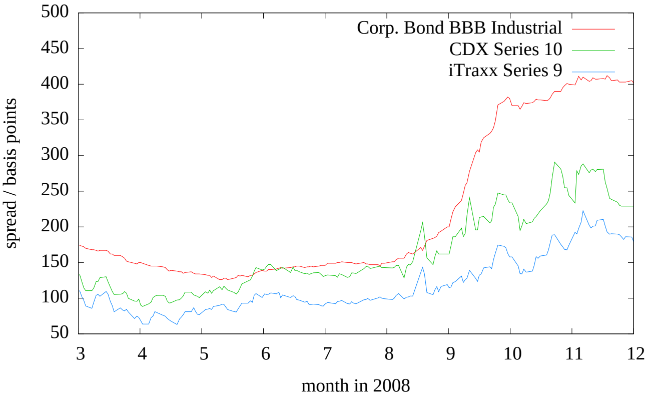


Figure: Evolution of credit spreads in 2008.

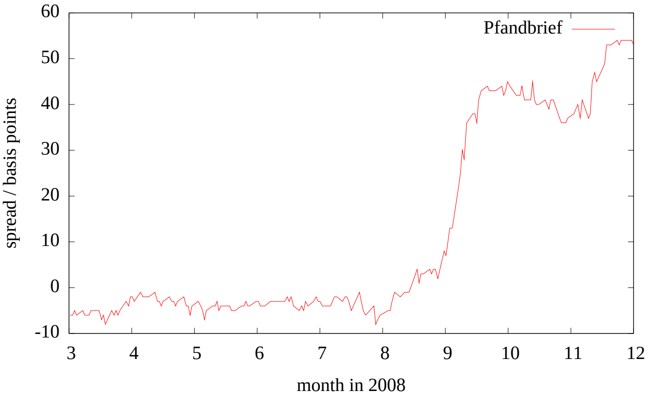


Figure: Evolution of Pfandbrief spreads in 2008.

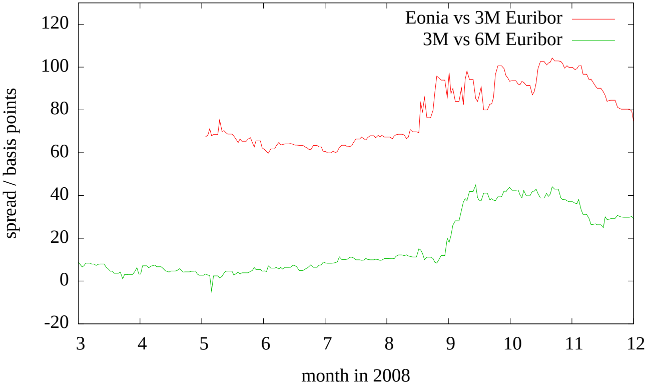


Figure: Evolution of EUR basis spreads in 2008.

Exercises I

1. FRN: Generalize the pricing formula

$$\Pi(t) = N\,L(t_0,t_1)\,\delta(t_0,t_1)\,P(t,t_1) + N\sum_{i=2}^{n}\left(P(t,t_{i-1}) - P(t,t_i)\right) + N\,P(t,t_n)$$

to varying notional.

2. Instantaneous forward rate: Derive the relation between Forward rate $F(t;T)$ and discount curve $P(t,T)$ for an infinitesimally short forward period starting at future time T

3. Piecewise flat: Show why log-linear interpolation of discount factors leads to piecewise flat instantaneous forward rates (following a step-like forward curve), see the EONIA forwards in figure 9.

4. Basis Spread: Let $P_{6M}(t,\tau)$ and $P_{3M}(t,\tau)$ be the discount curves built from 6M and 3M tenor instruments. Derive a formula for the fair basis spread $s(T)$ in a basis swap with maturity T (exchanging 3M Euribor plus spread $s(T)$ against 6M Euribor flat payments) as a function of $P_{6M}(t,\tau)$ and $P_{3M}(t,\tau)$.

Exercises II - QuantLib

1. Download and install QuantLib Addin (http://www.quantlibxl.org), for help regarding Mac OS X, please contact roland.lichters@web.de
2. Curve Building I: Familiarize with curve building using QuantLib Addin (example Excel sheets provided for 3M and 6M tenor EUR curves).
3. Vanilla Swap I: Familiarize with Swap valuation using QuantLib Addin (example Excel sheet provided).
4. Vanilla Swap II: Build curves for 1M and 6M tenor USD instruments as of 27 January 2009 and compute the fair rates for (a) a 6M tenor Swap with maturity 18M, (b) a 6M tenor Swap with maturity 11Y, (c) a 1M tenor Swap with maturity 21M (USD market data provided).
5. Advanced and optional: Download and install QuantLib-0.9.7 from source, following the steps at http://www.quantlib.org.