5634 - Stochastic Methods
Michelmas Term - 2016-2017
Homework 3 - Due January 23rd, 2017

1. Consider one-dimensional integrals of the form

$$\langle f \rangle = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x) \mathrm{e}^{-x^2} dx$$

   (a) Code the Metropolis algorithm to evaluate such integrals for a given function f. The states in the Markov chain are the real numbers x. Propose a new x' that is at most a distance Δ away from x, where Δ is a parameter of your algorithm.

   (b) consider $\langle \cos(x) \rangle = 0.7788007831$ and $\langle x^2 \rangle = 0.5$; plot the Monte-Carlo history (i.e. the values in the Markov chain vs. Monte-Carlo time) in each case for 3 values of Δ. What do you observe?

   (c) Measure the acceptance rate (i.e. the number of accepted proposals over the total number of proposals) as a function of Δ. What would be a good parameter choice?

   (d) Measure the true variance by calculating the variance of the binned data as a function of bin size and looking for a plateau. Obtain the integrated autocorrelation times for both functions from the naive variance divided by the true variance.

   (e) calculate the same integrals using n statistically independent Gaussian random variables. Compare the error thus obtained with the error from a Markov chain of length N (excluding thermalization) for the same *effective* statistics (i.e. $N_{\text{eff}} = N/(2\tau_f) = n$).