M.Sc. in High-Performance Computing
5633 - Numerical Methods
Assignment 2

Marina Krstic Marinkovic
(mmarina@maths.tcd.ie)
School of Mathematics, TCD

Rules

To submit, make a single tar-ball with all your code and a pdf of any written part you want to include. Submit this via `msc.tchpc.tcd.ie` or via email to `mmarina@maths.tcd.ie` by the end of **Friday November 18th**. Instead of R, you may use Matlab or Python for the numerical/plotting part. Late submissions without prior arrangement or a valid explanation will result in reduced marks.

Question

1. Expand
$$f(x) = 1 - x^4$$
as an even function over (-1,1) in a finite Fourier series. Compare its values at several points with the value of this expansion and also the value of the infinite Fourier series truncated to the same number of terms.

2. Write a R script that uses the bisection method to find the root of a given function on a given interval, and apply this program to find the roots of the functions bellow on the indicated intervals. Be careful to avoid unnecessary function calls in a single iteration. Use a relationship derived in class to determine *a priori* the number of steps necessary for the root to be accurate within 10^{-6}

   (a) $x^3 - 2, [a, b] = [0, 2]$,
   (b) $e^x - 2, [a, b] = [0, 1]$.

   Apply Newton's method to find the root of the second function, $f(x) = e^x - 2$, for which the exact solution is $\alpha = \ln 2$. Perform the following experiments:

   (c) Compute the ratio
$$R_n = \frac{\alpha - x_{n+1}}{(\alpha - x_n)^2}$$
   and observe whether or not it converges to the correct value as $n \to \infty$.

(d) Compute the modified ratio

$$R_n = \frac{\alpha - x_{n+1}}{(\alpha - x_n)^p},$$

for various $p \neq 2$, but near 2. What happens? Comment, in the light of the definition of order of convergence.

Apply the secant method to the same function, using x_0, x_1 equal to the end points of the given interval. Stop the iteration when the error as estimated by $|x_n - x_{n-1}|$ is less than 10^{-6}. Compare to your results for Newton and bisection method.

Have your codes print out the entire sequence of iterates to the following files `bisection1.txt` (for function a)), `bisection2.txt`, `newton.txt`, and `secant.txt` (for function b)). In the case of bisection method, the file should have following five columns:

$$n \quad x_n \quad b_{n-1} - a_{n-1} \quad a_n \quad b_n,$$

the file for the Newton's method should contain the following four columns:

$$n \quad x_n \quad \alpha - x_n \quad log_{10}(\alpha - x_n),$$

and the file for the secant method should contain the following three columns:

$$n \quad x_n \quad \alpha - x_n.$$