5634 - Stochastic Methods
Michelmas Term - 2016-2017
Homework 2 - Due December 7th, 2016

1. Consider a bank with 1000 customers. On average there are 50 withdrawal requests per month, while the number of withdrawals in a single month is Poisson distributed. On average, the amount of each withdrawal is 800€ and the amounts are exponentially distributed. Calculate the probability that the sum total of withdrawals in a given month exceeds 50,000€.

2. Consider the integral
$$\int_0^3 \frac{e^{-s}}{1+\frac{s}{9}}\,\mathrm{d}s \approx 0.873109\,.$$
   (a) Estimate the value of this integral by generating uniform random numbers in the interval $[0,3]$. Tabulate your results for $10^2, 10^3, 10^4, 10^5$ random values. Make sure to estimate the error using the standard deviation.
   (b) Repeat the previous part using exponentially distributed random variables.

3. Suppose that a student's diet for dinner consists of either a pizza or a burrito. If he has pizza on one day the probability to have another pizza on the next day is 15 percent. If he has a burrito on one day the probability to have pizza the next day is 70 percent.
   (a) Define suitable states of the system and the Markov matrix.
   (b) If the student had pizza on a Sunday, what is the probability of him having pizza on the following Tuesday?
   (c) Disregarding health aspects, what can you say about the long term relative pizza or burrito consumption of this student?

4. Consider the population dynamics of a predator-prey system. At a given time, there are x predators and y prey animals. The time evolution of the system is modeled by a Markov Chain such that
$$P(X_t = x', Y_t = y' | X_{t-1} = x, Y_{t-1} = y) = \frac{1}{c(x,y)} \times \begin{cases} s, & (x',y') = (x,y), \\ \alpha y, & (x',y') = (x,y+1), \\ \beta xy, & (x',y') = (x,y-1), \\ \gamma xy, & (x',y') = (x+1,y), \\ \delta x, & (x',y') = (x-1,y), \\ 0, & \text{otherwise}, \end{cases}$$
   where $s, \alpha, \beta, \gamma, \delta > 0$.

(a) What are the states of the system?

(b) Is this Markov chain irreducible?

(c) Calculate the normalization factor $c(x, y)$.

(d) Describe the long-time behaviour of the each of the populations if $\beta = \gamma = 0$.

(e) Implement this Markov chain and calculate the average number of time steps until either of the species becomes extinct. Use $s, \alpha, \beta, \gamma, \delta = 0.1$ and initial populations of $x = 100$, $y = 1000$. Run many simulations and estimate the error on your result.