5634 - Stochastic Methods
Michelmas Term - 2016-2017
Homework 1 - Due November 14th, 2016

1. Download and compile (or install the package as part of the GNU Scientific Library) the `DieHarder` battery of random number tests from `http://www.phy.duke.edu/~rgb/General/dieharder.php`. Compare the test results for

   (a) The Mersenne Twister

   (b) RANDU

   and detail your findings.

2. Install Martin Lüscher's RANLUX random number generator, obtainable (together with documentation and the link to his original paper) from his webpage `http://luscher.web.cern.ch/luscher/`

3. Write a programme to estimate integrals of the type

   $$E[g(X)] = \int_0^1 g(x)dx$$

   using uniform random numbers on the interval (e.g. from RANLUX). Then

   - estimate the integrals of the functions

     $$g(x) = x, \qquad g(x) = x^2, \qquad g(x) = \sqrt{x},$$

     by averaging over N samples. Produce a plot of the results vs. N. How large does N have to be to reproduce the analytic result to 2,3, or 4 digits, respectively? Can you estimate how large N would have to be to obtain 8 digit accuracy?

   - estimate $\text{Var}[g(x)] = E[(g(x) - \mu)^2]$, once using the analytic result for $\mu = E[g(x)]$ and then using the estimate of μ as sample mean instead. Do you notice any difference? How is this variance related to the accuracy of your previous estimates for $E[g(x)]$?

   - One may be tempted to estimate π numerically by choosing

     $$g(x) = \frac{2}{\sqrt{1 - x^2}}.$$

     Why is this not such a great idea?