Quaternion
Risk Management

Derivatives Pricing, Funding and Capital
Case Study with Open Source Risk Engine

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Trinity College Dublin, 30 March - 6 April 2017

Agenda

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Agenda and Overview

Part I

- Swaps, Swaptions
- Monte Carlo Simulation
- Exposure Evolution
- Default Risk
- Credit Value Adjustment
- Capital Requirements

Assignment I: KVA

Part II

- Netting
- Variation Margin
- Initial Margin Background
- Forecasting Initial Margin
- Margin Value Adjustment

Assignment II: MVA

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Agenda

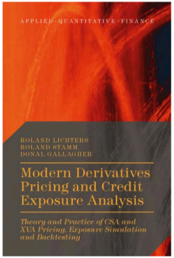
- 2-day financial engineering workshop using **Open Risk Engine** (ORE)
- Refresh and sketch some financial maths foundations
- Run and discuss examples in ORE
- Extend ORE in assignments

Open Risk Engine Overview

Open Source project, first release October 2017, second to come in April

Framework for sophisticated pricing and risk analysis

Based on QuantLib, the open-source library for quantitative finance, and



Modern Derivatives Pricing and Credit Exposure Analysis

Theory and Practice of CSA and XVA Pricing, Exposure Simulation and Backtesting

Roland Lichters, Roland Stamm, Donal Gallagher

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Open Risk Engine - Motivation

Background

- **Financial crisis** following Lehman's default has led to tighter regulation for financial institutions (banks, asset managers, insurance companies, ...)
- Regulators keep coming up with new **capital requirements** to do business
- New rules strongly encourage accurate **risk analysis and risk mitigation**
- Banks struggle with **growing complexity** and **computational challenges**
- Transparency in methodologies and software solutions needed

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Open Risk Engine - Motivation

ORE serves as transparent reference

- to benchmark/validate vendor/bank solutions
- as foundation for new development

Quaternion sponsors the development of ORE, offers ORE extensions and consulting services

Open Risk Engine - Scope

Initial scope of ORE:

- **Valuation** of Interest Rate and FX derivatives
- Monte Carlo **simulation** of portfolio values **through time**
- **Exposure simulation** for portfolios with **netting and collateral**
- Derivative **value adjustments** (“CVA”, “DVA”, “FVA”, “MVA”)
- Market risk analysis and **Initial Margin**
- Basic **front end** for launching ORE and visualization of results

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Open Risk Engine - Roadmap

Roadmap:

- More asset classes: Inflation, Equity and Credit derivatives (see April release)
- More products: Loans and Bonds (see April release)
- More Market Risk: Sensitivities, stress testing (see April release)
- More value adjustments (“KVA”, “TVA”)
- Extend ORE from pricing/risk analysis to portfolio optimisation
- Front end, Excel integration
- ORE Book

How to get and build ORE

Web site, FAQ, Forum:

`http://www.opensourcerisk.org`

Code base (release tarball or git access):

`https://github.com/OpenSourceRisk/Engine`

Build instructions: See user guide in the release tarball or at the ORE web site

How to run ORE

Usage:

```
./ore path/to/ore.xml
```

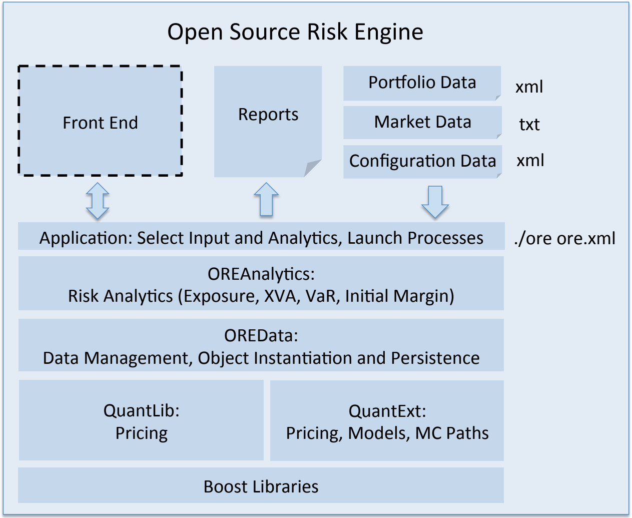
Inputs

File Name	Description
`ore.xml`	Master input file, selection of further inputs below and selection of analytics
`portfolio.xml`	Trade data
`netting.xml`	Collateral (CSA) data
`simulation.xml`	Configuration of simulation model and market
`market.txt`	Market data snapshot
`fixings.txt`	Index fixing history
`curveconfig.xml`	Curve and term structure composition from individual market instruments
`conventions.xml`	Market conventions for all market data points
`todaysmarket.xml`	Configuration of the market composition, relevant for the pricing of the given portfolio as of today (yield curves, FX rates, volatility surfaces etc)
`pricingengines.xml`	Configuration of pricing methods by product

Outputs

File Name	Description
npv.csv	NPV report
flows.csv	Cashflow report
curves.csv	Generated yield (discount) curves report
xva.csv	XVA report, value adjustments at netting set and trade level
exposure_trade_*.csv	Trade exposure evolution reports
exposure_nettingset_*.csv	Netting set exposure evolution reports
rawcube.csv	NPV cube in readable text format
netcube.csv	NPV cube after netting and colateral, in readable text format
*.dat	Intermediate storage of NPV cube and scenario data in binary format
*.pdf	Exposure graphics produced by the python script run.py after ORE completed

ORE Components



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Swap

Agreement to exchange fixed for floating cash flows

Recall that, mathematically, its present value is computed as

$$
\begin{aligned}
NPV &= \overbrace{c\sum_{i=1}^{n}\delta_i\,P(t_i)}^{\text{fixed leg value}} - \overbrace{\sum_{i=1}^{n} f(t_{i-1},t_i)\,\delta_i\,P(t_i)}^{\text{floating leg value}} \\
&= c\sum_{i=1}^{n}\delta_i\,P(t_i) - \sum_{i=1}^{n}\left(P(t_i) - P(t_{i-1})\right) \\
&= c\sum_{i=1}^{n}\delta_i\,P(t_i) - \left(P(t_0) - P(t_n)\right)
\end{aligned}
$$

Needs: Contract details, discount curve $P(t_i)$ and forwards $f(t_{i-1},t_i)$

See ORE Swap example

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European Swaption

One party has the right (no obligation) to enter into a predefined Swap at some future point in time.

Asymmetric situation: The option owner will decide depending on the swap's value on the 'notice' date whether to 'exercise' or not.

From today's perspective, the swap will end up 'in-the-money' in some 'scenarios' (owner exercises), and 'out-of-the-money' in others (owner does not exercise).

Payoff at expiry T is hence

$$\max(NPV_{Swap}(T), 0) = NPV^{+}_{Swap}(T)$$

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Risk-Neutral Pricing Summary

Consider some payoff $V(T)$ at a future time T.

Its value at an earlier time t is an expectation of the discounted payoff:

$$V(t) = \mathbb{E}_t \left[D(t,T)\, V(T) \right]$$

This follows from the assumption of *no-arbitrage* ('no free lunch').

Discounting can be done with different choices of 'numeraire assets' (cash account, bond, etc), so more generally:

$$\frac{V(t)}{N(t)} = \mathbb{E}_t^N \left[\frac{V(T)}{N(T)} \right]$$

Relative asset prices are Martingales under the measure associated with the numeraire asset N.

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European Swaption

The present value of the option is an average over possible 'payoffs', discounted to today

$$NPV_{Swaption} = \mathbb{E}\left[D(0,T) \cdot NPV^{+}_{Swap}(T)\right]$$

where $D(0,T)$ is the stochastic discount factor from time T to today.

A **European Swaption** has only a single exercise date. This allows some analytical pricing methods, see next slides.

A **Bermudan Swaption** has several exercise dates, typically on period start dates. These can be priced only with numerical methods (trees, lattices, simulation).

European Swaption, Analytical Pricing

Step 1: Write Swap PV in terms of 'annuity' and 'forward swap rate'

$$
\begin{aligned}
NPV_{Swap}(T) &= c \sum_{i=1}^{n} \delta_i \, D(T, t_i) - \left(D(T, t_0) - D(T, t_n)\right) \\
&= A(T) \; (c - S(T))
\end{aligned}
$$

where

$$
\begin{aligned}
A(T) &= \sum_{i=1}^{n} \delta_i \, D(T, t_i) \\
S(T) &= \frac{D(T, t_0) - D(T, t_n)}{A(T)}
\end{aligned}
$$

European Swaption, Analytical Pricing

Step 2: Choose the 'annuity measure' and assume log-normal dynamics for the forward swap rate S. This yields

$$
\begin{aligned}
NPV_{Swaption}(0) &= \mathbb{E}\left[D(0,T)\cdot NPV^{+}_{Swap}(T)\right] \\
&= A(0)\cdot\mathbb{E}^{A}\left[\frac{A(T)\ (c-S(T))^{+}}{A(T)}\right] \\
&= A(0)\cdot\mathbb{E}^{A}\left[(c-S(T))^{+}\right] \\
&= A(0)\int_0^{\infty}(c-s)^{+}\,\rho(s)\,ds
\end{aligned}
$$

where the latter integral can be solved in closed form (the famous Black76 formula).

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Monte Carlo Simulation

Compute prices with Monte Carlo simulation using the risk-neutral pricing formula

$$V(t) = N(t)\,\mathbb{E}^N\left[\frac{V(T)}{N(T)}\right]$$

Recipe:

1. Generate many paths of numeraire N and V through time (including any ’risk factor’ needed to compute V)
2. Finally take an average.

Exposure Evolution

Having the paths of $NPV(t)$, we can also compute the evolution of several relevant statistics, i.e.

$$\begin{aligned}
EPE(t) &= \mathbb{E}\left[D(0,t)\,NPV^{+}(t)\right] \equiv EE(t) \\
ENE(t) &= \mathbb{E}\left[D(0,t)\;\left(-NPV(t)\right)^{+}\right] \\
PFE_q(t) &= \text{quantile } q \text{ of the distribution of } D(0,t)\,NPV(t) \\
ES_q(t) &= \text{tail average over the distribution of } D(0,t)\,NPV(t) \text{ beyond quantile } q
\end{aligned}$$

These are key quantities to analyse credit risk in derivative positions/portfolios.

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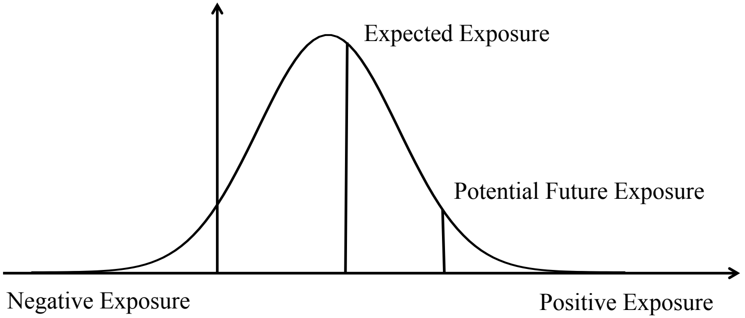
Exposures

NPV distribution and the location of expected exposure EE and potential future exposure (peak exposure) PFE.



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“Basel Comittee” Exposures, closely related to EE

Expected Exposure

$$EE_B(t) = \mathbb{E}[(NPV(t) - C(t))^+] \qquad \text{in ORE: } EE_B(t) = EE(t)\;/\;P(t)$$

Expected Positive Exposure

$$EPE_B(T) = \frac{1}{T}\sum_{t<T} EE_B(t)\cdot\Delta t, \qquad T \text{ is one year}$$

Effective Expected Exposure, recursively defined as running maximum

$$EEE_B(t) = \max(EEE_B(t-\Delta t), EE_B(t))$$

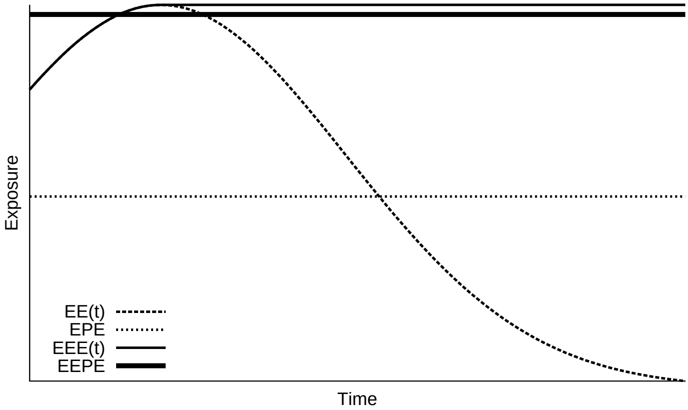
Effective Expected Positive Exposure

$$EEPE_B(T) = \frac{1}{T}\sum_{t<T} EEE_B(t)\cdot\Delta t, \qquad T \text{ is one year}$$

$EEPE$ in particular is a key quantity in “credit capital” calculations, see below.

Exposures

Evolution of the expected exposure through time, $EE_B(t)$, and corresponding $EPE_B(t)$, $EEE_B(t)$, and $EEPE_B(t)$.



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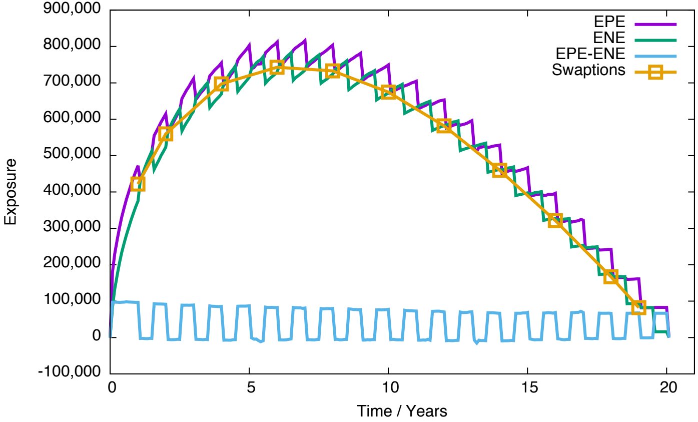
Swap Exposure

Run ORE example 1



Default Risk

Expressed in terms of cumulative probability of default or survival probability

$$SP(t) = 1 - PD(t)$$

for each credit 'name'.

A second key variable is the loss given default or recovery rate

$$R = 1 - LGD$$

depending on kind and quality of 'collateral' provided.

The estimation of these is the subject of rating models. Alternatively, R and $S(t)$ can be derived from credit default swap prices, as we will see below.

In the following we will assume for now that R is not time-dependent, and that both R and $S(t)$ are deterministic.

Credit Value Adjustment

It is century old practice in loan business to charge for the lender's risk of not getting his/her money back if the debtor defaults.

If we lend 1 Euro and expect it back with interest r after t years, we have to add a credit margin/spread Δ to the interest rate such that

$$e^{(r+\Delta)\,t} \times S(t) = e^{r\,t}$$

The smaller the debtor's survival probability $S(t)$ the larger is Δ

This is a bit simplified, ignoring the possibility of recovery.

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Credit Value Adjustment

Conversely, without default risk, the value of 1 Euro at time t is the interest rate induced discount factor $NPV_{rf} = P(t)$.

With default risk, this is further reduced:

$$
\begin{aligned}
NPV &= NPV_{rf} \times \left(\underbrace{1 - PD(t)}_{S(t)} + R \cdot PD(t) \right) \\
&= NPV_{rf} - PD(t) \times (1 - R) \times NPV_{rf} \\
&= NPV_{rf} - \underbrace{PD(t) \times LGD \times NPV_{rf}}_{CVA}
\end{aligned}
$$

In loan business this is called credit risk charge.

Credit Value Adjustment

In interbank derivatives business, people started considering the possibility of counterparty default in the mid 2000s, and it became obvious latest with Lehman Brothers' default in 2008.

When a derivative trade is one-sided i.e. has always positive NPV (like long option positions with expiry t), then the CVA is computed just as on the previous slide:

$$CVA = PD(t) \times LGD \times NPV_{rf}$$

Credit Value Adjustment

When the derivative's value can change sign during its life (like Swaps), it is more complicated.

There is a striking asymmetry:

- When the NPV is positive at the time of counterparty default, we loose the value and can expect to get recovery $R \times NPV$ only
- When the NPV is negative at the time of default, the counterparty's insolvency trustee will claim the value NPV from us, unfortunately

This changes the derivative value from our perspective by

$$\begin{array}{ll} 0 & \text{if } NPV < 0 \\ (1-R) \times NPV & \text{if } NPV > 0 \end{array}$$

or $LGD \times NPV^{+}$ when the default occurs.

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Credit Value Adjustment

We now weight this with the probability of default which takes us to the value change

$$PD \times LGD \times NPV^+.$$

The value change above is a *payoff* in some small time interval in the future,

$$PD(t_{i-1}, t_i) \times LGD \times NPV^+(t)$$

To get the full CVA, we add over time slices until derivative maturity, discount payoffs to today and average over scenarios:

$$CVA = \sum_i PD(t_{i-1}, t_i) \times LGD \times \underbrace{\mathbb{E}\left[D(0,t)\, NPV^+(t)\right]}_{EPE(t)}$$

Debit Value Adjustment

The counterparty will rightly consider the possibility of our default and come up with the corresponding value change

$$DVA = \sum_i PD_{Bank}(t_{i-1}, t_i) \times LGD_{Bank} \times \underbrace{\mathbb{E}\left[D(0,t)\,(-NPV(t))^+\right]}_{ENE(t)}$$

It is hence fair to assume that the derivative fair value is

$$NPV = NPV_{rf} - CVA + DVA$$

This is slightly simplified - we need to take the order of defaults into account which modifies the formula somewhat.

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Capital Requirements

A bank needs to hold capital to protect its investors against unexpected losses from its business.

Regulators require a minimum “Risk Capital” RC that must exceed the sum of the following components

$$RC > RC_{MarketRisk} + RC_{CreditRisk} + RC_{OperationalRisk}$$

Let’s focus on how the minimum credit risk capital $RC_{CreditRisk}$ is determined.

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Capital Requirements

Credit risk capital has these ingredients:

- Exposure at default (EAD): imagine a loan's notional
- Probability of Default (PD)
- Loss given default (LGD)
- Effective maturity (M)

There are several approaches prescribed by the regulator, varying in complexity and conservatism.

We consider the complex end ("Internal Ratings Based approach"), where all four ingredients are estimated by the bank.

Under this approach

$$RC_{Credit} = EAD \times K(PD, LGD, M)$$

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Capital Requirement - Unexpected Loss

Capital requirement defined as **difference between “99.9% unexpected loss” and expected loss over a one year horizon**.

Regulators’ choice: Simple “Asymptotic Single Risk Factor” (ASRF) or “Large Homogeneous Pool” model for computing the probability of experiencing an unexpected loss.

One year default probability $F(G)$ conditional on a common economic factor G:

$$K(PD, LGD, M) = LGD \times (F(G_{99.9\%}) - PD) \times g(M, PD)$$

with maturity or “transition risk adjustment”

$$g(x,y) = \frac{1 + (x - 2.5) \cdot b(y)}{1 - 1.5 \cdot b(y)}, \quad b(y) = (0.11852 - 0.05478 \ln y)^2.$$

ASRF Model

Probability of an unexpected loss ("Gaussian copula" form):

$$F(G) = \Phi\left(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\,G}{\sqrt{1-\rho}}\right)$$

where $\Phi(x)$ is the cumulative standard normal distribution and $G \sim \Phi(0,1)$.

Set $-G = \Phi^{-1}(99.9\%)$ for an extreme scenario, so that

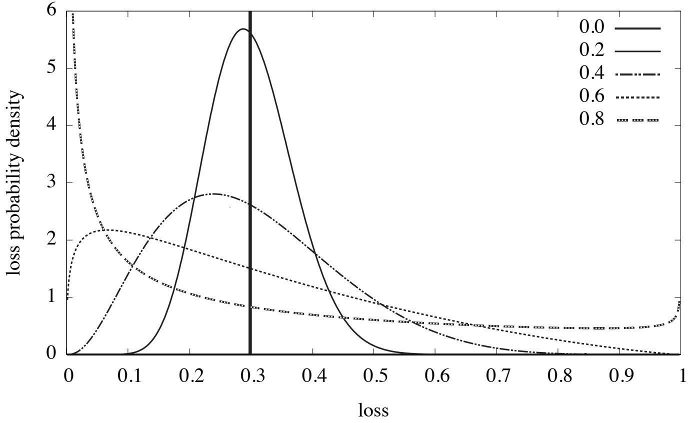
$$K(PD, LGD, M) = LGD \times \left(\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(99.9\%)}{\sqrt{1-\rho}}\right) - PD\right) \times g(M, PD)$$

Correlation ρ determined as a function of PD, somewhat simplified

$$\rho(PD) = 0.24 - 0.12 \times (1 - \exp(-50 \cdot PD)) + \text{size adjustment for corporates}$$

ASRF Model

Large Homogeneous Pool model loss distributions as a function of default correlation, $PD = 0.3$



Derivative EAD

To fit derivatives into the unexpected loss framework, regulators define

$$EAD = \alpha \times \max(EEPE_B,\, EEPE_B^{stressed})$$

- Calculation per netting set
- $\alpha = 1.4$ by default (to cover wrong way risk, granularity)
- α can be determined by the bank subject to regulatory approval, α floored at 1.2
- $RC_{Credit} = EAD \times K$, with K as above, but with “effective maturity”

$$M = \max\left(1, \min\left(5, \frac{\sum_{t_k \le 1} EEE_B(t_k)\, P(t_k)\, \Delta t_k + \sum_{t_k > 1} EE_B(t_k)\, P(t_k)\, \Delta t_k}{\sum_{t_k \le 1} EEE(t_k)\, P(t_k)\, \Delta t_k}\right)\right)$$

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Assignment I: KVA

1. Add the calculation of Credit Risk capital (RC_{Credit}) to Example 1, simplifications:
   - probability of default $PD = 0.01$
   - ignore size adjustment to $\rho(PD)$
   - use default $\alpha = 1.4$
   - assume $EEPE_{stressed} = EEPE$
   - recall $EE_B(t)\,P(t) = EE(t) = EPE(t)$ in ORE

   Options: Coding in C++ in ORE, or “post processing” and re-using data in output files
   ORE extensions would be much appreciated (appear in release 3)

2. What is the running cost of capital (per annum) assuming capital has to be funded at “capital rate” $r_c = 5\%$? How would you forecast $RC(t)$? Compute a simple **Capital Value Adjustment (KVA)**, ignoring discounting, i.e. $KVA = \sum_i RC(t_i) \times \delta_i \times r_c$

3. Suggest more adequate forecasting of the credit risk capital!

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