TRINITY COLLEGE DUBLIN, SCHOOL OF MATHEMATICS

Assignment #3, Module: MA5635

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1 PROB1

The code for this part is in the Python script prob1/program.py.
After simulation, the resulting image for part a, i.e. multiple paths for Brownian motion up to time $t = 2$, is in directory prob1/part_A/. Image rnd_walk_500_2.png contains the result for 40 random walks.
In directory prob1/part_B/ two histograms can be found; the first one is for $t = 1$ and the second for a value of $t = 2$.
It is important to notice that the simulations here are for the scaled random walk, which is the discretized version of Brownian motion. Ideally, when $n \to \infty$, scaled random walk tends to Brownian motion.

2 PROB2

Given the data from the problem description, the analytic solution to the differential equation takes the form:

$$X(t) = X(0) \cdot \exp\left\{\left(a - \frac{1}{2}b^2\right)t + bW(t)\right\} = \exp\{t + W(t)\} \Rightarrow X(t_k) = \exp\{t_k + W(t_k)\}$$

In the directory ./prob2/ there is a Python script (program.py) that that simulates previous solution, for values $n = [4, 8, 16, 32, 64]$; the plot corresponding to that simulation can be found within that same directory (i.e. analyt_solution_2.png). The simulation was performed for $t = 2$.

3 PROB3

The Python script for the full implementation of this simulation is located in the directory ./prob3/.
In the file forward_1.txt there are some numerical results for the error (the ones plotted). The plot of those numerical errors is displayed in the file log-log-errors.png.
The order of convergence is the slope of the curve plotted in that file. Therefore, the slope is: $\approx 4/3 \sim 1$, i.e. order of convergence is of order 1.
The error used here (and in next problem) is of the form:

$$E = \max_{k \leq n} |y(t_k) - t_k|$$

4 PROB4

For this part, in the Python script ./prob4/program.py, there are multiple important parts to mention: first, a random walk is generated, then, a numerical solution using Euler-Maruyama method is obtained, and finally, the analytic solution is obtained and put in a Python list, to finally compare both solutions; those three steps are performed for each value of n (= $[2,4,8,16,32]$).

The plots for both kind of errors (strong and weak), can be found in the directory ./prob4/ (as two .png files).

The order of convergence for both errors is roughly the same, approximately $15/2$ (~7).