Lecture Notes
Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, Jan 25th, 2:00pm - 3:00pm

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1 FTOC - Part I continued

Recall that FTOC - Part I states:

Let $a, b \in \mathbb{R}, a < b$.

IF f is continuous on $[a, b]$, and F is any antiderivative of f on $[a, b]$

THEN $\displaystyle\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$

ALSO $\displaystyle\int_a^b F'(x)dx = \int_a^b f(x)dx$

Proof

Let $P = \{x_i\}_{i=0}^n$ be a Riemann partition of $[a, b]$ such that

$\Delta x = \dfrac{b-a}{n}$, $x_i = a + i\Delta x$

So by Riemann Definition, $\displaystyle\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=0}^n f(x_i^*)\Delta x, x_i^* \in [x_{i-1}, x_i]$

Now let's start the proof

$\displaystyle\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=0}^n f(x_i^*)\Delta x, x_i^* \in [x_{i-1}, x_i]$	Recall $F'(x) = f(x), \forall x \in [a, b]$ (F is differentiable on [a,b]) differentiable implies continuity, thus F is also continuous. In particular, F is diff. on each $(x_{i-1}, x_i) \subset [a, b]$ also F is continuous on each $[x_{i-1}, x_i] \subset [a, b]$ that this satisfies the conditions for the Mean Value Theorem By MVT, $\exists c_i \in (x_{i-1}, x_i), \ni$: $F'(c) = \dfrac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}$ $F'(x) \cdot (x_i - x_{i-1}) = F(x_i) - F(x_{i-1})$ $f(c_i) \cdot \Delta x = F(x_i) - F(x_{i-1})$
Since we know that $c_i \in (x_{i-1}, x_i)$ Chose $x_i^* = c_i$ $\displaystyle\lim_{n\to\infty}\sum_{i=0}^n f(c_i)\Delta x, c_i \in [x_{i-1}, x_i]$ $\displaystyle\lim_{n\to\infty}\sum_{i=0}^n (F(x_i) - F(x_{i-1}))$	

Now we have

$$\begin{aligned}
\int_a^b f(x)dx = &\quad \lim_{n\to\infty}(F(x_1) - F(x_0)) + (F(x_2) - F(x_1)) \cdots + (F(x_{n-1} - F(x_{n-2})) + (F(x_n) - F(x_{n-1}))) \\
= &\quad \lim_{n\to\infty} -F(x_0) + F(x_n) \\
= &\quad F(x_n) - F(x_0) \\
\int_a^b f(x)dx = &\quad F(a) - F(b) \text{ By our Riemann Partition}
\end{aligned}$$

QED

2 FTOC - Part II

see Pg 390

Let $a, b \in \mathbb{R}, a < b$

IF f is cont on $[a, b]$, define $F(x) = \int_a^x f(t)dt, x \in [a, b]$

THEN F is cont on $[a, b]$ and F is differentiable on (a, b), Moreover $F'(x) = f(x), \forall x \in [a, b]$

This means F is an antiderivative of F on $[a, b]$

$\frac{dF}{dx} = \frac{d}{dx}(\int_a^x f(t)dt) = f(x)$

This theorem is used whenever we are asked to differente a function defined by an integral

Example

Compute

$\frac{d}{dx}(\int_x^4 \sqrt{1+t^4}dt)$

SOLN: $f(t) = \sqrt{1+t^4}$ is cont on $\mathbb{R}$ because $1 + t^4 \geq 0, \forall t \in \mathbb{R}$

In particular F is cont on $[x, 4] \subset \mathbb{R}$

So $\frac{d}{dx}(\int_x^4 \sqrt{1+t^4})dt = \frac{d}{dx}(\int_4^x (-1)\sqrt{1+t^4})dt = -f(x) = -\sqrt{1+x^4}$