

CSCC63 TUT 0002

Tutorial 11

Assignment Help & Review

Prove that EQTM is decidable, recognizable, co-recognizable or neither

suppose that M is a TM, $L(M) = \{ x \mid M \text{ accepts } x \}$

$\text{EQTM} = \{ \langle M, N \rangle \mid L(M) = L(N) \}$ not recognizable

reduce from HALT-c

P on input $\langle M, w \rangle$:

define M' on input $\langle x \rangle$:

run M on w

accept

define N'

reject

return $\langle M', N' \rangle \# \langle M', N' \rangle$ in EQTM if M' in HALT-c

$\text{EQTM-c} = \{ \langle M, N \rangle \mid L(M) \neq L(N) \}$ not co-recognizable

reduce from HALT-c

P on input $\langle M, w \rangle$:

define M' on input $\langle x \rangle$:

run M on w

accept

define N'

accept

return $\langle M', N' \rangle \# \langle M', N' \rangle$ in EQTM-c if M' in HALT-c

$\text{DECIDABLE} = \{ \langle M \rangle \mid L(M) \text{ is decidable} \}$

$L(M)$ is decidable if there is a decidable TM N such that $L(N) = L(M)$.

in the last problem we talked about given two TMs, whether we can compare if they are equal.

[given x, y , asking if $x = y$]

in this problem we are given a TM, asking if there is a decidable TM N such that $L(M) = L(N)$.

[given x , is there y such that $x = y$]

Look at language forms to see if they are harder than languages you've seen before.

$L = \{ \langle M \rangle \mid L(M) = \{1\} \}$ not recognizable

$L-c = \{ \langle M \rangle \mid L(M) \neq \{1\} \}$ here either M rejects 1 or M accepts something other than 1. not co-recognizable.

say we have a TM that loops on all.

$L(M) = \text{empty} \neq \{1\}$

both can be proven with reduction from HALT-c

P on input $\langle M, w \rangle$:

define M' on input $\langle x \rangle$:

accept if $x = 1$ # if M, w in halt-c then M' accepts only 1

run M on w

accept # if M, w in halt, we want M' to accept more than just $\{1\}$

P on input $\langle M, w \rangle$:

define M' on input $\langle x \rangle$:

run M on w

accept if $x = 1$ # we want $L(M')$ to be $= \{1\}$

$L = \{ \langle G \rangle \mid G \text{ is a CFG such that } L(G) \text{ contains a palindrome} \}$

$L-c = \{ \langle G \rangle \mid G \text{ is a CFG such that } L(G) \text{ contains no palindromes} \}$

recognizable via the following recognizer:

R on input $\langle G \rangle$:

 for w in Σ^*

 if $w + w^R$ in G , accept

PCP to L

PCP not co-recog \Rightarrow L not co-recog

Say the input pcp takes in the form of $(t_1, b_1), (t_2, b_2), \dots (t_n, b_n)$

where t_i is the top string of the i -th tile, b_i is the bottom string of the i -th tile

G: $S \rightarrow t_1 A b_1^R \mid t_2 A b_2^R \mid \dots \mid t_n A b_n^R$

$A \rightarrow \# \mid t_1 A b_1^R \mid t_2 A b_2^R \mid \dots \mid t_n A b_n^R$

G generates the following string: $t_1 t_3 t_5 \# b_5^R b_3^R b_1^R$

if the PCP has a solution, say tile 1, 3, 5

$t_1 + t_3 + t_5 = b_1 + b_3 + b_5$, and if such a combination exists then $t_1 t_3 t_5 \# b_5^R b_3^R b_1^R$ is a palindrome

if the PCP has no solutions, this means that no concatenation of any t_i 's will match the concatenation of the same b_i 's if this is the case, then the grammar cannot generate a palindrome since the grammar contains t_i on one end and b_i^R on the other. this grammar generates $s\#y$, only when an instance of a PCP is a yes instance can $s = y^R$.

Q5: given oracle for NAE-3SAT, solve F-NAE-3SAT in polytime.

let NAE-3SAT-ORACLE be the oracle for NAE-3SAT
F on input $\langle \phi \rangle$:

for each variable:

if setting that variable to true, still yields a ϕ' for which it has a remaining NAE 3SAT assignment, then in the final variable assignment, this variable can be set to true.

for each variable consider adding a set of clauses to fix variable assignment:
 $(x_i \text{ or } a \text{ or } b) \text{ AND } \dots \text{ AND } \dots$

Input: 3CNF ϕ

Output: a satisfying truth assignment if one exists, otherwise null

F-3CNF is an oracle for 3SAT

M on input $\langle \phi \rangle$:

if F-3CNF(ϕ) returns false, return null

let $\phi' = \phi$, $t = []$, # where $t[i] =$ the truth assignment of x_i

for each variable x_i in ϕ' :

let $\phi'' = \phi' \text{ AND } (x_i \text{ or } x_i \text{ or } x_i)$

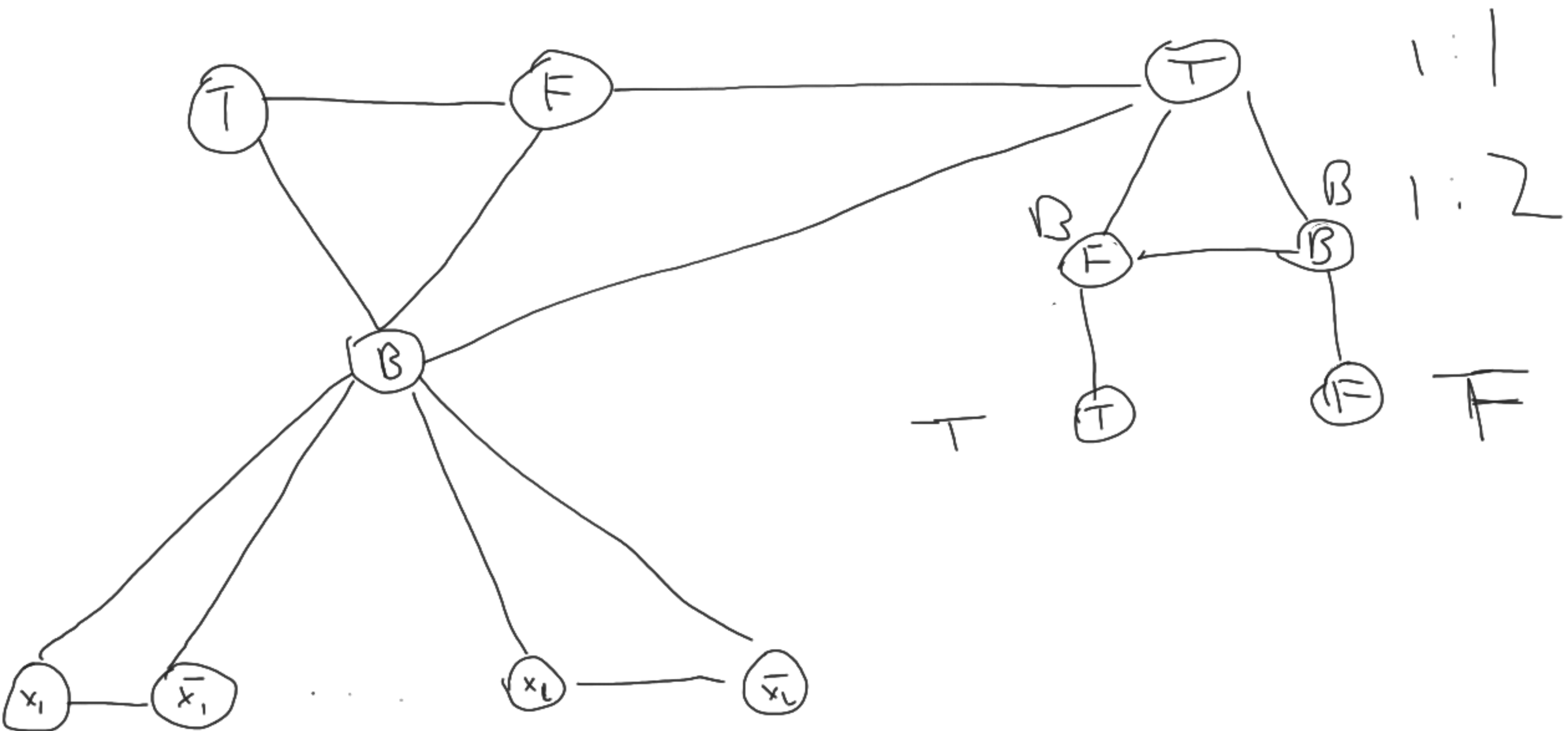
run F-3CNF(ϕ'')

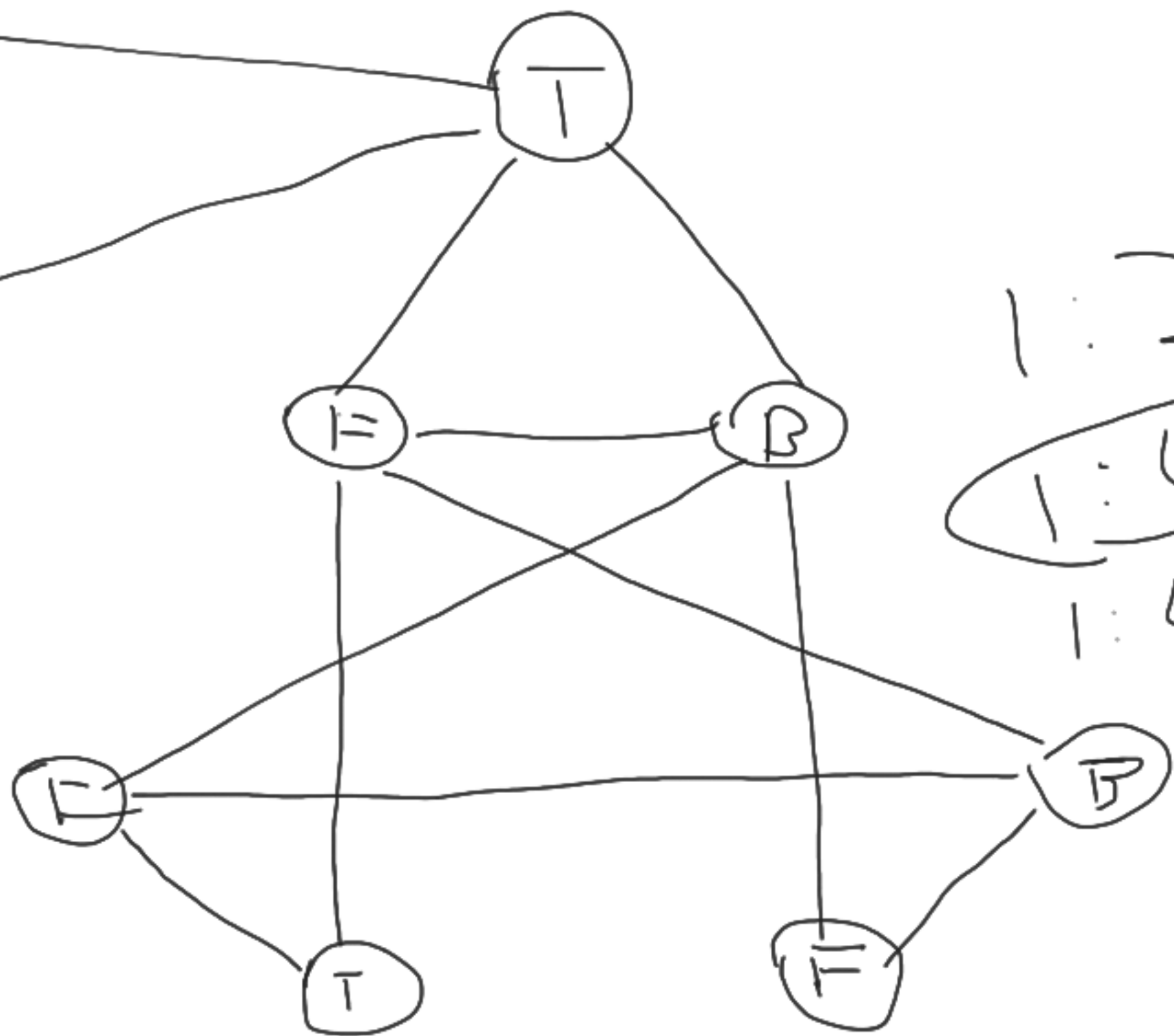
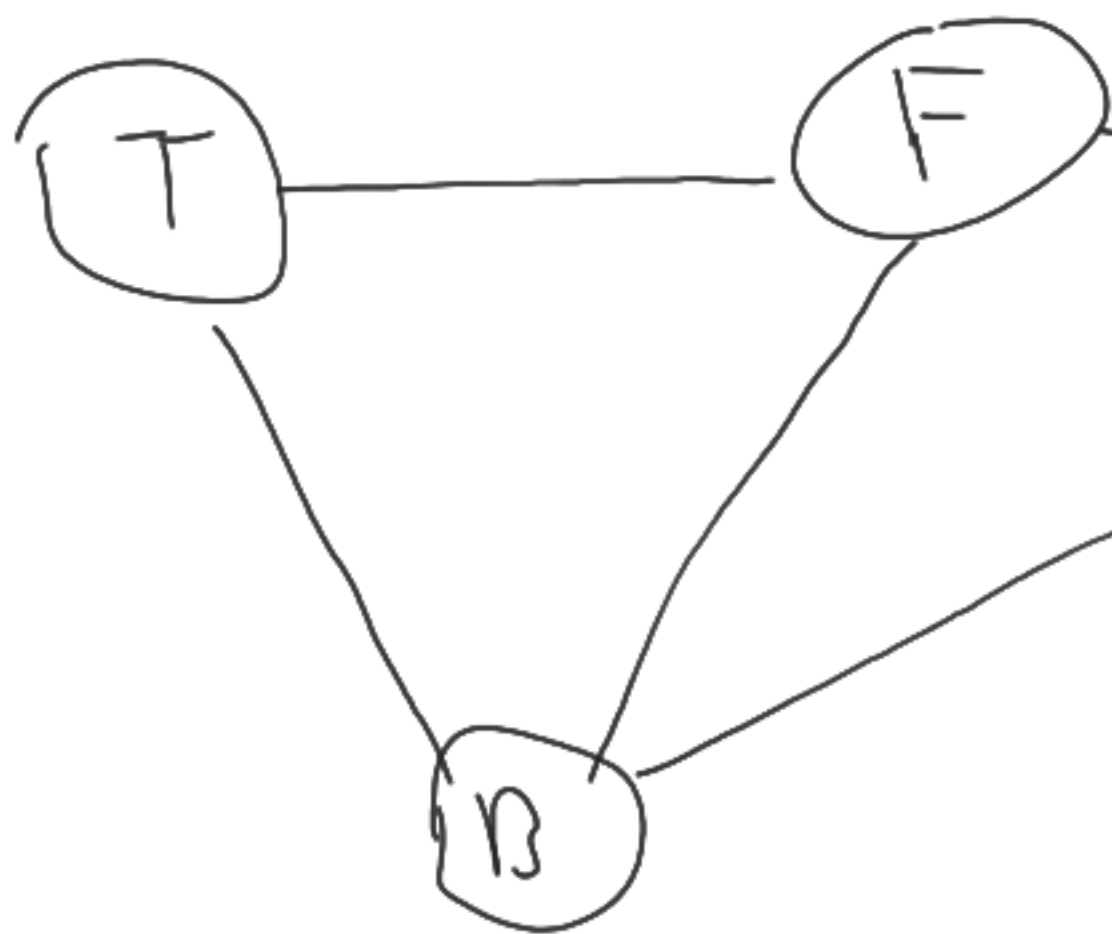
if returns true, $\phi' = \phi''$ and $t[i] = \text{true}$

otherwise $\phi' = \phi'' \text{ AND } (\text{not } x_i \text{ or not } x_i \text{ or not } x_i)$ and $t[i] = \text{false}$

return t

Show that #3COL is #P complete by finding a parsimonious reduction from #3SAT
 we have 3 colors, {T, F, B}





1:2
 1:4
 1:4:3!

T F T