MAT A29 TUT0018, Tutorial 5 (Week 7)
Tuesdays 7 - 9pm (We will start at 7:10pm)

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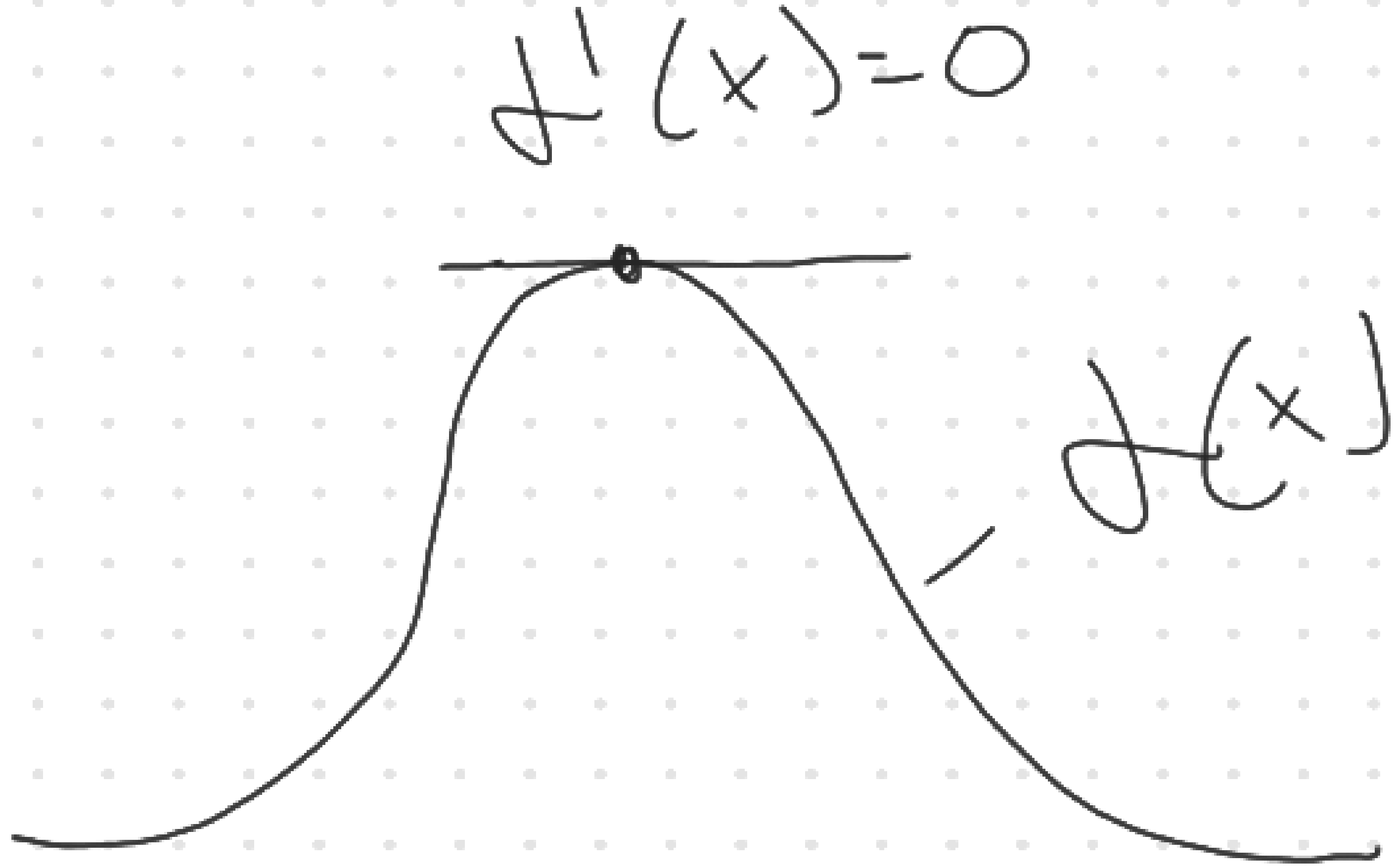
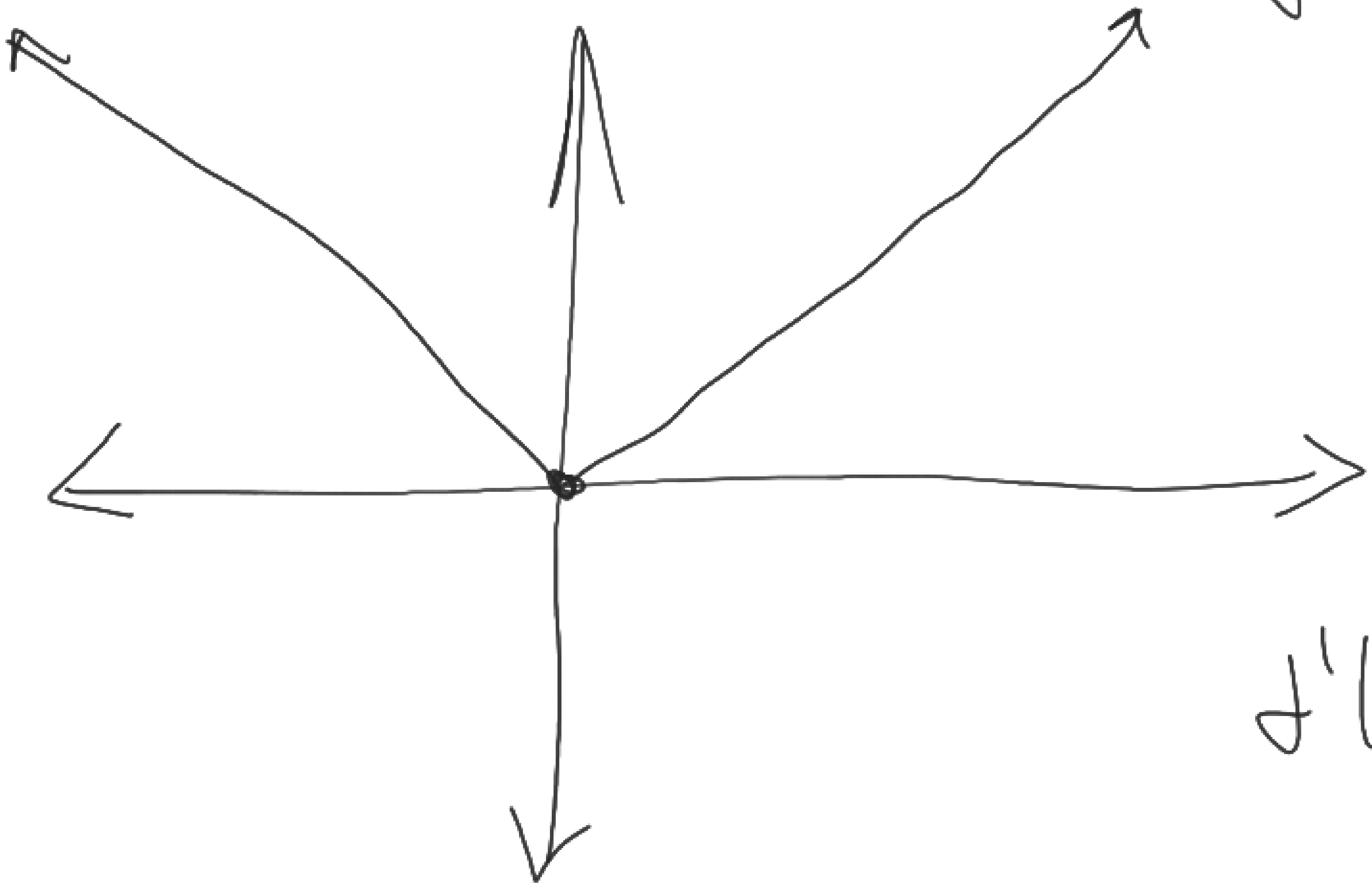


local & absolute maxima/minima of a function f(x)
f'(x) = 0 / undefined to find POI (points of interest)
Then, if
f''(x) < 0 -> x is a local maximum
f''(x) > 0 -> x is a local minimum
f''(x) = 0 -> cannot tell

2nd Derivative test

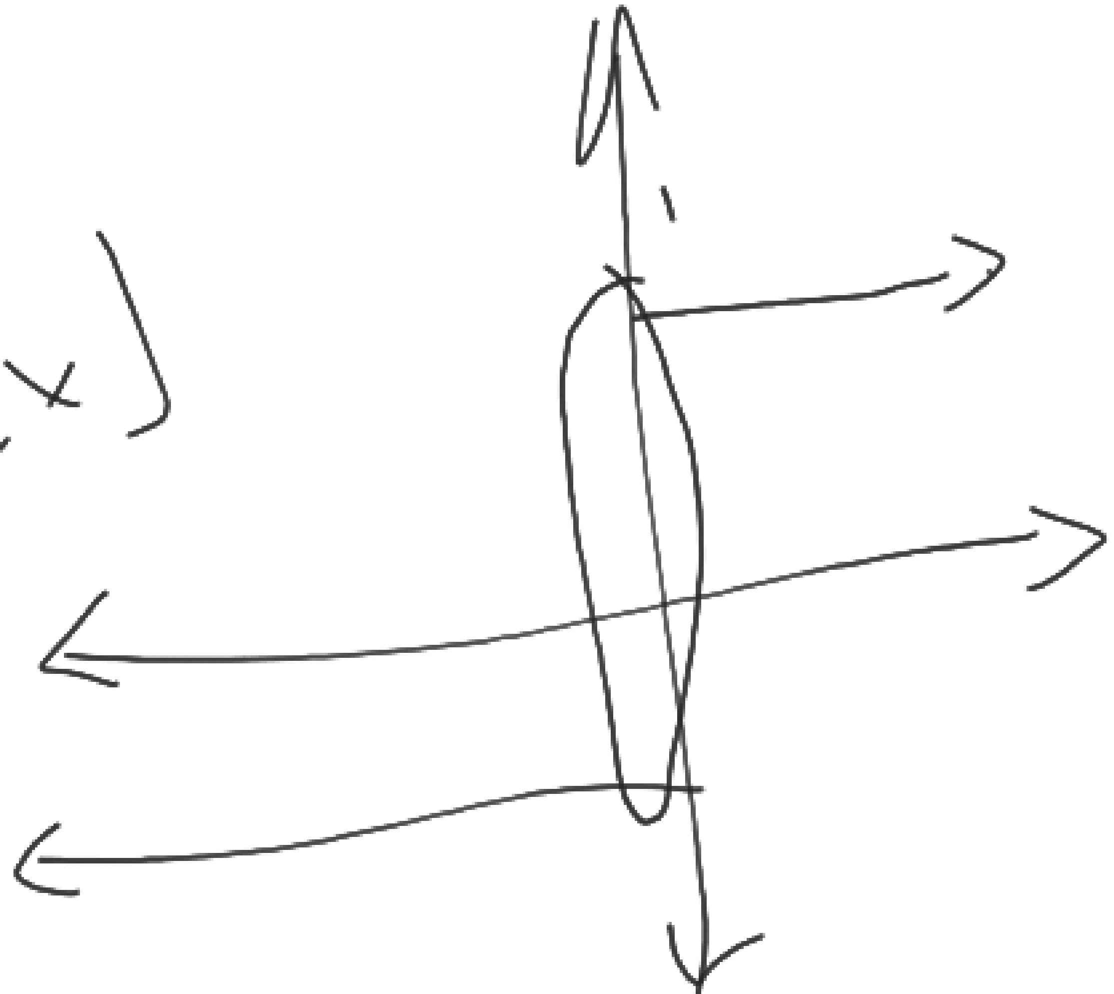
Question

When you are checking for critical points, explain why you also need to determine points where $f'(x)$ is undefined. Draw a graph to support your conclusion.



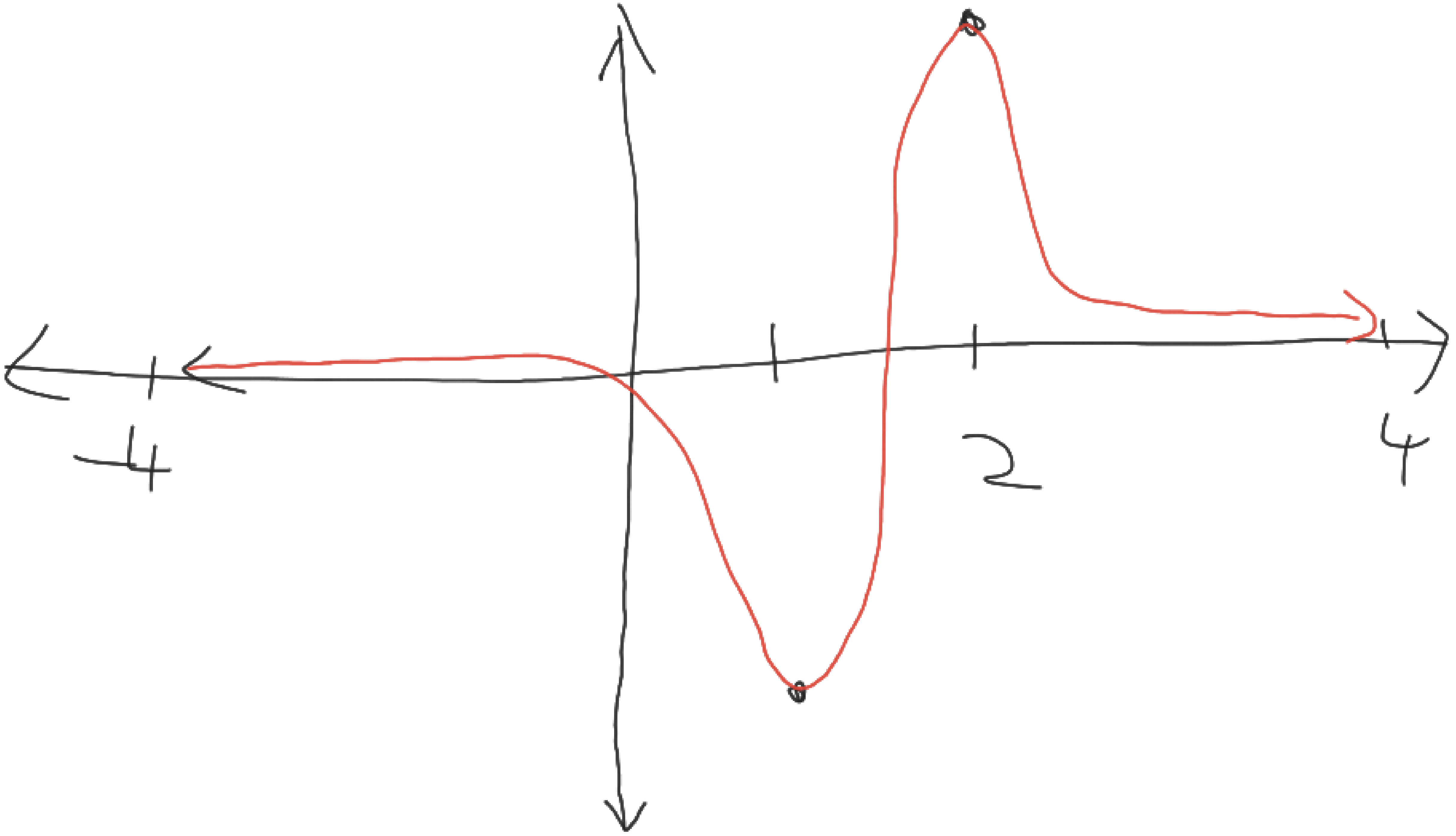
$f(x) = |x|$

$f'(x)$



Question

Draw a graph of a continuous function on $[-4, 4]$ *with an absolute minimum at* $x = 1$ *and an absolute maximum at* $x = 2$.



Question

Find the local/absolute maxima for the function $f(x) = x^2 + \frac{2}{x}$ on the domain $[1,4]$.

f'(x) = 2x - 2/x^2
f'(x) = undefined, x = 0

f'(x) = 0 =>
x = 0, x = 1

f(1) = 1 + 2 = 3
f(4) = 16 + 1/2 = 16.5

f(4) > f(1) x = 4 must yield an absolute maxima on the domain [1,4]

$$f'(x) = 2x - \frac{2}{x^2}$$

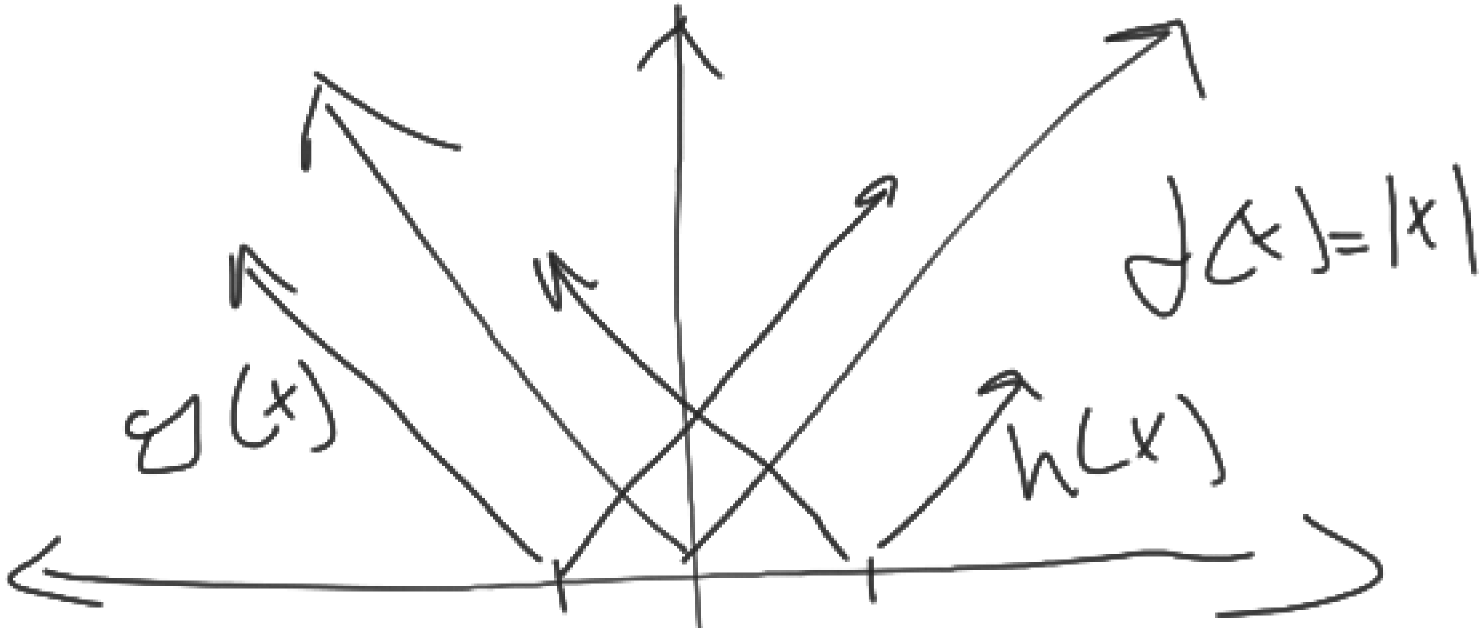
$$= 2x\left(1 - \frac{1}{x^3}\right) = 0$$

$$x = 0, \quad 1 - \frac{1}{x^3} = 0$$

$$\frac{1}{x^3} = 1 \Rightarrow x = 1$$

Question

Find the local/absolute maxima for the function $f(x) = |x+1| + |x-1|$ on the domain $[-3, 2]$.



$g(x) = |x + 1|, h(x) = |x - 1|$

$$g'(x) = \begin{cases} 1 & x > -1 \\ -1 & x < -1 \end{cases}$$

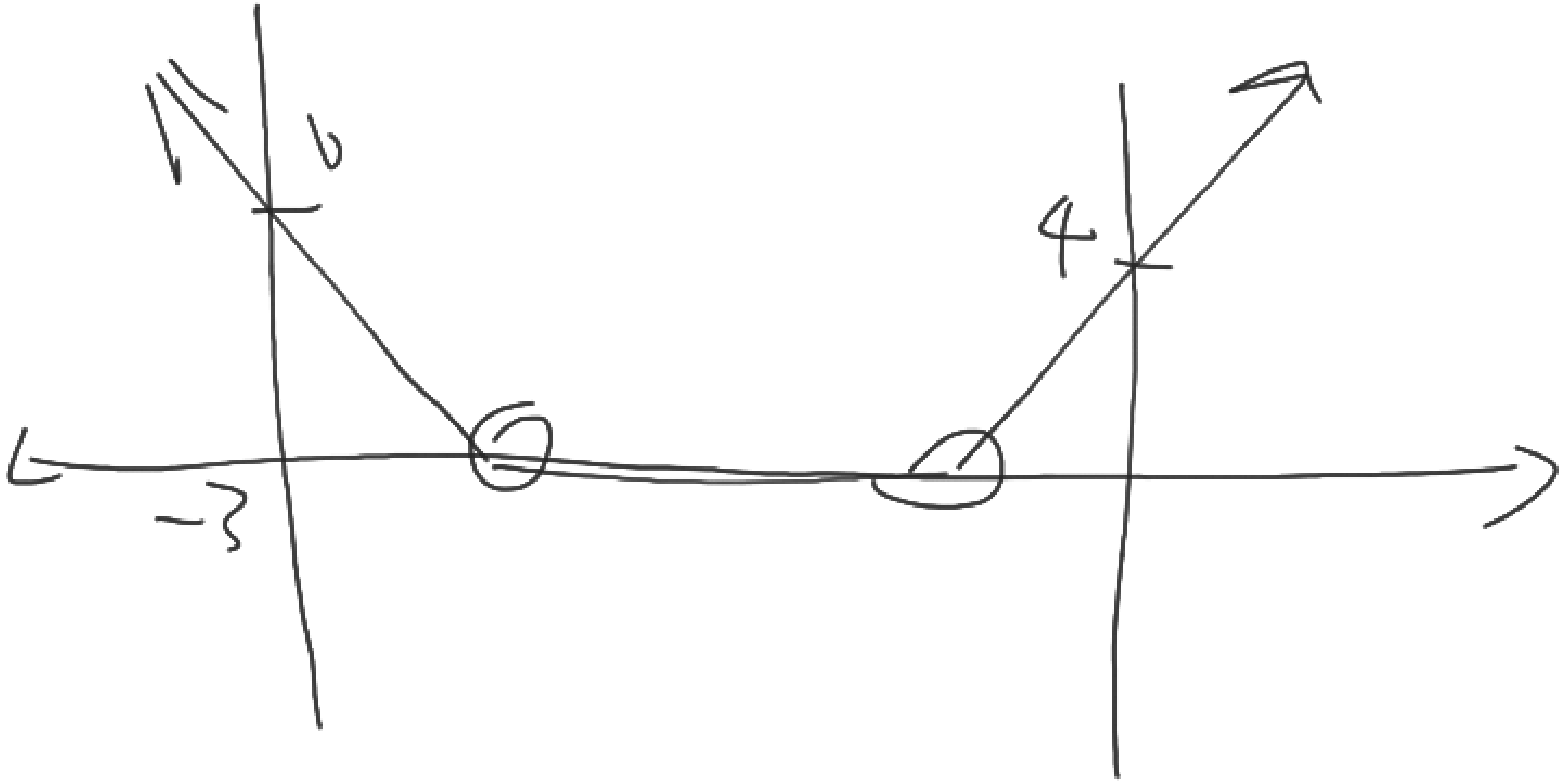
$f'(x)$

$(-\infty, -1)$ $(-1, 1)$, $(1, -1)$

$$h'(x) = \begin{cases} 1 & x > 1 \\ -1 & x < 1 \end{cases}$$

$$f'(x) = \begin{cases} -2 & x < -1 \\ 0 & -1 < x < 1 \\ 2 & x > 1 \end{cases}$$

poi = -1, 1, -3, 2



f(-1) = 2
f(1) = 2
f(-3) = 6
f(2) = 4

function has absolute maximum at f(-3) = 6

Question

Find the positive integer that minimizes the sum of the number and its reciprocal.

$1 = \frac{1}{x^2}$

f(x) = x + 1/x

f'(x) = 1 - 1/x^2, f'(x) = undefined => x = 0, f'(x) = 0 => x = 1
f''(x) = 1/x^3

f''(1) = 1 > 0, then 1 must be a minimum

x = 1 is the positive integer that minimizes f(x)

Question

You need to construct a fence around an area of $1600 ft^2$. What are the dimensions of the rectangular pen to minimize the amount of material needed?

let w be the width of the fence
let l be the length of the fence
l * w = 1600, w = 1600/l, l = 1600/w
m(w, l) = 2w + 2l, m(w) = 2w + 2(1600/w)

$$m'(w) = 2\left(1 - \frac{1600}{w^2}\right) = 0$$

$$\frac{1600}{w^2} = 1 \iff w = \sqrt{1600} = 40$$

w = 0, w = 40

$$m''(w) = 2\left(\frac{3200}{w^3}\right)$$

m''(40) > 0, thus a width of 40 minimizes the amount of materials needed

l = 1600 / 40 = 40

thus w = 40, l = 40 minimizes the amount of materials needed.