CSCC63 Winter 2022 Tutorial 2

Rice Theorem

Reductions

Tape TMs

Assignment Help

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Rice's Theorem

Any language dictating non-trivial functional properties of TMs is not decidable
- functional property means relation between input output
- nontrivial means not all posess this property and not all lack it

for example L = { <M> | L(M) = { 1, 2, 3 } }

$A = \{\langle M \rangle : |\mathcal{L}(M)| \geq 3\}$

is this decidable? if not recognizable/co-recognizable?

```
let xi's be an enumeration over all strings
R on input <M>:
  for i = 0... inf:
    for j = 0 ...i:
       run M on xi ... xj for j steps
       if M accepts 3 or more times, accept
```

Use a mapping reduction to show that A is not co-recognizable
A = { <M> | L(M) >= 3 }
A-c = { <M> | L(M) < 3 }, HALT-c = { <M, w> | M loops on w }

WTS HALT-c <= A-c

if <M, w> is in HALT-c, M' should be in A-c
if M loops on w, M' should accept less than 3 strings
if <M, w> is not in HALT-c, M' should not be in A-c
if M halts on w, M' should accept 3 or more strings

```
R on input <M, w>:
  define M' on input <x>:
    run M on w
    accept
  return M'
```

<M, w> in HALT-C iff M' in A-c
HALT-c <= A-c, HALT-c is not recognizable, A-c is not recognizable, A not corec

$$B = \{\langle M\rangle : |\mathcal{L}(M)| = 3\}$$

B-c = { <M>| L(M) != 3 }

is this decidable? if not rec/co-rec?

HALT-c <= B

if M loops on w, M' should accept exactly 3
if M halts on w, M' should accept any number but 3

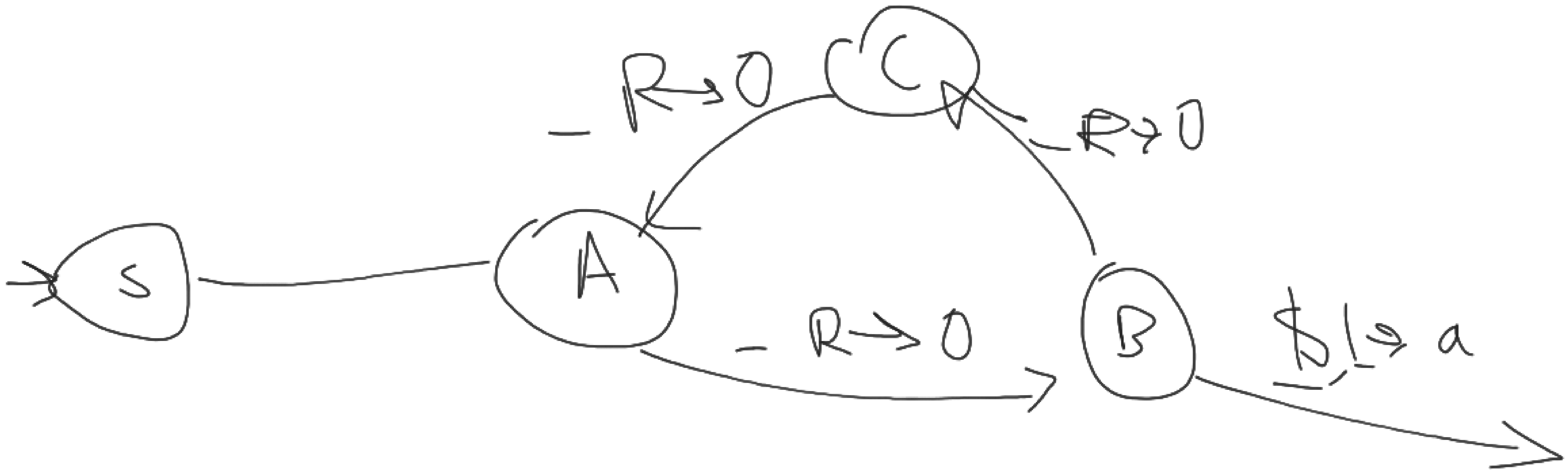
```
R on input <M, w>:
   define M' on input <x>:
      accept x if x in [0, 00, 000]
      run M on w
      accept
   return M'
```

Consider the language “writes symbol on empty tape”

$WS = \{\langle M, a\rangle : \text{TM } M \text{ on empty tape writes symbol } a\}$

Is WS decidable? If not, is it recognizable?

$HET = \{\langle M\rangle : \text{TM } M \text{ halts on empty tape}\}$



$WNB = \{\langle M \rangle$: TM M on empty tape writes some non−blank symbol$\}$

given a TM with k states, if on the k + 1th step, nothing is written, then 2 of the configurations would've been the same.

q_1 —

q_1 — — —

q_2 - — —

q_3 — - . _ _ — q_1 —

.

.

q_k - - - -