Lecture Notes
Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, Feb 4th, 2:00pm - 3:00pm

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1 Integration by Parts

Section 5.2, Theorem on Pg 421, 423

IF $u = f(x), v = g(x)$ are differentiable

THEN:

1. Indefinite form
$$\int u \cdot du = u \cdot v - \int v \cdot du$$

2. Definite form
$$\int_a^b u \cdot du = uv\Big|_a^b - \int_a^b v \cdot du$$

We must first choose/identify; u, dv
then compute du, v

Proof of indefinite form
Suppose $u = f(x), v = g(x)$ are differentiable
We know $(f(x)g(x))' = f(x)g'(x) + g(x)f'(x) \Leftrightarrow (uv)' = u'v + uv'$
Now $uv' = (uv)' - vu' \Leftrightarrow udv = (uv)' - vdu$
If we integrate both sides, we have
$\displaystyle\int udv = \int((uv)' - vdu) = \int(uv)' - \int vdu$, By $\displaystyle\int$ properties
$\displaystyle\int udv = uv - \int vdu$, QED

2 Example

1. Compute: $\int(x+1)e^{-x}dx$

   Solution: choose $u = x + 1, dv = e^{-x}dx$

   There are two checks for this technique. if du, v aren't easy, you made a mistake.

   $du = 1dx, v = -e^{-x}$

   if $\int vdu$ is harder than the original, you made a mistake.

   Now we have $\displaystyle(x+1)(-e^{-x}) - \int -e^{-x}dx = -(x+1)e^{-x} - e^{-x} + C$

2. Compute: $\displaystyle\int_1^2 x \cdot \ln(x)dx$

   Solution: choose $u = \ln x, dv = xdx$, $du = \dfrac{1}{x}dx, v = \dfrac{x^2}{2}$

   Now we have $\displaystyle\int_1^2 x \cdot \ln(x)dx = \ln x\frac{x^2}{2}\Big|_1^2 - \int_1^2 \frac{x^2}{2} \cdot \frac{1}{x} = 2\ln(2) - \frac{1}{4}(4-1) = 2\ln(2) - \frac{3}{4}$

   Choose u to be the polynomial unless there's a trig or ln

3. Compute: $\int t^2 \cdot e^t dt$

   We must use double integration by parts here.

   Solution: choose $u_1 = t^2, dv_1 = e^t dt,\ du_1 = 2t, v_1 = e^t$

   Now we have $\int t^2 \cdot e^t dt = t^2 e^t - \int e^t(2t \cdot dt) = t^2 e^t - 2\int te^t dt$

   Now choose $u_2 = t, dv_2 = e^t dt,\ du_2 = 1dt, v = e^t$

   Again, we have $\int t^2 e^t - 2\int te^t dt = t^2 e^t - 2\left[te^t - \int e^t\right] = t^2 e^t - 2 \cdot te^t + 2e^t + C$

4. Compute: $\int_0^1 \arctan(x)dx$

   Solution: choose $u = \arctan(x), dv = 1dx, du = \dfrac{1}{1+x^2}dx, v = x$

   Now we have $\int_0^1 \arctan(x)dx = \arctan(x) \cdot x\Big|_0^1 - \int_0^1 \dfrac{x}{1+x^2}dx = \arctan(x) \cdot x\Big|_0^1 - \left[\dfrac{\ln(1+x^2)}{2}\right]_0^1$

   $= \arctan(1) - \dfrac{1}{2}[\ln 2 - \ln 1] = \dfrac{\pi}{4} - \dfrac{\ln 2}{2}$

5. $\int e^x \sin(x)dx$

   Choose $u = \sin(x), dv = e^x dx,\ du = \cos(x)dx, v = e^x$

   Now we have $\sin(x)e^x - \int e^x \cos(x)dx$

   Choose $u_1 = \cos(x), dv_1 = e^x dx,\ du_1 = -\sin(x)dx, v_1 = e^x$

   Again, we have $\sin(x)e^x - \int e^x \cos(x)dx = \sin(x)e^x - \left(\cos(x)e^x - \int e^x(-\sin(x))dx\right)$

   $\int e^x \sin(x)dx = \sin(x)e^x - \cos(x)e^x - \int e^x \sin(x)dx$

   $2\int e^x \sin(x)dx = \sin(x)e^x - \cos(x)e^x$

   $\int e^x \sin(x)dx = \dfrac{1}{2}\left(\sin(x)e^x - \cos(x)e^x\right) + C$

Exercises: Compute

1. $\int \dfrac{\ln(x)}{x^2}dx$
2. $\int x\sec^2(x)dx$
3. $\int (x^2+1)e^{-3x}dx$
4. $\int \cos(x)\ln(\sin(x))dx$
5. $\int \ln(x)dx$

3 Partial Fraction Decomposition

Purpose: to rewrite the integrand of an integral $f(x) = \dfrac{Q(x)}{P(x)}$ where $P(x), Q(x)$are proper rational functions.

Recall proper ratioanl function is arational function such that deg(numerator) ¡ deg(denominator)

Motivation: $f(x) = \dfrac{5x+11}{(x+3)(x+2)} = \dfrac{4}{x+3} + \dfrac{1}{x+2}$

$\displaystyle\int f(x)dx = \int \left(\frac{4}{x+3} + \frac{1}{x+2}\right)$

Examples Write the form only of the partial fraction decompositions

1. $f(x) = \dfrac{8x-42}{(x+6)(x-1)} = \dfrac{A}{x+3} + \dfrac{B}{x+2}$
2. $f(x) = \dfrac{9-9x}{(x^2-1)(2x-7)x} = \dfrac{Ax+D}{x^2-1} + \dfrac{B}{2x-7} + \dfrac{C}{x}$