MAT B24 TUT0018, Tutorial 3 (Week 4)
Thursdays 3 - 5pm

XiangQian Hong (Richard)
richard.hong@mail.utoronto.ca

Proving linear independence of a set X

1. System of equations
2. if Span(X) = F, say XL is a known Linearly independent set where Span(XL) = F, if |X| = |XL| => X is Linearly independent
3. Wronskian Matrix, say {f(x), g(x), h(x)} is linearly independent
   exists x,
   af(x) + bg(x) + ch(x) = 0
   af'(x) + bg'(x) + ch'(x) = 0
   af''(x) + bg''(x) + ch''(x) = 0
   = > a = b = c = 0,

Proving linear dependence of a set X

1. X = {x1, x2, x3} show that x1 = a x2 + b x3
2. span(X) subseteq F = span(XL), |XL| < |X|

3.2 #36

Prove that if W is a subspace of some n-dimensional vector space V and the dim(W) = n, then W = V

WTS W = V,

we know W subseteq V, W = span({w1, ... , wn}), dim(V) = n => V spanned by n elements

Suppose exists v in V, such that v is not in W. thus v is no linear combination of w1, ... , wn which means {w1, ... , wn, v} is a linear independent set.

Additional Problem 3
Suppose V is a vector space of dimension n, V = span({v1, v2, ... vn}), WTS any set X in V of size greater than n is linearly dependent.

X subseteq V, span (X) subseteq V.

Let X be a subset of V, such that |X| = n + 1 WLOG, assume for a contradiction that X is a linearly independent subset. If this is true then span(X) = W superset of V. since W is a vector space with dimension n + 1. Therefore X is cannot be a subset of V, and any subset of V that has more than n elements are linearly dependent.

3.2 Question 11. Determine whether the given set of vectors is dependent or independent
X = { x^2 - 1, x^2 + 1, 4x, 2x - 3 } in P
Span(X) subseteq P_2 = Span({x^2, x, 1}),
argue if Span(X) is a subset of Span(XL) where |XL| < |X| then X is not linearly independent.

since |X| = 4, and span(X) subseteq P_2 = Span({x^2, x, 1}). Then elements of X are in a 3 dimensional vector space.