Lecture Notes
Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, March 1st, 1:00pm - 2:00pm

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1 Improper Integrals

Example Does $\displaystyle\int_{-\infty}^{1} \frac{1}{x-2}dx$ Converge?

Sol'N $\displaystyle\int_{-\infty}^{1} \frac{1}{x-2}dx = \int_{-\infty}^{0} \frac{1}{x-2}dx + \int_{0}^{1} \frac{1}{x-2}dx$

Consider $\displaystyle\int_{-\infty}^{0} \frac{1}{x-2}dx = \lim_{a\to-\infty}\int_{a}^{0} \frac{1}{x-2}dx = \lim_{a\to-\infty}(\ln|-1| - \ln|a-1|) \approx -\infty$

Notice $x \in [0,1], x-1 < 0 \Rightarrow \dfrac{1}{x-1} < 0$ and thus $\displaystyle\int_{0}^{1} \frac{1}{x-2}dx$ can never yield infinity.

Thus, $\displaystyle\int_{-\infty}^{1} \frac{1}{x-2}dx$ diverges.

Motivation Does $\displaystyle\int_{0}^{\infty} \frac{\arctan^5(x))}{(1+x^6)^{6/13}}dx$ Converge? What of $\displaystyle\int_{1}^{\infty} e^{x^2}dx$?

2 Comparison Theorem

Comparison Theorem (CT) for Type I

let $a \in \mathbb{R}$ Suppose g, f are continuous on $[a,\infty)$

$f(x) \geq g(x) \geq 0, \forall x \in [a,\infty) \wedge \displaystyle\int_{a}^{\infty} f(x)dx$ Converges $\Longrightarrow \displaystyle\int_{a}^{\infty} g(x)dx$ Converges

$h(x) \geq g(x) \geq 0, \forall x \in [a,\infty) \wedge \displaystyle\int_{a}^{\infty} h(x) = \infty \Longrightarrow \int_{a}^{\infty} g(x)dx = \infty$ (Diverges)

MIDTERM - TEST COVERS A1 - A6, Solving Improper Integrals by Evaluation