Week 8, LEC02 - November 9th
Discrete Mathematics, Fall 2018

CSCA67 - Lecture Notes

Current Instructor: Dr. Richard Pancer

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$S(n)$: Given n lines in the plane such that no two are parallel, and no three intersect at a single point, we have $\binom{n}{3}$ Triangles.

Claim: $\forall n \in \mathbb{N}, n \geq 3, s(n)$

Base $n = 3$, by observation(inspection), given three lines only one triangle can be formed.

$\binom{3}{3} = 1$, $S(n)$ for n= 3.

Inductive Hypothesis: $k \in \mathbb{N}, k \geq 3, S(k)$

Inductive Step: $S(k) \rightarrow S(k+1)$

With k lines in the plane, we have $\binom{k}{3}$ triangles, by Inductive Hypothesis.

When the $(k+1)^{st}$ line is added, it must cross every existing pair of lines in the plane.

When this happens, a new triangle is formed.

we have $\binom{k}{2}$ ways of selecting 2 lines from k lines.

$\therefore$ Total number of Triangles after the $(k+1)^{st}$ line is added is $\binom{k}{3}$(IH) $+\binom{k}{2}$(New triangle from $(k+1)^{st}$ line).

$= \frac{k!}{(k-3)!3!} + \frac{k!}{(k-2)!2!} = \frac{(k+1)!}{(k+1-3)!3!} = \binom{k+1}{3}$. $S(k+1-)$

Conclusion: Since k is arbitrary, $\forall n \in \mathbb{N}, n \geq 3, S(n)$

This question will re-appear on assignment 2!

1 Counting with Repetitions

The genetic code of an organism stored in DNA molecules consist of 4 nucleotides:

Adenine, Cytosine, Guanine and Thymine

- It is possible to sequece short strings of molecules.

- One way to sequence the nucleotides of a longer string of DNA is to split the string into shorter sequences.

- A C-enzyme will split a DNA-sequence at each C. This means that each fragment will end at a C except possibly the last fragment.

- Similarly for A-enzymes, G-enzymes and T-enzymes.

- If the original nucleotide is split on each of C, A, G and T then it can be sequenced as it is most likely a unique sequence that can be constructed by each of the four sets of fragments.

Example. Given a 20-nucleotide string split at the Cs, one might have the fragments:

AC, AC, AAATC, C, C, C, TATA, TGGC

Q. How many different 20-nucleotide strings could have given rise to the above set of fragments? In other words, how many different arrangements are there of these fragments?

A.