Lecture Notes
Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, Jan 7th, 1:00pm - 3:00pm

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1 Chapter 4; Sigma Notation

Definition.
$\sum$ - Notation:
Sigma notation is convenient way to express a sum of a collection of objects that take on a common form.
If we have a_k and b_k that are real-valued fcns(functions), then $a_m + a_{m+1} \cdots + a_k \cdots + a_n = \displaystyle\sum_{k=m}^{n} a_k$
a_k - General Term
k - index(letter doesn't matter)
m, n - inital index value and final index value

Example.
Express $-1 + 2 - 3 + 4 - 5 + \cdots - 21$ in $\sum$ - notation.
$\displaystyle\sum_{k=1}^{21} (-1)^k$ will produce the alternating signs.
$\displaystyle\sum_{k=1}^{21} (-1)^k \cdot k = -1 + 2 - 3 + 4 - 5 + \cdots - 21$

Sums are NOT unique, and there will be multiple ways of expressing the same sum.
$\displaystyle\sum_{k=1}^{21} (-1)^k \cdot k = \sum_{k=0}^{20} (-1)^{k+1} \cdot (k+1)$

Theorem. Pg(319 - 320)
Properties of $\sum$ - Notation
consider $m, l, k \in \mathbb{Z}^+, \ni: 1 \leq k \leq m, l < m.$ and let $c \in \mathbb{R}$
If we have a_k and b_k that are real-valued fcns(functions), then:

$$\text{(i)} \sum_{k=1}^{m} (a_k + b_k) = \sum_{k=1}^{m} a_k + \sum_{k=1}^{m} b_k$$
$$\text{(ii)} \sum_{k=1}^{m} c \cdot a_k = c \cdot \sum_{k=1}^{m} a_k$$
$$\text{(iii)} \sum_{k=1}^{m} a_k = \sum_{k=1}^{l-1} a_k + \sum_{k=l}^{m} a_k$$

Proofs.

(i) Suppose that a_k and b_k are real valued functions $(a_k, b_k \in \mathbb{R})$, let $m, k \in \mathbb{Z}^+$

WTS $\sum_{k=1}^{m}(a_k + b_k) = \sum_{k=1}^{m} a_k + \sum_{k=1}^{m} b_k$

Consider $\sum_{k=1}^{m}(a_k + b_k)$

$$\sum_{k=1}^{m}(a_k + b_k) = a_1 + a_2 \cdots + a_m + b_1 + b_2 \cdots + b_m$$

$$\sum_{k=1}^{m}(a_k + b_k) = a_1 + b_1 + a_2 + b_2 \cdots + a_m + b_m = \sum_{k=1}^{m} a_k + \sum_{k=1}^{m} b_k$$

(ii) Suppose that a_k is a real valued function $(a_k \in \mathbb{R})$, let $c \in \mathbb{R}, m, k \in \mathbb{Z}^+$

WTS $\sum_{k=1}^{m} c \cdot a_k = c \cdot \sum_{k=1}^{m} a_k$

Consider $\sum_{k=1}^{m} c \cdot a_k$

$$\sum_{k=1}^{m} c \cdot a_k = c \cdot a_1 + c \cdot a_2 \cdots + c \cdot a_m = c(a_1 + a_2 \cdots + a_m) = c \cdot \sum_{k=1}^{m} a_k$$

(iii) Suppose that a_k is a real valued function $(a_k \in \mathbb{R})$, let $l, m, k \in \mathbb{Z}^+, \ni: 1 \leq l < m < k$

WTS $\sum_{k=1}^{m} a_k = \sum_{k=1}^{l-1} a_k + \sum_{k=l}^{m} a_k$

Since $1 \leq l < m < k$,

$$\sum_{k=1}^{m} a_k = a_1 + a_2 + \cdots + a_{l-1} + a_l + a_{l+1} \cdots + a_m$$

$$\sum_{k=1}^{m} a_k = \sum_{k=1}^{l-1} a_k + \sum_{k=l}^{m} a_k$$

Example

1. evaluate $\sum_{i=0}^{203}(2i-1)$.

$$\begin{aligned}
\sum_{i=0}^{203}(2_{i-1}) &= \sum_{i=1}^{203}2i - \sum_{i=1}^{203}1 \text{ by } \sum \text{ - property (i)} \\
&= 2\cdot\sum_{i=1}^{203}i - \sum_{i=1}^{203}1 \text{ by } \sum \text{ - property (ii)} \\
&= \frac{2(203)(204)}{2} - 203 \text{ By definition of geometric series} \\
&= 41209
\end{aligned}$$

2. evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{5}{n^4}(k^3+1)$ using Sigma notation properties

Sol'n

$$\sum_{k=1}^{n}\frac{5}{n^4}(k^3+1) = \frac{5}{n^4}\cdot\sum_{k=1}^{n}(k^3+1) = \frac{5}{n^4}\cdot(\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}1)$$

$$\begin{aligned}
\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}1 &= \frac{n^2(n+1)^2}{4}+n \\
\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}1 &= \frac{n^2(n+1)^2+4n}{4} \\
\frac{5}{n^4}\cdot(\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}1) &= \frac{5}{n^4}\cdot(\frac{n^2(n+1)^2+4n}{4}) \\
\lim_{n\to\infty}\sum_{k=1}^{n}\frac{5}{n^4}(k^3+1) &= \lim_{n\to\infty}\frac{5}{n^4}\cdot(\frac{n^2(n+1)^2+4n}{4})
\end{aligned}$$

Skipping over some algebra...

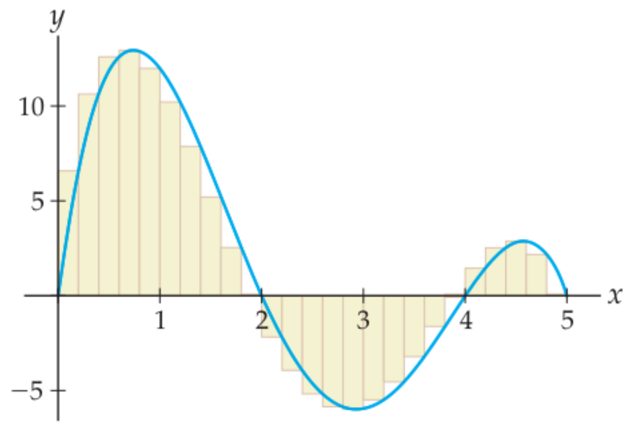
$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{5}{n^4}(k^3+1) = \frac{5}{4}$$

Summation Formulas

$$\begin{aligned}
\sum_{i=1}^{n}i &= \frac{n(n+1)}{2} \\
\sum_{i=1}^{n}i^2 &= \frac{n(n+1)(2n+1)}{6} \\
\sum_{i=1}^{n}i^3 &= \frac{n^2(n+1)^2}{4} \\
\sum_{i=1}^{n}r^i &= \frac{1-r^{n+1}}{1-4}, r\neq 1
\end{aligned}$$

2 Chapter 4.2; Riemann Sum

Definition.
Let $a, b \in \mathbb{R}, a < b$ Suppose $[a, b] \in Dom(f)$
A partition P is a collection of n + 1 points,
P = $\{x_0, x_1, x_2 \ldots x_n\}$,$\ni$: $x_0 = a < x_1 < x_2 \ldots x_n = b$



Example. Consider I = [0,1],
then P = $\{x_0 = 0, x_1 = 0.1, x_2 = 0.5, x_3 = 1\}$

Example. Consider [a,b], $a, b \in \mathbb{R}, a < b$,
then P = $\{x_0, x_1, x_2 \ldots x_n\}$, $\ni$: $x_i = a + i\triangle x, i \in [0, n]$
The sum of the areas of these partitions is called a **Riemann Sum**