

CSCC63 TUT 0002

Tutorial 12

- Assignment Help (Q7 and more)
- Exam Contents/Review

What I suspect will be on the exam

Almost Guaranteed:

1 Recognizable/Co-recognizable/Both/Neither question

(could also be one simple one, and one more difficult one that requires dovetailing)

2 NP-complete questions (verifier & reduction)

Generalized geography reduction, given the TQBF, write out the GG tree

Highly Likely:

PCP/CFG reduction

Oracle TM question (given an oracle for some language, write a TM that uses the oracle to decide another)

P-Space question (write an algorithm, justify why it runs in PSPACE)

Not as Likely:

Approximation question, show that something cannot be approximated with a constant ratio

Counting Problem ($\#P$ question)

Bonus Question Candidates:

Parsimonious Reduction, PCP/CFG reduction, $\#P$ question, approximation question

Tips for the exam:

Finish all the NP Verifier Questions first, write the polytime verifiers
Do the TQBF to Generalized geography question

After, do all of the questions you already know (if there isn't any that's okay)

Prioritize recognizability ones. $L = \{ \langle M \rangle \mid L(M) = \{ 1 \} \}$, $L = \{ \langle M \rangle \mid L(M) \subseteq \{ 1 \} \}$,

Approximation Question,

Given a language, Show that it cannot be approximated better than a ratio of $1/4$

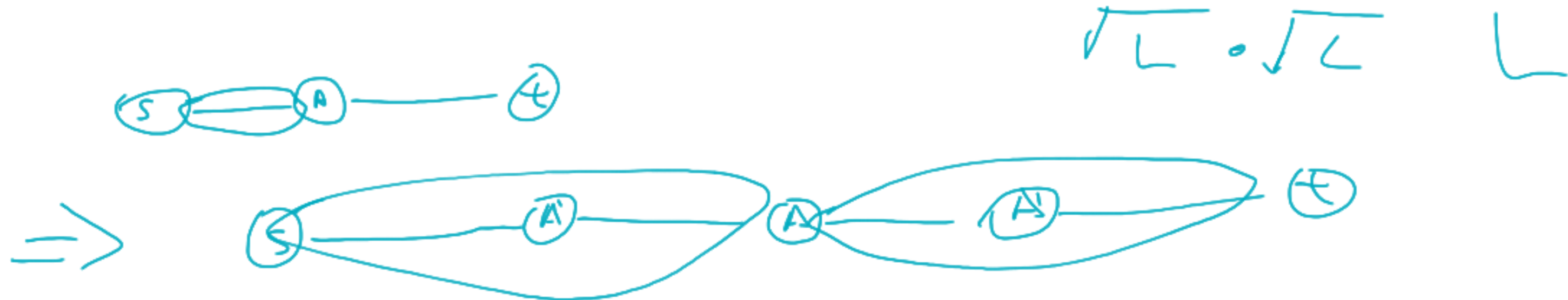
1. Assume that such an approximation exists, that is to say a function can approximate this language with a result that is better than $1/4$ th.
 2. Write a function using this approximation function and approximate the problem to a ratio better than $1/2$.
 3. Your function should look like this
- P on input $\langle x \rangle$
- y = square x in some way
 - run the approximation on y, receive a better approximation than $1/2$

LONGEST

Input A direct graph G , nodes s, t

Output the length of the longest path from s to t in G

Assume that a function named `approx` can compute this length up to a ratio of $1/4$,



When we run `approx` on G' , any path of length l in G' can be mapped to a path of length \sqrt{l} in G

P on input $\langle G, s, t \rangle$

$G' \leftarrow \text{Modify}(G)$

get the results of `approx(G')`

suppose that G has a longest path length of l , then G' has longest path l^2

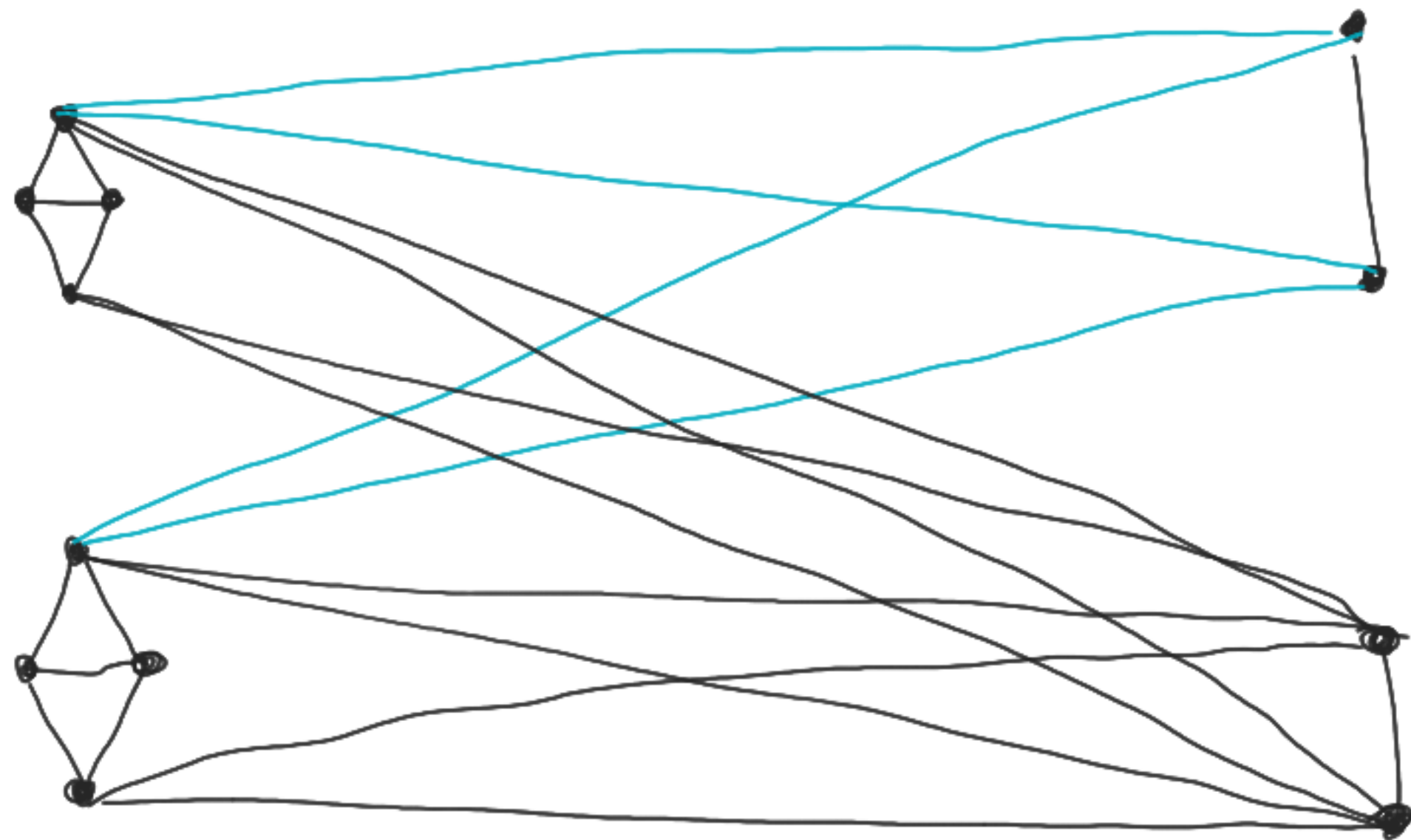
the result of `approx(G')` gives us a path length of $(l^2/4)$

this result maps to a path length of $(\sqrt{l^2/4}) = l/2$

(x or y)

x

y



C1

C1 extra

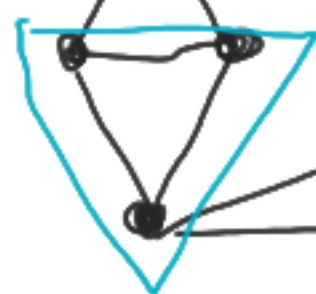
(x or y)

$$x = T, \quad y = T$$

x



y

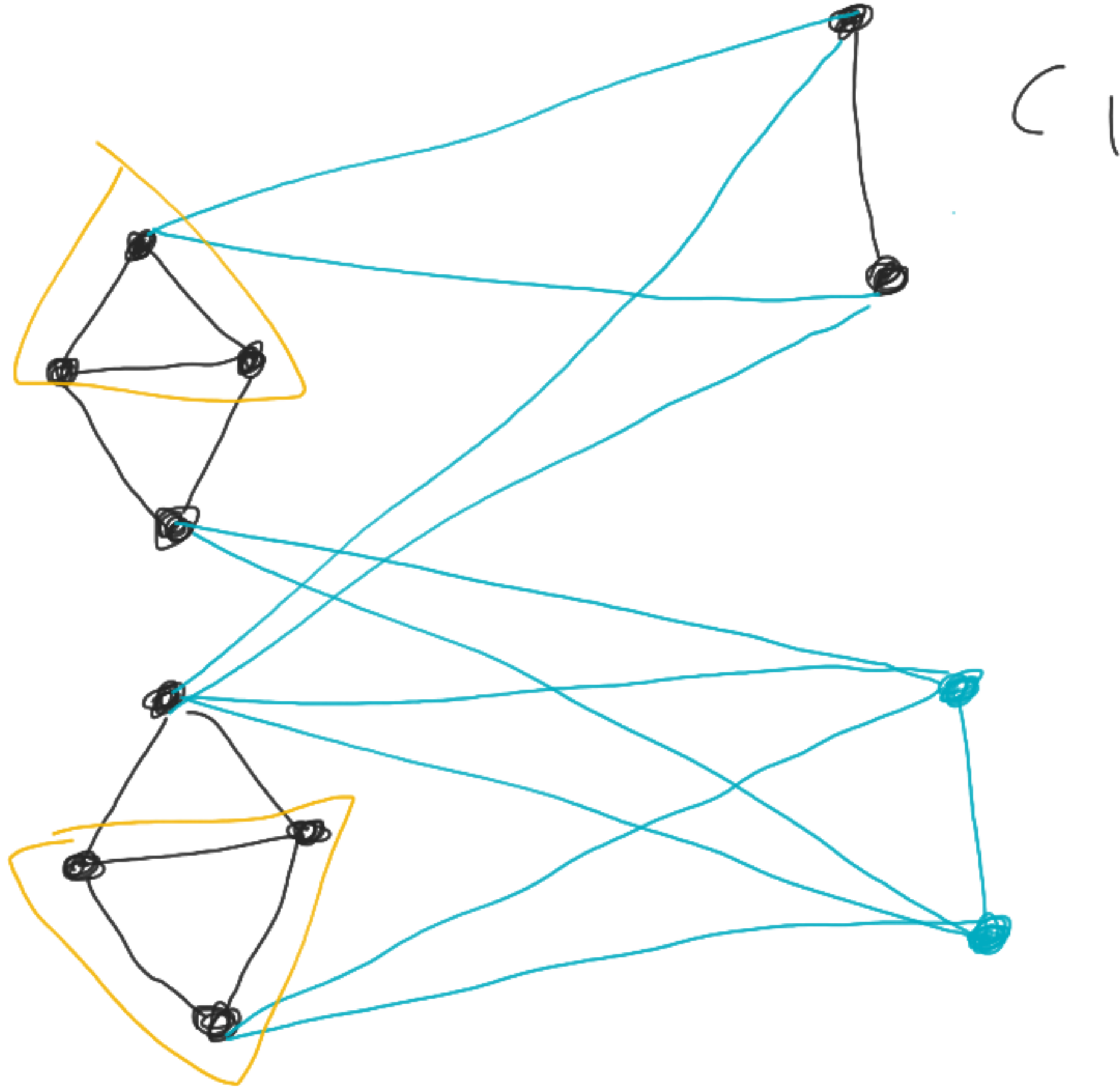


C_1

c_1 extra

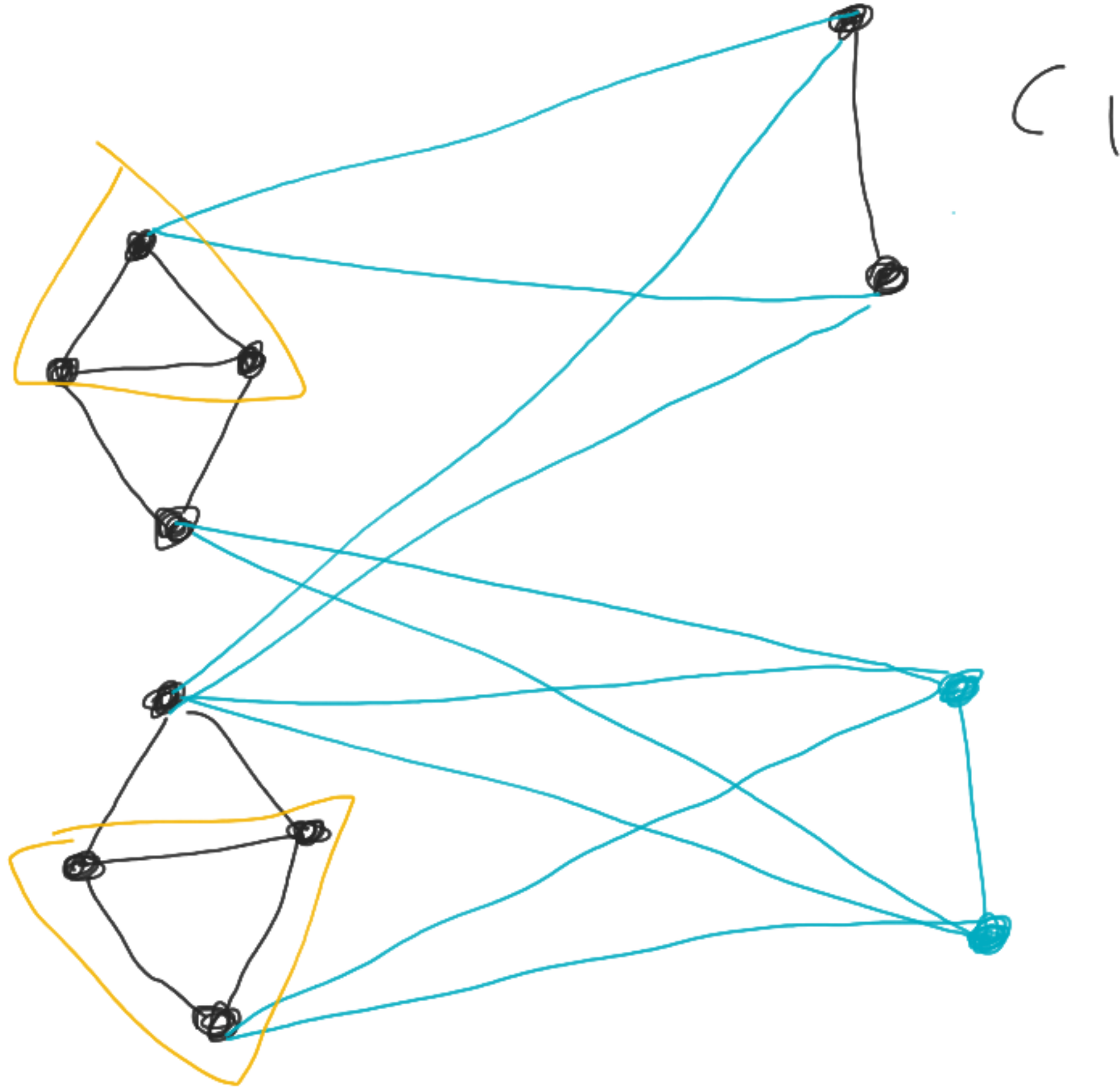
(x or y)

$$x = \bar{F} \quad , \quad y = T$$



(x or y)

$$x = \bar{F} \quad , \quad y = T$$



6.
Given oracle for HALF-CLIQUE or IS, and G
Return the clique/independent set that proves G in HALF-Clique or IS

You can simply return the nodes that belong to this clique/IS

P on input $\langle G \rangle$
if ORACLE(G) returns false, return null
let G' be G with modifications
use the ORACLE to determine whether the graph contains a HALF-CLIQUE or IS.

for each node in G remove and see if Oracle still passes

Clique Solver given Oracle $\langle G = (V, E) \rangle$
for node n in G :
if Oracle($V - \{n\}$) let $V = V - \{n\}$
return V

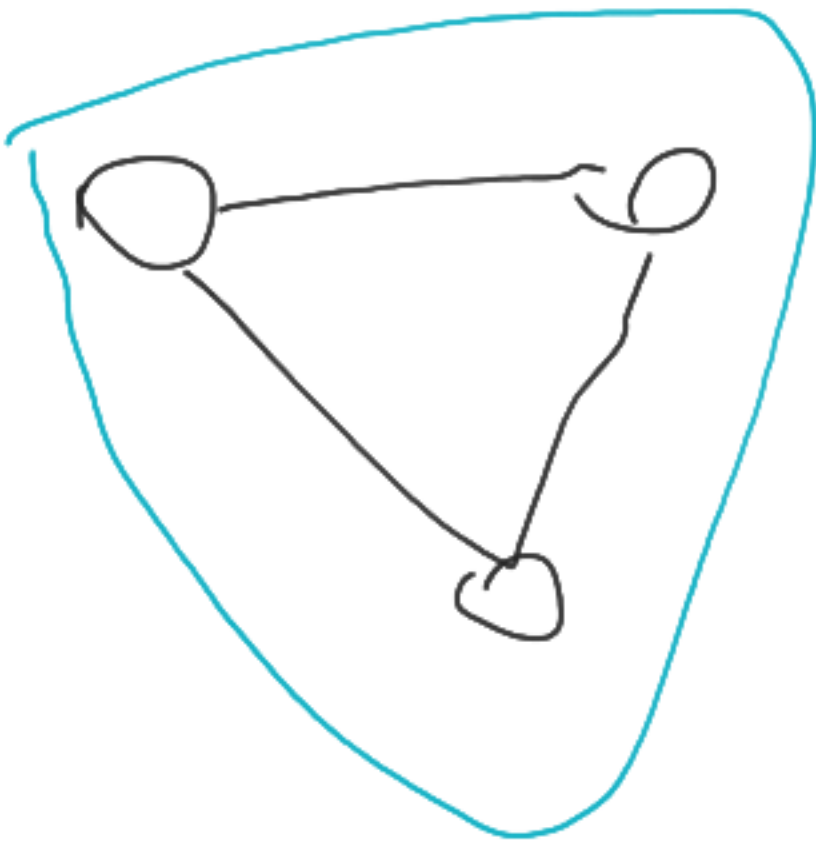
3b

Show many Many-Clique is co-NP-HARD (show Co-Many-Clique is NP Hard)

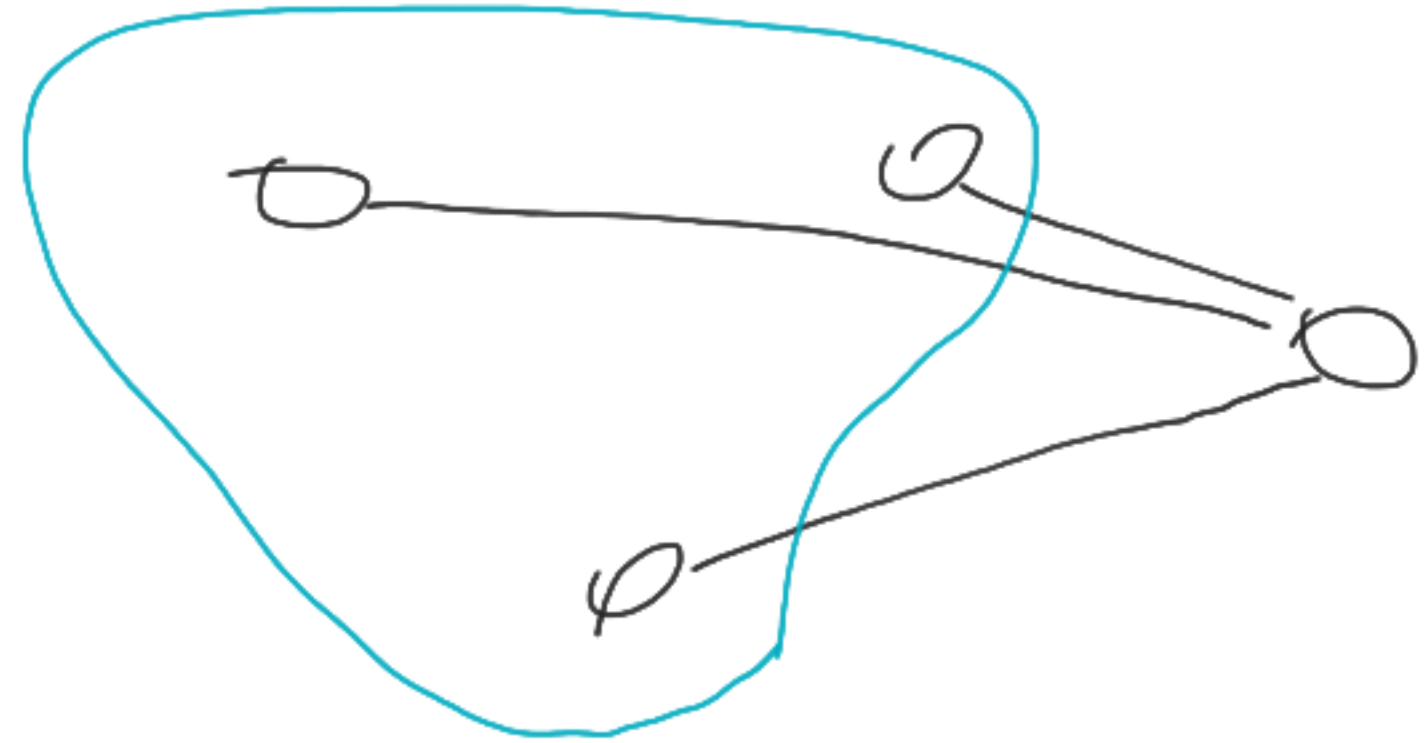
Clique = NP hard, Co-Many-Clique. Given an instance of Clique, output a graph G such that exists size r subgraph of G does not contain a k -clique

Co-Many-Clique $\{ \langle G, r, k \rangle \mid \text{Exists size } r \text{ subgraph of } G \text{ does not contain a } k\text{-clique} \}$

Given $\langle G, k \rangle$, if the input contains a k -clique, we want our output graph to have a size r subgraph that does not contain a k -clique



\cup



if such a k clique exists in the original graph, then a size k subset, exists in the complement where no nodes are connected, if no nodes are connected, then there is no 2-clique

Q5

NAE-3SAT

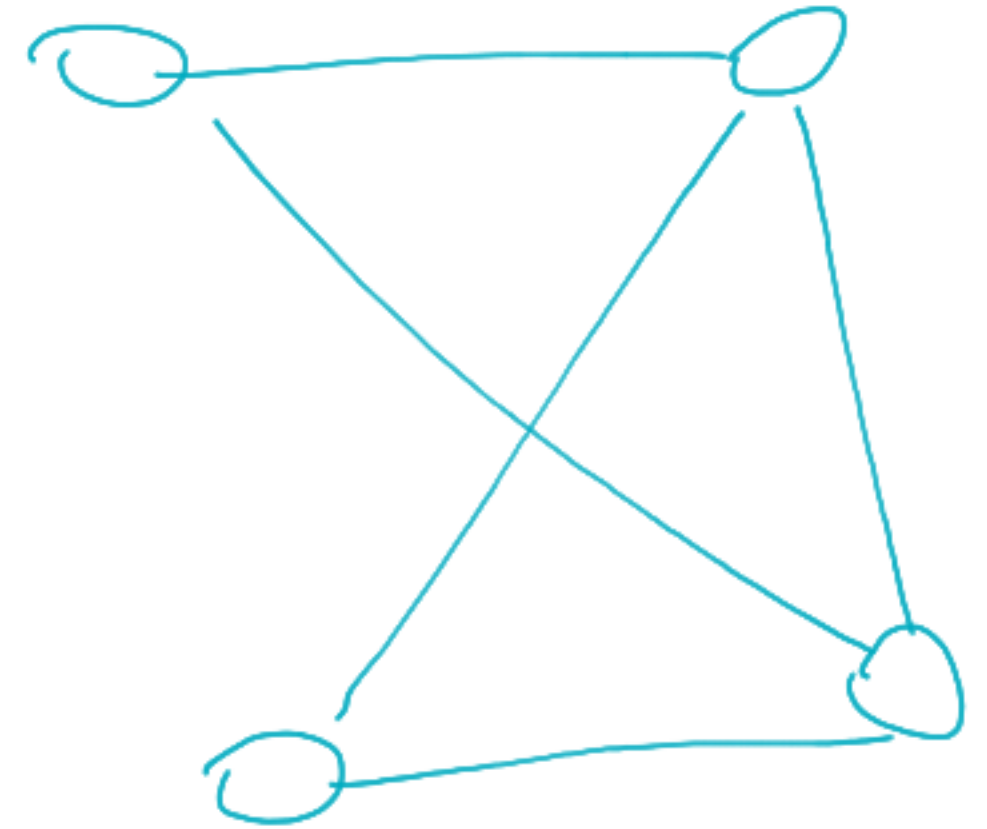
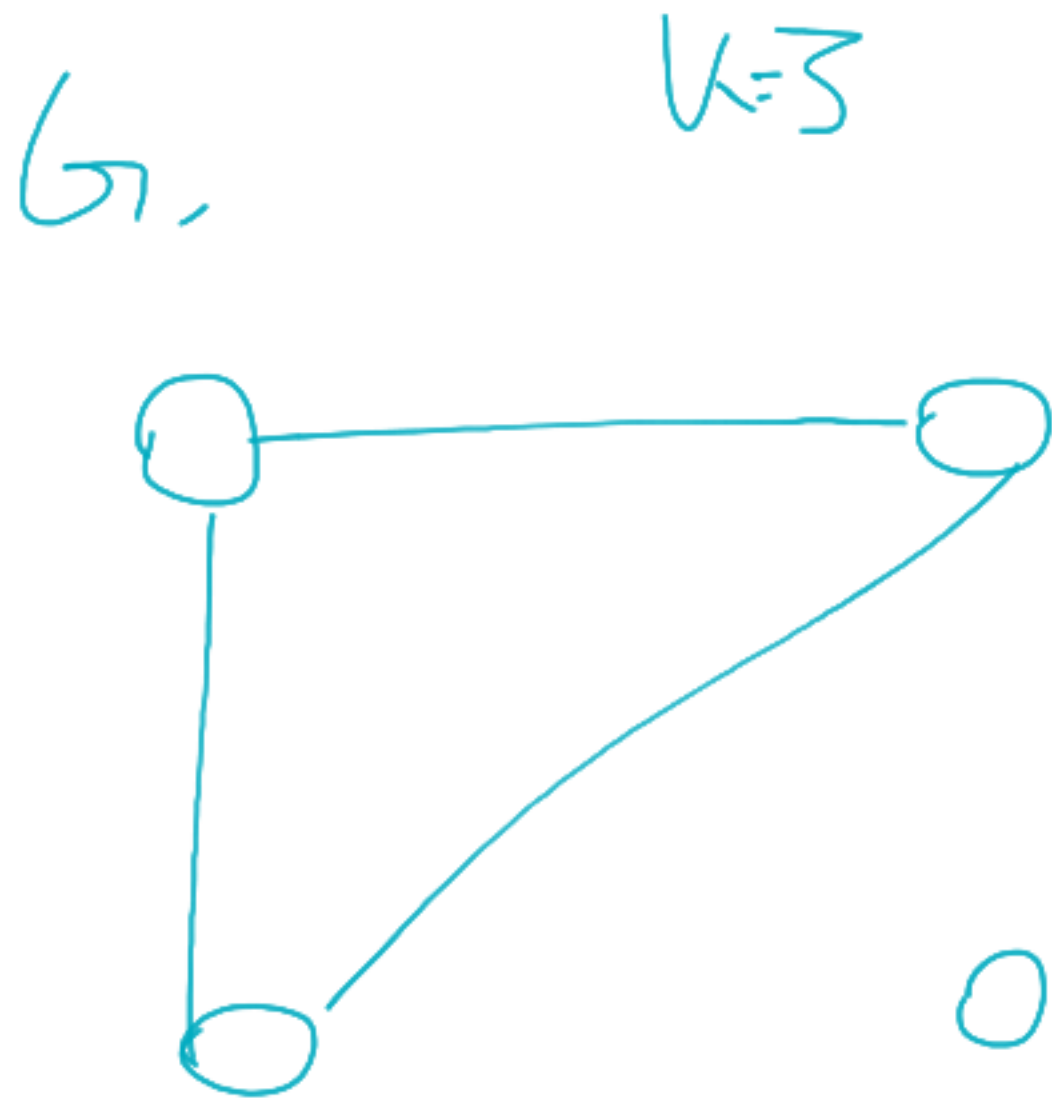
P on input $\langle \phi \rangle$:

 for each variable x

 run oracle on ϕ and $(x \text{ or } x \text{ or } x_{\text{aux}})$ and (x_{aux}, \dots)

 if oracle returns true, variable needs to be true

 else variable needs to be false



If an input graph lacks a k clique, then there is no K sized subgraph in the complement such that no edges are in between the nodes.