Pigeonhole Principle

If there are x pigeons, and n holes, since all of the pigeons needs to go into holes.
Thus there will be y holes containing k pigeons each.

Example:
Given an array of n + 1 elements: A[0...n]. Each of these elements is a natural number between [1, n] (1, 2, 3, ... n).
WTP atleast two elements in A are equal (one pair of duplicates).

Phrased to use Pigeonhole Principle:
n holes, n + 1 pigeons ... 2 pigeons in the same hole
by the PHP, two elements in A are equal.

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1. Suppose that we have 20 different classes and we need to get three people from one of these classes to help with a learning survey. If we go to any particular class to advertise for volunteers we know from past experience that the chances of getting 3 volunteers is low. Instead we advertise on all classes piazza discussion boards. How many responses do we need to
guarantee that we get 3 volunteers from one class?

WTS: number of pigeons (responses)

If there are (2x20) + 1 = 41 pigeons, and 20 holes, since all of the pigeons needs to go into holes. Thus there will be atleast 1 hole containing 3 pigeons.

Ans: 41

2. Suppose there are 20 people at a party and there are 48 pairs of people who know each other. Prove that there must be some person who knows at most 4 people.

Equivalent to: Not everybody can know 5+

Assume everybody knows 5, 20 x 5 / 2 = 50 pairs
Since 50 pairs > 48 pairs. This contradicts the assumption, which is everybody knows 5+.
If not everybody knows 5+, somebody know 4.

3. Over a 44 day period, Gary will train for triathlons at least once per day, and a total of 70 times in all. Show that there is a period of consecutive days during which he trains exactly 17 times.

A [1...44] be an array of natural, increasing, numbers each number A[i] in [1, 70]
A[44] = 70, A[1] >= 1
WTS there exists indices i < j, such that A[i] + 17 = A[j]

1 <= x1 < x2 < x3 ... < x43 < x44 = 70
xi + 17 = xj, i < j

x1, x2, x3, ... , x43, x44 no two numbers are equal in this series, xi in [1, 70]
x1+17, x2+17, x3+17 ..., x44 + 17 no two numbers are equal in this series, xi + 17 in [17, 87]

x1, x2, x3, ... , x43, x44, x1+17, x2+17, x3+17 ..., x44 + 17 each entry in [1, 87]

by php, atleast 2 of the elements above msut be equal