**Lecture Notes**
Winter 2019

# MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

**LEC03**, Feb 8th, 2:00pm - 3:00pm

UNIVERSITY OF TORONTO SCARBOROUGH


**Instructor:**

**Dr. Kathleen Smith**
**Email:** smithk@utsc.utoronto.ca
**Office:** IC458
**Office Hours:** TBA

# 1 Partial Fraction Decomp

**Exercises**

1. Write in PFD form for:

   To tell how many partial fractions there are, simply look at the total number of ireducible factors.

   (a) $f(x) = \dfrac{4x^2+\sqrt{2}}{(x-1)(x-2)^2(x^4+x+7)} = \dfrac{A}{x-1} + \dfrac{F}{(x-2)} + \dfrac{G}{(x-1)^2} + \dfrac{Bx^3+Cx^2+Dx+E}{(x^4+x+7)}$

   Note that $\dfrac{1}{(x-2)^2} = \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2}$

   (b) $f(x) = \dfrac{25}{(x^2+1)^2x^3} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2} + \dfrac{E}{x} + \dfrac{F}{x^2} + \dfrac{G}{x^3}$

   (c) Find $\displaystyle\int \frac{5x+11}{x^2+5x+6}dx = \int \frac{5x+11}{(x+2)(x+3)} = \int \frac{A}{x+2} + \frac{B}{x+3}dx = \int \frac{A(x+3)+B(x+2)}{(x+2)(x+3)}dx$

   $A(x+3)+B(x+2) = 5x+11 = Ax+3A+Bx+2B = x(A+B)+3A+2B$

   Coeff of highest power of x: $5 = (A+B)$

   Coeff of second highest, $x^0$: $11 = 3A+2B$

   We have $A = 1, B = 4$

   Thus, $\displaystyle\int \frac{5x+11}{x^2+5x+6}dx = \int \frac{1}{x+2} + \frac{4}{x+3} = \ln|x+2| + 4\ln|x+3| + C$

   (d) Find $\displaystyle\int \frac{5x^3-3x^2+2x-1}{x^4+x^2}dx = \int \frac{5x^3-3x^2+2x-1}{x^2(x^2+1)}dx$

   Pfd:$\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1} = \dfrac{Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2}{x^4+x^2}$

   $5x^3+3x^2-2x-1 = Ax^3+Ax+Bx^2+B+Cx^3+Dx^2 = (A+C)x^3+(B+D)x^2+Ax+B$

   Coeff of $x^3$: $5 = A+C$

   Coeff of $x^2$: $-3 = B+D$

   Coeff of $x$: $2 = A$

   Coeff of $x^0$: $-1 = B$

   We have $A = 2, B = -1, C = 3, D = -2$

   Thus, $\displaystyle\int \frac{5x+11}{x^2+5x+6}dx = \int \frac{2}{x} - \frac{1}{x^2} + \frac{3x-2}{x^2+1}dx = 2\ln x + \frac{1}{x} + \int \frac{3x-2}{x^2+1}dx = 2\ln x + \frac{1}{x} + \int \frac{3x}{x^2+1} - \frac{2}{x^2+1}dx$