Discrete Mathematics, Fall 2015

CSCA67 - FINAL EXAM & ANSWERS

UNIVERSITY OF TORONTO SCARBOROUGH

GOOD LUCK EVERYONE!
-Richard

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1. Your local grocery store just received a large shipment of apples, oranges, pears, and bananasmore than 100 pieces each. You are shopping at the store and will purchase your fruit for the week.

   (a) How many ways can you select 10 pieces of fruit from your stores supply of apples, oranges, pears, and bananas?

   let f_i denote a kind of fruit

   $f_1 + f_2 + f_3 + f_4 = 10, f_i \geq 0, i \in [1, 4]$

   Total = $\binom{10 + (4 - 1)}{4 - 1}$

   (b) How many ways can you select 10 pieces of fruit from your stores supply of apples, oranges, pears, and bananas if you need at least one piece of each kind of fruit?

   let f_i denote a kind of fruit

   $f_1 + f_2 + f_3 + f_4 = 10, f_i \geq 1, i \in [1, 4] \equiv f_1 + f_2 + f_3 + f_4 = 6, f_i \geq 0, i \in [1, 4]$

   Therefore, Total = $\binom{6 + (4 - 1)}{4 - 1}$

2. In how many ways can six 3s and four 2s be arranged in a row so that the 2s are always apart?

   Let's write this as □3□3□3□3□3□3□

   We see that we have 7 □s to fill our four 2s in, so that the 2s are always apart.

   ∴ ways to arrange six 3s and four 2s be arranged in a row so that the 2s are always apart is $\binom{7}{4}$

3. Your Toronto Maple Leafs won 30 of 82 games last season (i.e., the 2014-2015 season), giving them a winning percentage of 37%. If we assume this means the probability of the Leafs winning any given game is 0.37, then we can predict how they would have done in a playoff series.

   Answer the following questions to determine the probability that the Leafs would have won a best of 7 playoff series (i.e., won 4 games) had they made the playoffs last season.

   (a) Rephrase this question in terms of sequences of 0s and 1s.

   What is the shortest length of a sequence? 4. (4 wins)

   What is the longest length of a sequence? 7. (7 combination of wins and loses)

   (b) Calculate the number of sequences which correspond to the Leafs winning the series. (Note that the answer is not C(7, 4).)

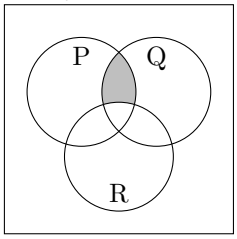
   (c) Calculate the number of sequences as they relate to this problem. (Note that the answer is not 2 7 as not all series would last 7 games.)

   (d) Calculate the probability that the Leafs would win the series.

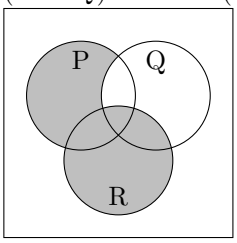
   (e) What is your best guess for the probability that the Leafs will ever win the Stanley Cup again (the ultimate prize in the NHL)?

4. For each of the following statements, list the numbered region(s) of the venn diagram that represent(s) when the statement is true.

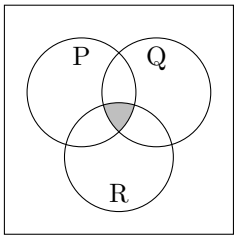
(a) $P \wedge Q \wedge \neg R$



(b) $P \to Q \to R$
Assuming this is $(P \to Q) \to R$.
$(P \to Q) \to R \equiv \neg(\neg P \vee Q) \vee R \equiv (P \wedge \neg Q) \vee R$



(c) $\neg((P \wedge Q) \to \neg R)$
$\neg((P \wedge Q) \to \neg R) \equiv \neg(\neg(P \wedge Q) \vee \neg R) \equiv \neg(\neg P \vee \neg Q \vee \neg R) \equiv P \wedge Q \wedge R$



5. Logical equivalence and contrapositive.

(a) Circle the statements below that are equivalent to $(a \wedge b) \to c$.

1. $(\neg a \vee \neg b) \to \neg c$
2. $\neg c \to (\neg a \vee \neg b)$ ✓

Justification: $(a \wedge b) \to c \equiv \neg(\neg a \vee \neg b) \to c \equiv \neg c \to (\neg a \vee \neg b)$

3. c is neccesary for a and b ✓

Justification: $a \to b \equiv$ b is neccesary for a

4. c is sufficient for a and b
5. a and b are sufficient for c ✓

Justification $a \to b \equiv$ a is sufficient for b

6. a and be are neccesary for c
7. all of the above
8. none of the above

(b) If the implication $a \to b$ is difficult to prove directly, we can attempt a proof by contrapositive. Use a truth table to show that the contrapositive of $a \to b$ is equivalent to $a \to b$.
Show $a \to b \equiv \neg b \to \neg a$

$a \to b$	Given
$\neg a \vee b$	Conditional or ($\to$) Law
$b \vee \neg a$	Commutative Property
$\neg b \to \neg a$	Conditional or ($\to$) Law

a	b	$a \to b$	$\neg b \to \neg a$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$\therefore a \to b \equiv \neg b \to \neg a$