STAB52

Summary Of

Probability and Statistics

The Science of Uncertainty

Second Edition

By: **Michael J. Evans and Jerey S. Rosenthal**

University of Toronto

Instructor:	**Dr. Daniel Roy**
Email:	droy@utsc.utoronto.ca
Office:	IC462
Office Hours:	TU 11:00 - 12:00

1 Probability Basics

1.1 Probability Models

Sample space, often written S. This is any set that lists all possible outcomes of some unknown experiment. Collections of events are subsets of S, to which probabilities can be assigned.
Finally, a probability model requires a probability measure, usually written P. This must assign to each event A, a probability $P(A)$ with the following properties:

1. $P(A)$ is always a non-negative real number, between 0 and 1 inclusive.
2. $P(\varnothing) = 0$
3. $P(S) = 1$
4. P is countably additive, where for disjoint events $A_1, A_2, A_3, \dots$
   we have $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$

1.2 Venn Diagrams and Subsets

The complement of a set A, denoted set $A^c = \{s | s \notin A\}$
The intersection of two sets A, B, denoted $A \cap B = \{s | s \in A \wedge s \in B\}$
The union of two sets A, B, denoted $A \cup B = \{s | s \in A \wedge s \in B\}$

We also have properties $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$

1.3 Properties of Probability Models

For any event A, A and A^c are always disjoint.
Furthermore, their union is always the entire sample space: $A \cup A^c = S$
And since we have $P(S) = 1$. $P(A^c) = 1 - P(A)$

Suppose that $A_1, A_2, \dots$ are disjoint events that form a partition of the sample space i.e., $A_1 \cup A_2 \cup + \dots = S$.
For any event B, $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \dots$

Principle of inclusion-exclusion, Let A, B be two events. Then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

1.4 Uniform Probability on Finite Spaces

If a sample space S is finite, then one possible probability measure on S is the uniform probability measure, which assigns probability $\dfrac{1}{|S|}$ to each outcome. By additivity, we see that for any event $A, P(A) = \dfrac{|A|}{|S|}$

1. Multiplication Principle

   With m in A and n elements in B, there are $m \times n$ total possible ordered pairs of elements from both sets, $C = \{(a_i, b_j) | a_i \in A, b_j \in B\}, |C| = m \times n$

2. Permutation Principle

   Ordered arrangement of k objects, chosen without replacement from n possible objects.

   The number of these ordered arrangements is $P_k^n = \dfrac{n!}{(n-k)!}$

3. Combination Principle

   Unordered arrangement of k objects, chosen without replacement from n possible object.

   The number of these unordered arrangement is $C_k^n = \binom{n}{k} = \dfrac{P_k^n}{k!} = \dfrac{n!}{k!(n-k)!}$

1.5 Conditional Probability and Independence

Given two events A, B with $P(B) > 0$, the conditional probability of A given B written $P(A|B)$ denotes the fraction of time that A occurs once we know that B has occured.

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Then the law of total probability can be rewritten: Let $A_1, A_2, \ldots$ be events that form a partition of the sample space S, each of positive probability.
Then for any event B, $P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \ldots$