MAT A29 TUT0018, Tutorial 1 (Week 2)
Tuesdays 7 - 9pm (We will start at 7:10pm)

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Domain and Range

Question

Find the domain and range of $f(x) = \dfrac{1}{2+x^2}$.

$D = \{x \mid x \in \mathbb{R}\}$

For no values of x is f(x) undefined
therefore D = R

$$x = \frac{1}{2+f(x)^2} \Rightarrow \frac{1}{x} = 2 + f(x)^2$$

$$\Rightarrow \frac{1}{x} - 2 = f(x)^2 \Rightarrow f(x) = \sqrt{\frac{1}{x} - 2}$$

$$f(x)=\sqrt{\frac{1}{x}-2}$$

$\sqrt{x}$ dep $x \geq 0$

$$\frac{1}{x}-2 \geq 0$$

Range $(0, \frac{1}{2}]$

$$\Rightarrow \frac{1}{x} \geq 2 \Rightarrow 0 \leq x \leq \frac{1}{2}$$

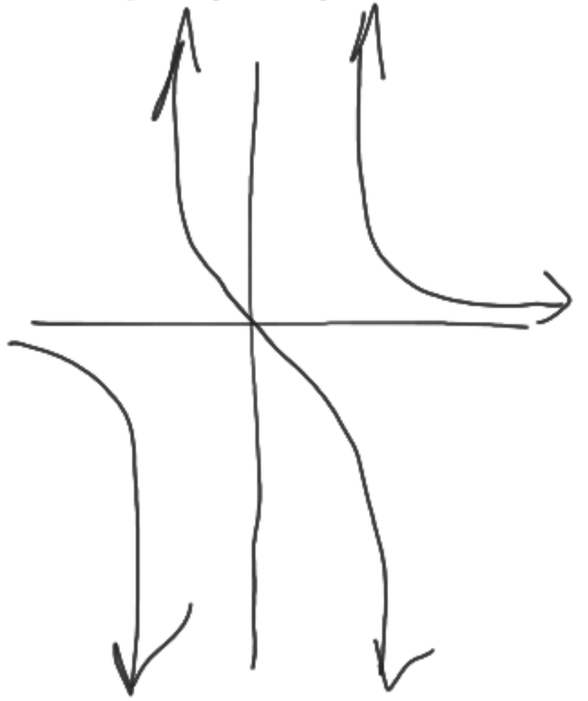
OpenStax §1.1 Q 14

Question

Find the domain and range of $f(x) = \dfrac{x}{x^2 - 16}$.

f(x) not defined when x^2 - 16 = 0
=> x^2 = 16 => x = sqrt(16) => ~~x~~ ±4

f(x) looks
something like



Range = ℝ

OpenStax §1.1 Q 17

Question

Find the zeroes of $y = -1 + \sqrt{x+2}$.

$$0 = -1 + \sqrt{x+2}$$

$$\Rightarrow 1 = \sqrt{x+2}$$

$$\Rightarrow 1 = x+2$$

$$\Rightarrow x = -1$$

OS §1.1 Q55

Question

A certain bacterium grows in culture in a circular region. The radius of the circle, measured in centimeters, is given by $r(t) = 6 - \frac{5}{t^2+1}$ *where* t *is time measured in hours since a circle of a 1-cm radius of the bacterium was put into the culture.*

- *Express the area of the bacteria as a function of time.*
- *Find the exact and approximate area of the bacterial culture in 3 hours.*

$A = \pi r^2$

$A(t) = \pi\, r(t)^2$

$= \pi \left(6 - \frac{5}{t^2+1}\right)^2$

$A(3) = \pi \left(6 - \frac{5}{3^2+1}\right)^2$

$= \pi \left(6 - \frac{5}{10}\right)^2$

$= \pi \cdot 5.5^2 \approx 31.30$

93

OpenStax §1.2 Q 59

Question

Find the slope of a line through $(-2, 4)$ *and* $(1, 1)$.

(x_1, x_2) (y_1, y_2)

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

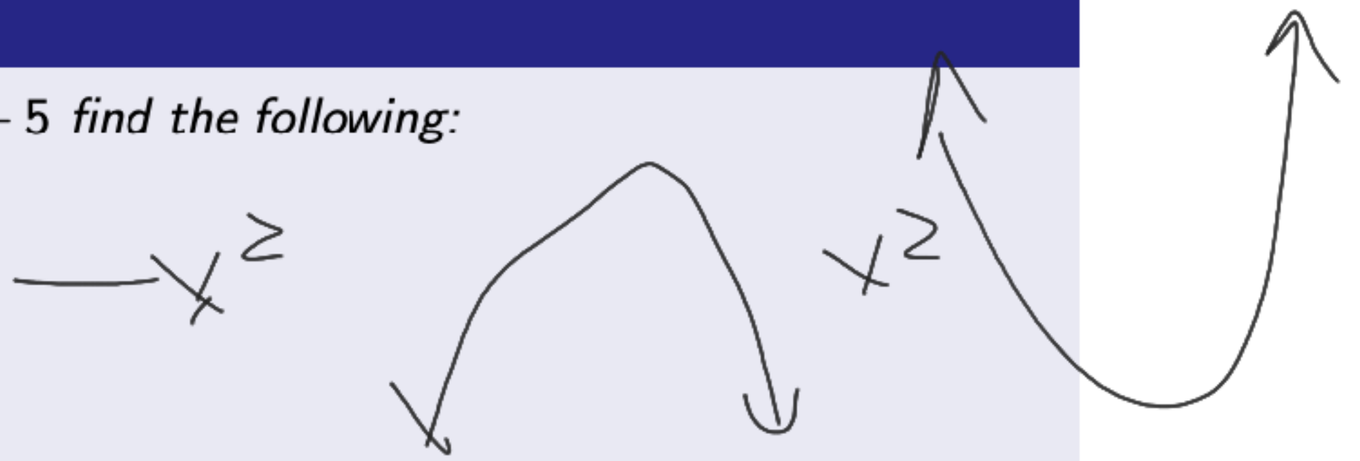
$$\frac{4-1}{-2-1} = \frac{3}{-3} = -1$$

OpenStax §1.2 Q 83

Question

For the polynomial $f(x) = 2x^2 - 3x - 5$ *find the following:*

- *Degree d*
- *Any zeroes*
- *y-intercepts*
- *End Behaviour*
- *Symmetry: even / odd / neither*



- Degree: 2 (2x^2 has the highest exponent)
- Zeroes: f(x) = (x + 1)(2x - 5) = 0 => at x = -1, 5/22
- Y intercepts: f(0) = 1 * (-5) = -5
- End Behaviour: as x -> -infty, y -> infty and as x -> infty, y -> infty
- Symmetry: None (parabola is shifted)

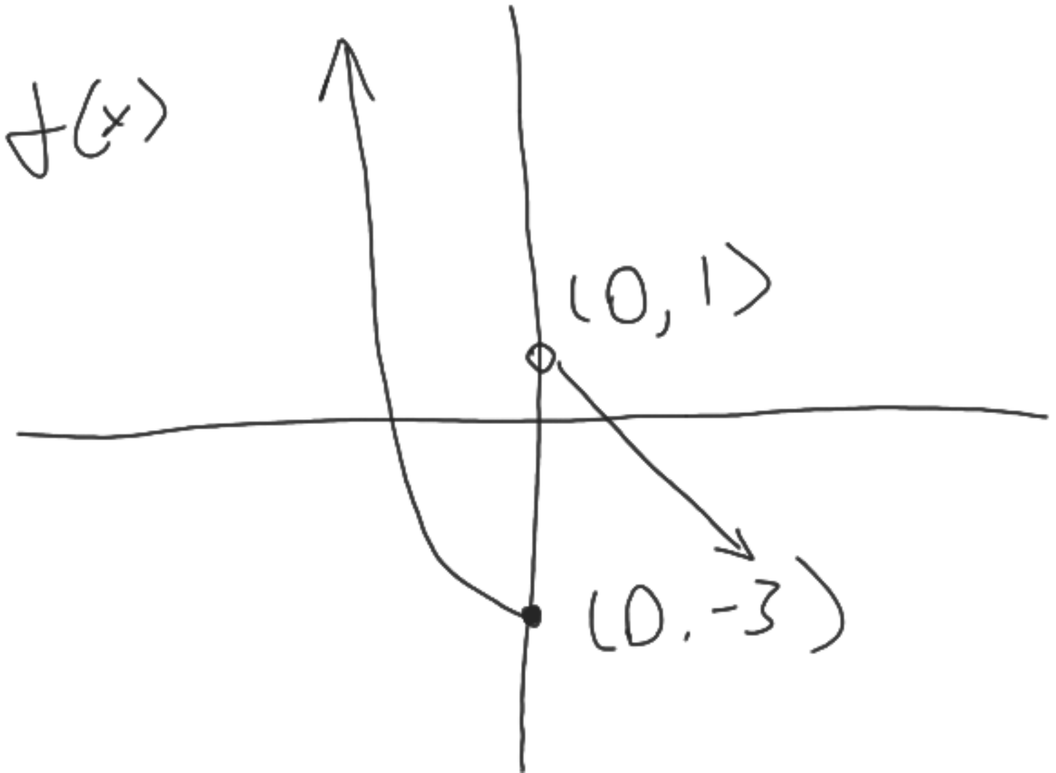
OpenStax §1.2 Q 95

Question

Sketch the function:

$$f(x) = \begin{cases} x^2 - 3 & x \leq 0 \\ -x + 1 & x > 0 \end{cases}$$

Calculate the values $f(-3)$, $f(0)$, and $f(2)$.



$f(-3) = (-3)^2 - 3 = 9 - 3$

$= 6$

$f(0) = (0^2) - 3 = -3$

$f(2) = -2 + 1 = -1$