

Week 7, LEC02 - November 1st
Discrete Mathematics, Fall 2018

CSCA67 - Lecture Notes

Current Instructor: Dr. Richard Pancer



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1 Introduction to Counting

Counting Pizza Toppings*

The commercial's deal was:

- 2 pizzas
- up to 5 toppings on each
- 11 toppings to choose from
- all for \$7.98 (back in 1997).

Important: duplicate toppings not allowed, order of toppings does not matter, and CHEESE IS NOT A TOPPING.

The commercial's math kid claimed there are 1,048,576 possibilities.

Let's do the calculation ourselves.

Q. How many ways can we order a pizza with 0 toppings?

A. 1

Q. How many ways can we order a pizza with 1 topping?

A. 11

Q. How many ways can we order a pizza with 2 toppings?

A. $\frac{11 \cdot 10}{2} = 55$

Q. How many ways can we order a pizza with 3 toppings?

A. $\frac{11 \cdot 10 \cdot 9}{3!}$

Combinations of $x, y, z = 6 = 3! = 3 \cdot 2 \cdot 1$

Q. How many ways can we order a pizza with 4 toppings?

A. $\frac{11 \cdot 10 \cdot 9 \cdot 8}{4!}$

5 toppings: $\frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5!}$, 6 toppings: $\frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{6!}$

Therefore, the total number of ways to order a single pizza with up to 5 toppings when choosing from 11 toppings is:

$$1 + 11 + \frac{11 \cdot 10}{2} + \frac{11 \cdot 10 \cdot 9}{3!} + \frac{11 \cdot 10 \cdot 9 \cdot 8}{4!} + \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5!} = 1024$$

Note: On the final, you will not be asked to resolve this calculation.

Q. How did they get 1,048,576 in the commercial?

A. $1024 \cdot 1024 = 1,048,576$, This is not the correct answer!

Q. What was their mistake? and how do we correct for it?

A. The order of the pizza toppings does not matter, (pizza A, pizza B) is the same thing as (pizza B, pizza A) so, $\frac{1024 \cdot 1024}{2!} = 524,288$

Q. This is still not quite correct. Why?

A. We don't have 2 orders of (pizza A, pizza A), but we still have to divide by 2.

How many ways can we order two identical pizzas? 1024 So, $\frac{1024 \cdot 1024}{2!} + \frac{1024}{2!} = 524,800$