Lecture Notes
Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, Mar 22, 2:00pm - 3:00pm

Computer & Mathematical Sciences
UNIVERSITY OF TORONTO
SCARBOROUGH

Instructor:	**Dr. Kathleen Smith**
Email:	smithk@utsc.utoronto.ca
Office:	IC458
Office Hours:	TBA

1 Geometric series

Recall, a series $\sum_{i=1}^{n} a_n$ converges to sume $s \in \mathbb{R}$ implies $\lim_{n\to\infty} a_n = \infty$

A series of the form $a + ar + ar^2 + ar^3 \cdots = \sum_{n=0}^{\infty} ar^n$ where $a \in \mathbb{R} - \{\varnothing\}, r \in \mathbb{R}$ is a **Geometric Series**

Example

What $r \in \mathbb{R}$ values does $\sum_{n=0}^{\infty} ar^n$ converge/diverge?

SOL'N Proceed by def, computing $\lim_{n\to\infty} S_n$, recall S_n is the n-th partial sum. $\left(S_n = a + ar + ar^2 \cdots + ar^n\right)$
If we multiply both sides by r,
$r \cdot S_n = ar + ar^2 + ar^3 \cdots + ar^{n+1}$, $S_n - rS_n = a - ar^{n+1}$, $S_n(1-r) = a(1-r^{n+1})$

1. If $r \neq 1 \Rightarrow S_n = \frac{a}{1-r}(1 - r^{n+1})$

   $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a}{1-r}(1 - r^{n+1}) = \frac{a}{1-r} \cdot 1 \cdot \lim_{n\to\infty} r^{n+1}$

   Notice $\lim_{n\to\infty} r^{n+1} = \begin{cases} 0 & \text{If } r \in (-1,1) \\ \infty & r > 1 \\ DNE & r \leq -1 \end{cases} = \begin{cases} \frac{a}{1-r} & \text{If } |r| < 1 \\ DNE & |r| > 1 \vee r = 1 \end{cases}$

2. If $r = 1 \Rightarrow S_n = (n+1) \cdot a$

   $\lim_{n\to\infty} S_n = \lim_{n\to\infty} (n+1)a = \pm\infty = DNE$

Thus $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ when $|r| < 1$, and diverges when $|r| \geq 1$
This is called the GS test.

Exercise

$\sum_{n=2}^{\infty} \frac{(-1)^n}{\pi^n} e = e \cdot \sum_{n=2}^{\infty} \frac{(-1)^n}{\pi^n}$