1. For the following questions, suppose you have a trick die (still 6 sided) where the even numbers have probability p of being rolled and the odd numbers have probability $3p$ of being rolled.

   (a) Which property of probability will help you determine the value of p?

   Total Probability

   (b) Determine p.

   If you were unable to solve this ask your TA for help.

   (c) If one die is rolled, what is the probability of *not* getting a 5?

   (d) If two dice are rolled what is the probability of getting doubles?

   (e) Two dice are rolled, what is the probability of getting a sum of 7?

b) P(i) i in [1, 6]. P(1) + P(2) + ... + P(6) = 1

3p + p + 3p + p + 3p + p = 12p = 1 => p = 1/12

c) 1 - P(5) = 1 - 3p = 1 - 3/12 = 9/12 = 3/4

d) (1/4) * (1/4) * 3 + (1/12) * (1/12) * 3 = 5/24

e) Ways to sum to 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

6 * (1/4) * (1/12) = 1/8

2. Suppose E is the event that a randomly generated bit string of length four begins with a 1 and F is the event that this bit string contains an even number of 1s. Assume all four bit strings are equally likely.

   (a) Solve this using counting principles (not by writing out the sample space and finding the number of occurrences of E and the number of occurrences of F).

   (b) What is the probability of E or F occurring? Again use counting principles.

$$\underline{1}\ \ \underline{\quad}\ \ \underline{\quad}\ \ \underline{\quad}$$

$$P(E) = \frac{2^{\wedge}3}{2^{\wedge}4} = \frac{1}{2}$$

$$P(F) = \frac{1+1+6 \leftarrow \binom{4}{2}}{2^{\wedge}4} = \frac{1}{2}$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$P(E \cap F) = \frac{1 + \binom{3}{1}}{2^4} = \frac{1}{4}$$

$$\frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$$

3. Consider the experiment of throwing 3 standard die, one after the other. The outcome of this experiment is the sum of the die.

(a) Write out or otherwise specify the elemental sample space for this experiment.

(b) Write out or otherwise specify an appropriate compound sample space for this experiment. How many elements are in the compound sample space?

(c) Using either the elemental or compound sample space, calculate the probability of throwing a 7 in this experiment. For ease of calculation, is (a) or (b) preferred? Explain.

$x_1 + x_2 + x_3 = 7$

$x_i \geq 1$

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a) S = { (x1, x2, x3) | xi in {1, 2, 3, 4, 5, 6} } |S| = 6^3

b) C = { sum(x1, x2, x3) | xi in {1, 2, 3, 4, 5, 6 } }

C = { 3, 4, 5 ... 18 } = { x in N | x in [3, 18] }

|C| = 18 - 3 + 1 = 16

c) E = { (x1, x2, x3) | sum(x1, x2, x3) = 7, xi in {1, 2, 3, 4, 5, 6 } }

|E| = 6 choose 2 = 15

P(E) = |E| / |S| = 5 / 72

4. Consider the experiment of flipping a fair coin 100 consecutive times. Let H represent "heads", and T represent "tails".

(a) Write out or otherwise specify the elemental sample space for this experiment.

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5. What are the complement events in each of 4(b) and 4(c)? Would using the complement events make the|calculations easier? Explain.

No

(b) Using either the sample space in (a), or counting principles, calculate the probability of flipping exactly 29 heads in this experiment.

(c) Calculate the probability of flipping at least 97 tails.

a) S = { (x1, x2, ... x100) | xi in {H, T} }
|S| = 2^100

b) E = set of all 100 flips with 29 heads
|E| = 100 C 29
P(E) = |E| / |S| = 100 C 29 / 2^100

c) P(97+T) = P(97T) + P(98T) + P(99T) + P(100T)
P(97+T) = P(3-H) = P(0H) + P(1H) + P(2H) + P(3H)
|3-H| = |1H| + |2H| + |3H|
= 1 + 100 C 1 + 100 C 2 + 100 C 3

P(97+T) = (1 + 100 C 1 + 100 C 2 + 100 C 3)/2^100