Week 7, LEC02 - November 1st
Discrete Mathematics, Fall 2018

CSCA67 - Lecture Notes

Current Instructor: Dr. Richard Pancer

Instructors:

	Dr. Anna Bretscher	**Dr. Richard Pancer**
Email:	bretscher@utsc.utoronto.ca	pancer@utsc.utoronto.ca
Office:	IC493	IC490
Office Hours:	Monday 12:10 - 1:30 Wednesday 1:10 - 2:00 Friday 1:10 - 2:00 (will change after week 6)	Monday 11:10 - 12:30 Friday 1:30 - 3:00

1 Introduction to Counting

Counting Pizza Toppings*

The commercial's deal was:
- 2 pizzas
- up to 5 toppings on each
- 11 toppings to choose from
- all for $7.98 (back in 1997).

Important: duplicate toppings not allowed, order of topppings does not matter, and CHEESE IS NOT A TOPPING.

The commercial's math kid claimed there are $1,048,576$ possibilities.

Let's do the calculation ourselves.

Q. How many ways can we order a pizza with 0 toppings?

A. 1

Q. How many ways can we order a pizza with 1 topping?

A. 11

Q. How many ways can we order a pizza with 2 topping?

A. $\frac{11 \cdot 10}{2} = 55$

Q. How many ways can we order a pizza with 3 toppings?

A. $\frac{11 \cdot 10 \cdot 9}{3!}$

Combinations of $x, y, z = 6 = 3! = 3 \cdot 2 \cdot 1$

Q. How many ways can we order a pizza with 4 toppings?

A. $\frac{11 \cdot 10 \cdot 9 \cdot 8}{4!}$

5 toppings: $\frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5!}$, 6 toppings: $\frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{6!}$

Therefore, the toatl number of ways to order a single pizza with up to 5 toppings when choosing from 11 toppings is:

$$1 + 11 + \frac{11 \cdot 10}{2} + \frac{11 \cdot 10 \cdot 9}{3!} + \frac{11 \cdot 10 \cdot 9 \cdot 8}{4!} + \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5!} = 1024$$

Note: On the final, you will not be asked to resolve this calculation.

Q. How did they get 1,048,576 in the commercial?

A. $1024 \cdot 1024 = 1,048,576$, This is not the correct answer!

Q. What was their mistake? and how do we correct for it?

A. The order of the pizza toppings does not matter, (pizza A, pizza B) is the same thing as (pizza B, pizza A) so, $\frac{1024 \cdot 1024}{2!} = 524,288$

Q. This is still not quite correct. Why?

A. We don't have 2 orders of (pizza A, pizza A), but we still have to divide by 2.

How many ways can we order two identical pizzas? 1024. So, $\frac{1024 \cdot 1024}{2!} + \frac{1024}{2!} = 524,800$

Q. How do we know that 524,800 is the correct answer?

A. One way to convince ourselves is to try and find another way to count he same problem.

we counted all the possible orders and then removed duplicates.

Exercise. Recount using a different method.

The 2 pizzas are either different or the same.

Different: $\frac{1024 \cdot 1023}{2!}$

Same: 1024

Total: $\frac{1024 \cdot 1023}{2!} + 1024 = 524,800$

Potentially on E6:

Exercise: What if double-toppings were allowed? or triple, or quadruple... (ie., two of the toppings are the same)?

Create a secure password