

CSCC63 TUT 0002

Tutorial 2

how many iterations to get the string "aaabbc"

aaaaa\_:  $3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5$

aaaba\_:  $3^2$

aaabb\_: 3

aaabbc: 3

aaabaa  $\rightarrow$  bab  $\rightarrow$  bac

aaabba  $\in$  a, b, c

$\Sigma$ , a, b, c, aa, ab, ...

$M = (Q, \Sigma, \Gamma, d, s, q_A, q_R)$

$L = \{ \langle M, w \rangle \mid M \text{ doesn't move its head beyond the end of the input} \}$

Decidable: Recognizable and Co-Recognizable

Recognizable: Halts on yes instances and accepts

Co-Recognizable: Halts on no instances and rejects

$L$  is decidable if a TM  $M$  accepts its yes instances, and rejects its no instances

$L = \{ \langle M, w \rangle \mid M \text{ doesn't move its head beyond the end of the input} \}$

$$(|\Gamma|^{|w|}) * |Q| * |w|$$

$M_1$  on input  $\langle M, w \rangle$ :

run  $M$  on  $w$  for  $|\Gamma|^{|w|} * |Q| * |w|$

reject if any configurations goes beyond the end  
of the input

otherwise accept

We say A reduces to B if there exists a function P such that for all x, x belongs to A iff P(x) belongs to B

If A reduces to B:

- and A is undecidable, B is undecidable

$HALT = \{ \langle M, w \rangle \mid M \text{ halts on } w \}$

$L = \{ \langle M, w \rangle \mid M \text{ tries to move its head off the left side of the input} \}$

$$HALT \leq_p L$$

P on input  $\langle M, w \rangle$ :

defined M' on input  $\langle x \rangle$ :

1. []
2. run M on w
3. [move left on x if  $x = w$ ]

return  $\langle M', w' = w \rangle$

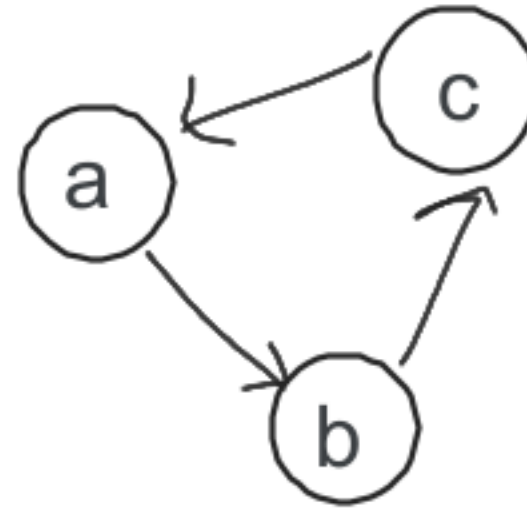
show that if  $\langle M, w \rangle$  in HALT  $\langle M', w' \rangle$  in L

If  $\langle M, w \rangle$  in HALT, M halts on w, M' will move left on w, therefore  $\langle M', w' \rangle$  in L

show that if  $\langle M', w' \rangle$  not in L, then  $\langle M, w \rangle$  not in HALT

If  $\langle M', w' \rangle$  not in L, line 3 was never ran, line 2 must have looped. If M loops on w, then  $\langle M, w \rangle$  is not in halt.

Graph:  $G = (V, E)$



For every node  $v$ , build the tiles

|            |    |
|------------|----|
| #v (s = v) | #  |
| #          | v# |

the idea is to enforce that there is at least one tile in between the augmented start and end tiles.

|           |          |          |          |    |
|-----------|----------|----------|----------|----|
| #a(s = a) | b(s = a) | b(s = a) | a        | #  |
| #         | a(s = a) | c(s = a) | c(s = a) | a# |