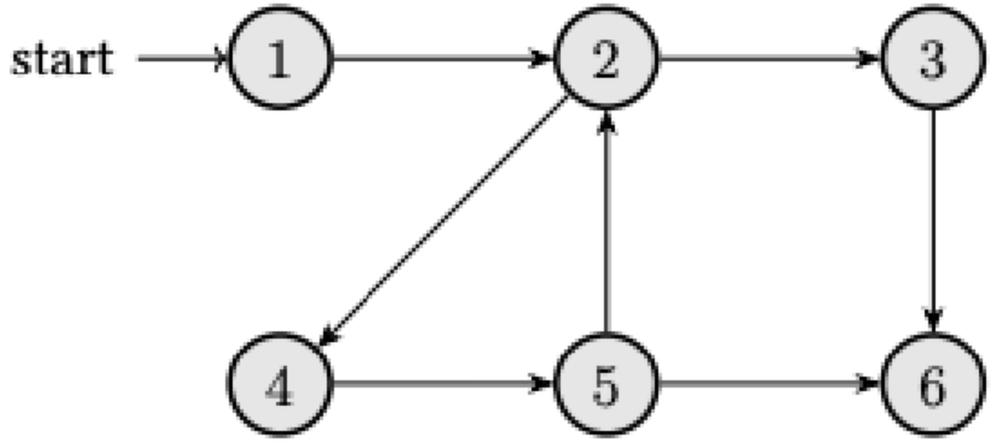
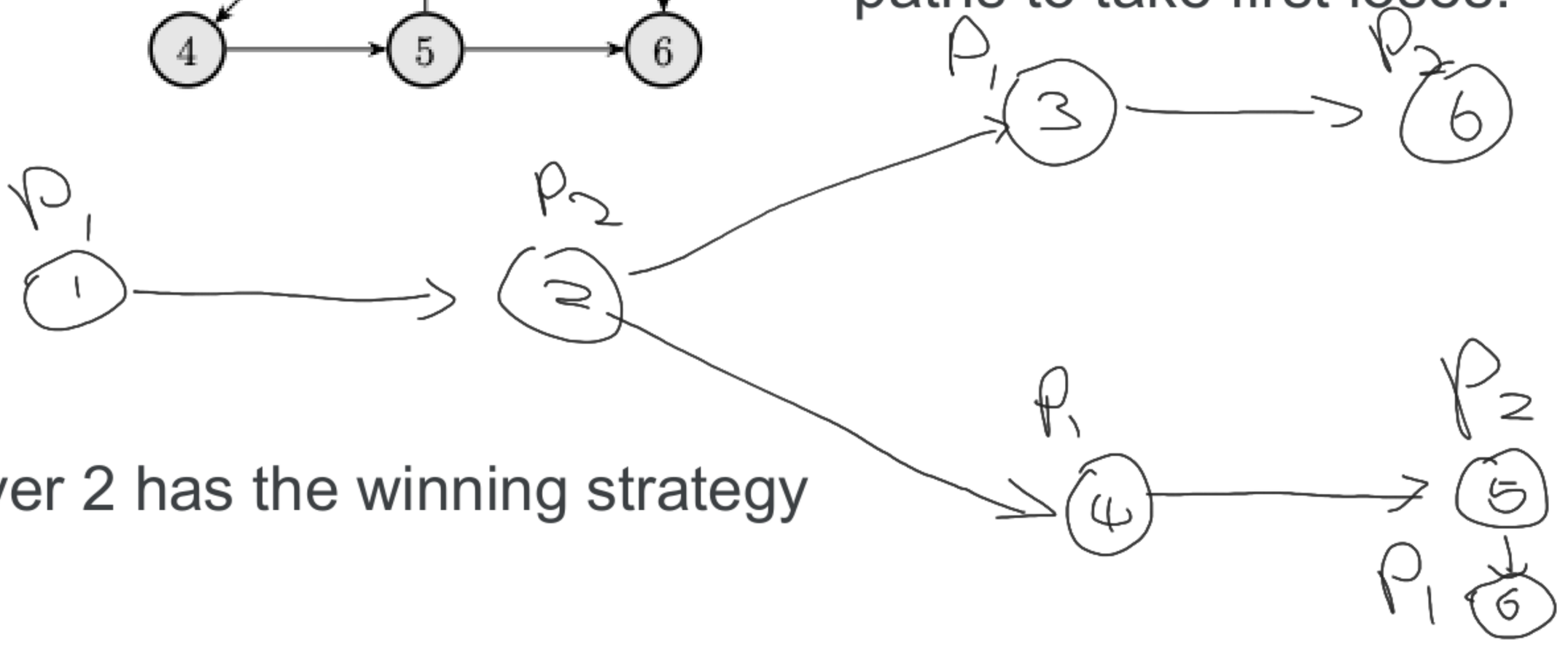
CSCC63 TUT 0002
Tutorial 9

Generalized Geography Example
NP Oracles

1. Generalized Geography Game



Player 1 goes first, whoever runs out of paths to take first loses.



Player 2 has the winning strategy

Clique = { <G, k> | G is an undirected graph with a k clique }
Max Clique:
  Input: An undirected graph G
  Output: Largest clique of G (set of nodes that make up the max-clique)

WTS given oracle for CLIQUE, we can solve MAX-CLIQUE in polytime and vice versa.

Suppose we have an oracle solver SOLVE-MAX-CLIQUE (takes input G, returns set of nodes) that solves Max-Clique in one step, write a function that solves Clique in polytime.

P on input <G = (V, E), k>:
  max = |SOLVE-MAX-CLIQUE(G)| # O(|V|)
  accept if max >= k, otherwise reject # O(1)

Suppose we have an oracle solver SOLVE-CLIQUE (takes input G, number k, returns whether G has a k sized CLIQUE) that solves Clique in one step, write a function that solves Max-Clique in polytime.

```
P on input <G=(V, E)>: # We want to return some set of nodes that form the max clique
  max = 0
  for i = |V| ... 0: # O(|V|)
     if SOLVE-CLIQUE(<G, i>):
        max = i
        break
  let V' be a copy of V # O(|V|) duplicate the set of nodes
  let G' = (V', E') be a copy of G
  for v in V': # O(|V|)
    let V'' = V' - { v } # O(|V|)
    let E'' = E' - { (u', v') | u' = v | v' = v } # O(|E|)
    if SOLVE-CLIQUE(<(V'', E''), max>): # O(1)
       V' = V''
       E' = E''
  return V'
```

Q2 Help:
  Multiset: set that contains duplicates, your sum(S') will probably take in the form of `a + a^2`

Consider 1-2 Subset
  Input: A multiset S
  Question: Whether or not S can be divided into subsets A and B where 2 sum(A) = sum(B)

Consider the following reduction from subset sum
P on input <S, t>:
  return S' = S union { sum(S), 3t + sum(S) } # sum (S') takes the `a + 2a`

Suppose that <S, t> in Subset-Sum
there must exists S' subset S such that sum(S') = t
consider A = S' union {sum(S)}, B = (S - S') union {3t + sum(S)}
sum(A) = sum(S') + sum(S) = t + sum(S)
sum(B) = sum(S) - sum(S') + (3t + sum(S)) = sum(S) - t + 3t + sum(S) = 2t + 2sum(S)
Suppose that S' in 1-2 Subset (How come <S, t> has to be in subset sum?)

Q1 Help:

Colorability

We say that a graph is k-Colorable, if every node can colored with one of the k colors with no two neighbouring nodes sharing the same color.

Example: k = 3