MAT B24 TUT0018, Tutorial 2 (Week 3)
Thursdays 3 - 5pm

XiangQian Hong (Richard)
richard.hong@mail.utoronto.ca

Definitions

Given vectors $\vec{v}_i \in K$, $r_i \in \mathbb{R}$

- Linear Combination $\sum_{i=0} r_i \vec{v}_i$

- Dependence
independent: LC = 0 <=>each r_i = 0

- Span(X), set of all LC
- Subspace
A subset W of a vector space V is a subspace if W is a vector space itself.

Section 3.2 #3

Determine whether the set of all functions f, such that $f(0) = 1$ in the vector space F of all functions mapping R to R is a subspace.

Not a subspace because the zero vector is not in the subset.

Section 3.2 #4

Determine whether the subset S of all functions f, such that f(1) = 0 in the vector space F of all functions mapping R to R is a subspace.

let $f(1) = g(1) = 0$, $h(x) = f(x) + g(x)$,
$h(1) = f(1) + h(1) = 0$, $(f(x) + g(x))$ in S

let $f(1) = 0$, r in R, $h(x) = r * f(x)$
$h(1) = 0 * r = 0$, $(r * f(x))$ in S

let $f(1) = g(1) = 0$, r, k in R, $h(x) = rf(x) + kg(x)$
$h(1) = r * f(1) + k * g(1) = r * 0 + k * 0 = 0$, $(rf(x) + kg(x))$ and S is closed under +, *

$\sin^2 x + \cos^2 x = 1$

Section 3.2 #7

Let F be the set of all functions mapping R into R, show that

a. span($\sin^2 x$, $\cos^2 x$) contains all constant functions, i.e. $f(x) = c$, forall x

$c = c(\sin^2 x + \cos^2 x)$

b. span($\sin^2 x$, $\cos^2 x$) contains the function $\cos(2x)$

$\cos(2x) = -1 \sin^2 x + \cos^2 x$

c. span(7, sin^2 2x) contains the function 8 cos (4x)

$$
\begin{aligned}
8\cos(4x) &= 8(\cos^2(2x) - \sin^2(2x)) \\
&= 8((1 - \sin^2(2x)) - \sin^2(2x)) \\
&= 8(1 - 2\sin^2(2x)) \\
&= 8 - 16\sin^2(2x) \\
&= (8/7) * 8 - 16\sin^2(2x)
\end{aligned}
$$

Section 3.2 #8

Let P be the vector space of polynomials. Prove that span(1, x) = span(1 + 2x, x)
[Hint: show that these subspaces is subset of the other]

given X, Y WTS X = Y, X subseteq Y, Y subseteq X

$X \subseteq Y,\ Y \subseteq X$

WTS span(1, x) subset span(1 + 2x, x)
1 = 1 + 2x + (-2)x
x = 0(1 + 2x) + (1)x

$\mathrm{span}(1, x) \subseteq \mathrm{sp}(1+2x, x) \Rightarrow X = Y$

WTS span(1 + 2x, x) subset span(1, x)
1 + 2x = 1 + 2x
x = 0 * 1 + x

Therefore span(1+2x, x) = span(1, x)

Problem one. Suppose that X is a nonempty subset of the vector space V. Show that the span of X is a subspace of V.

X = { x1, x2, ... , xn }, a1, a2, ..., an in R, b1, b2, ... bn in R, c in R
let v1 be an arbitrary element in span(X)
v1 = a1 x1 + a2 x2 + ... + an xn
v2 = b1 x1 + b2 x2 + ... + bn xn

v1 + v2 = (a1 + b1) x1 + (a2 + b2) x2 + ... + (an + bn) xn in span(X)
Therefore span(X) closed under addition

c * v1 = c (a1 x1 + a2 x2 + ... + an xn) = (c * a1) c1 + (c * a2) x2 + ... in span(X)
Therefore span(X) closed under multiplication

Thus span(X) is a subspace of V.

Problem two. Give an example of a subset of R^2 such that its closed under addition but not scalar multiplication

$$\left\{ [x, y] \mid x, y \in \mathbb{R} \right\}$$

$$\left\{ x \mid x \in \mathbb{R} \right\}$$

$$\backslash\backslash \quad x \geq 0, \quad y \geq 0$$

Section 3.2 #4 ation has the form

$$f_n(x)y^{(n)} + f_{n-1}(x)y^{(n-1)} + \cdots + f_2(x)y'' + f_1(x)y' + f_0(x)y = 0.$$

Show that the set of all solutions for this equation that lie in the space F of all functions mapping R into R is a subspace of F

let y1, y2 be solutions of the equation,

f(x)y1 + f(x)y2 = 0
f(x)(y1 + y2) = 0

Show span({}) = {0}

$$\sum r_i x_i = 0 \quad , \quad x_i \in \{\}$$

$$\text{span}(X) = \left\{ \sum r_i v_i \,\middle|\, x_i \in X, r \in \mathbb{R} \right\}$$

$$= \{0\}$$

[0, 1], [1,2] generates R^2
x^1, x^2, ... x^n generates P^n

F, set of all functions R to R

assume for contradiction this is finitely generated,
p1, to, pn, p^n + 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$