Lecture Notes
Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, Feb 1st, 2:00pm - 3:00pm

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1 Indefinite Proof

Prove:

IF f and g' are cont on [a, b]

THEN $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

Where $u = g(x), du = g(x)dx$

Suppose that f and g' are cont on [a, b]

Let F be an antiderivative of f on [a, b]

Claim: $F(g(x))$ is an antiderivative of f(g(x)) f'(x) on [a, b]

Check: Let $x \in [a, b]$ be arbitrary, $(F(g(x)))' = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(x)$

And so $\int_a^b f(g(x))g'(x) = F(g(x))|_a^b$ By FTOC I

Thus we have $F(g(b)) - F(g(x))$

Recall $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

$\int_{g(a)}^{g(b)} f(u)du = F(u)|_{g(a)}^{g(b)} = F(g(b)) - F(g(a))$ By FTOC I QED

Examples

1. $\int_0^1 \sqrt{2-x} = \int_{u(0)}^{u(1)} \sqrt{u} \cdot (-du) = \int_2^1 u^{\frac{1}{2}} \cdot (-du) = \int_1^2 u^{\frac{1}{2}} du = [\frac{2}{3}u^{\frac{1}{2}}]_1^2 = \frac{2}{3}[1 - \sqrt{8}]$

   let $u = 2 - x$

2. $\int \sin^5(x) \cdot \cos^3(x)dx = \int \sin^3(x) \cdot \cos^2(x) \cdot \cos(x)dx = \int \sin^5(x) \cdot (1 - \sin^2(x)) \cdot \cos(x)dx$

   let $u = \sin(x), du = \cos(x)dx$

   $\int \sin^5(x) \cdot \cos^3(x)dx = \int u^5(1-u^2)du = \int u^5 - u^7 du = \frac{u^6}{6} - \frac{u^8}{8} + C = \frac{\sin^6(x)}{6} - \frac{\sin^8(x)}{8} + C$

   For an exercise, do this with $u = \cos(x)$

3. $\int \sqrt{3+x^2}x^5dx = \int \sqrt{3+x^2}x^{2^2} \cdot x \cdot dx$

   Let $u = 3 + x^2, \frac{du}{2} = x \cdot dx, x^2 = u - 3$

   $\int \sqrt{3+x^2}x^5dx = \int \sqrt{u}(u-3)^2\frac{du}{2} = \frac{1}{2}\int \sqrt{u} \cdot (u^2 - 6u + 9)du = \frac{1}{2}\int u^{\frac{5}{2}} - 6u^{\frac{3}{2}} + 9u^{\frac{1}{2}} du$