MAT B24 TUT0018, Tutorial 5 (Week 6) Thursdays 3 - 5pm

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Theorem 3.10

Let V and V' be finite-dimensional vector spaces with basis B = (b1, b2 ... bn) and B' = (b'1, b'2, ... b'm) respectively.

Let T: V -> V'.

Let T': R^n -> R^m, T'([v]B) -> [T(v)]B'

Then the standard matrix representation T'

$$A = \begin{bmatrix} T(\vec{b}_1) B' & T(\vec{b}_2) B' & - - - & T(\vec{b}_n) B' \\ & & & \end{bmatrix}$$

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24. Let $T: P_3 \to P_2$ be defined by T(p(x)) = p'(2x + 1), where p'(x) = D(p(x)), and let $B = (x^3, x^2, x, 1)$ and $B' = (x^2, x, 1)$.

a. Find the matrix A.

b. Use A to compute $T(4x^3 - 5x^2 + 4x - 7)$.

$$T'([v]B) = T(v)B'$$

$$T(x^3) = 3(2x + 1)^2 = 3(4x^2 + 4x + 1)$$
 $T(x^2) = 2(2x + 1) = 4x + 2$
 $T(x) = 1$
 $T(1) = 0$

$$T(x^3)B' = [12, 12, 3]$$
 $T(x^2)B' = [0, 4, 2]$
 $T(x)B' = [0, 0, 1]$
 $T(1)B' = [0, 0, 0]$

$$A = 12 4 0 0$$

$$T(v) = T'([v]B) = A[v]B$$

$$[v]B = [4, -5, 4, -7] => T(v) = A [4, -5, 4, -7]$$

26. Let
$$T: P_3 \to P_3$$
 be the linear transformation defined by $T(p(x)) = D^2(p(x)) - 4D(p(x)) + p(x)$. Find the matrix representation A of T, where $B = (x, 1 + x, x + x^2, x^3)$.

$$A = [[T(x)B], [T(1 + x)]B, [T(x + x^2)]B, [T(x^3)]B]$$

$$A = \begin{bmatrix} (-4 + x) \\ (-7 + x) \\ (-7$$

(x+1)2 (x+1)(x+)

Additional Problems

WTP

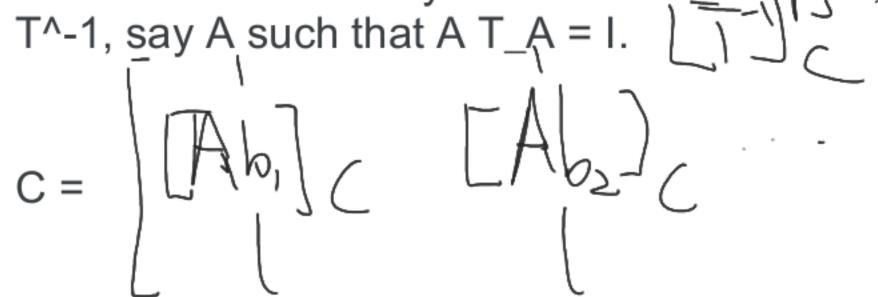
Problem one. Suppose that $B = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ is a basis for the vector space V, that $C = \{\mathbf{c}_1, \dots, \mathbf{c}_n\}$ is a basis for the vector space W, and that $T: V \to W$ is a linear transformation. Show that T is an invertible transformation if and only if the matrix representation $[T]_B^C$ (or denoted by $R_{B,C}$) of T with respect to the bases B and C is an invertible matrix.

R_B,C invertible <=> T is invertible

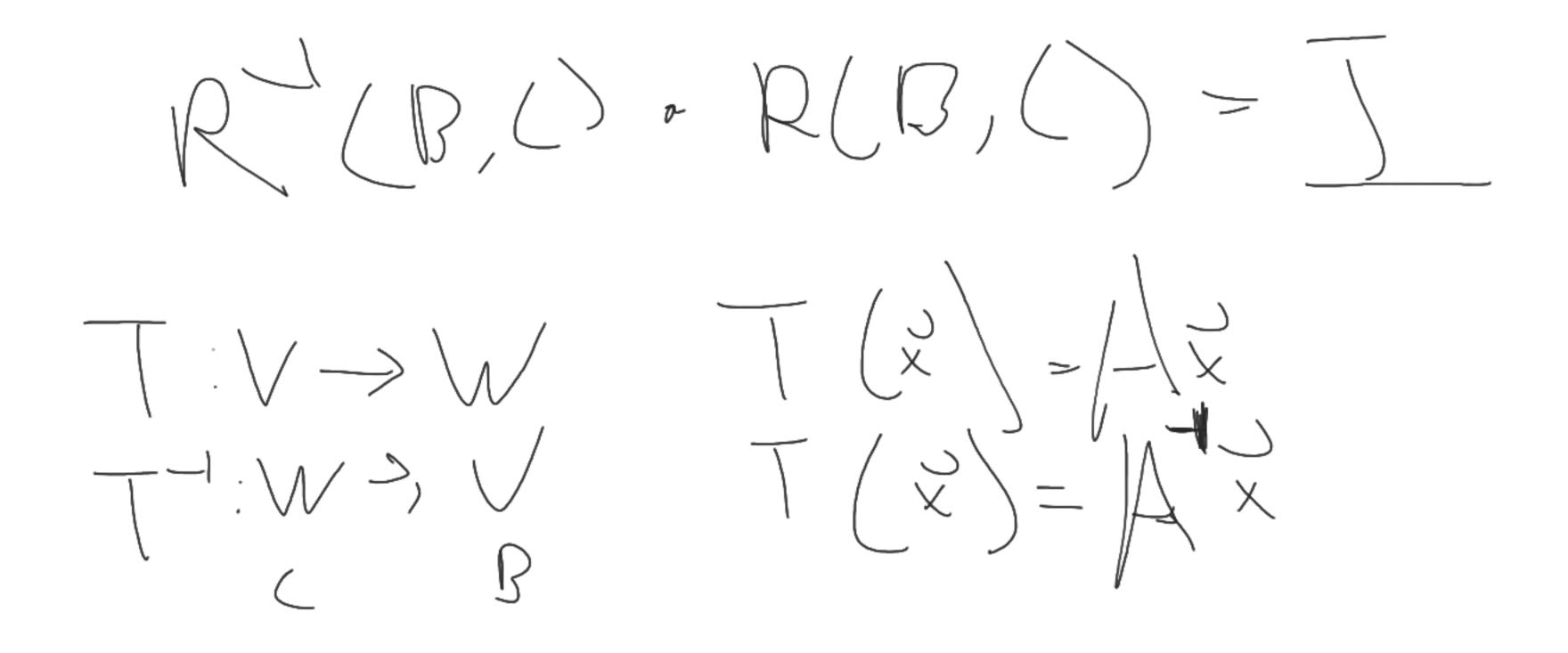
Assume that R_B,C is invertible, this means there exists some matrix A such that R_B,C A = I. Then let S: W -> V, and S(w) = Aw. Then S = T^{-1} S(T (v)) = R_B,C * A * v = I * v



Assume that T is invertible, if T is invertible then there is some T^-1: W -> V such that T o T^-1 is the identity. since this I^-1 exists, then there is some matrix representation of







TTTP= TATZB--- LATZB Problem four. Let $T: P_4 \to P_4$ be the linear transformation defined by T(p) = xp'(x) - p(x). Find a basis for the image of T and a basis for the kernel of T.

Img(T) = column space of A = sp([-1, 0, 0, 0, 0,], [0, 0, 1, 0, 0], [0, 0, 0, 2, 0], [0, 0, 0, 0, 3])

$$B = (1, x, x,^2, x^3, x^4)$$

basis for $im(T) = \{ -1, x^2, 2x^3, 3x^4 \}$

Let T be a linear transformation with standard matrix representation A

Kernel(T) = Nullspace of A Image(T) = Column space of A