MAT A29 TUT0018, Tutorial 6 (Week 7)
Tuesdays 7 - 9pm (We will start at 7:10pm)

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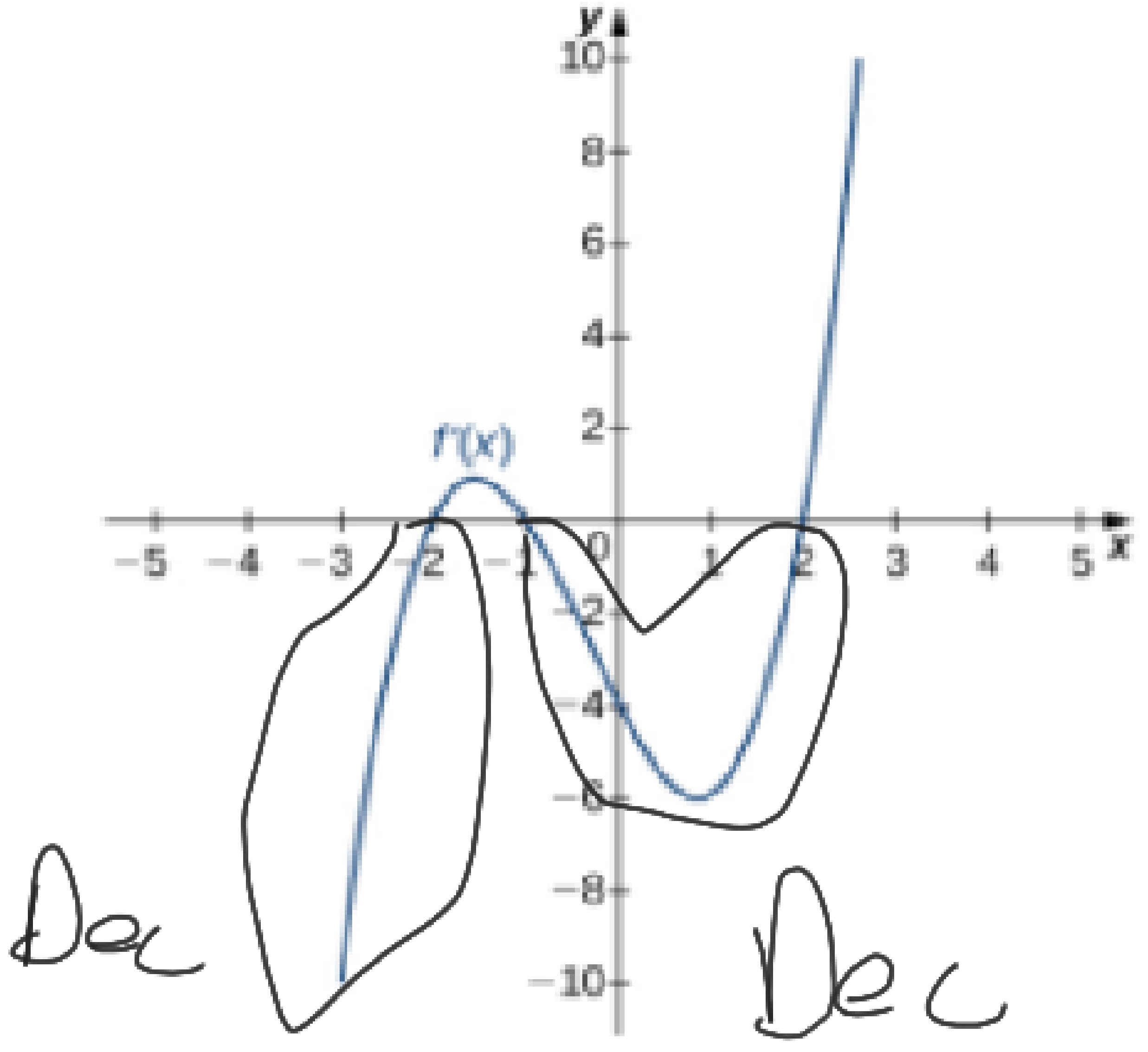
Question

Analyze the given graph of $f'(x)$ and determine the intervals where $f(x)$ is increasing and decreasing.

f(x) is increasing if f'(x) is > 0
f(x) is decreasing if f'(x) is < 0

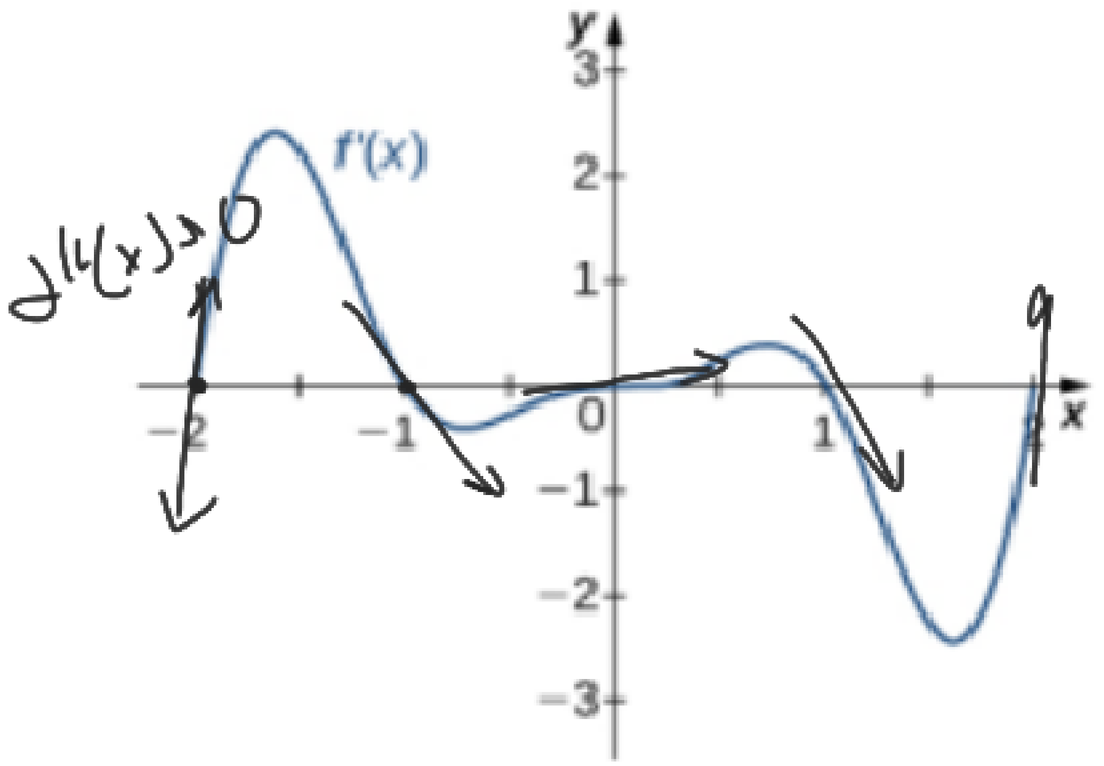


(-inf, -2) decreasing
(-2, -1) increasing
(-1, 2) decreasing
(2, inf) increasing

$f'(x) = 0$

Question

Analyze the given graph of $f'(x)$ and determine the maximum and minima of $f(x)$.



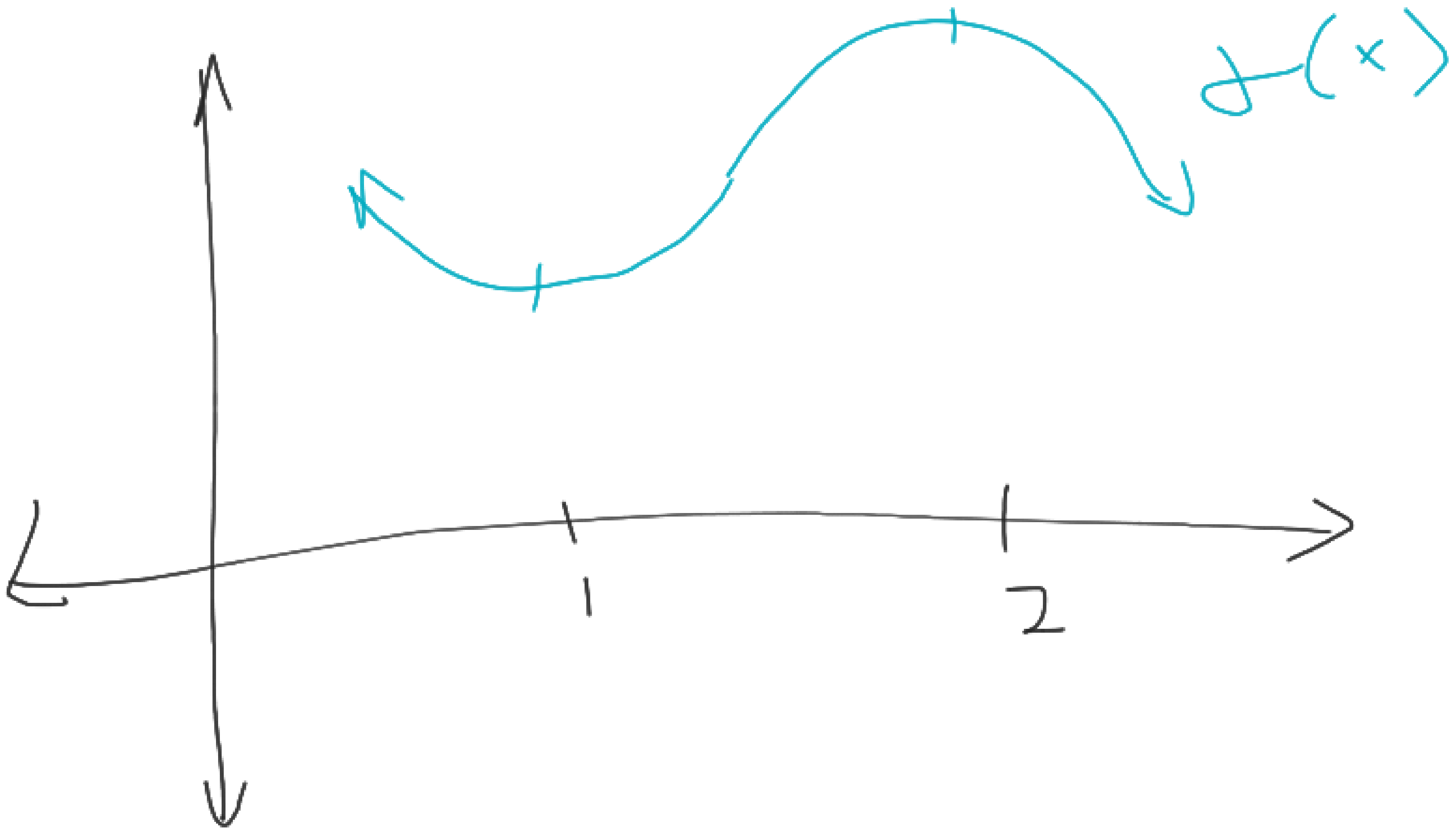
$f'(x) = 0$
$x = -2, -1, 0, 1, 2$

we want to know $f''(x)$ at these points
$f''(x) < 0$ -> x is a local maximum
$f''(x) > 0$ -> x is a local minimum

$f''(x) > 0$, $x = -2, 0, 2$ Minima
$f''(x) < 0$, $x = -1, 1$ Maxima

Question

Produce a graph with the following properties: there is a local maximum at $x = 2$, a local mimimum at $x = 1$, and the graph is neither concave up nor concave down.



Question

Consider $f(x) = x^3 - 6x^2$. Determine where $f(x)$ is increasing and decreasing.

$x < 0$ $\qquad$ $x > 4$

f(x) decreasing => f'(x) < 0

f(x) increasing => f'(x) > 0

f'(x) = 3x^2 - 12x

$$3x^2 - 12x > 0$$

$$3x(x-4) > 0$$

$\oplus / \ominus$ $\qquad$ $\oplus / \ominus$

$x > 4$ $\qquad$ $++$

$x < 0$ $\qquad$ $--$

$\oplus$ $\qquad$ $\ominus$

$$3x(x-4) < 0$$

$$x \in (0, 4)$$

f(x) is increasing when x in (-inf, 0) and (4, inf). f(x) is decreasing from (0, 4).

Question

Consider $f(x) = x^3 - 6x^2$. Determine the local minima and maxima of $f(x)$.

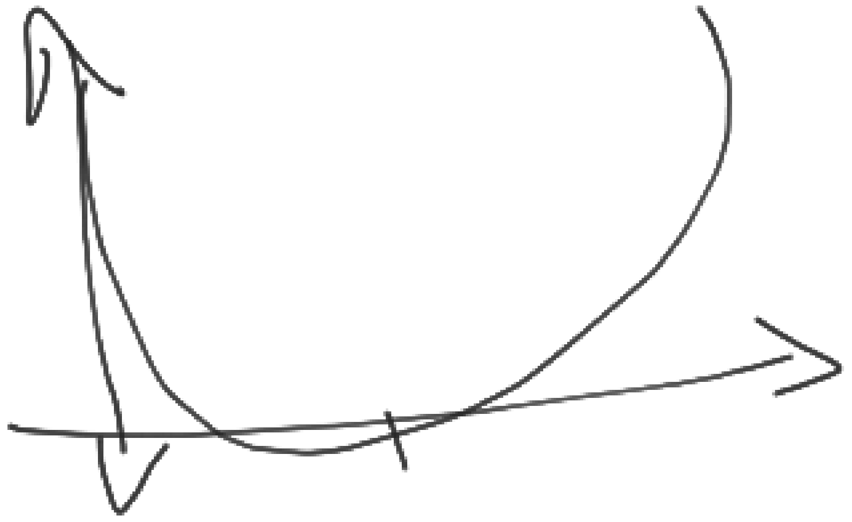
f'(x) = 3x^2 - 12x, f''(x) = 6x - 12
f'(x) = 0 when x = 0, 4

f''(0) = -12
f''(4) = 24 - 12 = 12

x = 0 f(x) is a local maxima
x = 4 f(x) is a local minima

Question

Consider $f(x) = x^3 - 6x^2$. Determine where $f(x)$ is concave up and concave down. Locate any inflection points of $f(x)$.

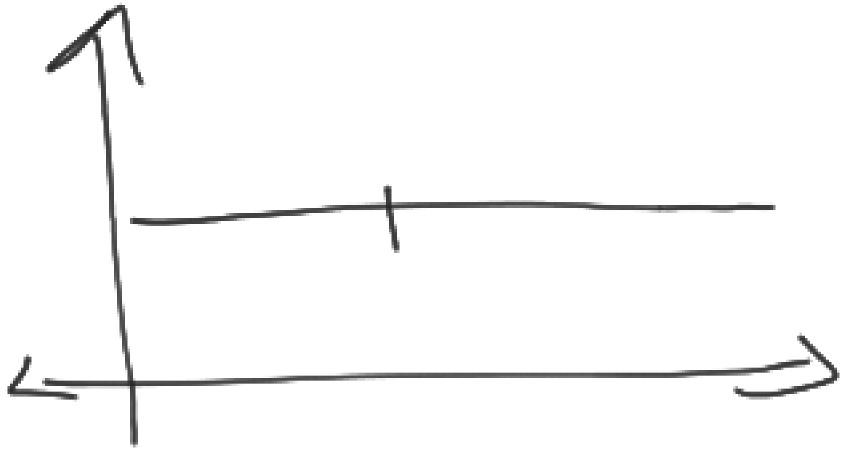


$f''(x) = 0$, x is a inflection point
$f''(x) > 0$, f(x) is concave up in this interval
$f''(x) < 0$, f(x) is concave down

$f''(x) = 6x - 12 = 0 \Rightarrow x = 2$

$f''(3) = 18 - 12 = 6$
$f''(1) = 6 - 12 = -6$

$x > 2$, $f''(x) > 0$ thus f(x) is concave up
$x < 2$, $f''(x) < 0$ thus f(x) is concave down



Question

Suppose that $f(t)$ represents the size of a population at time t. Interpret the sentence "the population is growing more slowly" using mathematical notation and $f(t)$, $f'(t)$, and $f''(t)$.

$f(t)$ is growing if $f'(t) > 0$
more slowly => $f'(t)$ is decreasing => $f''(t) < 0$

$f'(t) > 0$ and $f''(t) < 0$