MAT B24 TUT0018, Tutorial 4 (Week 5)
Thursdays 3 - 5pm

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Q12

Let

$$B = (1, 1 + 2x, 1 + 2x + 3x^2, 1 + 2x + 3x^2 + 4x^3)$$

be an ordered basis for the vector space P_3. Find the coordinate vector of the polynomial

$$p(x) = 2 + 12x^2 + 4x^3$$

relative to the ordered basis B.

4 - Correct [p(x)]B
5 - System of equations
1 - Used System of Equations

Q13

Suppose that U and W are subspaces of the vector space V. Show that the set $S = \{\mathbf{u} + \mathbf{w} : \mathbf{u} \in U \text{ and } \mathbf{w} \in W\}$ is a subspace of V.

Prove the following
3 - Inclusion of zero vector
3.5 - proof of additive closure
3.5 - proof of scalar multiplication closure

Linear Transformation is a function T: V -> W

1. T(u + v) = T(u) + T(v), forall u, v in V
2. T(ru) = rT(u) forall u in V, r in R

17. $T([x_1, x_2, x_3]) = [x_1 - x_2 + 3x_3, x_1 + x_2 + x_3, x_1]$

$$\begin{bmatrix} | & | & | \\ T(\vec{e}_1) & T(\vec{e}_2) & T(\vec{e}_3) \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

1. $T: F \to \mathbb{R}$ defined by $T(f) = f(-4)$

Linear Transformation is a function T: V -> W
1. T(u + v) = T(u) + T(v), forall u, v in V
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let f, g in F, r in R
T(f + g) = (f + g)(-4) = f(-4) + g(-4) = T(f) + T(g)
T(rf) = (r f)(-4) = rf(-4) = rT(f)

ker(T) = { f in F | f(-4) = 0 }

2. $T: F \to \mathbb{R}$ defined by $T(f) = f(5)^2$

Linear Transformation is a function T: V -> W
1. T(u + v) = T(u) + T(v), forall u, v in V
2. T(ru) = rT(u) forall u in V, r in R

(a + b)^2 != a^2 + b^2

f(x) = 1, g(x) = 2
T(f + g) = (1 + 2)^2 = 3^2
T(f) + T(g) = 1^2 + 2^2 = 5
T(f + g) != T(f) + T(g)

8. Let F be the vector space of all functions mapping $\mathbb{R}$ into $\mathbb{R}$, and let $T: F \to F$ be a linear transformation such that $T(e^{2x}) = x^2$, $T(e^{3x}) = \sin x$, and $T(1) = \cos 5x$. Find the following, if it is determined by this data.

a. $T(e^{5x})$

b. $T(3 + 5e^{3x})$

c. $T(3e^{4x})$

d. $T\left(\frac{e^{4x} + 2e^{5x}}{e^{2x}}\right)$

a.

b. T(3 + 5e^(3x)) = T(3) + T(5e^(3x)) = 3T(1) + T(5e^(3x)) = 3cos5x + 5 T(e^(3x))
= 3cos5x + 5sinx

c.

d. T(e^2x + 2e^3x) = x^2 + 2sinx

Problem one. Show that the composition $S \circ T$ of the linear transformations $S : V \to W$ and $T : U \to V$ is a linear transformation.

Linear Transformation is a function T: V -> W
1. T(u + v) = T(u) + T(v), forall u, v in V
2. T(ru) = rT(u) forall u in V, r in R

Let G = S o T, WTS G is a linear transformation G: U -> W
let u, v in U
G(u + v) = S(T(u + v)) = S(T(u) + T(v)) [BC T is a linear transformation]
= S(T(u)) + S(T(v)) = G(u) + G(v)

let r in R
G(rv) = S(T(rv)) = S(rT(v)) = rS(T(v)) = rG(v)

Problem two. Suppose that V is a vector space of dimension n, and that B is a linearly independent set in V with n elements. Show that B is a basis for V.

Suppose for a contradiction that B is not a basis for V. This means that there must be some v in V, such that v not in span(B). If this is the case, then B union { v } is a linearly independent set of n + 1 elements subset of V. This is a contradiction as a subset of n + 1 elements in V cannot be linearly independent, thus B must be a basis for V.

Problem three. Suppose that V is a vector space of dimension n, and that B is a subset of V with n elements which spans V. Show that B is a basis for V.

Suppose for a contradiction that B is a subet of V with n elements that span (V). WTS B is linearly independent.

Assume for a contradiction that b in B such that b in span (B - {b}), if this is the case then span(B - {b}) = V. V cannot have dimension n, if it is spanned by a set of size n - 1. |B| = n, |B - {b}| = n - 1.

Problem four. Suppose that $T : V \to W$ is a linear transformation between vector spaces, and that S is a subspace of W. Show that the inverse image $T^{-1}(S) = \{\mathbf{v} \in V : T(\mathbf{v}) \in W\}$ is a subspace of V.

T(0 v) = 0 * T(v) = 0

show that INVERSE = {v in V | T(v) in W} is a subspace of V
1. inclusion of zero vector
since V is a vector space, 0 in V. T(0) = 0, thus 0 in INVERSE
2. closure under addition

3. closure under scalar multiplication