CSCC63 TUT 0002
Tutorial 10

3SAT to Subset-Sum

3SAT = { <p> | p is a 3CNF formulat that can be satisfied }
Subset-Sum = { <S, t> | S is a multiset of integers, t is an integer, a subset of S sums to t }

3CNF Formula:
(x1' or x2' or x3) and (x1' or x2 or x3') and (x1 or x2' or x3') and (x1 or x2 or x3')

	x1	x2	x3	c1	c2	c3	c4
x1	1	0	0	0	0	1	1
x1'	1	0	0	1	1	0	0
x2	0	1	0	0	1	0	1
x2'	0	1	0	1	0	1	0
x3	0	0	1	1	0	0	1
x3'	0	0	1	0	1	1	0
	1	1	1	1	1	1	3
				1	0	0	0
				1	0	0	0
				0	1	0	0
				0	1	0	0
				0	0	1	0
				0	0	1	0
				0	0	0	1
				0	0	0	1
	1	1	1	3	3	3	3

not parsimonious, has many possible solutions

3CNF Formula:
(x1' or x2' or x3) and (x1' or x2 or x3') and (x1 or x2' or x3') and (x1 or x2 or x3')

	x1	x2	x3	c1	c2	c3	c4
x1	1	0	0	0	0	1	1
x1'	1	0	0	1	1	0	0
x2	0	1	0	0	1	0	1
x2'	0	1	0	1	0	1	0
x3	0	0	1	1	0	0	0
x3'	0	0	1	0	1	1	1
	1	1	1	2	2	2	1
				1	0	0	0
				2	0	0	0
				0	1	0	0
				0	2	0	0
				0	0	1	0
				0	0	2	0
				0	0	0	1
				0	0	0	2
	1	1	1	4	4	4	4

parsimonious, only has one solution

Q3 Help:

a) Give a reduction from some NP-hard language (consider Clique)

b) Give a reduction from some co-NP hard language (or reduce reduce NP-hard to co-Many-Cliques)

Co-Many-Cliques = { <G, r, k> | exists a size r subgraph of G that doesn't contain a k-clique }

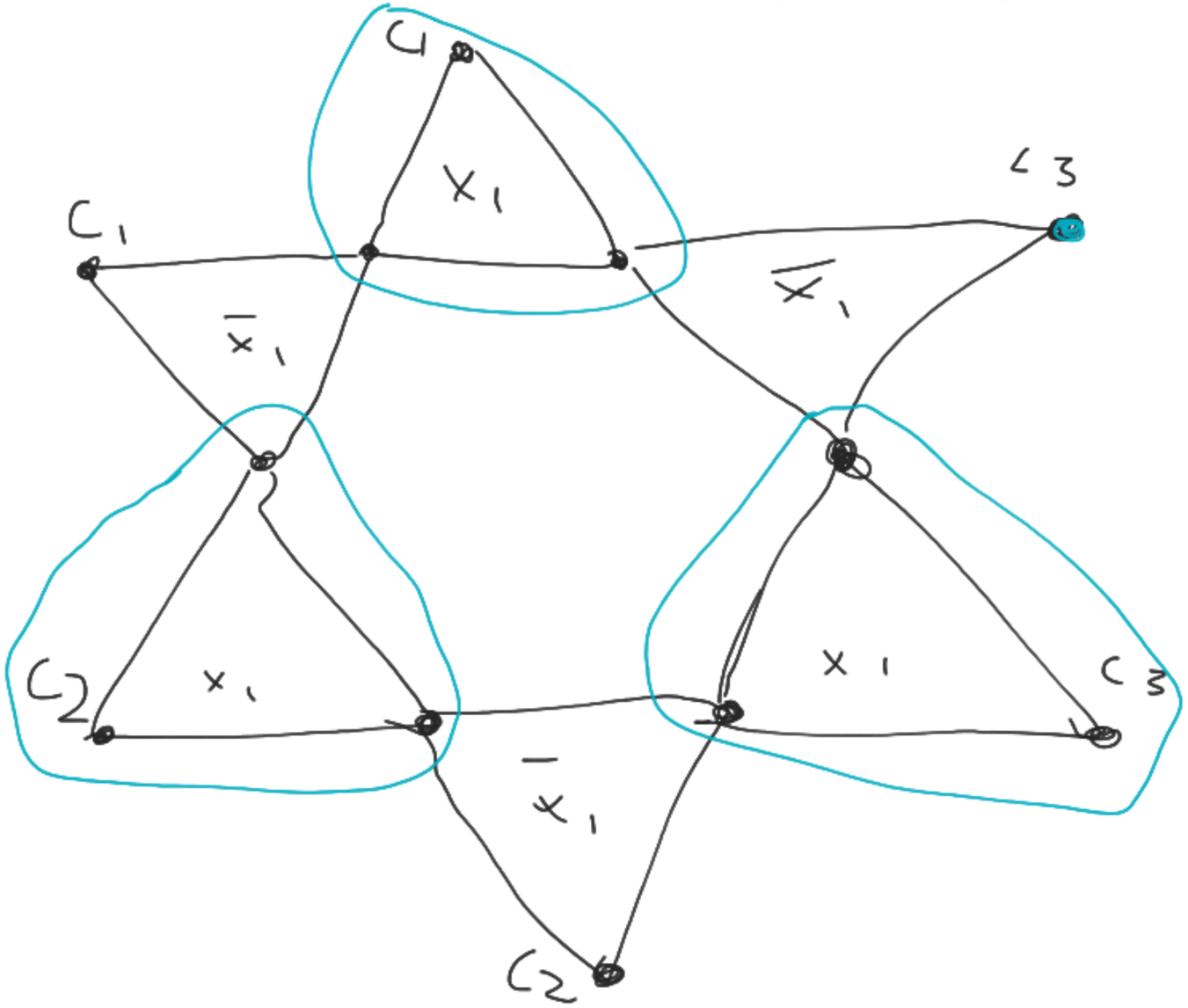
c) Write a solver, justify that it is ran in PSPACE (Decider), always halts, and returns whether a given input belongs ti Many-Cliques.

Example: 3COLOR, show that SOLVE-3COL is in PSPACE

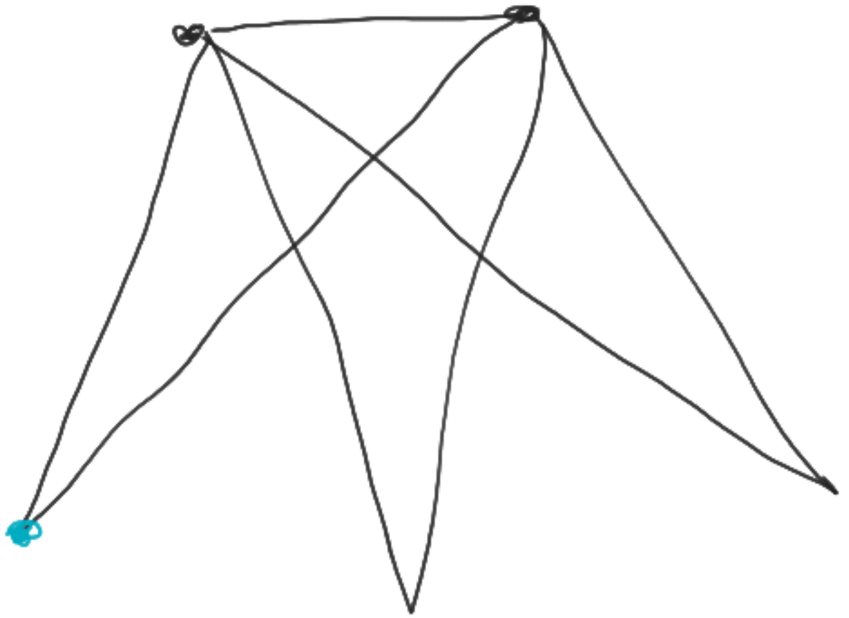
Input <G>, output: whether or not the graph is 3 colorable.

```
COLOR on input <G>:
  define COLOR-SOLVER input <G = (V, E), M>: # M is a hashmap mapping nodes to their color
    if |V| = |M| accept;
    for node v in V:
      if M[v] exists:
          continue
      for color in [r, g, b]
        if none of v's neighbours has the same color
        M[v] = color
        accept if COLOR-SOLVER(G, M) accepts
      M[v] = nil #none of the previous coloring choices lead to a colorable graph, we must backtrack
      return
In total |V| max recursive depth,
```

Partition-Into-Triangles
Input: A graph G
Question: Can G be split into disjoint triangles?

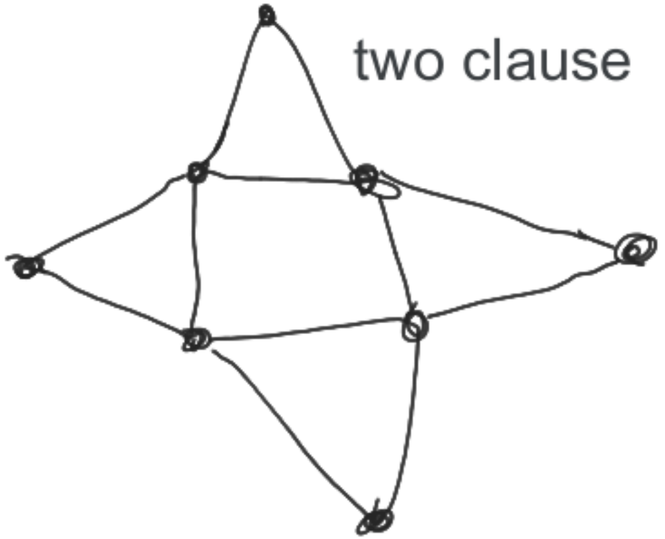


Clause



for each variable, encode a widget based off the number of clauses it belongs to,

one clause



two clause

. . .

clause widgets: suppose we have (x1 or x2 or x3)