MAT B24 TUT0018, Tutorial 6 (Week 7)
Thursdays 3 - 5pm

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Change of Basis Matrices

Suppose B, B' are basis for an n dimensional vector space V

$$C_{B\to B'}\,[v]_B = [v]_{B'} \qquad C_{B\to B'} = \begin{bmatrix} | & & | \\ [\vec{b}_1]_{B'} & \cdots & [\vec{b}_n]_{B'} \\ | & & | \end{bmatrix}$$

$$[\vec{v}]_B$$

$$C_B[\vec{v}]_B$$

$$[\vec{v}]_B = [v_1, \ldots, v_n]$$

$$\vec{v} = v_1 \cdot \vec{b}_1 + v_2 \cdot \vec{b}_2 \cdots v_n \cdot \vec{b}_n = \begin{bmatrix} \vec{b}_1 & \cdots & \vec{b}_n \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$$[\vec{v}]_{B'} = \left[C_B \cdot [\vec{v}]_B\right]_{B'} \qquad C_{B'}[\vec{v}]_{B'} = \vec{v} = C_B[\vec{v}]_B$$

$$C_{B'}[\bar{v}]_{B'} = C_B[\bar{v}]_B$$

$$[\bar{v}]_{B'} = C^{-1}_{B'}C_B[\bar{v}]_B$$

$$C^{-1}_{B'} \Rightarrow [C_{B'} \mid C_B] \sim [I \mid C]$$

$$C_{B\to B'}$$

by reducing the augmented matrix $[M_{B'} \mid \mathbf{b}_j]$. Thus all n coordinate vectors $(\mathbf{b}_j)_{B'}$ can be found at once by reducing the augmented matrix $[M_{B'} \mid M_B]$. But this is precisely what our boxed procedure for finding $C_{B,B'}$ calls for. We conclude that

$$C_{B,B'} = \begin{bmatrix} \vert & \vert & & \vert \\ (\mathbf{b}_1)_{B'} & (\mathbf{b}_2)_{B'} & \cdots & (\mathbf{b}_n)_{B'} \\ \vert & \vert & & \vert \end{bmatrix}. \qquad (9)$$

$T : V \to V \quad T(\vec{v}) = v$

$T_{\beta \to \beta'}(\vec{v}) = C_{\beta \to \beta'}(\vec{v})$ $\quad I$ std matrix

$[T(\vec{v}_\beta)]_{\beta'} = C_\beta I C_{\beta'}^{-1} [\vec{v}]_\beta \qquad [\vec{b}_i]_{\beta'}^T$

$C_\beta A C_\beta^{-1} \qquad [A\vec{b}_i]_{\beta'} = [T(\vec{b}_i)]_{\beta'}$

11. $T: P_2 \to P_2$ defined by $T(p(x)) = p(x+1) + p(x)$; $B = (x^2, x, 1)$, $B' = (1, x, x^2)$

$[0, 0, 2]_B$

$R_{B' \to B'} \vec{v} = [T([\vec{v}]_{B'})]_{B'}$

$= C^{-1} R_B C_{B' \to B}$

WTS: Matrices RB and RB' and an invertible matrix C, such that Rb' = inv(C) (RB) C for the linear transformation T

$R_B = R_{B \to B} = T_{B \to B}$

$R_{B' \to B'} = C^{-1} R_B C$

$R_B = \begin{bmatrix} T(\vec{b}_1)_B & \cdots & T(\vec{b}_3)_B \end{bmatrix}$

$C_{B' \to B} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$(x+1)^2$

x^2+2x+1

$(x+)+x$

$2x+1$

$= \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

$($+

$R_{B'} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

$p(x) = 1$

$T(p(x)) =$

T: V - > W
V has dimension m, then V is spanned by B, B = (b1, ... , bm)

$$A = \begin{bmatrix} | & & | \\ [b_1]_B & \cdots & [b_m]_B \\ | & & | \end{bmatrix}_{m \times m}$$

$$\mathrm{Ker}(T) = \mathrm{Null}(A)$$

$$\mathrm{Image}(T) = \mathrm{Colmn}(A)$$

$T_{B \to B'}$ $C_{B \to B'}$

A is diagonalizable if all of the eigen value's algebraic multiplicity sum to n
https://pub.math.leidenuniv.nl/~strengtc/la1na_2017/fraleigh.pdf