Lecture Notes

Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, Feb 25th, 1:00pm - 3:00pm

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1 Trig. Substitution

Guidelines for a Trig Substitution

1. Notice one of the following forms: $a^2 + u^2, a^2 - u^2, u^2 - a^2$
2. Simplify and integrate the resulting integral
3. If the integral is indefinite, then recover back the original variable of integration

Example
use trig substitution to evaluate $\int_0^4 \frac{x^3}{\sqrt{14+x^2}}dx$, let $x = 4\tan(\theta), \theta \in (\pi/2, \pi/2)$, $dx = 4\sec^2(\theta)d\theta$
$4 + x^2 = 4 + (4\tan\theta)^2 = 4^2\sec^2(\theta)$
$x = 0 \Rightarrow \theta = 0$
thus $\int_0^4 \frac{x^3}{\sqrt{14+x^2}}dx = \int_0^{\pi/4} \frac{(4\tan\theta)^3}{\sqrt{(4\sec(\theta))^2}} \cdot 4\sec^2(\theta)d\theta = \int_0^{\pi/4} \frac{(4\tan\theta)^3}{4\sec(\theta)} \cdot 4\sec^2(\theta)d\theta$
$= 4^3 \int_0^{\pi/4} \tan^3(\theta)\sec(\theta)d\theta = 4^3 \int_0^{\pi/4} \tan^2(\theta)\tan(\theta)\sec(\theta)d\theta = 4^3 \int_0^{\pi/4} (\sec^2(\theta) - 1)\tan(\theta)\sec(\theta)d\theta$
let $u = \sec(\theta), du = \sec(\theta)\tan(\theta)d\theta$
$= 4^3 \int_1^{\sqrt{2}} (u^2 - 1)du = 4^3 \left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}} = \frac{64}{3}(-\sqrt{2} + 2)$

Homework
Evaluate $\int \frac{1}{1+x^2}dx$

2 Improper Integrals

in the past, we have assumed that $\int_a^b f(x)dx$ is

1. $[a, b]$ is bounded
2. f has no Vertical Asymmptotes

if 1 or 2 fails,it's an improper integral. Our methods on integration does not hold, FTOC does not apply.
$\int_1^\infty \frac{\tan^{-1}(x)}{1+x^2}dx, \int_2^3 \frac{8}{\sqrt{x-2}}dx, \int_{\pi/2}^{\pi} \csc(x), \int_{-1}^{1} \frac{1}{x^2}dx, \int_{-\infty}^{0} \arctan(x)dx$
Example
Evaluate $\int_1^\infty \frac{1}{(3x+1)^2}dx = \lim_{A\to\infty} \int_1^A \frac{1}{(3x+1)^2}dx$, let $u = 3x + 1, \frac{du}{3} = dx$
we have $\lim_{A\to\infty} \int_4^{3A+1} \frac{1}{3u^2} = \lim_{A\to\infty} \frac{1}{3} \int_4^{3A+1} \frac{1}{u^2} = \lim_{A\to\infty} -\frac{1}{3}\left(\frac{1}{3A+1} - \frac{1}{4}\right) = \frac{1}{12}$ Converges to 1/12.

Evaluate $\int_0^5 \frac{\ln(x)}{x}dx = \lim_{A\to 0^+} \int_A^5 \frac{\ln(x)}{x}dx$
let $u = \ln x, du = \frac{1}{x}dx$, we have $\lim_{A\to 0^+} \int_{\ln A}^{\ln(5)} udu = \lim_{A\to 0^+} \left[\frac{\ln^2 x}{2}\right] = \lim_{A\to 0^+} \left[\frac{\ln^2 5}{2} - \frac{\ln^2 A}{2}\right] \approx -\infty$
thus $\int_0^5 \frac{\ln(x)}{x}dx$ does not converge.

Find $\displaystyle\int_{-1}^{1} x^{-2} = \lim_{A\to 0^\pm}\left(\int_{-1}^{A} x^{-2} + \int_{A}^{1} x^{-2}\right)$

Consider $\displaystyle\lim_{A\to 0^+}\int_{A}^{1} x^{-2} = \lim_{A\to 0^+} -\left[1-\frac{1}{A}\right] \approx \infty$, Note $x^{-2} > 0, x \in [-1, 0]$ thus $\displaystyle\int_{-1}^{0} x^{-2}dx \neq -\infty$

thus $\displaystyle\int_{-1}^{1} x^{-2}$ diverges.

Find $\displaystyle\int_{-\infty}^{1} \frac{1}{x-1}dx = \int_{-\infty}^{0} \frac{1}{x-1}dx + \int_{0}^{1} \frac{1}{x-1}dx$

Consider $\displaystyle\int_{0}^{1} \frac{1}{x-1}dx = \lim_{A\to 1^-}\int_{0}^{A} \frac{1}{x-1}dx = \lim_{A\to 1^-}[\ln|A-1| - \ln|1|] \approx -\infty$

note that $x \in (-\infty, 0], x-1 < 0 \Rightarrow \dfrac{1}{x-1} < 0$

So $\displaystyle\int_{-\infty}^{0} \frac{1}{x-1}dx \neq \infty$

thus $\displaystyle\int_{-\infty}^{1} \frac{1}{x-1}dx$ diverges.