Course Notes

Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

Instructor:

Dr. Kathleen Smith

Email: smithk@utsc.utoronto.ca

Office: IC458

Office Hours: TBA

1 Chapter 4; Sigma Notation

Definition.
$\sum$ - Notation
Sigma notation is convenient way to express a sum of a collection of objects that take on a common form.
If we have a_k and b_k that are real-valued fcns(functions),then $a_m + a_{m+1} \cdots + a_k \cdots + a_n = \sum_{k=m}^{n} a_k$

a_k - General Term
k - index(letter doesn't matter)
m, n - inital index value and final index value

Example.
Express $-1 + 2 - 3 + 4 - 5 + \cdots - 21$ in $\sum$ - notation.
$\sum_{k=1}^{21}(-1)^k \cdot k = \sum_{k=0}^{20}(-1)^{k+1} \cdot (k+1)$
Theorem. Pg(319 - 320)
Properties of $\sum$ - Notation
consider $m, l, k \in \mathbb{Z}^+, \ni: 1 \leq k \leq m, l < m.$ and let $c \in \mathbb{R}$
If we have a_k and b_k that are real-valued fcns(functions), then:

$$\text{(i)} \sum_{k=1}^{m}(a_k + b_k) = \sum_{k=1}^{m} a_k + \sum_{k=1}^{m} b_k$$
$$\text{(ii)} \sum_{k=1}^{m} c \cdot a_k = c \cdot \sum_{k=1}^{m} a_k$$
$$\text{(iii)} \sum_{k=1}^{m} a_k = \sum_{k=1}^{l-1} a_k + \sum_{k=l}^{m}$$

Proofs.

(i) Suppose that a_k and b_k are real valued functions $(a_k, b_k \in \mathbb{R})$, let $m, k \in \mathbb{Z}^+$

WTS $\displaystyle\sum_{k=1}^{m}(a_k + b_k) = \sum_{k=1}^{m} a_k + \sum_{k=1}^{m} b_k$

Consider $\displaystyle\sum_{k=1}^{m}(a_k + b_k)$

$$\sum_{k=1}^{m}(a_k + b_k) = a_1 + a_2 \cdots + a_m + b_1 + b_2 \cdots + b_m$$

$$\sum_{k=1}^{m}(a_k + b_k) = a_1 + b_1 + a_2 + b_2 \cdots + a_m + b_m = \sum_{k=1}^{m} a_k + \sum_{k=1}^{m} b_k$$

(ii) Suppose that a_k is a real valued function $(a_k \in \mathbb{R})$, let $c \in \mathbb{R}, m, k \in \mathbb{Z}^+$

WTS $\displaystyle\sum_{k=1}^{m} c \cdot a_k = c \cdot \sum_{k=1}^{m} a_k$

Consider $\displaystyle\sum_{k=1}^{m} c \cdot a_k$

$$\sum_{k=1}^{m} c \cdot a_k = c \cdot a_1 + c \cdot a_2 \cdots + c \cdot a_m = c(a_1 + a_2 \cdots + a_m) = c \cdot \sum_{k=1}^{m} a_k$$

(iii) Suppose that a_k is a real valued function $(a_k \in \mathbb{R})$, let $l, m, k \in \mathbb{Z}^+, \ni: 1 \leq l < m < k$

WTS $\displaystyle\sum_{k=1}^{m} a_k = \sum_{k=1}^{l-1} a_k + \sum_{k=l}^{m}$

Since $1 \leq l < m < k$, $\displaystyle\sum_{k=1}^{m} a_k = a_1 + a_2$

Example

evaluate $\sum_{i=0}^{203}(2i-1)$.

$$\begin{aligned}
\sum_{i=0}^{203}(2_{i-1}) &= \sum_{i=1}^{203}2i-\sum_{i=1}^{203}1 \text{ by } \sum \text{ - property (i)} \\
&= 2\cdot\sum_{i=1}^{203}i-\sum_{i=1}^{203}1 \text{ by } \sum \text{ - property (ii)} \\
&= \frac{2(203)(204)}{2}-203 \text{ By definition of geometric series} \\
&= 41209
\end{aligned}$$

evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{5}{n^4(k^3+1)}$ using Sigma notation properties

Sol'n

$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{5}{n^4}(k^3+1)=\frac{5}{n^4}\cdot\sum_{k=1}^{n}(k^3+1)=\frac{5}{n^4}\cdot(\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}1)$