

Lecture Notes
Winter 2019

MATA37 - CALCULUS II FOR THE MATHEMATICAL SCIENCES

LEC03, Jan 25th, 2:00pm - 3:00pm



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1 FTOC - Part I continued

Recall that FTOC - Part I states:

Let $a, b \in \mathbb{R}, a < b$.

IF f is continuous on $[a, b]$, and F is any antiderivative of f on $[a, b]$

THEN $\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$

ALSO $\int_a^b F'(x)dx = \int_a^b f(x)dx$

Proof

Let $P = \{x_i\}_{i=0}^n$ be a Riemann partition of $[a, b]$ such that

$$\Delta x = \frac{b-a}{n}, x_i = a + i\Delta x$$

So by Riemann Definition, $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=0}^n f(x_i^*)\Delta x, x_i^* \in [x_{i-1}, x_i]$

Now let's start the proof

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=0}^n f(x_i^*)\Delta x, x_i^* \in [x_{i-1}, x_i]$$

Recall $F'(x) = f(x), \forall x \in [a, b]$ (F is differentiable on $[a, b]$)

differentiable implies continuity, thus F is also continuous.

In particular, F is diff. on each $(x_{i-1}, x_i) \subset [a, b]$

also F is continuous on each $[x_{i-1}, x_i] \subset [a, b]$

that this satisfies the conditions for the Mean Value Theorem

By MVT, $\exists c_i \in (x_{i-1}, x_i), \ni: F'(c) = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}$

$$F'(x) \cdot (x_i - x_{i-1}) = F(x_i) - F(x_{i-1})$$

$$f(c_i) \cdot \Delta x = F(x_i) - F(x_{i-1})$$

Since we know that $c_i \in (x_{i-1}, x_i)$

Chose $x_i^* = c_i$

$$\lim_{n \rightarrow \infty} \sum_{i=0}^n f(c_i)\Delta x, c_i \in [x_{i-1}, x_i]$$

$$\lim_{n \rightarrow \infty} \sum_{i=0}^n (F(x_i) - F(x_{i-1}))$$

Now we have

$$\begin{aligned} \int_a^b f(x)dx &= \lim_{n \rightarrow \infty} (F(x_1) - F(x_0)) + (F(x_2) - F(x_1)) \cdots + (F(x_{n-1}) - F(x_{n-2})) + (F(x_n) - F(x_{n-1})) \\ &= \lim_{n \rightarrow \infty} -F(x_0) + F(x_n) \\ &= F(x_n) - F(x_0) \end{aligned}$$

$$\int_a^b f(x)dx = F(a) - F(b) \text{ By our Riemann Partition}$$

QED

2 FTOC - Part II

see Pg 390

Let $a, b \in \mathbb{R}, a < b$

IF f is cont on $[a, b]$, define $F(x) = \int_a^x f(t)dt, x \in [a, b]$

THEN F is cont on $[a, b]$ and F is differentiable on (a, b) , Moreover $F'(x) = f(x), \forall x \in [a, b]$

This means F is an antiderivative of f on $[a, b]$

$$\frac{dF}{dx} = \frac{d}{dx} \left(\int_a^x f(t)dt \right) = f(x)$$

This theorem is used whenever we are asked to differentiate a function defined by an integral

Example

Compute

$$\frac{d}{dx} \left(\int_x^4 \sqrt{1+t^4} dt \right)$$

SOLN: $f(t) = \sqrt{1+t^4}$ is cont on \mathbb{R} because $1+t^4 \geq 0, \forall t \in \mathbb{R}$

In particular F is cont on $[x, 4] \subset \mathbb{R}$

$$\text{So } \frac{d}{dx} \left(\int_x^4 \sqrt{1+t^4} dt \right) = \frac{d}{dx} \left(\int_4^x (-1) \sqrt{1+t^4} dt \right) = -f(x) = -\sqrt{1+x^4}$$