MAT A29 TUT0018, Tutorial 6 (Week 8)
Tuesdays 7 - 9pm (We will start at 7:10pm)

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$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

Question

Show that $F(x) = 5x^3 + 2x^2 + 3x + 1$ *is an antiderivative of* $f(x) = 15x^2 + 4x + 3$.

translates to show that the derivative of F(x) , is g(x)

F'(x) = 3 5x^2 + 4 x + 3 = f(x)

Question

Find all antiderivatives of $f(x) = e^x - 3x^2 + \sin(x)$.

$$\int e^x - 3x^2 + \sin(x) \quad dx$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\int e^x\,dx - 3\int x^2\,dx + \int \sin(x)\,dx$$

$$\frac{d}{dx} - \cos x = \sin x$$

$$\downarrow$$

$$e^x - 3\frac{x^{2+1}}{2+1} - \cos x = e^x - x^3 - \cos x + C$$

Question

Find all antiderivatives of $f(x) = (\sqrt{x})^3$.

$$f(x) = \left(x^{\frac{1}{2}}\right)^3 = x^{3/2}$$

$$\int f(x)\,dx = \frac{x^{3/2+1}}{3/2+1} = \frac{x^{5/2}}{\frac{5}{2}} = \frac{2}{5}x^{5/2} + C$$

Theorem

1. *If* $F(x) = \int_a^x f(t)dt$ *then* $F'(x) = f(x)$.
2. $\int_a^b f(t)dt = F(b) - F(a)$ *for any antiderivative* $F(x)$ *of* $f(x)$.

Question

Use the Fundamental Theorem of Calculus (Part 1) to find the derivative:

$$\frac{d}{dx}\int_4^x \frac{dt}{\sqrt{16-t^2}}$$

$$\frac{d}{dx}\int_4^x \frac{1}{\sqrt{16-t^2}}\,dt = \frac{1}{\sqrt{16-x^2}}$$

$$\frac{d}{dx}\left(F(x)+C\right)$$

$$= F'(x)$$

Question

Use the Fundamental Theorem of Calculus (Part 1) to find the derivative:

$$\frac{d}{dx}\int_0^{\sqrt{x}} t\,dt$$

Notice the $\sqrt{x}$ up top.

$$F(t) = \int t^1 \, dt = \frac{t^2}{2} + C$$

$$\frac{d}{dx}\int_0^{\sqrt{x}} t \, dt = \frac{d}{dx}\left(F(\sqrt{x}) - F(0)\right)$$

$$= \frac{(\sqrt{x})^2}{2} - \frac{0}{2} = \frac{x}{2}$$

Question

Calculcate the following:

$$\partial = \int_{-2}^{3} x^2 + 3x - 5dx$$

$$F(x) = \int x^2 + 3x - 5 \, dx$$

$$= \frac{x^3}{3} + 3 \cdot \frac{x^2}{2} - 5x + C$$

$$\int_{-2}^{3} \partial = F(3) - F(-2)$$

$$= \frac{35}{3} + \frac{15}{2} - 25 = -\frac{35}{6}$$