# Accelerating Haskell Array Codes with Multicore GPUs

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## Abstract

Current GPUs are massively parallel multicore processors optimised for workloads with a large degree of SIMD parallelism. Good performance requires highly idiomatic programs, whose development is work intensive and requires expert knowledge.

To raise the level of abstraction, we propose a domain-specific high-level language of array computations that captures appropriate idioms in the form of collective array operations. We embed this purely functional array language in Haskell with an online code generator for NVIDIA's CUDA GPGPU programming environment. We regard the embedded language's collective array operations as algorithmic skeletons; our code generator instantiates CUDA implementations of those skeletons to execute embedded array programs.

This paper outlines our embedding in Haskell, details the design and implementation of the dynamic code generator, and reports on initial benchmark results. These results suggest that we can compete with moderately optimised native CUDA code, while enabling much simpler source programs.

***Categories and Subject Descriptors*** D.3.2 [*Programming Languages*]: Language Classification—Applicative (functional) languages; Concurrent, distributed, and parallel languages; D.3.4 [*Programming Languages*]: Processors—Code generation

***General Terms*** Languages, Performance

***Keywords*** Arrays, Data parallelism, Dynamic compilation, GPGPU, Haskell, Skeletons

## 1. Introduction

The current generation of *graphical processing units (GPUs)* are massively parallel multicore processors. They are optimised for workloads with a large degree of SIMD parallelism and good performance depends on highly idiomatic programs with low SIMD divergence and regular memory-access patterns. Hence, the development of *general-purpose GPU (GPGPU)* programs is work intensive and requires a substantial degree of expert knowledge.

Several researchers proposed to ameliorate the status quo by either using a library to compose GPU code or by compiling a subset of a high-level language to low-level GPU code [1, 6, 19, 22, 24,

25]. Our work is in that same spirit: we propose a domain-specific high-level language of array computations, called *Accelerate*, that captures appropriate idioms in the form of parameterised, collective array operations. Our choice of operations was informed by the *scan-vector model* [11], which is suitable for a wide range of algorithms, and of which Sengupta et al. demonstrated that these operations can be efficiently implemented on modern GPUs [30].

We regard Accelerate's collective array operations as algorithmic skeletons that capture a range of GPU programming idioms. Our dynamic code generator instantiates CUDA implementations of these skeletons to implement embedded array programs. Dynamic code generation can exploit runtime information to optimise GPU code and enables on-the-fly generation of embedded array programs by the host program. Our code generator minimises the overhead of dynamic code generation by caching binaries of previously compiled skeleton instantiations and by parallelising code generation, host-to-device data transfers, and GPU kernel loading and configuration.

In contrast to our earlier prototype of an embedded language for GPU programming [23], Accelerate programs are not wrapped in a monad to represent sharing; instead sharing is recovered automatically (see Section 3.3). Moreover, Accelerate supports rank polymorphism in the manner of the *Repa* system [21] and uses a refined skeleton-based code generator, which we discuss in detail in the present paper.

In summary, our main contributions are the following:

- An embedded language, *Accelerate,* of parameterised, collective array computations that is more expressive than previous GPGPU proposals for Haskell (Section 2).

- A code generator based on CUDA skeletons of collective array operations that are instantiated at runtime (Section 3 & 4).

- An execution engine that caches previously compiled skeleton instances and host-to-device transfers as well as parallelises code generation, host-to-device data transfers, and GPU kernel loading and configuration (Section 5).

- Benchmarks assessing runtime code generation overheads and kernel performance (Section 6)

We discuss related work in Section 7. Our current implementation[1] targets CUDA, but the same approach would work for OpenCL.

## 2. Accelerated array code

In the following, we illustrate Accelerate at two simple examples after briefly introducing our array types.

---

[1] http://hackage.haskell.org/package/accelerate

```
data Z             = Z            — rank-0
data tail :. head = tail :. head — increase rank by 1

type DIM0 = Z
type DIM1 = DIM0 :. Int
type DIM2 = DIM1 :. Int
type DIM3 = DIM2 :. Int
⟨and so on⟩

type Array DIM0 e = Scalar e
type Array DIM1 e = Vector e
```

**Figure 1.** Types of array shapes and indices.

## 2.1 Arrays, shapes, and indexing

Parallelism in Accelerate takes the form of collective array operations over types `Array sh e`, where `sh` is the *shape* and `e` the element type of the array. Following the approach taken by the *Repa* array library, we represent both the shapes and indices of an array using an inductive notation of tuples as heterogenous *snoc* lists [21, Section 4.1] to enable rank-polymorphic definitions of array functions.

As shown in Figure 1, on both the type-level and the value-level, we use the constructor `Z` to represent the shape of a rank-0 array and the infix operator `(:.)` to increase the rank by adding a new dimension to the right of the shape. Thus, a rank-3 index with components `x`, `y` and `z` is written `(Z:.x:.y:.z)` and has type `(Z:.Int:.Int:.Int)`.

Overall, an array of type `Array (Z:.Int:.Int) Float` is a rank-2 array of single-precision floating point numbers. Figure 1 also defines synonyms for common types: a singleton array of shape `DIM0` represents a scalar value; an array of shape `DIM1` is a vector, and so on. It might appear as if the explicit mentioning of the type `Int` for each dimension is redundant. However, as explained in [21], we need indices over types other than `Int` for rank-polymorphic functions that replicate a given array into one or more additional dimensions or slice a lower-dimensional subarray out of a larger array.

## 2.2 Arrays on the host and device

Accelerate is an *embedded language* (aka *internal language*) [18] that distinguishes between vanilla Haskell arrays (in CPU host memory) and embedded arrays (in GPU device memory) as well as computations on both flavours of arrays. Embedded array computations need to be explicitly executed before taking effect; they are identified by types formed from the type constructor `Acc`, which is parameterised by a tuple of one of more arrays that are the result of the embedded computation. The following function embeds a Haskell array into an embedded array computation, implying a host-to-device memory transfer:

```
use :: (Shape sh, Elt e)
    => Array sh e -> Acc (Array sh e)
```

The type classes `Shape` and `Elt` characterise the types that may be used as array indices and elements, respectively. We already discussed the structure of array indices previously. Array elements can be signed & unsigned integers (8, 16, 32, and 64-bit wide) and floating point numbers (single & double precision). Moreover, `Char`, `Bool`, and tuples of all those; array indices formed from `Z` and `(:.)` may be used as array elements as well.

In the context of GPU programming, the distinction between regular arrays of type `Array sh e` and arrays of the embedded language `Acc (Array sh e)` has the added benefit that it differentiates between arrays allocated in host memory and arrays allocated in GPU device memory. Consequently, `use` implies host-to-device data transfer. Similarly, computations in `Acc` are executed on the device, whereas regular Haskell code runs on the host.

## 2.3 Computing a vector dot product

Consider computing the dot product of two vectors, using standard Haskell lists:

```
dotp_list :: [Float] -> [Float] -> Float
dotp_list xs ys = foldl (+) 0 (zipWith (*) xs ys)
```

The two input vectors are multiplied pointwise and the resulting products are summed, yielding a scalar result.

Using Accelerate, we implement this computation as follows:

```
dotp :: Vector Float -> Vector Float
     -> Acc (Scalar Float)
dotp xs ys = let xs' = use xs
                 ys' = use ys
             in
             fold (+) 0 (zipWith (*) xs' ys')
```

Here `fold` and `zipWith` are from `Data.Array.Accelerate`, rather than from `Data.List`. The Accelerate code differs from the list version in three respects: (1) the result is an Accelerate computation, indicated by `Acc`; (2) we lift the two plain vectors `xs` and `ys` into `Acc` with `use`; and (3) we use `fold` instead of `foldl`.

The first two points are artefacts of lifting the computation into the embedded language, effectively delaying it. Concerning point (3), `foldl` guarantees a left-to-right traversal, whereas `fold` leaves the order in which the elements are combined unspecified and requires an associative binary operation for deterministic execution.[2] This allows for a parallel tree reduction [11, 30].

## 2.4 Array computations versus scalar expressions

The signatures of the two operations `zipWith` and `fold`, used in `dotp`, are given in Table 1 (which omits the `Shape` and `Elt` context for brevity). They follow the signatures of the corresponding list functions, but with all arrays wrapped in `Acc`. In addition to `Acc`, which marks embedded array computations, we also have `Exp`, which marks *embedded scalar* computations; a term of type `Exp Int` represents an embedded expression yielding a result of type `Int` and `Exp Float -> Exp (Float, Float)` characterises an embedded function that takes an argument of type `Float` to a result of type `(Float, Float)`. Computations embedded in `Exp` are, just like those embedded in `Acc`, executed on the device.

Compared to regular Haskell, `Exp` computations are rather limited, to meet the restrictions on what can be efficiently executed on GPUs. For example, to avoid excessive SIMD divergence, we do not support any form of recursion or iteration in `Exp`, as `Acc` computations issue many parallel instances of an `Exp` computation.

Accelerate distinguishes the types of collective and scalar computations —`Acc` and `Exp`— to achieve a stratified language. Collective operations comprise many scalar computations that are executed in parallel, but scalar computations *cannot* contain collective operations. This stratification excludes *nested, irregular* data-parallelism statically; instead, Accelerate is limited to *flat data-parallelism* involving only regular, multi-dimensional arrays.

## 2.5 Sparse-matrix vector multiplication

As a second example, consider the multiplication of a sparse matrix with a dense vector. Let us represent sparse matrices in the *compressed row format (CSR)* [11]—i.e., a matrix consists of an array of matrix rows that only stores non-zero elements explicitly,

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[2] Although, floating-point arithmetic is not associative, it is common to accept the resulting error in parallel applications.

but pairs them with their column index. For example, the matrix

$$\begin{pmatrix} 7 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 3 \end{pmatrix} \text{ corresponds to } [[(0, 7.0)], [], [(1, 2.0), (2, 3.0)]]$$

in compressed row representation (note that we start indexing with 0). Since we don't directly support nested parallelism, we cannot represent the sparse matrix as an array of arrays. Instead, we use a second array to store the number of non-zero elements of each row; we call this a *segment descriptor*. Now, we can represent our example matrix as the pair of two flat vectors:

```
([1,0,2],                    -- segment descriptor
 [(0,7.0),(1,2.0),(2,3.0)])  -- index-value pairs
```

We use the following type synonyms for sparse matrices:

```
type Segments       = Vector Int
type SparseVector a = Vector (Int, a)
type SparseMatrix a = (Segments, SparseVector a)
```

`Segments` is the type of segment descriptors. A `SparseVector` is a vector of pairs of index positions and values. We combine both to represent a sparse matrix.

Now we can define a sparse-matrix vector product as follows:

```
smvm :: SparseMatrix Float -> Vector Float
     -> Acc (Vector Float)
smvm (segd', smat') vec'
  = let segd          = use segd'
        (inds, vals) = unzip (use smat')
        vec          = use vec'
        --
        vecVals =
          backpermute (extent inds)
                       (λi -> index1 $ inds ! i)
                       vec
        products = zipWith (*) vecVals vals
    in
    foldSeg (+) 0 products segd
```

The function `index1` has type `Exp Int -> Exp Z:.Int`, and converts an `Int` expression into an index expression for a rank-1 Accelerate array. The function `backpermute` extracts those values from the `vec'` that have to be multiplied with the corresponding non-zero matrix values in `smat'`. Then, `zipWith` performs the multiplications and `foldSeg` adds all the products that correspond to the values of one row. See [10] for a more detailed explanation of the algorithm.

### 2.6 Accelerate in a nutshell

Scalar Accelerate expressions of type `Exp e` support Haskell's standard arithmetic operations by overloading the standard type classes `Num`, `Integral`, etc, as well as bitwise operations from `Data.Bits`. Moreover, they support equality and comparison operators as well as logical connectives using the operators `==*`, `/=*`, `<*`, `<=*`, and so on. (We cannot overload on `Bool` in standard Haskell.) We also have conditional expressions of the form `c ? (t, e)`, which evaluate to `t` if `c` yields `True`; otherwise, to `e`. Moreover `arr!ix` indexes an array and `shape` queries an array's extent. Finally, we have tupling, untupling, and auxiliary functions to compute with indices.

Table 1 summarises the supported collective array operations. The operations are largely self-explanatory. The function `fold` is *shape polymorphic*—it requires an argument of at least rank-1. The two functions `replicate` and `slice` are shape polymorphic as well, but require more complex shape constraints that we characterise with the type families `FullShape` and `SliceShape` to stat-

ically track changing array dimensions. For details on the various forms of shape polymorphism, see [21]. Overall, the collective operations of `Accelerate` are a multi-dimensional variant of those underlying the scan-vector model [11, 30]

## 3. Embedding array computations

Next, we shall illustrate the embedding of Accelerate and its compilation to CUDA code at the `dotp` example. Subsequent sections will cover the CUDA backend in more detail. A more detailed discussion of the frontend is beyond the scope of this paper.

### 3.1 The surface language

The operations of the embedded language (of both the array and scalar sub-language) do not directly issue any computations; instead, they build term trees to represent embedded computations. These trees use *higher-order abstract syntax (HOAS)* to embed function-valued scalar expressions as well as typeclass overloading to reflect arithmetic expressions. For example, the body of the `dotp` function, `fold (+) 0 (zipWith (*) xs' ys')`, turns into

```
Fold add (Const 0) (ZipWith mul xs' ys')
where
  add = λx y -> PrimAdd (⟨elided type info⟩)
               `PrimApp`
               Tuple (NilTup `SnocTup` x
                            `SnocTup` y)
  mul = ⟨...as add, but using PrimMul...⟩
```

This is very much like the approach taken by Elliott [13], Gill et al. [14], and Mainland and Morrisett [24]. A difference of our approach is that we maintain full type information of the embedded language in the term tree (not shown above to favour conciseness) and use type-preserving transformations in the frontend.

### 3.2 The structure of the frontend

Figure 2 summarises the overall architecture of Accelerate. It comprises a frontend and multiple backends that can target a variety of architectures. In this paper, we are only concerned with the CUDA-generating GPU backend; in future work, we plan to tackle optimising multicore CPU backends exploiting SIMD instructions, backends for OpenCL, and for reconfigurable hardware, such as FPGAs.

The frontend reifies embedded programs using HOAS. Afterwards, it recovers sharing and turns the HOAS representation into a *nameless de Bruijn representation*. The frontend uses GADTs [27] and type families [9, 29] to preserve the embedded program's type information. The frontend is also responsible for all backend-independent program analysis and optimisation—only very little of the latter is performed in the current implementation. Each backend maps the nameless (de Bruijn) AST to low-level code for a particular architecture and performs architecture-specific optimisations.

### 3.3 Namelessness and sharing

The HOAS representation, while convenient for the human reader, is awkward for program transformations as it complicates looking under lambdas—i.e., inspecting and manipulating the bodies of function abstractions. Hence, we convert HOAS terms into a nameless representation using typed de Bruijn indices (similar to the dependently typed lambda terms of Altenkirch and Reus [3]). The type preserving conversion using GADTs is described elsewhere [8] and was simultaneously discovered by Atkey et al. [4].

Together with the conversion to a nameless representation, we recover the sharing introduced by let-bound subterms in the embedded program. For example, consider

```
let brr = map f arr
in  zipWith g brr brr
```

| | | |
|---|---|---|
| `use` | `:: Array sh e -> Acc (Array sh e)` | embed array |
| `unit` | `:: Exp e -> Acc (Scalar e)` | create singleton array |
| `reshape` | `:: Exp sh -> Acc (Array sh' e) -> Acc (Array sh e)` | impose a new shape |
| `generate` | `:: Exp sh -> (Exp sh -> Exp e) -> Acc (Array sh e)` | array from mapping |
| `replicate` | `:: Slice slix` | replicate… |
| | `=> Exp slix -> Acc (Array (SliceShape slix) e)` | …across new… |
| | `-> Acc (Array (FullShape slix) e)` | …dimensions |
| `slice` | `:: Slice slix` | remove existing… |
| | `=> Acc (Array (FullShape slix) e) -> Exp slix` | …dimensions |
| | `-> Acc (Array (SliceShape slix) e)` | |
| `map` | `:: (Exp a -> Exp b) -> Acc (Array sh a)-> Acc (Array sh b)` | map function over array |
| `zipWith` | `:: (Exp a -> Exp b -> Exp c) -> Acc (Array sh a) -> Acc (Array sh b)` | apply function to… |
| | `-> Acc (Array sh c)` | …pair of arrays |
| `fold` | `:: (Exp a -> Exp a -> Exp a) -> Exp a -> Acc (Array (sh:.Int) a)` | tree reduction along… |
| | `-> Acc (Array dim a)` | …innermost dimension |
| `scan{l,r}` | `:: (Exp a -> Exp a -> Exp a) -> Exp a -> Acc (Vector a)` | left-to-right & right- |
| | `-> (Acc (Vector a), Acc (Scalar a))` | …to-left pre-scan |
| `permute` | `:: (Exp a -> Exp a -> Exp a) -> Acc (Array sh' a) -> (Exp sh -> Exp sh')` | Forward… |
| | `-> Acc (Array sh  a) -> Acc (Array sh' a)` | …permutation |
| `backpermute` | `:: Exp sh' -> (Exp sh' -> Exp sh) -> Acc (Array sh a) -> Acc (Array sh' a)` | Backwards permutation |

⟨in addition, we have other flavours of folds and scans as well as segmented versions of array operations, such as `foldSeg`⟩

**Table 1.** Summary of Accelerate's core array operations, omitting `Shape` and `Elt` class contexts for brevity.
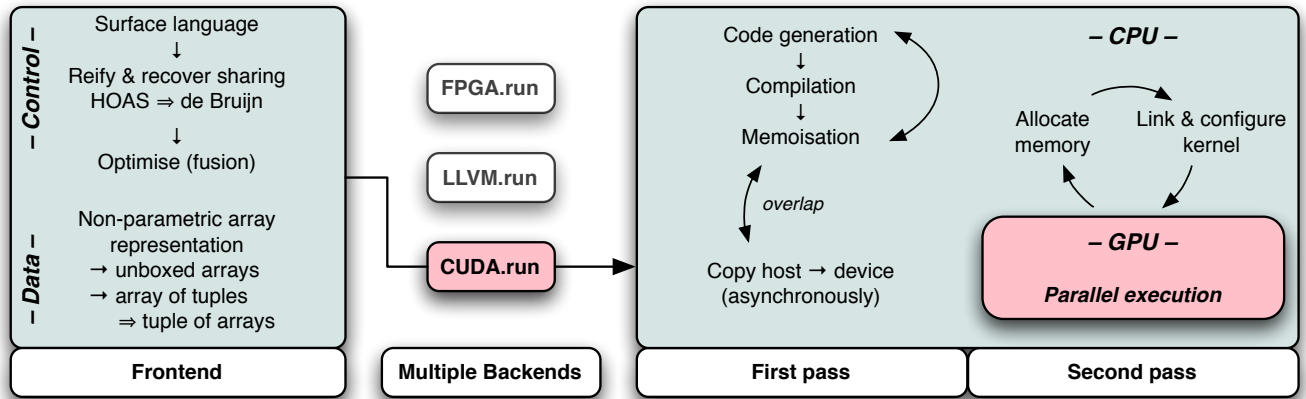


**Figure 2.** Overall structure of `Data.Array.Accelerate`.

If we don't take special care, we will translate this expression inefficiently as `zipWith g (map f arr) (map f arr)`. To recover sharing, we use a variant of Gill's technique [15]; in contrast to Gill's original work, we preserve types and produce a nested term with minimal flattening, instead of a graph.

Overall, the nameless form of `dotp` is

```
Fold add (Const 0) (ZipWith mul xs' ys')
where
  add = Lam (Lam (Body (
    PrimAdd (⟨elided type info⟩)
    'PrimApp'
    Tuple (NilTup 'SnocTup' (Var (SuccIdx ZeroIdx))
                'SnocTup' (Var ZeroIdx)))))
  mul = ⟨…as add, but using PrimMul…⟩
```

(The subterms bound to `add` and `mul` here are inline in the term, we only use a where-clause to improve readablity.) There is no sharing in this example; so, the only interesting change, with respect to the HOAS term, is the representation of functions. The `Lam` constructor introduces nameless abstractions and `Var` wraps a de Bruijn index.

At this point, the program may be further optimised by the frontend (e.g., by applying a fusion transformation), but we leave significant backend-independent optimisations for future work. As Figure 2 indicates, the frontend is also responsible for a representation transformation that is crucial for our support of arrays of $n$-ary tuples. This representation transformation has no significant impact on the `dotp` example and we defer its discussion until Section 4.

### 3.4 The structure of the CUDA backend

The right portion of Figure 2 displays the structure of our CUDA backend. It is based on the idea of parallel programming with *algorithmic skeletons* [12], where a parallel program is composed from one or more *parameterised skeletons*, or templates, encapsulating specific parallel behaviour. In our case, the various collective array computations of Accelerate constitute skeletons that are parameterised by types and scalar embedded expressions to be injected at predefined points. The skeletons for collective operations are hand-tuned to ensure efficient global-memory access and the use of fast on-chip shared memory for intra-block communication—all this is needed to ensure good use of a GPU's hardware resources [26].

```
__global__ void zipWith
(
    TyOut *          d_out,
    const TyIn1 *    d_in1,
    const TyIn0 *    d_in0,
    const int        length
){
    int ix   = blockDim.x * blockIdx.x + threadIdx.x;
    int grid = blockDim.x * gridDim.x;

    for (; ix < length; ix += grid) {
        d_out[ix] = apply(d_in1[ix], d_in0[ix]);
    }
}
```

**Listing 1.** CUDA skeleton for `zipWith` in `zipWith.inl`.

The code for `dotp` contains two such skeletons, namely `fold` and `zipWith`, which are parameterised by their non-array arguments.

As Figure 2 suggests, the CUDA backend operates in two passes, implemented as two separate traversals of the nameless AST: the first pass generates GPU device code, while simultaneously transferring the input arrays to the device[3]; and the second pass executes the various GPU kernels and manages intermediate storage on the device. A single Accelerate array computation —of type `Acc a` and executed with `CUDA.run`— will usually involve the execution of multiple GPU kernels that need to be coordinated.

### 3.5 Skeletons

Our code generator includes CUDA code implementing individual skeletons in such a manner that the code generator can instantiate each skeleton with a set of concrete parameters. As an example of this process, consider the (somewhat simplified) `zipWith` skeleton in Listing 1. The skeleton code implements a CUDA kernel [26], which encodes the behaviour of each thread for this computational pattern. The skeleton code fixes `zipWith`'s parallel algorithmic structure and contains placeholders for the element types of the input and output arrays, `TyIn0`, `TyIn1`, and `TyOut`, as well as for the function, `apply`, that is applied pairwise to the individual array elements.

The `zipWith()` function in Listing 1 is marked as `__global__` to indicate that the CUDA C compiler ought to compile it as a GPU kernel function. Each of many data-parallel GPU threads will execute the code in the listing simultaneously once the kernel is invoked. To accommodate arbitrary array sizes on hardware with varying capabilities, each GPU thread may be required to process multiple elements; this requirement is met by striding the total number of threads in each loop iteration of the `for` loop.

The use of `zipWith` in `dotp` requires the elements of the input and output arrays to be `float`. The applied function in nameless AST form is

```
Lam (Lam (Body (
   PrimMul (⟨elided type info⟩)
   'PrimApp'
   Tuple (NilTup 'SnocTup' (Var (SuccIdx ZeroIdx))
                'SnocTup' (Var ZeroIdx)))))
```

Our code generator translates this into a C function definition and bundles it with `typedef`s fixing the input and output types as follows:

```
typedef float TyOut;
typedef float TyIn1;
typedef float TyIn0;

static inline __device__
TyOut apply(const TyIn1 x1, const TyIn0 x0)
{
    return x1 * x0;
}

#include <zipWith.inl>
```

As `apply` is defined as an inline function, the CUDA C compiler, `nvcc` —invoked in the "Compilation" stage of the first pass in Figure 2— will inline it into the `zipWith` skeleton defined in the include file `zipWith.inl`; thus completing skeleton instantiation.

We use a simple memoisation technique to avoid the repeated compilation of the same skeleton instance. For each use of a collective array operation, we compute a hash value of the nameless AST and associate that hash value with the binary code generated by the CUDA C compiler from the instantiated skeleton code. If we encounter the same computation again, we reuse that binary. In the `dotp` example, we will compute a hash value for the typed AST representing `fold (+) 0` and a second hash value for the typed AST representing `zipWith (*)`. If the Haskell host program executes `dotp` multiple times, our CUDA backend will instantiate and compile the two skeletons only once. As part of the first pass, only the copying of arrays from the host to the device will be repeated. The second pass of the backend proceeds unaltered.

### 3.6 Invoking CUDA programs from Haskell

As can be seen in Listing 1, CUDA is an extension of the C programming language. It includes supports for defining GPU kernels, which is code executed in many data-parallel thread instances on multiple GPU cores. These threads are arranged in a multi-dimensional structure of *thread blocks* and *grids*—this is what `blockDim`, `blockIdx`, `threadIdx`, and `gridDim` in the listing refer to. The CUDA extension to C also includes support for compiling, loading, and executing these kernels as well as for thread synchronisation, device memory allocation, data transfer between host and device memory, and similar operations. To use these features from Haskell, we developed a binding to the CUDA C API using Haskell's foreign function interface. This binding is available as a separate Haskell library package.[4]

During the first pass of the CUDA backend, we use the Haskell CUDA binding to asynchronously transfer all input arrays to the GPU. These arrays are easily identified while traversing the nameless AST: whenever we encounter a `Use` node, we initiate the transfer of the associated array. In `dotp`, we have two occurrences of `Use`, one for each of the two input vectors. We use asynchronous data transfers to overlap code generation with host-device data transfer.

Once code generation and array transfer have completed, the CUDA backend traverses the de Bruijn AST once more. This second pass evaluates an entire array computation, invoked by `CUDA.run`, by executing the generated GPU kernels for each collective array operation of the AST bottom up. It uses functionality from the Haskell CUDA binding to allocate intermediate arrays on the device and to invoke kernels on the device passing the correct arrays, which were either transferred from the host or generated on the device by a previous kernel execution. Finally, the resulting arrays are transferred back from the device to the host.

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[3] GPUs typically have their own high-performance memory, which is separate from the host's main memory. Data transfer is by DMA (direct memory access) and needs to be explicitly managed via the CUDA runtime [26].

[4] `http://hackage.haskell.org/package/cuda`

### 3.7 Dynamic versus static code generation

An obvious difference between using Accelerate for general-purpose GPU programming and using CUDA directly is that, with Accelerate, GPU kernels are generated dynamically (i.e., at application runtime), whereas plain CUDA code will usually be pre-compiled. The main drawback of dynamically generating and compiling GPU kernels is the overhead at execution time — we will quantify that overhead in Section 6. We mitigate the overhead by memoising compiled GPU kernels, so that kernels that are invoked multiple times are only generated and compiled once.

In addition, it is only worthwhile to offload computations to a GPU if they are compute-intensive. This implies a significant runtime and usually also the use of significant amounts of input and output data. Hence, the overhead of dynamic kernel compilation is often not problematic in the face of long kernel runtimes and long data-transfer times between host and device memory. Especially, if those kernels are compiled once and executed on multiple data sets.

Finally, if all else fails, we are confident that we can use Template Haskell [31] to support precompiling Accelerate kernels. We haven't implemented this yet, but as Template Haskell executes arbitrary Haskell code at compile time, we can invoke the Accelerate CUDA backend at compile time, too.

Dynamic code generation also has significant advantages, especially for embedded languages. In particular, the code generator can query the capabilities of the hardware at the time of code generation and can optimise the generated program accordingly as well as specialise the generated code to the available input data. Finally, the host program can generate embedded programs on-the-fly.

## 4. Skeleton-based code generation

We will now expand on the general overview of the CUDA backend from the last section and discuss some of the more sophisticated features of our code generator. In particular, we will discuss support for heterogenous tuples as array elements—a feature that CUDA does not support efficiently. However, before we turn to tuples, we will have a brief look at generating CUDA C from a scalar Accelerate expression.

### 4.1 Scalar code

While instantiating skeletons, the code generator needs to translate scalar Accelerate expressions to plain C code. For the most part, this is a straightforward syntactic translation from the de Bruijn AST to C, where we use the Haskell `language-c` package[5] to first generate a C AST, which we subsequently pretty print into a file. However, the following features of the scalar fragment of Accelerate deserve particular attention: (1) lambda abstractions, (2) shapes, (3) references to arrays, and (4) tuples. We start by discussing lambda abstraction, and discuss the other three features in the following subsections.

Concerning lambda abstractions, we saw previously that from

```
Lam (Lam (Body (
    PrimMul (⟨elided type info⟩)
    ʻPrimAppʻ
    Tuple (NilTup ʻSnocTupʻ (Var (SuccIdx ZeroIdx))
                 ʻSnocTupʻ (Var ZeroIdx)))))
```

we generate the C function

```
static inline __device__
TyOut apply(const TyIn1 x1, const TyIn0 x0)
{
    return x1 * x0;
}
```

[5] http://hackage.haskell.org/package/language-c

This is possible as the two lambda abstractions are outermost; hence, we can translate them into a binary C function. Accelerate's scalar expression language is first-order in the sense that, although it includes lambda abstractions, it does not include a general application form. In other words, lambda abstractions of scalar expression can only be used as arguments to collective operations, such as `zipWith`. As a consequence, lambda abstractions are always outermost (in type correct programs) and we always translate them to plain C functions.

### 4.2 Shapes

As discussed in Section 2.2, Accelerate directly supports collective operations on multi-dimensional arrays. During code generation, we map multi-dimensional indices to `structs`.

```
typedef int32_t             Ix;
typedef Ix                  DIM1;
typedef struct { Ix a1,a0; } DIM2;
⟨and so on⟩
```

As CUDA supports the use of C++ features, such as templates and overloading, we simplify code generation by overloading functions operating on indices for the various index types—i.e., we have the following families of functions:

```
int  dim(DIMn sh);
int  size(DIMn sh);
Ix   toIndex(DIMn sh, DIMn ix);   // index into row-
                                   // major format
DIMn fromIndex(DIMn sh, Ix ix);   // invert 'toIndex'
```

The major advantage of this approach is that the CUDA skeletons are ad-hoc polymorphic; i.e., we have a single skeleton that during *instantiation* is specialised to a particular array element type and dimensionality.

### 4.3 Array references in scalar code

Accelerate includes two scalar operations that receive an array-valued argument, namely indexing (`!`) and determining the `shape` of an array. They are, for example, used in `smvm` from Section 2.5. Specifically, this code includes the following use of `backpermute`:

```
backpermute (shape inds)
            (λi -> index1 $ inds!i)
            vec
```

Here the array computation `inds :: Acc (Array DIM1 Float)` is used in the first and second argument of `backpermute`. In the code for `smvm`, `inds` is a previously let-bound variable. If instead, collective array operations would have been used in place of `inds` in the scalar function `λi -> index1 $ inds!i`, we would lift it out of the scalar function and let bind it. After all, we obviously don't want to execute an arbitrarily complex array computation once for every invocation of the scalar function. In fact, CUDA would not permit us to do that as the scalar function will turn into GPU kernel code, which cannot include further nested parallelism.

Nevertheless, even when `inds` is a let-bound variable, skeleton-based code generation is not straight forward. The prototype of the CUDA C code for the `backpermute` skeleton is as follows:

```
__global__ void backpermute (      ArrOut  d_out,
                              const ArrIn0  d_in0,
                              const DimOut  shOut,
                              const DimIn0  shIn0);
```

It has a fixed set of arguments: the output and input array as well as their shapes. To access `inds` inside the skeleton code after it has been instantiated with `λi -> index1 $ inds!i`, we would need to add `inds` as a new array-valued argument to the prototype

of `backpermute`. This is difficult to realise in our approach as the generated code simply includes the skeleton definitions using a C pre-processor `include` directive. We never rewrite skeleton code dynamically.

Fortunately, *texture references* —a CUDA feature that comes from its graphics heritage— enable an alternative solution. We can define textures (which are effectively read-only arrays) as global tables on a per skeleton instantiation basis and access them with the CUDA operation `tex1Dfetch()`. For `backpermute` in `smvm`, our code generator produces the following instantiation:

```
texture<int32_t, 1> tex0;
...
typedef DIM1 DimOut;
typedef DIM1 DimIn0;
static inline __device__
DimIn0 project(const DimOut x0)
{
    DimIn0 r = tex1Dfetch(tex0, x0);
    return r;
}
#include <backpermute.inl>
```

Here `tex0` represents the texture storing the contents of `inds` and $\lambda i \rightarrow index1 \$ inds!i$ is implemented by the function `project`, which is used at the appropriate place in the CUDA skeleton code for `backpermute` defined in `backpermute.inl`. (We omit the CUDA code for the `backpermute` skeleton due to space constraints.)

Another important reason for using texture references is that skeletons, such as `backpermute`, allow unconstrained indexing patterns, which will usually not follow CUDA's requirements for coalesced and aligned access to global memory. Global memory access can then incur severe performance penalties, whereas texture access is cached, and hence, may be more efficient in these circumstances [26].

### 4.4 Arrays of tuples

Accelerate arrays of primitive types, such as `Float` and `Int`, are easily represented in CUDA, using the corresponding floating point and integral types. More interesting are arrays of tuples. A naïve implementation of tuples in CUDA might use arrays of struct types—i.e., we might consider representing values of type `Array DIM1 (Int, Float)` in CUDA by values of type

```
typedef struct {int a; float b;} arrayIntFloat[];
```

This representation is in general not efficient as it easily violates the strict memory access rules imposed on CUDA devices, decreasing effective bandwidth by as much as an order of magnitude [26].

***Non-parametric.*** To avoid this inefficiency, Accelerate uses a non-parametric array representation: arrays of tuples are represented as tuples of arrays in CUDA. For example, values of type `Array DIM1 (Int, Float)` are represented by values of type

```
typedef struct {int a[]; float b[];} arrayIntFloat;
```

By virtue of this non-parametric array representation, Accelerate (1) maintains global memory access coalescing rules and (2) in some cases avoids redundant reads of elements that are never used.

***Getters and setters.*** While we want to represent arrays of tuples as tuples of arrays, we would still like the convenience of conventional tuples for scalars. Moreover, the CUDA skeleton code needs to abstract over array element types; otherwise, we would need large families of alike skeletons, one for each combination of argument types.

To abstract over the array representation in CUDA skeletons, we use getter and setter functions to read and write array elements.

These getter and setter functions are appropriately defined by the code generator when it generates a skeleton instance. As an example, consider this slightly contrived function:

```
samxpy :: Acc (Vector (Float, Float))
       -> Acc (Vector Float)
       -> Acc (Vector Float)
samxpy = zipWith (λx y -> 1.5 * uncurry min x + y)
```

The Haskell idiom `uncurry min` implements a minimum function on pairs. Overall, given a vector of pairs and a simple vector, `samxpy` elementwise multiplies the minimum of the pair by a constant and adds it to the corresponding element from the second array.

In this example, the first argument to `samxpy` will be represented by a pair of two float arrays and we need an appropriate getter function to read array elements from that array. During skeleton instantiation, our code generator generates the following getter function:

```
typedef struct { float  a1; float  a0; } TyIn1;
typedef struct { float* a1; float* a0; } ArrIn1;

static inline __device__
TyIn1 get1(const ArrIn1 d_in1, const Ix idx)
{
    TyIn1 x = { d_in1.a1[idx], d_in1.a0[idx] };
    return x;
}
```

In order to make use of this getter, we need to generalise the `zipWith` skeleton from Listing 1. More precisely, the body of the `for` loops needs to change from

```
d_out[ix] = apply(d_in1[ix], d_in0[ix]);
```

to

```
set(d_out, ix, apply(get1(d_in1, ix),
                     get0(d_in0, ix)));
```

which uses `get1` to access the elements of the first array—the getter `get0` and setter `set` are trivial as the second argument of `samxpy` and its result are simple arrays of `float`s. Setter and getter functions are inlined by the CUDA C compiler, eliminating any runtime overhead due to the parameterisation.

The code generator generates the following code for the function body `1.5 * uncurry min x + y`:

```
static inline __device__
TyOut apply(const TyIn1 x1, const TyIn0 x0)
{
    return (float) 1.5 * fminf(x1.a1, x1.a0) + x0;
}
```

## 5. Executing Accelerate computations

To complete the discussion of the CUDA backend, we will now have a closer look at those items of the right-hand side of Figure 2 that surround the actual code generation. In particular, we will cover (1) the management of the CUDA backend execution-state, (2) the extension of the execution state by host-to-device memory transfer and code generation, and finally (3) the actual execution of of an Accelerate computation on the host and device.

In Figure 2, these three items correspond to (1) the `CUDA.run` function, (2) the first pass, and (3) the second pass. We shall discuss each in turn.

## 5.1 `CUDA.run` and the execution state

The CUDA backend provides a single point of entry, which encapsulates the entire process of compiling and evaluating an embedded array program denoted in the Accelerate language. It is the function

```
CUDA.run :: Arrays a => Acc a -> a
```

Internally, running an Accelerate computation in the CUDA backend utilises the `StateT` monad[6] stacked over `IO`. This is necessary to use the Haskell foreign function interface (FFI) to communicate with the CUDA runtime, maintain the CUDA environment's device context, and to keep track of a range of internal resources that we discuss in the following subsections.

Nevertheless, that we are under the hood manipulating CUDA through the FFI and that we need to maintain internal state should not distract from the property that `CUDA.run` is a pure function at the user-level Accelerate API. In fact, Accelerate provides an alternative `Interpreter.run` implemented as a purely functional interpreter. We consider `Interpreter.run` as an executable specification of the denotational semantics of any Accelerate backend and use it to validate accelerated backends, such as `CUDA.run`.

## 5.2 First pass

In essence, the first pass (c.f. Figure 2) over the de Bruijn AST of a particular array computation is about setting up the execution environment; to this end, we need to ensure that (a) all data needed for evaluation is available on the device and (b) all GPU kernels needed to execute the involved collective operations are available as CUDA binaries. As both tasks can be time consuming, our backend overlaps data transfer and code generation as far as possible.

### 5.2.1 Data transfer & garbage collection

To determine the arrays whose contents need to be transferred from the host to the device, the CUDA backend extracts all `Use` subterms during the bottom-up sweep of the first pass. As part of the execution state, the CUDA backend maintains a hash table — we call it the *memory table*— that associates the host array with a reference to the device memory area that holds the copied data (once the asynchronous data transfer has been completed).

The second pass will use the memory table to obtain the appropriate device memory reference to provide as input data to the GPU kernels upon invocation. Moreover, as host-device data transfers are expensive, given the relatively high latency and low bandwidth of the PCI-E bus, we also utilise the memory table to minimise data transfer. Consider the following example:

```
square :: (Elt e, IsNum e, Shape dim)
        => Array dim e -> Acc (Array dim e)
square xs = zipWith (*) (use xs) (use xs)
```

Here we `use` a single array twice. However, device arrays can be safely shared because our skeletons do not mutate data, so we use the memory table to ensure `xs` is transferred only once.

Furthermore, the memory table uses a reference counting scheme that facilitates an efficient use of device memory. Additional `Use` occurrences simply increase the reference count of the associated device memory. When the array is consumed as input to a kernel computation, we decrement the reference count and free the device array once the reference count drops to zero.

### 5.2.2 Compilation & code memoisation

In unison with the AST traversal that extracts all `Use` subterms to initiate data transfers, the CUDA backend initiates code generation

---

[6] For details on `StateT`, see `http://hackage.haskell.org/package/monads-fd`

for each collective array operation it encounters. For each collective operation, skeleton instantiation and CUDA code generation proceeds as described in Section 4. After the CUDA code is generated, it is compiled with the external `nvcc` tool-chain to produce a CUDA binary for the instantiated skeleton.

The CUDA-backend execution state includes a second hash table —the *kernel table*— that associates a CUDA binary with the skeleton instantiation whose computation that binary implements. More precisely, CUDA binaries are keyed on a skeleton and the parameters of its instantiation—they are not keyed on the specific AST node representing that skeleton instantiation. As a result, we can easily reuse a binary when the same skeleton instantiation is required again, whether that is as the same Accelerate computation applied to a different set of input arrays, or within an entirely different computation. For example, instantiations such as `fold (+) 0` are very common, and it would be wasteful to dynamically generate the same code multiple times.

As with data transfer, compilation of the generated CUDA code proceeds asynchronously. Linking of compiled code is deferred until it is needed during execution in the second pass.

## 5.3 Second pass

The second pass implements an expression evaluator for array computations. Traversing the de Bruijn AST bottom-up, for each node it distinguishes between three cases:

1. If it is a `Use` node, the evaluator refers to the memory table to obtain a reference to the device memory holding the array data.

2. If it is a non-skeleton node (i.e., a let-binding, shape conversion, or similar), the evaluator executes it directly, by adjusting the environment or similar as required.

3. If it is a skeleton node, the evaluator refers to the kernel table to obtain the corresponding CUDA binary. It then allocates device memory to hold the result of skeleton evaluation, and finally invokes the one or more GPU kernels that implement the skeleton.

In summary, the second pass interleaves host-side evaluation and the invocation of GPU kernels, while keeping track of device memory, allocating intermediates, and releasing device arrays once no longer required (their reference count drops to zero).

### 5.3.1 Launch configuration & thread occupancy

As briefly mentioned in Section 3.6, GPU kernels are executed by a multi-dimensional hierarchy of threads. In particular, they are grouped into thread blocks, which are distributed over the available streaming multiprocessors. Unlike a CPU core, a GPU multiprocessor has no branch prediction and no speculative execution. In exchange they are capable of executing hundreds of threads concurrently; in particular, they execute small groups of threads —called *warps*— concurrently and by executing other warps when one warp is paused or stalled they can keep the ALUs busy. The ratio of active warps (i.e., warps that could run) to the maximum possible number of active warps is called *occupancy*.

Higher occupancy does not always equate to higher performance, but low occupancy always interferes with the ability to hide memory and instruction latency, resulting in suboptimal performance. Several factors influence the configuration required for maximum possible occupancy, but once a kernel has been compiled and its resource usage known, we can calculate an ideal launch configuration. Our CUDA backend does this for every kernel and it additionally limits the number of thread blocks per multiprocessor to that which can be physically resident, to make optimal use of physical resources which vary from GPU to GPU. Occupancy calculations for the `dotp` kernels are shown in Table 2.

|  | Compute Capability | | |
|---|---|---|---|
|  | 1.0, 1.1 | 1.2, 1.3 | 2.0 |
| zipWith (×) | 96 | 128 | 192 |
| fold (+) 0 | 128 | 128 | 256 |

**Table 2.** Optimum thread block size for the `dotp` kernels for different compute capabilities, and therefore hardware resources.

### 5.3.2 Kernel execution

The second pass traverses the de Bruijn AST in depth-first order. Our current implementation executes kernels on a single GPU, but the design is amenable to executing these multiple subtrees concurrently on separate GPUs, as is available in some NVIDIA Tesla configurations. Accelerate's collective operations have a purely functional semantics; as a result, such concurrent expression evaluation is always sound.

To prepare a single GPU kernel for execution —of which some collective operations comprise more than one— the array computation evaluator needs to do the following:

- Allocate device memory for the kernel result. The extent of kernel results may depend on the results of previous kernels, so allocation of intermediate storage needs to be dynamic.

- Link the CUDA binary implementing the kernel. As mentioned in Section 5.2.2, the first pass initiates both the data transfer of input arrays as well as the compilation of CUDA code asynchronously. Accordingly, the evaluator may need to wait until all inputs and the binary code for the kernel are available. The CUDA binary includes information about all resource requirements, which we use to determine the optimal configuration for the current architecture as discussed in the previous subsection.

- Invoke the CUDA runtime system. Using our Haskell binding mentioned in Section 3.6, we construct the launch configuration and function call stack, as well as bind references to free array variables represented as textures (c.f. Section 4.3).

GPU kernels execute asynchronously; hence, the evaluator can immediately begin to set up the next stage of the computation. If the result of an asynchronously executing kernel is needed before the kernel completes, the evaluator will block before invoking the kernel depending on that result.

## 6. Evaluation

In this section, we discuss the performance of some simple Accelerate programs and identify several areas for future improvement. Benchmarks were conducted on a single Tesla T10 processor (compute capability 1.3, $30 \times 1.3$GHz) backed by two quad-core Xeon E5405 CPUs (64-bit, 2GHz, 8GB RAM) running GNU/Linux.

### 6.1 Runtime overheads

Runtime code generation, kernel loading, data transfer, and so on can lead to a serious overhead for GPU computations. Our CUDA backend attempts to minimise these overheads, through caching and parallelisation, but we start by having a look at the worst case overheads in this subsection. Thereafter, we will focus exclusively on the execution time of the generated kernels.

***Data transfer.*** Communication with the GPU is limited by the relatively narrow PCI-E bus. It dominates the runtime of computationally simple operations: for a vector dot product of arrays of 18 million floats, the data transfer takes approximately 20ms per array, whereas the actual computation time is 3.5ms. Although the transfer times dominate, off-loading can still be worthwhile. Using the

| Operation | No. kernels | Code Generation (ms) | Compilation (ms) |
|---|---|---|---|
| **dotp** | 2 | 0.807 | 651 |
| **smvm** | 6 | 2.75 | 742 |
| **black-scholes** | 1 | 44.3 | 1344 |

**Table 3.** Execution time for code generation and compilation. Only the first invocation of a skeleton will incur these overheads.
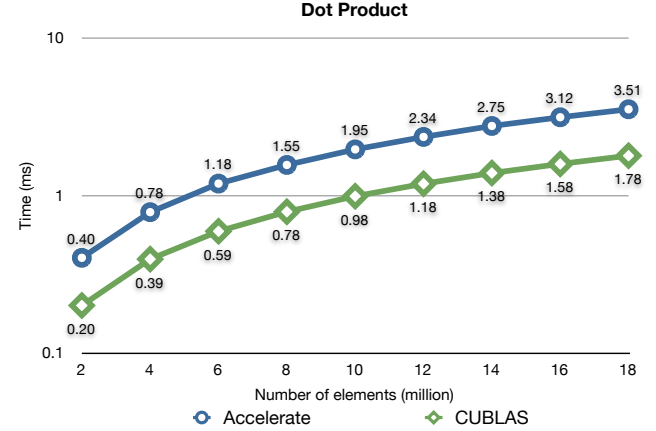


**Figure 3.** Kernel execution time for a dot product.

high-performance `vector` package,[7] a single-core CPU implementation of the same computation takes 71.3ms for 18 million floats.

***Code generation & compilation.*** For the three benchmarks discussed in the following, Table 3 displays the time needed by our CUDA backend for code generation as well as the time required by the NVIDIA CUDA compiler to generate a binary, where our backend minimises compilation time by executing several instances of the external CUDA compiler `nvcc` concurrently. The inflated times for `black-scholes` are due to the lack of sharing amongst scalar terms, which we will discuss later. It is important to remember that only the first invocation of a skeleton will incur these overheads; subsequent invocations use the cached binary.

### 6.2 Dot product

Figure 3 compares the computation of a dot product by the Accelerate CUDA backend with the CUBLAS library, a hand-written implementation of the Basic Linear Algebra Subprograms in CUDA by NVIDIA.

Accelerate takes almost precisely twice as long as CUBLAS, because CUBLAS implements the dot product as a single kernel. Accelerate uses two kernels: one for `zipWith (*)` and one for `fold (+) 0`. Consequently, it also needs to store this intermediate result in global memory on the device. The second kernel then reloads this intermediate array, which accounts for the additional overhead in his computationally simple, bandwidth-bound operation. To eliminate this overhead, Accelerate supposes to support automatic fusion of adjacent kernels; we leave this to future work.

### 6.3 Black-Scholes option pricing

The Black-Scholes algorithm is a partial differential equation for modelling the evolution of a stock option price under certain assumptions. Figure 4 compares the execution time for Accelerate with that of a reference implementation from NVIDIA's CUDA
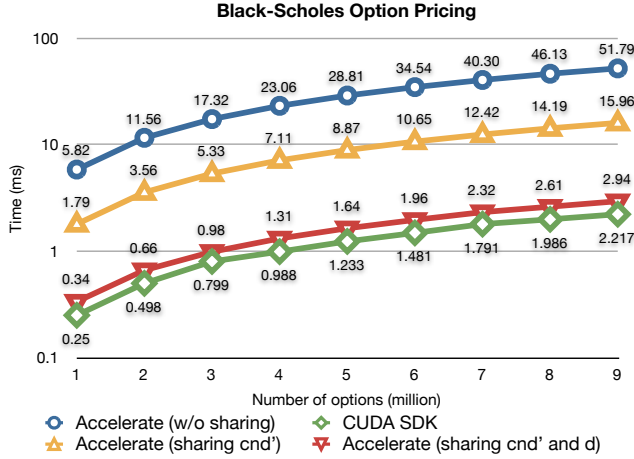
---

[7] `http://hackage.haskell.org/package/vector`

**Figure 4.** Kernel execution time of Black-Scholes option pricing, using varying amounts of sharing of sub-expressions.



**Figure 5.** Sparse-matrix vector multiplication (higher is better)

SDK. The three graphs for Accelerate demonstrate the impact of various amounts of sub-expression sharing. From the benchmark, the following computes a cumulative normal distribution:

```
cnd :: Exp Float -> Exp Float
cnd d =
    let poly = horner coeff
        k    = 1.0 / (1.0 + 0.2316419 * abs d)
        cnd' = rsqrt2 * exp (-0.5*d*d) * poly k
    in
    d >* 0 ? (1 - cnd', cnd')
```

The conditional expression d >* 0 ? (1 - cnd', cnd') results in a branch that only introduces the predicated execution of a small number of extra instructions if the computation of cnd' is shared between the two occurrences of that variable. Without sharing, the value of cnd' is computed twice, and worse, the growing number of predicated instructions leads to a large penalty on the SIMD architecture of a GPU.

Figure 4 gives three flavours of the Accelerate implementation: (1) without any sharing, (2) sharing only cnd', and (3) sharing cnd' and the argument d, used repeatedly within cnd. Flavour (2) avoids divergent branching in the conditional expression while flavour (3) additionally avoids the re-computation of d. The large discrepancy in runtime demonstrates the impact of differences in instruction count (2573 vs. 501) and of warp divergence, which serialises portions of the execution.

The currently released version of Accelerate recovers no sharing. However, we are presently completing the implementation of a variant of Gill [15]'s observable sharing—his paper also discusses the issue of sharing in embedded languages in greater detail.

### 6.4 Sparse-matrix vector multiplication

Figure 5 compares the sparse-matrix vector multiplication from Section 2.5 with the high-performance CUSP library [5]; a special purpose library for sparse-matrix operations, providing highly optimised algorithms that exploit properties in the layout of the non-zero elements. Using a 14 matrix corpus derived from a variety of application domains [34], we compare against several of the hand-optimised CUSP algorithms most similar to our implementation.

The COO format stores each non-zero element as a triple of row and column coordinate together with the data, with the kernel exhibiting complete memory access coalescing so is largely insensitive to irregularity in the underlying structure. The CSR (scalar)
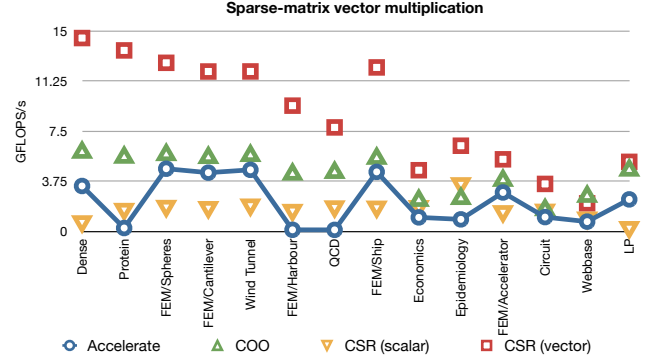
and CSR (vector) kernels store data in a compressed-row format similar to our implementation. The CSR (scalar) kernel uses one thread per matrix row, so memory access is rarely coalesced, while the CSR (vector) kernel uses a single warp per matrix row for partial access coalescing. The CSR kernels are sensitive to row length irregularity so may exhibit thread divergence. All CUSP kernels utilise the texture cache. Our implementation uses a segmented reduction method at its core that, similar to the CSR (vector) kernel, uses one warp per segment.

It comes as no real surprise that our high-level, general-purpose matrix code is slower than the manually tuned special-purpose library code of CUSP, which uses optimised data layouts and algorithms. Nevertheless, overall performance remains competitive, and is fairly consistent in terms of relative throughput across the range of matrices, generally placing between CUSP's COO and CSR (scalar). A notable exception are the Protein, FEM/Harbour, and QCD matrices, wherein our segmented reduction foldSeg is significantly slower than in the other test cases due to increased warp divergence. This arises from the skeleton component of the kernel, and we believe it can be further optimised. As in the case of the dot product, support for automatic fusion should further improve the performance of Accelerate for this benchmark.

## 7. Related work

***Haskell-based approaches.*** Vertigo, Obsidian, and Nikola are embedded languages for GPGPU programming in Haskell. Vertigo [13] is a statically compiled graphics language targeting the DirectX 8.1 shader model, whereas Obsidian [32] produces CUDA code as we do. Nikola [24] is both in its aim and approach to the embedding closest to Accelerate. Nikola's CUDA backend is, however, quite different. It explicitly schedules loops, whereas we use algorithmic skeletons. The expressiveness of Nikola's embedded language is more limited than Accelerate's as Nikola does not support generative functions, such as replicate, whose memory requirements can't be statically determined by Nikola's size inference. Moreover, Accelerate array computations can span multiple CUDA kernels, whereas both Nikola and Obsidian can only express array computations that can be implemented in a single GPU kernel. Algorithms requiring multiple kernels need to be explicitly scheduled by the application programmer and incur additional host-device data-transfer overhead—which can be rather significant as our benchmarks showed.

***C++-based approaches.*** Accelerator [6, 33] is a library-based approach with less syntactic sugar for the programmer, but support for bindings to functional languages. In contrast to our current system, it already targets multiple architectures, namely GPUs, FPAs, and multicore CPUs. However, the code generated for GPUs uses

the DirectX 9 API, which doesn't provide access to several modern GPU features, negatively effecting performance and portability. RapidMind [35], which targets OpenCL, and its predecessor Sh, are C++ meta programming libraries for data parallel programming. RapidMind was integrated into Intel's Array Buildings Blocks (ArBB) together with Intel's own Ct technology. However, the current release of ArBB seems to only support multicore CPUs, lacking GPU support. GPU++[19] employs a similar technique, but provides a more abstract interface than the other C++-based approaches listed here. Sato and Iwasaki [28] describe a C++ skeleton library including fusion optimisations. Finally, Thrust [17] is a library of algorithms written in CUDA with an interface similar to the C++ Standard Template Library.

***Others.*** PyCUDA [22] uses Python as a host language to facilitate GPU programming, while still providing access to the CUDA driver API. A similar approach is followed by CLyther [1], which targets OpenCL. Copperhead [7] uses PyCUDA and Thrust internally to provide a higher level of abstraction to compile, link, cache, and execute CUDA and C++ code.

The Jacket [2] extension to Matlab supports offloading of matrix computations to GPUs by introducing new data types for matrices allocated in device memory and overloading existing operations, such that application to GPU data types triggers the execution of suitable GPU kernels.

## 8. Conclusion

We introduced Accelerate, an embedded language of parameterised, collective array computations over multi-dimensional arrays. We outlined a skeleton-based, dynamic code generator for Accelerate that produces code that competes with moderately optimised native CUDA code in our benchmarks. We use a variety of techniques to optimise performance, such as memoising generated GPU kernels to avoid recompilation and overlapping code generation with host-to-device data transfer.

Our benchmarks indicate that a fusion optimisation combining adjacent kernels, where this is possible, would help to close the current gap between our code generator and hand-optimised CUDA code. Unfortunately, in the context of parallel programming, fusion needs to be applied with care to avoid eliminating too much of the available parallelism [10, 20]. Consequently, many standard fusion techniques cannot be applied directly. Fusion could be implemented by matching on a fixed set of combinations of collective operations, but this quickly leads to a combinatorial explosion of the required optimisation rules. We will investigate a suitable fusion framework for Accelerate in future work.

Although our current implementation specifically builds on CUDA, our approach is not limited to that framework. In particular, we are convinced that we could re-target our code generator to OpenCL [16] by rewriting the skeleton code and by replacing our binding to the CUDA runtime by a binding to the OpenCL runtime.

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