

# Perf Eval of Comp Systems

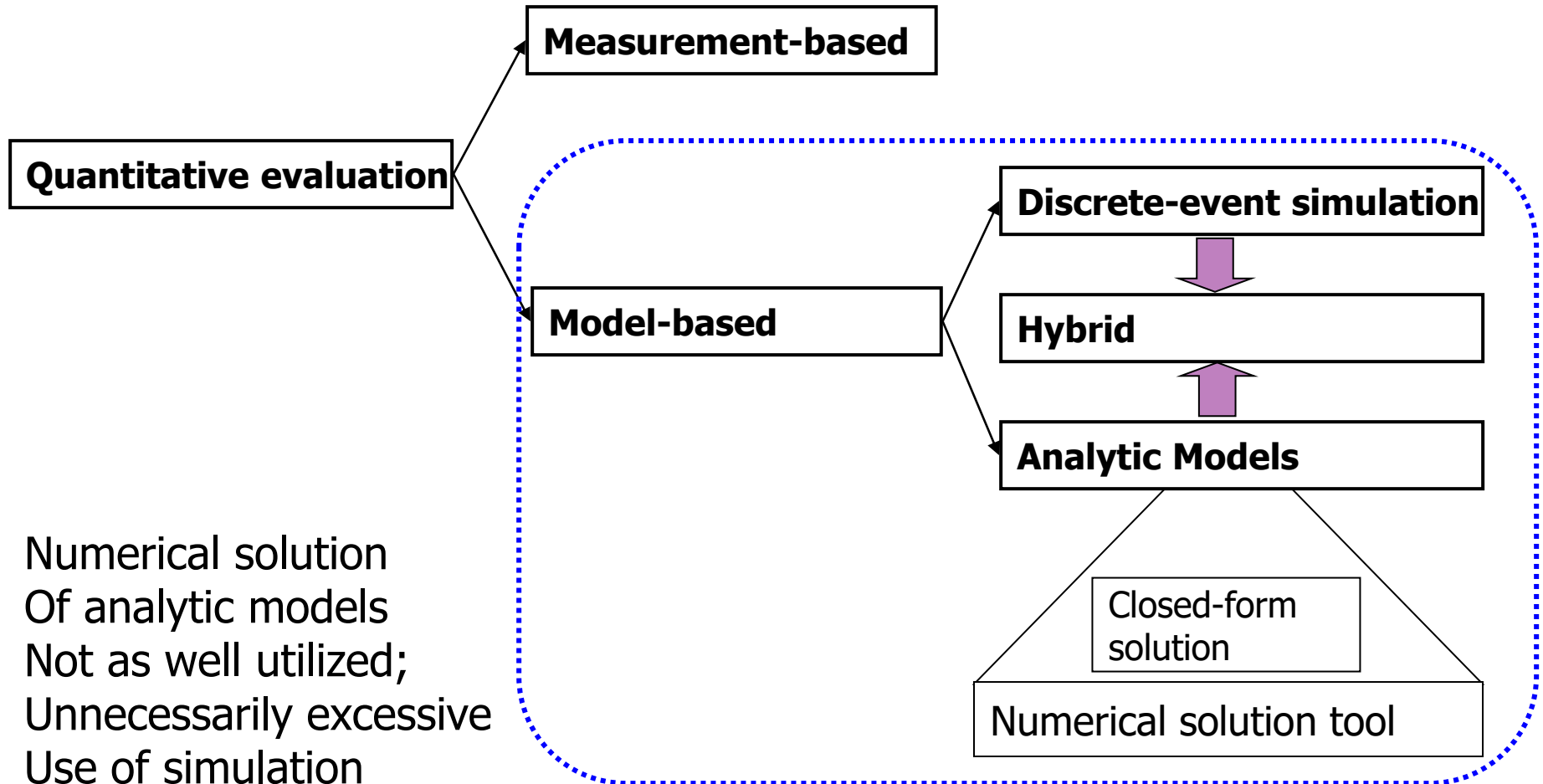
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Lect001 intro

# Evaluation Methods



# Reminder of Probability

## OUTLINE:

- Outcomes and events;
- Definitions of probability;
- Probability algebra;
  - Adding events
  - Conditional probability
  - Multiplication of events
- Measure of dependence between events;

# Reminder of Random Variables

## OUTLINE:

- Definitions
  - (measure theoretic)
  - Classic
- Full descriptors(PDF, pdf, pmf)
  - Discrete RV
  - Continuous RV
  - mixed RVs
- parameters (summaries):
  - mean;
  - variance;
  - skewness;
  - excess
- System of RVs: jointly distributed RVs
  - Conditional distributions and Mean (we saw Cond. Prob. Before)
  - Dependence and independence of RVs
  - Measure of dependence
  - Expectations of Sum and product of correlated RVs
  - Pdf of Sum of independent RVs
- Indicator RVs

## useful tools in prob.

- Functions of a Random Variable
- Transforms
  - Z-transform:
    - Definition;  $P_X(z) \triangleq E(z^X) = \sum_{k=0}^{\infty} P_k z^k \quad |z| \leq 1$
    - Properties;
    - Inversion.
  - Laplace transform:
    - Definition;  $\phi_X(s) = E[e^{-sX}] = \int_0^{\infty} f_X(x) e^{-sx} dx$
    - Properties;
    - Inversion.
  - Moment GF  $M_X(\theta) = E[e^{\theta X}]$
  - Characteristic function (Fourier –Stieltjes  $F_X(x)$ )  $\phi_X(\omega) = E[e^{j\omega X}]$   
 $= \int_{-\infty}^{\infty} e^{j\omega x} dF_X(x) \quad -\infty < \omega < \infty$

# Continuous Random Variables

- Laplace transform of a random sum
- important continuous dist.
  - Uniform distribution  $X \sim U(a, b)$
  - Exponential( $\lambda$ ) distribution
  - Erlang distribution  $X \sim \text{Erlang}(n, \lambda)$
  - Normal distribution  $X \sim N(\mu, \sigma^2)$
  - Multivariate Gaussian (normal) distribution
- Power Laws
  - Pareto distribution
  - Zipf's Law

# Discrete Random Variables

- Generating function of a random sum
- Compound RV and its expectation and its distr.
- The distribution of max and min of independent RVs
- ORDER STATISTICS
- Important distributions
  - Bernouli
  - Binomial
  - Negative binomial
  - Geometric
  - Poisson

# STOCHASTIC PROCESSES

Basic concepts

## Classification of Stochastic Processes

State space: the set of possible values of  $X_t$

Parameter (e.g. time) space



## Characterizing Stochastic processes: Highlight

Specifying RPs in terms of n-th order statistics : e.g.:

Gaussian Random process

Sinusoid with random phase

IID processes

Specifying RPs in terms of 1st order statistics:

Specifying full desc. Of RPs using 2<sup>nd</sup> order statistics :

Markov processes

Specifying RPs in terms of moment 1 and moment 2:

e.g. Poisson ((see Papoulis)): **note** : for Poisson we may find full desc. As well

1st order statistics:

$$\text{mean } E[X(t)] = E[n(0, t)] = \lambda t$$

$$\text{and variance } \sigma_x^2 = E[x^2(t)] - E[X(t)]^2 = \lambda t \quad \text{since } E[x^2(t)] = E[n^2(0, t)] = \lambda t + \lambda^2 t^2$$

$$\text{2<sup>nd</sup> order statistics } R_X(t_1, t_2) = E[n(0, t_1)n(0, t_2)] = E[n(0, t_1)n(t_1, t_2)] = \lambda^2 t_1(t_1 - t_2)$$

Specifying whether a RP process is stationary ( strict sense or WSS)

Specifying whether a RP is ergodic ( mean-ergodic , ergodic in correlation( covariance) , distribution ergodic ,..., Papoulis ch. 12)

## Examples of Stochastic process

Poisson Process

DTMC

CTMC

Semi-Markov Process

Birth-Death process

## System Analysis

- Classification of Queueing Networks
  - open networks
  - closed networks.
  - Interactive (terminal-driven)
  - Batch system
- Performance Metrics:
  - Response time
  - Throughput and Utilization
- Operational Laws
  - Little (open,closed)

## Queueing systems

- M/M/1 (distribution)
- M/G/1 (distribution)

## Simulation

- Discrete-event simulations
- Data collection and analysis
- Variance reduction techniques

## Simulation

- Discrete-event simulations
  - Why do we need simulations?
  - Step-by-step simulations;
  - Classifications;
  - Simulation program;
  - Basics of Discrete-event simulations;
  - Example: GI/G/1 queuing system;
  - Event advance design;
  - Unit-time advance design.

## Simulation

- Data collection and analysis
  - transient and steady-state simulations;
  - detecting the length of transient period;
  - characterizing central tendency;
  - characterizing variability;
  - data collection and analysis techniques;
  - comparison of methods;
  - estimations for transient simulations.

## Simulation

- Variance reduction techniques
  - Simulation with a given accuracy;
  - Variance reduction techniques;
  - Antithetic variates technique;
  - Control variates technique;
  - Validation of simulations.