

Linguagens de Programação

Cálculo Lambda

– Soluções –

1. $(\lambda x. \lambda y. x y)(\lambda x. x y)$

(a) 4 variáveis: x ligadora, y ligadora, x ligada, y ligada, x ligadora, x ligada, y livre

(b) $(\lambda a. \lambda b. a b)(\lambda c. c y)$

(c)
$$\frac{(\lambda a. \lambda b. a b)(\lambda c. c y) \rightarrow \lambda b. (\lambda c. c y) b \rightarrow \lambda b. b y}{\lambda b. b y}$$

2. (a) $(\lambda x. \lambda y. y (y x)) y (\lambda x. (\lambda x. x) x)$

i. 5 variáveis: x ligadora, y ligadora, y ligada, x ligada, y livre, x ligadora, x ligadora, x ligada, x ligada (-1)

ii. $(\lambda a. \lambda b. b (b a)) y (\lambda c. (\lambda d. d) c)$

iii.
$$\begin{aligned} & (\lambda a. \lambda b. b (b a)) y (\lambda c. (\lambda d. d) c) \rightarrow \\ & (\lambda b. b (b y)) (\lambda c. (\lambda d. d) c) \rightarrow \\ & (\lambda b. b (b y)) (\lambda c. c) \rightarrow \\ & (\lambda c. c) ((\lambda c. c) y) \rightarrow \\ & (\lambda c. c) y \rightarrow \\ & y \end{aligned}$$

(b) $(\lambda a. a a)(\lambda a. \lambda b. a b)$

i. 3 variáveis: a ligadora, a ligada, a ligada, a ligadora, b ligadora, a ligada, b ligada

ii. $(\lambda a. a a)(\lambda b. \lambda c. b c)$

iii.
$$\begin{aligned} & (\lambda a. a a)(\lambda b. \lambda c. b c) \rightarrow \\ & (\lambda b. \lambda c. b c)(\lambda b. \lambda c. b c) \rightarrow \\ & \lambda c. (\lambda d. \lambda e. d e) c \rightarrow \\ & \lambda c. \lambda e. c e \end{aligned}$$

3.

```
int f(int x) { return x * 2 + 1; }
int main() { return f(f(1)); }
```

(a)

```
let f = λx.x * 2 + 1 in
f (f 1)
```

(b)

$$\begin{aligned} & \frac{(\lambda f.f (f1)) (\lambda x.x * 2 + 1) \rightarrow}{(\lambda x.x * 2 + 1) ((\lambda x.x * 2 + 1) 1) \rightarrow} \\ & \frac{(\lambda x.x * 2 + 1) (1 * 2 + 1) \rightarrow}{(1 * 2 + 1) * 2 + 1 = 2 * 2 + 1 = 7} \end{aligned}$$

4.

```
fun f x = x * x;
fun g x = f x + f x;
f 2 + g 2;
```

(a)

```
let f = λx.x * x in
let g = λx.f x + f x in
f 2 + g 2
```

(b)

$$\begin{aligned} & (\lambda f.(\lambda g.f 2 + g 2)(\lambda x.f x + f x))(\lambda x.x * x) \rightarrow \\ & (\lambda f.f 2 + (\lambda x.f x + f x) 2)(\lambda x.x * x) \rightarrow \\ & (\lambda f.f 2 + (f 2 + f 2))(\lambda x.x * x) \rightarrow \\ & \frac{(\lambda x.x * x) 2 + ((\lambda x.x * x) 2 + (\lambda x.x * x) 2)}{(2 * 2) + ((2 * 2) + (2 * 2))} \rightarrow \\ & 4 + (4 + 4) = 12 \end{aligned}$$

5.

```
fun f x = x * x;
fun g x = f x + f x;
fun f (3 + g 2);
```

(a)

```
let f = λx.x * x in
let g = λx.f x + f x in
f (3 + g 2)
```

(b)

$$\begin{aligned} & (\lambda f.(\lambda g.f (3 + g 2))(\lambda x.f x + f x))(\lambda x.x * x) \rightarrow \\ & (\lambda f.f (3 + (\lambda x.f x + f x) 2))(\lambda x.x * x) \rightarrow \\ & (\lambda f.f (3 + (f 2 + f 2)))(\lambda x.x * x) \rightarrow \\ & \frac{(\lambda x.x * x)(3 + ((\lambda x.x * x) 2 + (\lambda x.x * x) 2))}{(\lambda x.x * x)(3 + ((2 * 2) + (2 * 2)))} \rightarrow \\ & \frac{(\lambda x.x * x)(3 + ((2 * 2) + (2 * 2)))}{(3 + ((2 * 2) + (2 * 2))) * (3 + ((2 * 2) + (2 * 2)))} \rightarrow \\ & 11 * 11 = 121 \end{aligned}$$

6. `fun f x = x * x;`
 `fun g x = f x + f x;`
 `g (f 2)`

(a)

`let f = λx.x * x in`
 `let g = λx.f x + f x in`
 `g (f 2)`

(b)

$$\begin{aligned}
 & (\lambda f.(\lambda g.g (f 2))(\lambda x.f x + f x))(\lambda x.x * x) \rightarrow \\
 & \frac{(\lambda g.g ((\lambda x.x * x) 2))(\lambda y.(\lambda x.x * x) y + (\lambda x.x * x) y))}{(\lambda y.(\lambda x.x * x) y + (\lambda x.x * x) y)) ((\lambda x.x * x) 2)} \rightarrow \\
 & \frac{(\lambda x.x * x) ((\lambda x.x * x) 2) + (\lambda x.x * x) ((\lambda x.x * x) 2)}{((\lambda x.x * x) 2) * ((\lambda x.x * x) 2)) + (\lambda x.x * x) ((\lambda x.x * x) 2)} \rightarrow \\
 & \frac{((\lambda x.x * x) 2) * ((\lambda x.x * x) 2)) + (((\lambda x.x * x) 2) * ((\lambda x.x * x) 2))}{((\lambda x.x * x) 2) * ((\lambda x.x * x) 2)) + (((\lambda x.x * x) 2) * ((\lambda x.x * x) 2))} \rightarrow \\
 & \frac{((2 * 2) * ((\lambda x.x * x) 2)) + (((\lambda x.x * x) 2) * ((\lambda x.x * x) 2))}{((2 * 2) * ((\lambda x.x * x) 2)) + (((\lambda x.x * x) 2) * ((\lambda x.x * x) 2))} \rightarrow \\
 & \frac{((2 * 2) * (2 * 2)) + (((\lambda x.x * x) 2) * ((\lambda x.x * x) 2))}{((2 * 2) * (2 * 2)) + (((\lambda x.x * x) 2) * ((\lambda x.x * x) 2))} \rightarrow \\
 & ((2 * 2) * (2 * 2)) + ((2 * 2) * (2 * 2)) = (4 * 4) + (4 * 4) = 32
 \end{aligned}$$

(c)

$$\begin{aligned}
 & (\lambda f.(\lambda g.g (f 2))(\lambda x.f x + f x))(\lambda x.x * x) \rightarrow \\
 & (\lambda f.(\lambda x.f x + f x) (f 2))(\lambda x.x * x) \rightarrow \\
 & \frac{(\lambda f.f (f 2) + f (f 2))(\lambda x.x * x)}{(\lambda x.x * x) ((\lambda x.x * x) 2) + (\lambda x.x * x) ((\lambda x.x * x) 2)} \rightarrow \\
 & \frac{(\lambda x.x * x) (2 * 2) + (\lambda x.x * x) ((\lambda x.x * x) 2)}{(\lambda x.x * x) (2 * 2) + (\lambda x.x * x) (2 * 2)} \rightarrow \\
 & \frac{((2 * 2) * (2 * 2)) + (\lambda x.x * x) (2 * 2)}{((2 * 2) * (2 * 2)) + (\lambda x.x * x) (2 * 2)} \rightarrow \\
 & ((2 * 2) * (2 * 2)) + ((2 * 2) * (2 * 2)) = (4 * 4) + (4 * 4) = 32
 \end{aligned}$$

7. `fun f y = y * y;`
 `fun g x = f x;`
 `f (g 5)`

(a)

`let f = λy.y * y in`
 `let g = λx.f x in`
 `f (g 5)`

(b)

$$\begin{aligned} & \frac{(\lambda f.(\lambda g.f (g 5))(\lambda x.f x))(\lambda y.y * y) \rightarrow}{(\lambda g.(\lambda y.y * y) (g 5))(\lambda x.(\lambda y.y * y) x) \rightarrow} \\ & \frac{(\lambda y.y * y) ((\lambda x.(\lambda y.y * y) x) 5)) \rightarrow}{((\lambda x.(\lambda y.y * y) x) 5) * (\lambda x.(\lambda y.y * y) x) 5)) \rightarrow} \\ & \frac{((\lambda y.y * y) 5) * (\lambda x.(\lambda y.y * y) x) 5)) \rightarrow}{((5 * 5) * (\lambda x.((\lambda y.y * y) x) 5) \rightarrow} \\ & \frac{((5 * 5) * ((\lambda y.y * y) 5)) \rightarrow}{((5 * 5) * (5 * 5)) = 25 * 25} \end{aligned}$$

(c)

$$\begin{aligned} & \frac{(\lambda f.(\lambda g.f (g 5))(\lambda x.f x))(\lambda y.y * y) \rightarrow}{(\lambda f.f ((\lambda x.f x) 5))(\lambda y.y * y) \rightarrow} \\ & \frac{(\lambda f.f (f 5))(\lambda y.y * y) \rightarrow}{(\lambda y.y * y) ((\lambda y.y * y) 5)) \rightarrow} \\ & \frac{(\lambda y.y * y) (5 * 5) \rightarrow}{(5 * 5) * (5 * 5) = 25 * 25} \end{aligned}$$

8. `fun f x = x + x;`
 `fun g f x = f x + 1;`
 `val h = g f;`
 `h 3;`

(a)

```
let f = λx.x + x in
let g = λf.λx.f x + 1 in
let h = g f in
h 3
```

(b)

$$\begin{aligned} & \frac{(\lambda f.(\lambda g.(\lambda h.h 3)(g f))(\lambda f.λx.f x + 1))(\lambda x.x + x) \rightarrow}{(\lambda f.(\lambda g.(g f) 3)(\lambda f.λx.f x + 1))(\lambda x.x + x) \rightarrow} \\ & \frac{(\lambda f.((\lambda j.λx.j x + 1) f) 3)(\lambda x.x + x) \rightarrow}{(\lambda f.((\lambda x.f x + 1) 3))(\lambda x.x + x) \rightarrow} \\ & \frac{(\lambda f.f 3 + 1)(\lambda x.x + x) \rightarrow}{((\lambda x.x + x) 3 + 1) \rightarrow} \\ & ((3 + 3) + 1) = (6 + 1) = 7 \end{aligned}$$

9. `fun c f g x = f (g x);`
 `fun h x = x + 1;`

c h h y;

(a)

let $c = \lambda f. \lambda g. \lambda x. f (g x)$ in
let $h = \lambda x. x + 1$ in
 $c h h y$

(b)

$$\begin{aligned} & \frac{(\lambda c. (\lambda h. c h h y) (\lambda x. x + 1)) (\lambda f. \lambda g. \lambda x. f (g x))}{(\lambda h. (\lambda f. \lambda g. \lambda x. f (g x)) h h y) (\lambda x. x + 1)} \rightarrow \\ & \frac{(\lambda f. \lambda g. \lambda x. f (g x)) (\lambda x. x + 1) (\lambda x. x + 1) y}{(\lambda g. \lambda x. (\lambda x. x + 1) (g x)) (\lambda x. x + 1) y} \rightarrow \\ & \frac{(\lambda x. (\lambda x. x + 1) ((\lambda x. x + 1) x)) y}{(\lambda x. x + 1) ((\lambda x. x + 1) y)} \rightarrow \\ & \frac{((\lambda x. x + 1) y) + 1}{(y + 1) + 1} \rightarrow \\ & (y + 1) + 1 = y + 2 \end{aligned}$$