Diffusion models math

Notation

$x_0 \sim p_{data}$

$\epsilon_0 \sim N(0,I)$

Forward process:

$x_t = x_t + D(x_t, t)$ — noise on timesteps

approximate, not invertible, by parameterization (by conditional mean)

Reverse process

$p(x_t|x_t)$ parameterized, to make sampling possible

$p(x_{t-1}|x_t) = \int p(x_t|x_t)\, p(x_t|x_t)\, dx_t$ — x_t the distribution over x_t

$x_t \sim N(\mu(x_t), \sigma_t I)$ — parameterization μ — σ

$= \int p(x_t|x_t) \left[\int p(x_t|x_t) \cdot D(x_t) \int \frac{1}{\sigma_t} \exp\left(-\frac{1}{2}\left(\frac{x_t - \mu(x_t)}{\sigma_t}\right)^2\right) \right] dx_t$

$p(x_t|x_t) = p(x_t|x_t)\, \frac{\sigma_t}{\sigma_t}$ — σ_t cancelling

$\frac{\sigma_t}{\sigma_t} = \frac{\sigma_t}{\sigma_t} = \frac{\sigma_t}{\sigma_t}$

$p(x_t|x_t) = p(x_t|x_t)\, \frac{\sigma_t}{\sigma_t}$ — σ cancelling, from

$\int p(x_t|x_t) \cdot D(x_t) \int \frac{1}{\sigma_t} \exp\left(-\frac{1}{2}\left(\frac{x_t - \mu(x_t)}{\sigma_t}\right)^2\right) dx_t$ — σ cancelling, from σ cancelling

$\int p(x_t|x_t) \cdot D(x_t) \int \frac{1}{\sigma_t} \exp\left(-\frac{1}{2}\left(\frac{x_t - \mu(x_t)}{\sigma_t}\right)^2\right) dx_t$

$\frac{\sigma_t}{\sigma_t} = \frac{\sigma_t}{\sigma_t}$