

2019-01-03

Nankai-Baidu Joint Laboratory

Parallel and Distributed Software Technology Lab





CPU Usage

2%

CPU: 6 vCore

RAM: 16384 MB

Storage: 200 GB SSD

Bandwidth: 0.31 GB of 5000 GB

关于HEAAN的总结

```
tree@tree: ~
 File Edit View Search Terminal Help
                                           Tasks: 22, 11 thr; 7 running
                                                                                                                                                                                                                                                                                                                        Load average: 6.20 6.17 4.74
                                                                                                                                                                                                                                                                                                                              Uptime: 01:01:03
nc NLGD time = 28550.7 \text{ ms}
                         ncWData.logq after: 242
                        !!! STOP 6 ITERATION !!!
                      !!! START 7 ITERATION !!!
                       ncWData.logq before: 242
                   Start Enc NLGD
                  Enc NLGD time = 24961.3 ms
                        ncWData.logq after: 79
                      !!! STOP 7 ITERATION !!!
                    041,-0.011796,-0.00698489,0.00437142,0.0333612,-0.0246181,0.0195439,0.00324137,0.0162196,-0.00800723,-0.00609741,-0.0253816,-0.00410
                 726,-0.0193205,0.00119126,0.0360199,0.0431541,0.00920888,-0.025231,-0.000594751,0.0116909,-0.000834049,-0.00183872,0.000804182,0.00
                   1451, 0.00663709, 0.0136006, 0.000449622, 0.00715692, -0.00119842, -0.00276409, 0.00692596, 0.00255863, 0.00644382, -0.000643898, -0.0224966, -0.0025666, -0.00255863, 0.00644382, -0.00643898, -0.0024966, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.006443898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.00643898, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.0064389, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.006489, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.00689, -0.0068, -0.0068, -0.0068, -0.0068, -0.0068, -0.0068, -0.0068, -0.0068, -0.0068, -0
                      00123865, 0.00167096, -0.000178631, 0.00576916, 0.0199624, -0.00231451, 0.0150486, 0.0145144, -0.0188257, -0.00354363, -0.0117113, -0.000245123, -0.0001466, -0.0001466, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.000146, -0.00
                         -0.00733171, -0.0192445, 0.0077089, 0.010513, 0.0108393, 0.00385812, -0.015641, -0.0145321, 0.0113047, -0.0109755, -0.00388248, -0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.000760774, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.00076074, 0.000
                       0133055, 0.00339216, 0.00257104, 0.0143263, 0.00291485, 0.0154321, -0.00565885, 0.0231084, 0.00586879, 0.00491765, -0.00547436, 0.00739722, 0.00586879, 0.00586879, 0.00491765, -0.00547436, 0.00739722, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.0058679, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.00586879, 0.005868
                   160957, -0.00761852, -0.0169673, 0.000845321, -0.00322828, -0.00875158, 0.0252244, -0.00137244, 0.00849906, -0.00545511, 0.00643193, -0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.0001659811, 0.00016598110, 0.00016598111, 0.00016598111, 0.000165911, 0.0001659111, 0.000165911, 0.0001659111, 0.000
                   29,-0.00299338,-0.029969,0.0187531,0.0183589,0.0329634,0.0103479,0.00563936,0.010644,
                   Correctness: 54.2857 %.
                   AUC: 0.608268
                   604896,-0.0123591,-0.00778621,0.00352665,0.0247787,-0.0222425,0.0192013,0.0027827,0.0152482,-0.00862207,-0.00669233,-0.0258352,-0.00
                  489067,-0.0196852,0.000668965,0.0350206,0.042419,0.00838237,-0.0253378,-0.00137873,0.00422837,0.00141878,-0.00186224,-0.000284045,0
                    44,-0.00902051,0.00365676,-0.000120347,0.0049936,0.0187372,-0.00322742,0.0145275,0.0137381,-0.0191393,-0.00429582,-0.0121181,-0.000
                  75986,-0.00786588,-0.0195447,0.00708575,0.00995374,0.00289399,0.00566061,-0.0152887,-0.0146475,0.0107319,-0.0111199,-0.00428033,-0.0
                   0158697, 0.0123018, 0.0030173, 0.00155763, 0.013463, 0.00204687, 0.0146836, -0.00607847, 0.0221053, -0.00229739, 0.00707741, -0.00510883, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.006581, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.00681, 0.0
                   5329,0.015379,-0.00814613,-0.0170278,9.25131e-05,-0.00361483,-0.00891884,0.0238204,-0.00205995,0.00778649,-0.00613448,0.00602609,-0
                   00107197,-0.0110664,-0.0272277,0.0190325,0.0175581,0.0317513,0.00942297,0.00491432,0.00990335,
                   Correctness: 53.6508 %.
                    AUC: 0.61505
                       !!! STOP 5 FOLD !!!
                   Average Encrypted correctness: 263.175%
                  Average Encrypted AUC: 3.02143
                   Average True correctness: 260.317%
                  Average True AUC: 3.03519
                   root@vultr:/home/sly/IDASH2017/IDASH2017/Debug#
```

Nankai-Baidu Joint Laboratory



Parallel and Distributed Software Technology Lab





密文下的比较操作

➤ 密文之间的大小 >> 对应的明文之间的大小 (X)

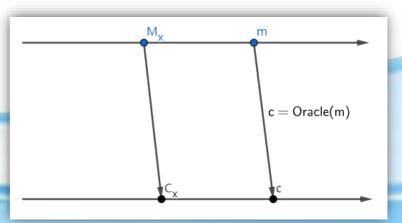
假设: 可以根据密文的大小推断出对应明文之间的大小,

则 : 一个对手可以在O(n)内确定一个密文所加密的明文。

公开密钥系统E: pk, sk

用户B: $C_x = Enc(pk, M_x)$

对手A: 根据已有的信息构建一个预言机 $Oracle(m) \approx Enc(pk, m)$



- -输入任意明文输出系统E下对应的密文
- $求用户B的<math>M_x$

-应该可以设计出O(lgn)的算法

Nankai-Baidu Joint Laboratory





密文下的比较操作

一种解决方案:多方计算涉及到密钥的传输设计实验比较麻烦、比较困难多方计算存在若干问题

同态加密的优势: 在计算过程中不对密文进行解密, 保证数据的安全

也有人研究多方计算





密文下的比较操作

➤ 把明文的二进制的每一位加密
IBM的HElib库实现了binaryCompare.h compareTwoNumbers(...)

Doing Real Work with FHE: The Case of Logistic Regression

5.4 Comparing Two Integers

The procedure for integer comparison is somewhat similar to integer addition. We have two integers in binary, $a=(a_{t-1},\ldots,a_1,a_0)$ and $b=(b_{t-1},\ldots,b_1,b_0)$, and we want to compute the two integers $x=\max(a,b)=(x_{t-1}\ldots x_0)$ and $y=\min(a,b)=(y_{t-1}\ldots y_0)$, as well as the two indicator bits $\mu=(a>b)$ and $\nu=(b>a)$ (note that when a=b, both μ,ν are zero).

We begin by computing for every i < t the bits $e_i := a_i + b_i + 1$ (which is 1 iff $a_i = b_i$) and $g_i := a_i + a_i b_i$ (one iff $a_i > b_i$). We then compute the products $e_i^* = \prod_{j \geq i} e_j$ and $g_i^* = g_i \cdot \prod_{j > i} e_j$, and the bits $\tilde{g}_i = \sum_{j \geq i} g_1^*$ (one iff $a_{t-1...i} > b_{t-1...i}$). Computing the products e_i^*, g_i^* is done using a recursive procedure somewhat similar to ComputeAllProducts from Section 4. Finally we compute the results by setting $\mu := \tilde{g}_0, \nu := 1 + \tilde{g}_0 + e_0^*$, and for $i = 0, \ldots, t-1$ we set $x_i := (a_i + b_i)\tilde{g}_i + b_i$ and $y_i := x_i + a_i + b_i$.

Note that we use all the g_i^* 's but only e_0^* for computing the output results, hence we somewhat optimized our procedure for computing these products by skipping the computation of e_i^* 's that are never used.

We remark that the last product $(a_i+b_i)\tilde{g}_i$ means that our procedure may use depth one more than the minimum possible. Using the absolutely smallest possible depth is challenging, straightforward solutions would take $O(t^2)$ multiplications (vs. O(t) multiplications in the procedure above). While getting minimal depth with O(t) multiplications is possible in theory, the procedure for doing this is overly complex (and extremely hard to parallelize), so we opted for a simpler procedure with slightly non-optimal depth. (Also, as opposed to the addition procedure from above, the simple procedure that we implemented here does not vary depending on the level of the input ciphertexts for a_i, b_i .) 明文空间{0,1} 明文模数为2

模2运算:加法、乘法

$$1+1+1=1 \pmod{2}$$

max(a,b) min(a,b)

$$u = (a > b)$$

$$v = (a > b)$$

Nankai-Baidu Joint Laboratory



Parallel and Distributed Software Technology Lab