INF244 - Second Project

October 25, 2013

Computing the Maximum Independent Set of a Graph by Message-Passing

Please read MessPassIS.pdf for background discussion. Please also look at `http://www.ii.uib.no/~matthew/INF244/NonBipMessPass.pdf` and `http://www.ii.uib.no/~matthew/INF244/SPASimple.pdf`.

The task is to write code to implement a message-passing algorithm to compute a maximum independent set of an undirected simple graph. You will implement it in two ways. The first way uses a 'binary graph', i.e. decodes a binary nonlinear code. The second way uses an '$\mathbb{F}_4$ graph', i.e. decodes an $\mathbb{F}_4$-additive code. For the second method you can make the message-passing dynamic by randomly updating the graph using randomly applied local complementation operations, where I suggest using function update rule no. 1. However the dynamic part will not be graded, so is an optional extra.

Specification:

- INPUT: a text file named 'IS.txt'. The format of 'IS.txt' is as follows:
- Line 1: Number of vertices (n). - (non-negative integer)
- Lines 2 to $n+1$: Binary adjacency matrix, (Γ), one length n binary string per per line (in ascii).
- Line $n+2$: Enter character to select algorithm:
  - '2': Decode on 'binary graph'.
  - '4': Decode on '$\mathbb{F}_4$-additive' graph.

  - 'D': Dynamic decoding on '$\mathbb{F}_4$-additive' graph. The dynamic option will not be graded - I leave it as a challenge for your interest only, where I suggest using function update rule no. 1.

- Line $n+3$ onwards: The function biases you shall use to find your answer (e.g. for a length-3 node you choose values b, and c, for a one or two black solution, respectively). The set of biases chosen depend on the maximum node degree of your input graph. Write your biases like this - the numbers chosen are just arbitrary examples (note that the numbers don't have to add up to 1 as normalisation isn't necessary):
  - 2: 3
  - 3: 4,7
  - 4: 5,6,8
  - 5: 5,12,31,75
  - ...etc ...

Output:

- A maximum independent set (e.g. $1, 3, 11, 17$ if a maximum independent set is of size 4 and is found at vertices 1,3,11,17).
- Number of message-passing iterations used to find the answer - this is a somewhat arbitrary figure as it depends on the type of message-passing schedule you choose.

Example Input:

```
5
01001
10101
01011
00101
11110
4
2: 3
3: 2,5
4: 1,4,7
```

Example Output:

```
0,3
20
```

Eventually you might want to search over various choices of bias assignments and, for bigger graphs, choose the biases according to some formula rather than having to input them explicitly.