3 Optimization tool NOISHHH

3.1 Introduction

This chapter will discuss the original software tool used to optimize the flight trajectories, NOISHHH. The program consists of an intermediate point-mass aircraft model including operational constraints, a geographical model, a noise model and a dose-response relationship. These four models are used to calculate the optimized flight trajectory within the International Standard Atmosphere using a dynamic optimization algorithm. The structure of the program looks as follows:

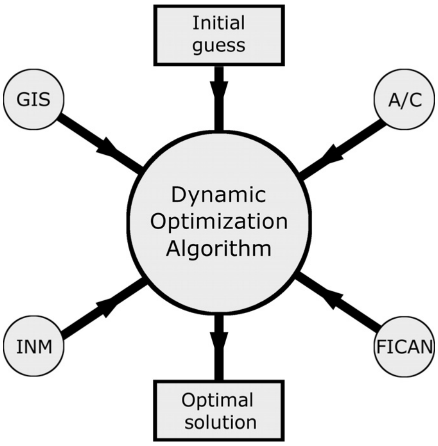

Figure 3.1: Structure of NOISHHH

3.2 The aircraft model

3.2.1 Point-mass model

NOISHHH uses an intermediate point-mass aircraft model based on the following assumptions:

- No wind vector
- A flat, non-rotating Earth
- Coordinated flight

This model defines the dynamics of the aircraft as a function of control variables and time. The model presented in this section is not yet constrained for RNAV flight procedures. The original version of NOISHHH uses 5 aircraft states and 3 controls:

$$\vec{x} = \begin{pmatrix} x \\ E \\ h \\ y \\ \chi \end{pmatrix} \quad ; \quad \vec{u} = \begin{pmatrix} \Gamma \\ \gamma \\ \mu \end{pmatrix} \qquad 3.1$$

Here x and y define the lateral position, both in meters in the "Rijksdriehoek" (RD)-coordinate system. The altitude h is also expressed in meters and the heading χ of the aircraft in radians is defined as the true compass heading with North at 0º and South at 180º. E is the energy height in meters which is defined as:

$$E = h + \frac{1}{2}\frac{v^2}{g_0} \tag{3.2}$$

From the energy height and the altitude the true airspeed (TAS) v can be derived.

The controls consist of the normalized throttle setting Γ which ranges from 0 to 1. From the throttle setting the thrust is calculated:

$$T = \left(T_{\max} - T_{\min}\right)\Gamma + T_{\min} \tag{3.3}$$

Furthermore γ defines the flight path angle in radians, and μ defines the bank angle in radians, where a positive bank angle defines a right turn.

Using these states and controls the time derivatives can be defined as follows:

$$\dot{x} = v\cos\gamma\sin\chi \tag{3.4}$$

$$\dot{y} = v\cos\gamma\cos\chi \tag{3.5}$$

$$\dot{h} = v\sin\gamma \tag{3.6}$$

The expression for the time derivative of the heading χ can be found from the force equilibrium in a horizontal steady turn, using Figure 3.2:

$$W = L\cos\mu \tag{3.7}$$

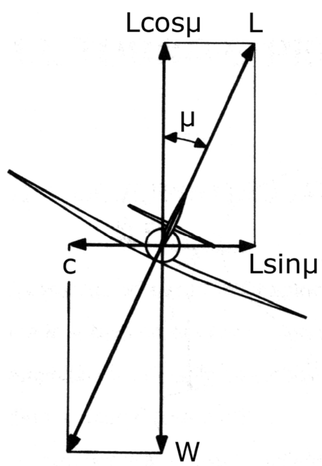

Figure 3.2: Force equilibrium in a turn [Ref. 27]

During the turn the following equations apply:

$$v = \omega R = \dot{\chi} R \quad 3.8$$

$$m\frac{v^2}{R} = L\sin\mu \quad 3.9$$

These equations result in an expression for the time derivative of the heading:

$$\dot{\chi} = \frac{g_0}{v}\tan\mu \quad 3.10$$

This equation is only valid for turns in the horizontal plane. However, since the flight path angle γ is small (γ < 15º), this equation will still result in a good approximation in 3-dimensional flight.

By taking the time derivative of equation 3.2 the time derivative of the energy height can be defined as:

$$\dot{E} = \frac{d}{dt}\left[h + \frac{1}{2}\frac{v^2}{g_0}\right] = \dot{h} + \frac{v\dot{v}}{g_0} \quad 3.11$$

Now using Figure 3.3 the following equation can be derived:

$$\dot{v} = \frac{g_0}{W}\left(T - D - W\sin\gamma\right) \quad 3.12$$

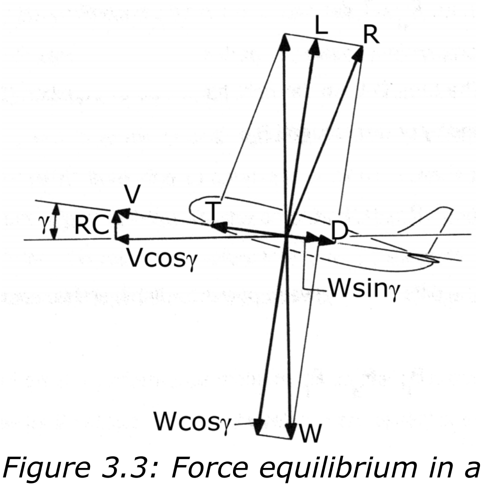

Figure 3.3: Force equilibrium in a climb [Ref. 27]

Now combining equations 3.11 and 3.12 the time derivative of the energy height becomes:

$$\dot{E} = \frac{v}{W}\left(T - D\right) \qquad 3.13$$

With equations 3.4, 3.5, 3.6, 3.10 and 3.13 the complete system of equations of motion has been defined.

3.2.2 Aircraft model

For this thesis the Boeing 737-300 has been used to find the optimized flight trajectories. The Boeing 737 is a short to medium range, single aisle, narrow-body jet airliner. The Boeing 737 entered service in 1968, and now has become the most ordered and produced commercial passenger jet in the world. Since 1967 7,800 737's have been ordered, and over 5,600 have been delivered.

The specific type of the 737 used in this thesis, the 737-300, made its first flight in 1984 and designated the start of the 737 'Classic' series. The aircraft has a typical capacity of 128 passengers in a two class configuration. The last 737-300 was delivered in 1999. By then 1104 Boeing 737-300's were delivered. The Boeing 737-400 and -500 are stretched and shortened version of the 737-300. In Table 3.1 some typical data as used for this thesis have been stated. Figure 3.4 shows a Ryanair Boeing 737-300.

Boeing 737-300		
Capacity		
	Crew	2
	Passengers (typical configuration)	128
Dimensions		
	Length	33.4 m
	Wing span	28.9 m
	Height	11.1 m
	Wing area	105.4 m^2
Weights		
	Operating empty weight	33,881 kg
	Maximum Take Off Weight	56,740 kg
	Take Off Weight used for this thesis	56,118 kg
Performance		
	Typical cruise speed	450 kts
	Typical range	1815 nm
	Engines	2 x CFM56-3B1

Table 3.1: Boeing 737-300 characteristics [Ref. 32]

Figure 3.4: Ryanair Boeing 737-300 [Ref. 3] © Jan Mogren

3.3 The Geographical Information System

The second part of the NOISHHH program is the Geographical Information System (GIS). The GIS contains population density information on grid points defined in the RD-coordinate system. These data are available for The Netherlands as a whole and were provided by the Dutch Central Bureau of Statistics (CBS). The data were provided on a 500 m square grid, but to benefit the calculation speed the grid was made coarser for the use in NOISHHH. An example of the population density around the airport is shown in Figure 3.5, plotted over a satellite image of the area.

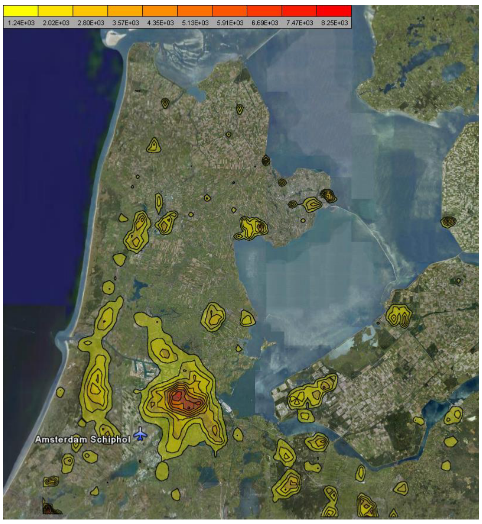

Figure 3.5: Population density in the vicinity of Schiphol [Ref. 15]