

DATA130004: Homework 5

Due in class on November 6, 2019

1. Exercises 6.4, 6.9, and 6.10.
2. Suppose X_1, \dots, X_n are i.i.d. samples from a normal distribution $N(\mu, \sigma^2)$, $n \geq 2$. Prove that $\sum_{i=1}^n (X_i - \bar{X})^2 / \sigma^2$ follows a $\chi^2(n-1)$ distribution and it is independent with the sample mean \bar{X} .
3. In Example 6.4, to construct a $(1 - \alpha) \times 100\%$ confidence interval for the variance parameter σ^2 , we assume that the lower bound is 0 and the upper bound corresponds to a quantity involving the α -quantile of a χ^2 distribution, we now consider using $\alpha/2$ and $(1 - \alpha/2)$ -quantiles of the same χ^2 distribution to construct another confidence interval. It certainly will excludes 0.
 - (1) Give the explicit form of the new confidence interval and justify its validity by showing the theoretical confidence level is $1 - \alpha$.
 - (2) Repeat the experiments in Example 6.5 with the same parameter set-up. Compare the two types of confidence interval, such as empirical coverage probability and average confidence interval width.
 - (3) Repeat the experiments in Example 6.6 with the same parameter set-up. Compare the two types of confidence interval, such as empirical coverage probability and average confidence width.
 - (4) Which confidence interval would you recommend in practice? Explain why.