Hands-On

Task: Compute the first derivative of an arbitrary function $f(x)$ using a centered finite difference scheme. Your program should read the *number of intervals* n you want to use to discretize the domain $\mathbb{R} \ni x \in [a, b]$ where a and b are the endpoints of the interval on the real line. For example,

```
./dfdx 5000
```

will compute the derivative $\frac{df}{dx}(x)$ of $f(x)$ using 5000 divisions for the interval $[a, b]$. You evaluate the function at the points $x_i = a + i\Delta x$ for all $i = 0, 1, \ldots, n$, where

$$\Delta x = \frac{b - a}{n}.$$

The central finite difference scheme we are using *approximates* the values of $\frac{df}{dx}(x_i)$ according to the rule

$$\frac{df}{dx}(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x}.$$

Breakdown of tasks

1. Write a program called `dfdx` that takes an integer argument n
2. The program computes the derivative of the function

$$f(x) = e^{-\frac{1}{2}x} \sin(x)\big(\cos(x)\big)^2$$

3. The program should write the result into a file where each line contains the value of x_i, $f(x_i)$ and $\frac{df}{dx}(x_i)$ (3 numbers) separated by white space, for all $i = 1, 2, \ldots, n-1$. The numbers should be formatted in Engineering notation with 6 decimal places, for example:

```
0.000000e+00  0.000000e+00  0.000000e+00
0.100000e+00  1.237482e+00  6.374822e+00
0.200000e+00  1.458696e+00  8.112393e+00
...
```

4. Write your code in the skeleton file `dfdx.cpp`. Refer to the additional comments given in the file.