CUED - Engineering Tripos Part IIA 2018-2019 **Module Laboratory Report**

Student name:	Harvey Hughes	CRSID:	hh458	Module number:	3C6

Feedback to the student

☐ **See also comments in the text**

		Very good	Good	Needs improvmt
CONTENT	**Completeness, quantity of content:** Has the report covered all aspects of the lab? Has the analysis been carried out thoroughly?			
	Correctness, quality of content Is the data correct? Is the analysis of the data correct? Are the conclusions correct?			
	Depth of understanding, quality of discussion Does the report show a good technical understanding? Have all the relevant conclusions been drawn?			
	Comments:			
PRESENTATION	**Attention to detail, typesetting and typographical errors** Is the report free of typographical errors? Are the figures/tables/references presented professionally?			
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Raw report mark	/5
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3F8 Digital Vibration Analysis

Harvey Hughes • Emmanuel College
Lab Date: 20/2/19 • Partner: wh307
hh458@cam.ac.uk

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Abstract

During the course of this lab three excitation techniques were used to identify resonances and transfer functions of two coupled cantilever beams. The beams were then uncoupled and individual transfer functions were measured. These individual transfer functions were accurately used to predict the coupled behaviour. Sinusoidal, noise and impulse inputs were the techniques used. Each method agreed with each other but could be performed at various speeds. An additional resonant peak is observed when exciting with a hammer. Sensitivity to noise at low frequency or anti-resonance was prominent across all experiments.

I. Summary

Following the sequence outlined in the lab handout, a series of measurements were made on a system of two coupled cantilever beams. Excitation force was applied using a moving-coil shaker and an instrumented impulse hammer. Response was measured using an accelerometer, but the displayed signal represented vibration velocity because an integration stage was included in the amplifier.

The driving-point response was measured near the point at which the coupling link was inserted between the two beams. Three types of input force were employed: (1) sinusoidal excitation, in which the frequency was adjusted by hand and single-frequency measurements combined to give a response plot; (2) band-limited pseudo-random noise was applied using the shaker; (3) short impulses were applied using the hammer, fitted with a soft rubber tip.

Two additional exercise were also performed. First, calibration factors for the measurement set-up were determined by measuring the response of a freely-suspended known mass and using Newtons law. Finally, the coupling link was removed and the responses of the two separate beams were measured using the hammer method. The two were combined using the theoretical formula for point coupling, and the result compared with the true coupled response.

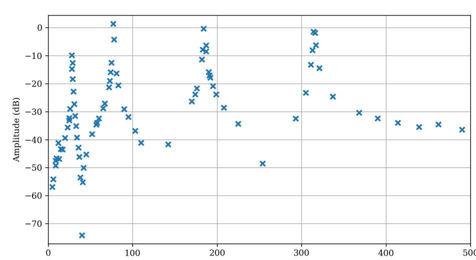

Figure 1: *Transfer Function of a Sinusoidal input at frequencies up to 500Hz for the coupled beams*

II. Results and Discussion

i. Sinusoidal Input

Using a shaker and accelerometer the response to inputs at given frequencies can be measured and plotted see figure 1. Care must be taken to wait for the transient response to decay before each measurement. This is time consuming and difficult to accurately capture the whole behaviour about resonances and anti-resonances. Four resonances were discovered at frequencies 25, 80, 190 and 310Hz.

ii. Random Noise Input

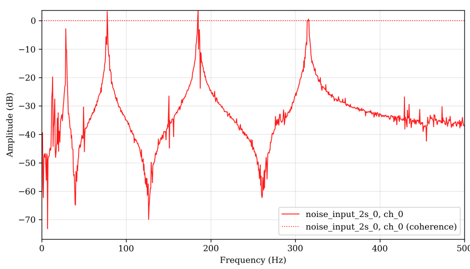

Figure 2: *Transfer Function from a random noise signal generated for 2 seconds for coupled beams*

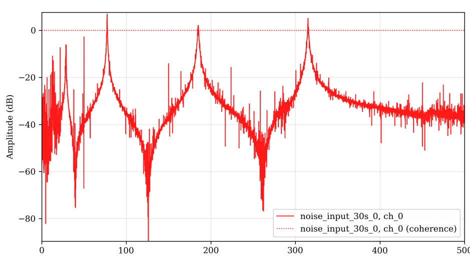

Figure 3: *Transfer Function from a random noise signal generated for 30 seconds for coupled beams*

A faster method of obtaining the transfer function over a frequency range is to input a random noise force and sample over a time period, using Fourier analysis to separate the frequency responses. In order to represent the desired frequencies the sampling period must be long enough such that the frequencies of interest have been generated in the random noise input. Figures 2 and 3 show the transfer functions for 2s and 30s time periods. Both of these show the same four resonant peaks with the 30s time being slightly noisier. With the 30s time period peaks at 50Hz intervals can be seen which is the mains frequency. The low frequency 0-50Hz is far noisier due to accelerations being lower magnitude at low frequency, therefore sensitivity to measurement noise is high.

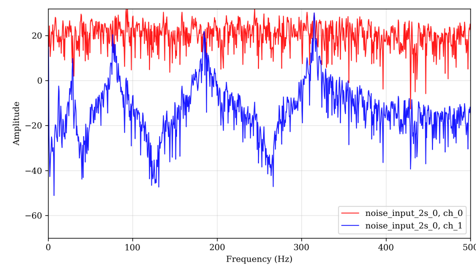

Figure 4: *Fast Fourier Transform from a random noise signal generated for 2 seconds for coupled beams*

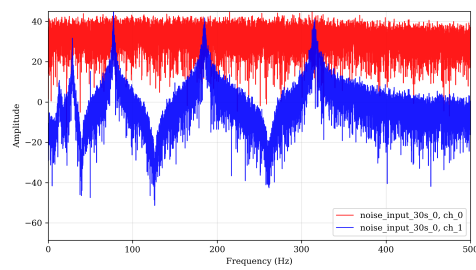

Figure 5: *Fast Fourier Transform from a random noise signal generated for 30 seconds for coupled beams*

Figures 4 and 5 show the FFT of the noise inputs (red) and response (blue). The larger sampling period includes more frequencies in the input. However, the variance in magnitude across these frequencies is larger than the smaller interval thus creating the noisier transfer function.

The noise present in the transfer function can be reduced by splitting up the whole interval of 30s into n intervals, performing a FFT on each mini interval and then averaging. The result of this for n=1, 30 and 100 can be seen in figures 3 , 6 and 7 respectively. Care must

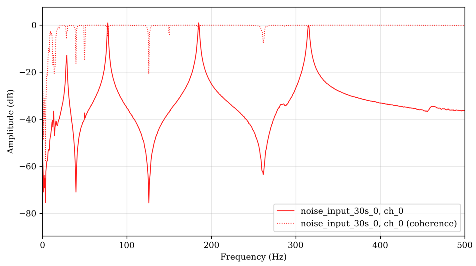

Figure 6: *Transfer function with a noise input with 30, 1s time intervals averaged*

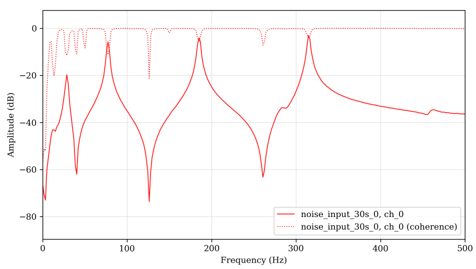

Figure 7: *Transfer function with a noise input with 100, 0.3s time intervals averaged*

be taken as using too many intervals means that the presence of each frequency within each interval is likely to be lower. This introduced incoherency between averaged results which can be seen at resonances in figure 7 for n=100. When n=30 the transfer function has good coherency at points of interest such as resonance, and the noise has been eliminated as desired. Incoherent results are to be expected at anti-resonances and very low frequencies as these have lower accelerations so are more sensitive to measurement noise.

iii. Hammer Input

A third method to obtain the transfer function is to excite the structure with an impulse and measure the response. The frequencies being tested can be found by doing a Fourier transform on the shape of the force input. For an ideal impulse this will include all frequencies. However, a hammer strike produces a more

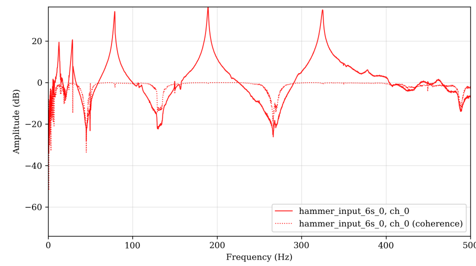

Figure 8: *Transfer Function due to an impulse from a hammer for coupled beams*

rounded force input due to a finite contact time therefore the higher frequencies aren't present. A longer hammer pulse causes lower frequencies to not be present. Figure 8 shows the transfer function of five hits averaged together. Five peaks can be seen in this plot, this additional peak is due to the whole structure bending. This occurs at a low frequency of 20Hz as more mass is moving therefore a low frequency is expected. This mode was less prominent in other measurements as the shaker rod added additional bending stiffness and restricted motion. Coherency between tests needs to be high at frequencies of interest about the resonances. The coherence is lower at very low frequencies and around antiresonance's due to lower accelerations.

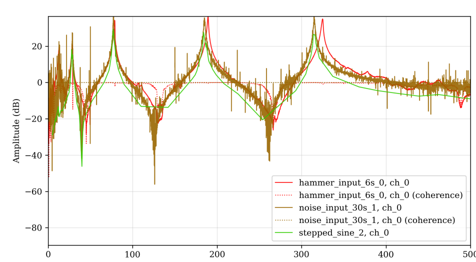

Figure 9: *Transfer functions for all three methods after the amplitudes have been matched*

By matching the amplitude of each methods transfer function a comparison can be made between each method, this is shown in figure 9. All three methods are consistent for frequencies 50-300Hz. At low frequencies the sine and noise method don't measure the first resonant peak. At the resonant peaks, especially the 5th peak, the hammer method shows resonance occurring at a slightly higher frequency. This is due to a lower mass in the system as the shaker rod and magnet had been removed. Less mass gives a higher resonant frequency as $\omega_n \propto \sqrt{\frac{k}{m}}$. The measurement speed increase of the noise and hammer excitation method over a sweeped sine excitation while maintaining high accuracy makes these methods far more powerful. The addition of being able to calibrate the hammer excitation (section iv) to gain true values for acceleration is once again very useful and powerful.

iv. Calibration

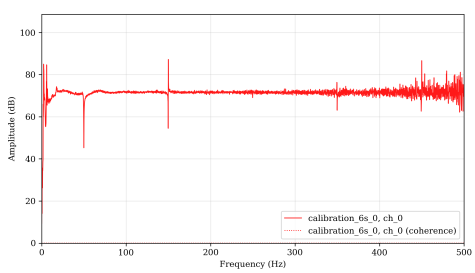

Figure 10: *Transfer function relating to acceleration of the hanging mass due to a hammer impulse*

The hammer force readings can be calibrated by measuring on a system with known transfer function to obtain true accelerations. The system chosen is a suspended mass. The transfer function is shown in equation 1.

$$\frac{a(s)}{F(s)} = m = \frac{1}{0.369} = 2.71 \tag{1}$$

This is a constant transfer function against acceleration and is shown experimentally in figure 10 as an almost constant line at 72dB across frequencies (except for peaks at 50Hz intervals from the mains). This corresponds to a linear value of 3981 which must match the true value of $\frac{a}{F} = 2.71$. This gives a calibration factor of $\frac{2.71}{3981} = 6.6x10^{-4}$.

v. Comparison of calibrated Responses

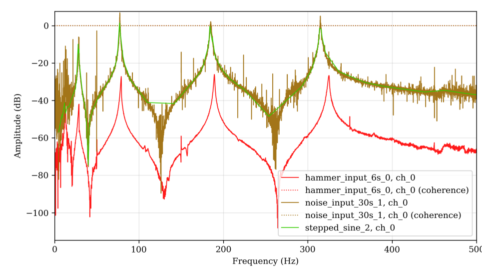

Figure 11: *Transfer functions for each measurement technique before calibration*

By using the calibration factor calculated in section iv the hammer excitation transfer function can be rescaled to represent true accelerations. The other techniques can then have their amplitudes matched to this calibrated hammer transfer function. This transforms figure 11 into figure 12. From this you can see that resonances reach a peak ratio between a and F of approximately -25dB = 0.056. The lower and higher frequencies aren't shown as they are too noisy to be matched accurately between methods.

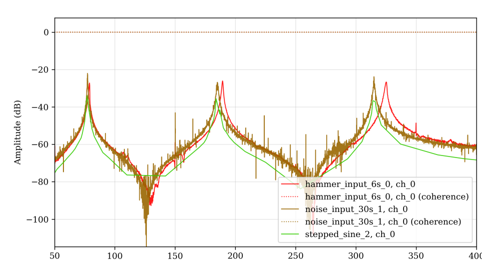

Figure 12: *Transfer functions for each measurement technique after calibration*

vi. Coupled System Response

When the couple connecting the two cantilever beams is removed you can measure the response of each beam independently then a prediction of the coupled response can be made.

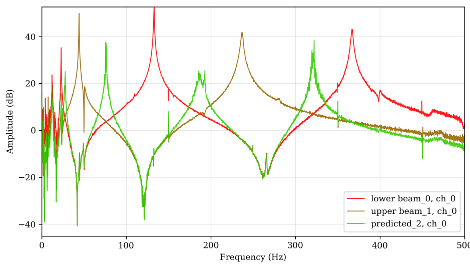

Figure 13: *Transfer functions using a hammer on the two beams uncoupled, with a prediction for the coupled behaviour*

This prediction uses the fact that when coupled the attachment points have the same velocity and amplitude see equation 2. Figure 13 shows these transfer functions. At low frequencies there is still lots of noise however the 1st resonant is just visible. The upper and lower beams alternate resonant peaks, with the coupled prediction peak residing near the mid points of these peaks. The definition of the predicted peaks is low and more noise is present. The only resonant peak that is shared between the cantilevers is the lowest one. This further shows that the vibration mode involves the whole structure bending and therefore can be excited by applying a force to either beam.

$$\frac{1}{Y_{coupled}(\omega)} = \frac{1}{Y_{top}(\omega)} + \frac{1}{Y_{bot}(\omega)} \qquad (2)$$

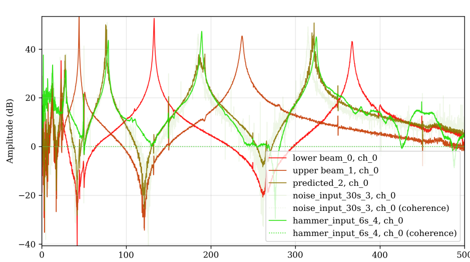

Figure 14: *Transfer functions of the predicted coupled response compared to the response when a hammer is applied to the coupled system*

Figure 14 shows the predicted coupled behaviour compared to the hammer excitation on the coupled system. The resonant peaks agree closely with more noise present in the predicted curve. However, anti-resonances appear more defined and lower in the predictive case. This could be due to the coupling transferring energy between beams and therefore each anti-resonance in the coupled case is higher, which may not be accounted for in our predictive model. This energy transfer could happen because the beams are never in anti-resonance at the same time.

III. Conclusion

- Sinusoidal input produces accurate results, but is very time consuming and hard to get defined peaks.
- Using random noise as the input allows for a very quick method to measure the transfer function. This is provided that the sampling period is long enough to ensure frequencies of interest will be generated.
- Noise in the transfer function from measuring a noise input can be reduced by splitting up the whole time period into smaller ones and then averaging their transfer functions. Once again this relies on intervals long enough to capture frequencies of interest or coherency will be lost.
- A hammer can be used as an excitation to approximate an impulse. Due to a finite contact time the high frequencies won't be produced.
- Using the hammer allows for an additional resonance peak to be discovered. This relates to the whole structure bending at a lower frequency due to a high mass.
- Noise in the transfer functions at low frequencies and often anti-resonances occurs as the measured acceleration is low at these points. Therefore sensitivity is also low and measurement noise is prominent.
- Decoupling the two beams and measuring individual transfer functions can be used to predict the behaviour of the coupled system accurately.