1 Homework 3

1.1 Problem 1

$G(s) = \frac{\frac{K}{s(s+2)}}{1+\frac{K}{s(s+2)}} = \frac{K}{s^2+2s+K}$, $K > 0$. 二阶系统.

$\omega_n = \sqrt{K}$, $\zeta = \frac{1}{\sqrt{K}}$. 欠阻尼 $\implies K > 1$.

$M_p = e^{-\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = e^{-\frac{\pi}{\sqrt{K-1}}} < 10\%$.

$\implies \sqrt{K-1} < \frac{\pi}{\ln 10} \implies K < 1 + (\frac{\pi}{\ln 10})^2$.

$\therefore 1 < K < 1 + (\frac{\pi}{\ln 10})^2$.

1.2 Problem 2

$t_r \approx \frac{1.8}{\omega_n} \leq 0.6 \implies \omega_n \geq 3$.

$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \leq 17\% \implies \zeta \geq -\frac{\ln 0.17}{\sqrt{(\ln 0.17)^2+\pi^2}} \approx 0.49$.

$t_s \approx \frac{4.4}{\sigma} \leq 8.8 \implies \sigma \geq 0.5$.

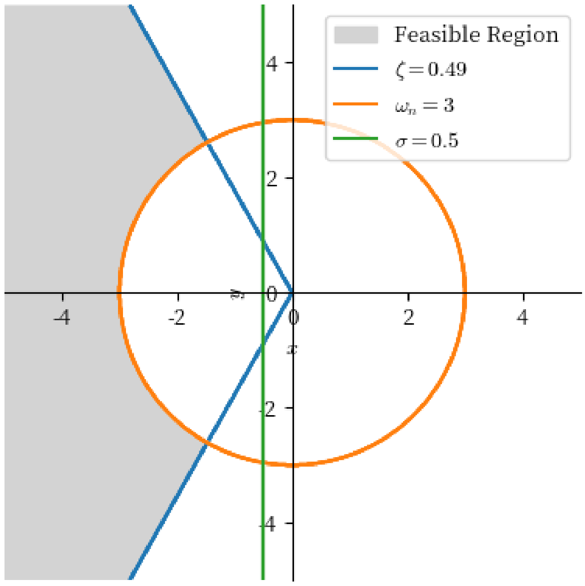

图 1: Range of poles

1.3 Problem 3

1. $J_m s\Omega_m(s) + (b + \frac{K_t K_e}{R_a})\Omega_m(s) = \frac{K_t}{R_a} V_a(s)$.

   $\implies \frac{\Omega_m(s)}{V_a(s)} = \frac{K_t}{R_a J_m s + R_a b + K_t K_e} = \frac{0.2}{s+0.104}$.

2. $V_a(s) = \frac{10}{s}$, $\Omega_m(s) = \frac{2}{s(s+0.104)} = \frac{250}{13}\left(\frac{1}{s} - \frac{1}{s+0.104}\right)$.

   $\xrightarrow{\mathcal{L}^{-1}} \omega(t) = \frac{250}{13}(1 - e^{-0.104t})1(t)$ rad/s.

   $\therefore \omega(+\infty) = \frac{250}{13}$ rad/s.

3. $J_m s^2\Theta_m(s) + (b + \frac{K_t K_e}{R_a})s\Theta_m(s) = \frac{K_t}{R_a} V_a(s)$.

   $\implies \frac{\Theta_m(s)}{V_a(s)} = \frac{K_t}{R_a J_m s^2 + R_a b + K_t K_e s} = \frac{0.2}{s^2+0.104s}$.

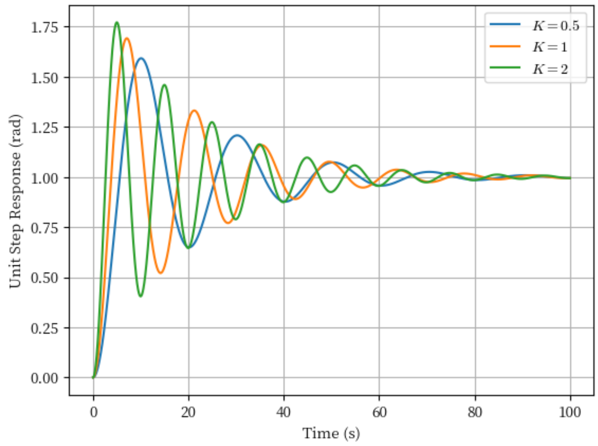

图 2: Range of K

4. $\frac{\Theta_m(s)}{K(\Theta_r(s)-\Theta_m(s))} = \frac{0.2}{s^2+0.104s}$.
   $\implies \frac{\Theta_m(s)}{\Theta_r(s)} = \frac{0.2K}{s^2+0.104s+0.2K}$.

5. 二阶系统, $\omega_n = \sqrt{0.2K}$, $\zeta = \frac{0.052}{\sqrt{0.2K}}$.
   $M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} < 20\% \implies \zeta > -\frac{\ln 0.2}{\sqrt{(\ln 0.2)^2+\pi^2}} \approx 0.456$.
   $\implies K < 0.065$, i.e. K 的最大值为 0.065.

6. $t_r \approx \frac{2.16\zeta+0.60}{\omega_n} < 4\text{ s} \implies K > 0.334$.

7. 单位阶跃响应:

	$K=0.5$	$K=1$	$K=2$
RiseTime	3.688	2.503	1.72
SettlingTime	72.715	72.24	75.113
SettlingMin	0.649	0.521	0.404
SettlingMax	1.592	1.692	1.772
Overshoot	59.231	69.227	77.169
Undershoot	0.0	0.0	0.0
Peak	1.592	1.692	1.772
PeakTime	10.072	7.073	4.984
SteadyStateValue	1.0	1.0	1.0

$K > 0.334$, 超调量 $M_p > 20\%$, 上升时间 $t_r < 4$ s. 与前两小问的计算结果一致.

1.4 Problem 4

劳斯阵列:

$$\begin{bmatrix} 1 & 10 & 5 & 0 & 0 & 0 \\ 5 & 10 & K & 0 & 0 & 0 \\ 8 & \frac{25-K}{5} & 0 & 0 & 0 & 0 \\ \frac{K+55}{8} & K & 0 & 0 & 0 & 0 \\ \frac{-K^2-350K+1375}{5(K+55)} & 0 & 0 & 0 & 0 & 0 \\ K & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

系统稳定 $\iff K > 0 \wedge K + 55 > 0 \wedge -K^2 - 350K + 1375 > 0.$

$\iff 0 < K < 80\sqrt{5} - 175.$

K 不满足取值范围:

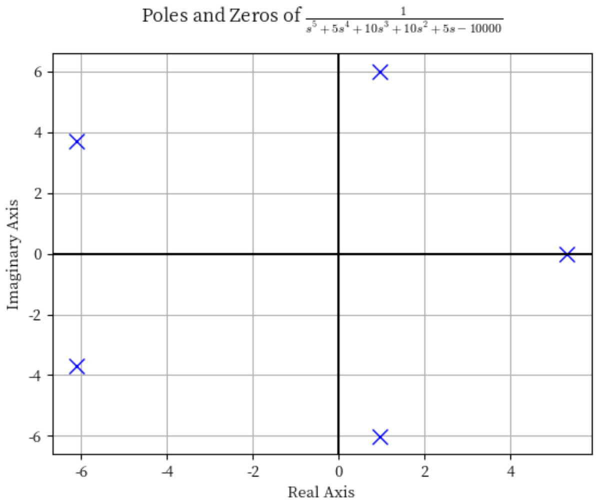

图 3: $K = -10000$

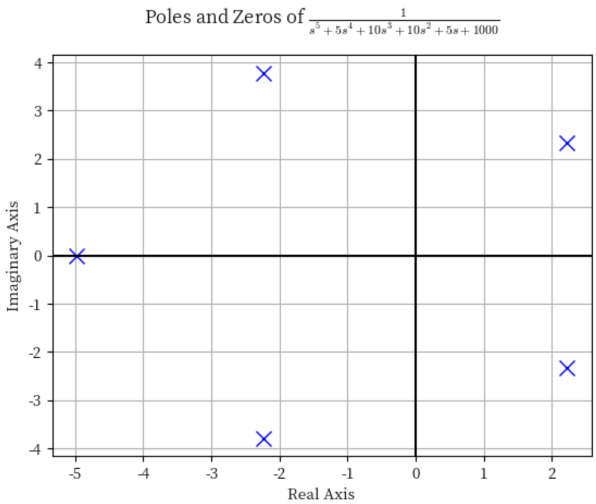

图 4: $K = 1000$

K 满足取值范围:

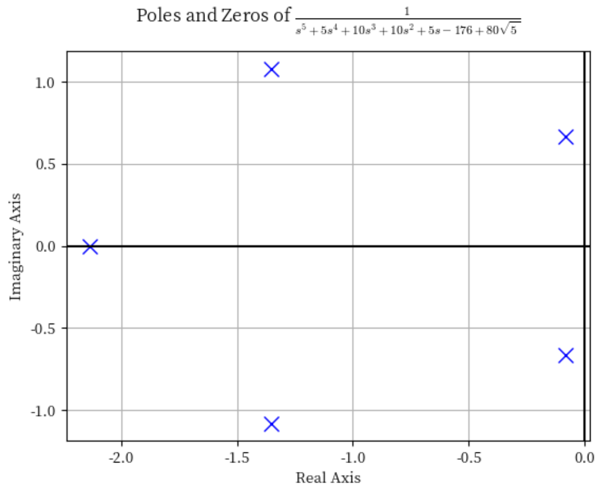

图 5: $K = 80\sqrt{5} - 176$

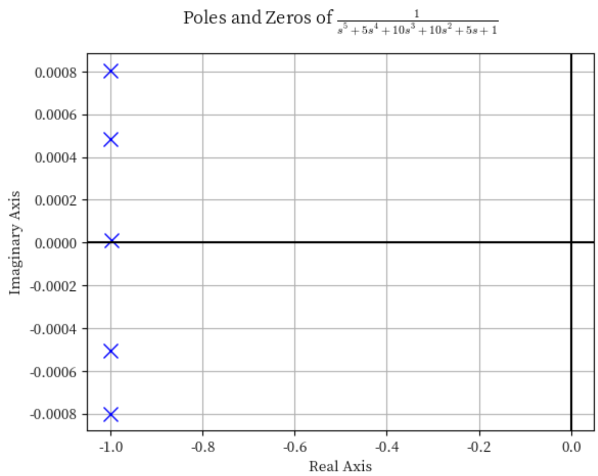

图 6: $K = 1$

1.5 Problem 5

$G(s) = \frac{KK_o(s+z)}{(s+p)(s^2-a^2)+KK_o(s+z)}$.

劳斯阵列:

$$\begin{bmatrix} 1 & KK_o - a^2 & 0 & 0 \\ p & KK_oz - a^2p & 0 & 0 \\ \frac{KK_o(p-z)}{p} & 0 & 0 & 0 \\ KK_oz - a^2p & 0 & 0 & 0 \end{bmatrix}.$$

系统稳定 $\iff p > 0 \ \wedge \ KK_o(p-z) > 0 \ \wedge \ KK_oz - a^2p > 0$.

1.6 Problem 6

$G_o(s) = G_c(s)G_p(s) = \frac{10(s+2)}{s^2(s+5)}$, 2 型系统.

加速度误差常数 $K_a = \lim_{s\to 0} s^2G_o(s) = 4$.

1.7 Problem 7

$G_o(s) = \frac{K}{s(4s+1)}$, 1 型系统.
速度误差常数 $K_v = \lim_{s\to 0} sG_o(s) = K = 1$.

1.8 Problem 8

$G(s) = \frac{3K}{4(s+1)}$.
$E(s) = (1 - G(s))R(s) = \frac{4s+4-3K}{4s(s+1)}$.
$e_{\text{ss}} = \lim_{s\to 0} sE(s) = 1 - \frac{3K}{4} = 0$.
$\implies K = \frac{4}{3}$.

1.9 Problem 9

1. $G_o(s) = \frac{K_1K_2/Is}{s(1+K_1K_2K_3/Is)} = \frac{K_1K_2}{s(25s+K_1K_2K_3)}$, 1 型系统.
   $e_{\text{ss}} = \frac{1}{\lim_{s\to 0} sG_o(s)} = K_3 < 0.01$.

2. $G(s) = \frac{G_o(s)}{1+G_o(s)} = \frac{\frac{K_1K_2}{25}}{s^2+\frac{K_1K_2K_3}{25}s+\frac{K_1K_2}{25}}$.
   二阶系统, $\omega_n = \frac{\sqrt{K_1K_2}}{5}$, $\zeta = \frac{\sqrt{K_1K_2}K_3}{10}$.
   $M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} < 10\% \implies \zeta > -\frac{\ln 0.1}{\sqrt{(\ln 0.1)^2+\pi^2}} \approx 0.591$.
   $K_1K_2 = \frac{100\zeta^2}{K_3^2} > \frac{100\times 0.591^2}{0.01^2} \approx 349464$.