

Homework 4

1. 被控对象 Z 传递函数

$$\begin{aligned} G(z) &= \mathcal{Z} \left\{ \frac{1 - e^{-TS}}{s^2(s+1)} \right\} = (1 - z^{-1}) \mathcal{Z} \left\{ \frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2} \right\} \\ &= \frac{1 - z^{-1}}{1 - e^{-T}z^{-1}} - 1 + \frac{Tz^{-1}}{1 - z^{-1}} \end{aligned}$$

开环 Z 传递函数

$$W_o(z) = KG(z) = K \left(\frac{1 - z^{-1}}{1 - e^{-T}z^{-1}} - 1 + \frac{Tz^{-1}}{1 - z^{-1}} \right)$$

闭环 Z 传递函数

$$\begin{aligned} W_c(z) &= \frac{W_o(z)}{1 + W_o(z)} = \frac{K \left(\frac{1 - z^{-1}}{1 - e^{-T}z^{-1}} - 1 + \frac{Tz^{-1}}{1 - z^{-1}} \right)}{1 + K \left(\frac{1 - z^{-1}}{1 - e^{-T}z^{-1}} - 1 + \frac{Tz^{-1}}{1 - z^{-1}} \right)} \\ &= \frac{K (T + e^{-T} - 1) z^{-1} + K (-Te^{-T} - e^{-T} + 1) z^{-2}}{1 + (K (T + e^{-T} - 1) - e^{-T} - 1) z^{-1} + (K (-Te^{-T} - e^{-T} + 1) + e^{-T}) z^{-2}} \end{aligned}$$

特征方程

$$1 + (K (T + e^{-T} - 1) - e^{-T} - 1) z^{-1} + (K (-Te^{-T} - e^{-T} + 1) + e^{-T}) z^{-2} = 0$$

令 $z^{-1} = \frac{1-w}{1+w}$, 整理得

$$\begin{aligned} &(K (-Te^{-T} - T - 2e^{-T} + 2) + 2e^{-T} + 2) w^2 \\ &+ (K (2Te^{-T} + 2e^{-T} - 2) - 2e^{-T} + 2) w + KT (-e^{-T} + 1) = 0 \end{aligned}$$

劳斯阵列

$$\begin{aligned} w^2 : & \quad K (-Te^{-T} - T - 2e^{-T} + 2) + 2e^{-T} + 2 \quad KT (-e^{-T} + 1) \\ w^1 : & \quad K (2Te^{-T} + 2e^{-T} - 2) - 2e^{-T} + 2 \\ w^0 : & \quad KT (-e^{-T} + 1) \end{aligned}$$

考虑 $K, T > 0$, 则 $KT (-e^{-T} + 1) > 0$. 要使系统稳定, 应有

$$K (-Te^{-T} - T - 2e^{-T} + 2) + 2e^{-T} + 2 > 0$$

$$K (2Te^{-T} + 2e^{-T} - 2) - 2e^{-T} + 2 > 0$$

i. 当 $T = 0.1$ 时,

$$K < \frac{2e^{-0.1} + 2}{2.1e^{-0.1} - 1.9} \approx 24024.000$$

$$K < \frac{e^{-0.1} - 1}{1.1e^{-0.1} - 1} \approx 20.339$$

$$\therefore 0 < K < 20.339$$

ii. 当 $T = 1$ 时,

$$K < \frac{2e + 2}{3 - e} \approx 26.397$$

$$K < \frac{e - 1}{2 - e} \approx 2.392$$

$$\therefore 0 < K < 2.392$$

2. i. 被控对象 Z 传递函数

$$\begin{aligned} G(z) &= \mathcal{Z} \left\{ \frac{10(1 - e^{-TS})}{s^2(s+1)} \right\} = 10(1 - z^{-1}) \mathcal{Z} \left\{ \frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2} \right\} \\ &= 10 \left(\frac{1 - z^{-1}}{1 - e^{-T}z^{-1}} - 1 + \frac{Tz^{-1}}{1 - z^{-1}} \right) \end{aligned}$$

开环 Z 传递函数

$$\begin{aligned} W_o(z) &= (1.5 - 0.5z^{-1})G(z) \\ &= \frac{(15T + 15e^{-T} - 15)z^{-1} + (-15Te^{-T} - 5T - 20e^{-T} + 20)z^{-2} + (5Te^{-T} + 5e^{-T} - 5)z^{-3}}{(1 - z^{-1})(1 - e^{-T}z^{-1})} \end{aligned}$$

分母含有因子 $(1 - z^{-1})$, 故系统为 1 型系统. 速度误差常数为

$$\begin{aligned} K_v &= \lim_{z \rightarrow 1} (1 - z^{-1}) W_o(z) \\ &= \frac{(15T + 15e^{-T} - 15) + (-15Te^{-T} - 5T - 20e^{-T} + 20) + (5Te^{-T} + 5e^{-T} - 5)}{1 - e^{-T}} \\ &= \frac{10T - 10Te^{-T}}{1 - e^{-T}} = 10T \end{aligned}$$

当 $T = 0.2$ 时, $K_v = 2$.

ii. 对于单位位置信号 $1(t)$, 稳态误差为 0; 对于单位速度信号 $t \cdot 1(t)$, 稳态误差为 $\frac{T}{K_v} = 0.1$.

故对于 $r(t) = (1 + 2t) \cdot 1(t)$, 稳态误差 $e_{ss} = 0 + 2 \times 0.1 = 0.2$

3. 系统的特征方程

$$\begin{vmatrix} z & -1 \\ 4.8 & z - 1.4 \end{vmatrix} = z^2 - 1.4z + 4.8 = 0$$

特征根 $z_{1,2} = 0.7 \pm j2.076$, $|z_{1,2}| = 2.191 > 1$, 位于单位圆外, 系统不稳定.