EE3006.01 计算机控制基础

第 2 次作业

2022 年秋季学期

截止日期: 2022-10-10 **允许讨论, 禁止抄袭**

1. 求下列函数的采样信号的拉氏变换 $F^*(s)$:

   (1) $f(t) = \mathrm{e}^{-(t-2T)}$;

   (2) $f(t) = t\sin(at)$, 式中 a 为正常数.

2. 求下列函数的 Z 变换:

   (1) $f(t) = \cos(at)$, 式中 a 为正常数;

   (2) $f(t) = t\mathrm{e}^{-at}$, 式中 a 为常数.

3. 求以下拉氏变换像函数的 Z 变换:

   (1) $G(s) = \frac{k}{T_1 s+1}\mathrm{e}^{-Ts}$, 式中 k, T_1 为常数;

   (2) $G(s) = \frac{k}{T_1 s+1} \cdot \frac{1-\mathrm{e}^{-Ts}}{s}$, 式中 k, T_1 为常数;

   (3) $G(s) = \frac{k}{s(s^2+s+1)}$, 式中 k 为常数;

4. 已知下列 Z 变换式, 求取对应的离散序列 $f(nT)$:

   (1) $F(z) = \frac{z+2}{(z-1)(z-2)}$;

   (2) $F(s) = \frac{z}{z-\mathrm{e}^{aT})(z-\mathrm{e}^{bT})}$, 式中 a, b 均为常数.

5. 分别用递推法、状态空间方法和 Z 变换求解差分方程

   (1)

$$y(kT+2T) + 3y(kT+T) + 2y(kT) = 0$$

   其中, 初始条件: $y(0) = 0$, $y(1) = 1$;

(2)

$$y(kT+2T)+3y(kT+T)+2y(kT)=\mathrm{e}^{-kT}$$

其中, 初始条件: $y(0)=0$, $y(1)=1$.

6. 已知 Z 传递函数

$$G(z)=\frac{3z^2-2z+2.5}{z^2-0.9z+0.2}$$

写出对应的离散时间状态空间表达式.

7. 已知离散系统的状态空间表达式为

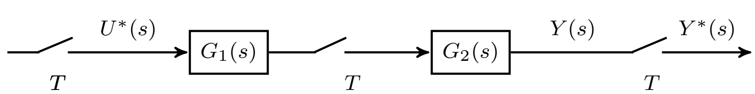
$$x(k+1)=\begin{bmatrix}-3 & 0\\ 1 & 0\end{bmatrix}x(k)+\begin{bmatrix}1\\ 0\end{bmatrix}u(k)$$
$$y(k)=\begin{bmatrix}0 & 1\end{bmatrix}x(k)$$

(1) 求 $X(z)$, $Y(z)$, 以及 $u(k)$ 到 $y(k)$ 的 Z 传递函数;

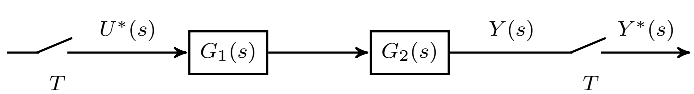
(2) 当 $u(k)=0$, $x(0)=[1\ \ 0]^T$, 求 $y(k)$;

(3) 当 $u(k)$ 为单位阶跃序列, $x(0)=[1\ \ 0]^T$, 求 $y(k)$.

8. 若 $G_1(s)=\frac{1}{s}$, $G_2(s)=\frac{10}{s+10}$, 计算图 1(a) 和 1(b) 所示系统的 Z 传递函数.



(a)



(b)

图 1. 题 8 系统

9. 如图 2 所示系统, 其中 $G_h(s)=\frac{1-\mathrm{e}^{-Ts}}{s}$ 为零阶保持器, $G_0(s)=\frac{\mathrm{e}^{-Ts}}{s+1)(s+2)}$.

(1) 分别计算 $Y(s)$ 和 $Y(z)$;

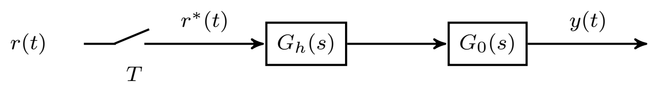


图 2. 题 9 系统

(2) 当 $r(k)$ 是单位阶跃序列时, 求 $y(k)$.

10. 已知连续被控对象的状态空间表达式为

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Br(t)\\ y(t) &= Cx(t)\end{aligned}$$

其中, $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$. 若用计算机控制并用零阶保持器恢复控制信号 $r(t)$, 求该被控对象的等效离散化状态空间表达式, 及其 Z 传递函数 $G(z)$.

11. 求图 3 所示系统的 $Y(s)$ 和 $Y(z)$;

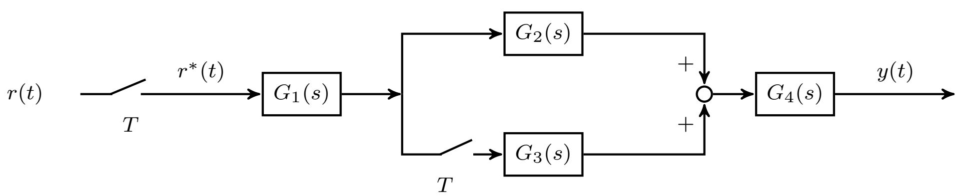


图 3. 题 11 系统

12. 求图 4(a) 和图 4(b) 所示计算机控制系统的闭环传递函数 $W(z)$.

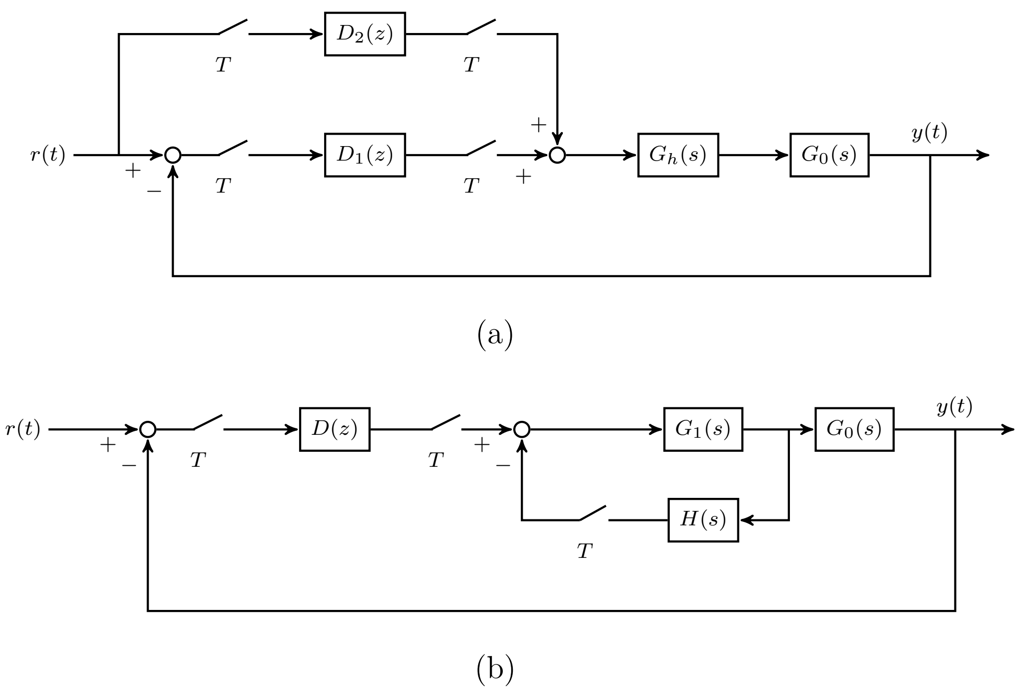


图 4. 题 12 系统

1. (1) $f(t)=e^{-(t-2T)} \longrightarrow F(z)=\dfrac{z^{-2}}{1-e^{-T}z^{-1}} \longrightarrow F^*(s)=\dfrac{e^{-2sT}}{1-e^{-T}e^{-sT}}$

(2) $f(t)=t\sin(at)=-\dfrac{d}{da}\cos(at) \longrightarrow F(z)=-\dfrac{d}{da}\dfrac{1-z^{-1}\cos aT}{1-2z^{-1}\cos aT+z^{-2}}$

$$=\frac{Tz^{-1}(1-z^{-2})\sin aT}{(1-2z^{-1}\cos aT+z^{-2})^2} \rightarrow F^*(s)=\frac{Te^{-sT}(1-e^{-2sT})\sin aT}{(1-2e^{-sT}\cos aT+e^{-2sT})^2}$$

2. (1) $f(t)=\cos(at) \longrightarrow F(z)=\dfrac{1-z^{-1}\cos aT}{1-2z^{-1}\cos aT+z^{-2}}$

(2) $f(t)=te^{-at}=-\dfrac{d}{da}e^{-at} \rightarrow F(z)=-\dfrac{d}{da}\dfrac{1}{1-e^{-aT}z^{-1}}=\dfrac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

3. (1) $G(s)=\dfrac{k}{T_1s+1}e^{-Ts} \rightarrow g(t)=\dfrac{k}{T_1}e^{-\frac{t-T}{T_1}}u(t-T) \longrightarrow G(z)=\dfrac{kz^{-1}}{T_1(1-e^{-T/T_1}z^{-1})}$

(2) $G(s)=\dfrac{k}{T_1s+1}\cdot\dfrac{1-e^{-Ts}}{s} \rightarrow G(z)=\mathcal{Z}\left\{\dfrac{k}{s}-\dfrac{kT_1}{T_1s+1}\right\}(1-z^{-1})=\left(\dfrac{k}{1-z^{-1}}-\dfrac{k}{1-e^{-T/T_1}z^{-1}}\right)(1-z^{-1})$

$$=\frac{k(1-e^{-T/T_1})z^{-1}}{1-e^{-T/T_1}z^{-1}}$$

(3) $G(s)=\dfrac{k}{s(s^2+s+1)}=\dfrac{-k(s+\frac{1}{2})}{(s+\frac{1}{2})^2+\frac{3}{4}}+\dfrac{-\frac{\sqrt{3}}{3}k\cdot\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}+\dfrac{k}{s}$

$$\longrightarrow G(z)=-k\frac{1-e^{-\frac{T}{2}}z^{-1}\cos\frac{\sqrt{3}}{2}T}{1-2e^{-\frac{T}{2}}z^{-1}\cos\frac{\sqrt{3}}{2}T+e^{-T}z^{-2}}-\frac{\sqrt{3}}{3}k\frac{e^{-\frac{T}{2}}\sin\frac{\sqrt{3}}{2}T}{1-2e^{-\frac{T}{2}}z^{-1}\cos\frac{\sqrt{3}}{2}T+e^{-T}z^{-2}}+\frac{k}{1-z^{-1}}$$

4. (1) $F(z)=\dfrac{z+2}{(z-1)(z-2)}=\left(\dfrac{4}{1-2z^{-1}}-\dfrac{3}{1-z^{-1}}\right)z^{-1} \rightarrow f(nT)=(2^{n+1}-3)u(nT-T)$

(2) $F(z)=\dfrac{z}{(z-e^{aT})(z-e^{bT})}=\left(\dfrac{e^{aT}}{e^{aT}-e^{bT}}\dfrac{1}{1-e^{aT}z^{-1}}-\dfrac{e^{bT}}{e^{aT}-e^{bT}}\dfrac{1}{1-e^{bT}z^{-1}}\right)z^{-1}$

$$\longrightarrow \frac{e^{anT}-e^{bnT}}{e^{aT}-e^{bT}}u(nT-T)$$

5. (1) Recurrence relation:

$y(kT+2T)+2y(kT+T)=-[y(kT+T)+2y(kT)]$, $y(T)+2y(0)=1$

$\Rightarrow y(kT+T)+2y(kT)=(-1)^k \Rightarrow y(kT+T)-(-1)^{k+1}=-2[y(kT)-(-1)^k]$, $y(0)-1=-$

$\Rightarrow y(kT)-(-1)^k=-(-2)^k \Rightarrow y(kT)=(-1)^k-(-2)^k$

State space:

$$x(kT+T)=\begin{pmatrix}0 & 1\\ -2 & -3\end{pmatrix}x(kT), \quad x(0)=\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$y(kT)=(1 \ \ 0)\,x(kT)$$

$$\Rightarrow x(kT)=\begin{pmatrix}0 & 1\\ -2 & -3\end{pmatrix}^k\begin{pmatrix}0\\ 1\end{pmatrix}=\mathcal{Z}^{-1}\left\{\begin{pmatrix}1 & -z^{-1}\\ 2z^{-1} & 1+3z^{-1}\end{pmatrix}^{-1}\right\}\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}2(-1)^k-(-2)^k & (-1)^k-(-2)^k\\ -2(-1)^k+2(-2)^k & -(-1)^k+2(-2)^k\end{pmatrix}\Big($$

$$=\begin{pmatrix}(-1)^k-(-2)^k\\ -(-1)^k+2(-2)^k\end{pmatrix}, \quad y(kT)=(-1)^k-(-2)^k.$$

Z transform:

$$z^2Y(z)-z^2y(0)-zy(T)+3zY(z)-3zy(0)+2Y(z)=0$$

$$\Rightarrow Y(z)=\frac{z}{z^2+3z+2}=\frac{1}{1+z^{-1}}-\frac{1}{1+2z^{-1}}$$

$$\rightarrow y(kT)=(-1)^k-(-2)^k.$$

(2) Recurrence relation:

$$y(kT+2T)=-3y(kT+T)-2y(kT)+e^{-kT}$$

$$y(2T)=-3y(T)-2y(0)+e^{-0}=-2$$

$$y(3T)=-3y(2T)-2y(T)+e^{-T}=4+e^{-T}$$

$$y(4T)=-3y(3T)-2y(2T)+e^{-2T}=-8-3e^{-T}+e^{-2T}$$

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State space:

$$x(kT+T)=\begin{pmatrix}0 & 1\\ -2 & -3\end{pmatrix}x(kT)+\begin{pmatrix}0\\ 1\end{pmatrix}e^{-kT}, \quad x(0)=\begin{pmatrix}0\\ 1\end{pmatrix}, \quad y(kT)=(1 \ \ 0)\,x(kT)$$

$$\Rightarrow x(kT)=\begin{pmatrix}0 & 1\\ -2 & -3\end{pmatrix}^k\begin{pmatrix}0\\ 1\end{pmatrix}+\sum_{n=0}^{k-1}\begin{pmatrix}0 & 1\\ -2 & -3\end{pmatrix}^n\begin{pmatrix}0\\ e^{nT}\end{pmatrix}=\begin{pmatrix}(-1)^k-(-2)^k\\ -(-1)^k+2(-2)^k\end{pmatrix}+\sum_{n=0}^{k-1}\begin{pmatrix}(-1)^n-(-2)^n\\ -(-1)^n+2(-2)^n\end{pmatrix}e^{-(k-1-n)}$$

$$\Rightarrow y(kT)=(-1)^k-(-2)^k+e^{-(k-1)T}\sum_{n=0}^{k-1}\left[(-1)^ne^{nT}-(-2)^ne^{nT}\right]=(-1)^k-(-2)^k+\frac{e^{-kT}-(-1)^n}{1+e^{-T}}-\frac{e^{-kT}-(-2)^n}{2+e^{-T}}$$

$$=\frac{e^{-kT}}{(1+e^{-T})(2+e^{-T})}+\frac{e^{-T}}{1+e^{-T}}(-1)^k-\frac{1+e^{-T}}{2+e^{-T}}(-2)^k$$

z transform:

$$z^2Y(z) - z + 3zY(z) + 2Y(z) = \frac{1}{1-e^{-T}z^{-1}}$$

$$\Rightarrow Y(z) = \frac{1}{1+z^{-1}} - \frac{1}{1+2z^{-1}} + \left(\frac{e^{-T}}{(1+e^{-T})(2+e^{-T})}\cdot\frac{1}{1-e^{-T}z^{-1}} + \frac{1}{1+e^{-T}}\cdot\frac{1}{1+z^{-1}} - \frac{2}{2+e^{-T}}\cdot\frac{1}{1+2z^{-1}}\right)z^{-1}$$

$$\rightarrow f(kT) = (-1)^k - (-2)^k + \left(\frac{e^{-kT}}{(1+e^{-T})(2+e^{-T})} - \frac{(-1)^k}{1+e^{-T}} + \frac{(-2)^k}{2+e^{-T}}\right)u(kT-T)$$

$$= (-1)^k - (-2)^k + \frac{e^{-kT}}{(1+e^{-T})(2+e^{-T})} - \frac{(-1)^k}{1+e^{-T}} + \frac{(-2)^k}{2+e^{-T}}$$

$$= \frac{e^{-kT}}{(1+e^{-T})(2+e^{-T})} + \frac{e^{-T}}{1+e^{-T}}(-1)^k - \frac{1+e^{-T}}{2+e^{-T}}(-2)^k$$

6.

$$x(k+1) = \begin{pmatrix} 0 & 1 \\ -0.2 & 0.9 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = (1.9 \quad 0.7)\, x(k) + 3u(k)$$

7. (1) $zX(z) - zX(0) = \begin{pmatrix} -3 & 0 \\ 1 & 0 \end{pmatrix} X(z) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} U(z)$

$$\Rightarrow X(z) = \begin{pmatrix} z+3 & 0 \\ -1 & z \end{pmatrix}^{-1}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} U(z) + zX(0)\right] = \begin{pmatrix} \frac{1}{z+3} \\ \frac{1}{z(z+3)} \end{pmatrix} U(z) + \begin{pmatrix} \frac{z}{z+3} & 0 \\ \frac{1}{z+3} & 1 \end{pmatrix} X(0)$$

$$Y(z) = (0 \quad 1)\, X(z) = \frac{U(z)}{z(z+3)} + \left(\frac{1}{z+3} \quad 1\right) X(0)$$

$$\left.\frac{Y(z)}{U(z)}\right|_{X(0)=0} = \frac{1}{z(z+3)}$$

(2) $Y(z) = \left(\frac{1}{z+3} \quad 1\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{z+3} \rightarrow y(k) = 3^{k-1} h(k-1)$ ($h(\cdot)$ denotes unit step sequence)

(3) $Y(z) = \frac{\frac{z}{z-1}}{z(z+3)} + \frac{1}{z+3} = \frac{1}{4}\cdot\frac{1}{z-1} + \frac{3}{4}\cdot\frac{1}{z+3} \rightarrow y(k) = \frac{1+3^k}{4} u(k-1)$

8. (a) $Y^*(s) = \left[(U^*(s) G_1(s))^* G_2(s)\right]^* = U^*(s) G_1^*(s) G_2^*(s)$

$$\Rightarrow \frac{Y(z)}{U(z)} = G_1(z) G_2(z) = \frac{10}{(1-z^{-1})(1-e^{-10T}z^{-1})}$$

(b) $Y^*(s) = [U^*(s) G_1(s) G_2(s)]^* = U^*(s) G_1G_2^*(s)$

$$\Rightarrow \frac{Y(z)}{U(z)} = G_1G_2(z) = \mathcal{Z}\left\{\frac{10}{s(s+10)}\right\} = \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-10T}z^{-1}}$$

9. (1) $Y(s) = R^*(s) G_h(s) G_0(s)$

$$= R^*(s) \frac{e^{-Ts}(1-e^{-Ts})}{s(s+1)(s+2)}$$

$$Y(z) = R(z) z^{-1}(1-z^{-1}) \mathcal{Z}\left\{\frac{1}{s(s+1)(s+2)}\right\}$$

$$= R(z) z^{-1}(1-z^{-1}) \left(\frac{\frac{1}{2}}{1-z^{-1}} - \frac{1}{1-e^{-T}z^{-1}} + \frac{\frac{1}{2}}{1-e^{-2T}z^{-1}}\right)$$

$$= R(z)\left(\frac{1}{2}z^{-1} - \frac{z^{-1}(1-z^{-1})}{1-e^{-T}z^{-1}} + \frac{z^{-1}(1-z^{-1})}{2(1-e^{-2T}z^{-1})}\right)$$

(2) $R(z) = \frac{1}{1-z^{-1}}$

$$Y(z) = \left(\frac{1}{2}\cdot\frac{1}{1-z^{-1}} - \frac{1}{1-e^{-T}z^{-1}} + \frac{1}{2}\cdot\frac{1}{1-e^{-2T}z^{-1}}\right) z^{-1}$$

$$\rightarrow y(k) = \left(\frac{1}{2} - e^{-kT} + \frac{1}{2}e^{-2kT}\right) u(k-1)$$

10. $F = e^{AT} = \exp\left\{\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} T\right\} = \begin{pmatrix} -e^{-2T}+2e^{-T} & e^{-2T}-e^{-T} \\ -2e^{-2T}+2e^{-T} & 2e^{-2T}-e^{-T} \end{pmatrix}$

$$G = \int_0^T e^{At} B\, dt = \int_0^T \begin{pmatrix} e^{-2t}-e^{-t} \\ 2e^{-2t}-e^{-t} \end{pmatrix} dt = \begin{pmatrix} -\frac{1}{2}e^{-2T}+e^{-T}-\frac{1}{2} \\ -e^{-2t}+e^{-t} \end{pmatrix}$$

$x(k+1) = F x(k) + G r(k)$

$y(k) = C x(k)$

$$G(z) = C(zI-F)^{-1}G = (0\ 1)\begin{pmatrix} z+e^{-2T}-2e^{-T} & -e^{-2T}+e^{-T} \\ 2e^{-2T}-2e^{-T} & z-2e^{-2T}+e^{-T} \end{pmatrix}^{-1} \begin{pmatrix} -\frac{1}{2}e^{-2T}+e^{-T}-\frac{1}{2} \\ -e^{-2T}+e^{-T} \end{pmatrix}$$

$$= \frac{a-1}{z-a} - \frac{a^2-1}{z-a^2}$$

or $G(z) = \mathcal{Z}\left\{\frac{1-e^{-Ts}}{s}\cdot\frac{s}{s^2+3s+2}\right\} = (1-z^{-1})\mathcal{Z}\left\{\frac{1}{s^2+3s+2}\right\} = \frac{a-1}{z-a} - \frac{a^2-1}{z-a^2}$

11. $Y(s) = R^*(s)\left[G_1(s)G_2(s) + G_1^*(s)G_3(s)\right]G_4(s)$

$\quad = R^*(s)\left[G_1G_2G_4(s) + G_1^*(s)G_3G_4(s)\right]$

$Y^*(s) = R^*(s)\left[G_1G_2G_4^*(s) + G_1^*(s)G_3G_4^*(s)\right]$

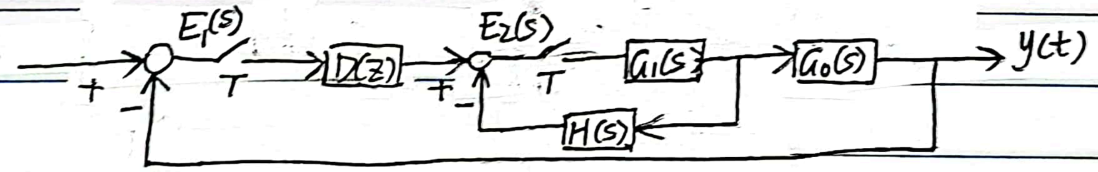
$Y(z) = R(z)\left[G_1G_2G_4(z) + G_1(z)G_3G_4(z)\right]$

12. (a) $Y(s) = \left[(R(s)-Y(s))^* D_1^*(s) + R^*(s)D_2^*(s)\right]G_hG_0(s)$

$$\Rightarrow \frac{Y^*(s)}{G_hG_0^*(s)} = R^*(s)\left(D_1^*(s)+D_2^*(s)\right) - Y^*(s)D_1^*(s)$$

$$\Rightarrow \frac{Y^*(s)}{R^*(s)} = \frac{D_1^*(s)+D_2^*(s)}{D_1^*(s)+\frac{1}{G_hG_0^*(s)}} \Rightarrow W(z) = \frac{(D_1(z)+D_2(z))\,G_hG_0(z)}{1+D_1(z)G_hG_0(z)}$$

(b)



$$\begin{cases} E_1(s) = R(s) - Y(s) \\ E_2(s) = E_1^*(s)D(s) - E_2^*(s)G_1(s)H(s) \\ Y(s) = E_2^*(s)G_1(s)G_0(s) \end{cases}$$

$$\Rightarrow \begin{cases} E_1^*(s) = R^*(s) - Y^*(s) \\ E_2^*(s) = E_1^*(s)D^*(s) - E_2^*(s)G_1H^*(s) \\ Y^*(s) = E_2^*(s)G_1G_0^*(s) \end{cases}$$

$$\Rightarrow Y^*(s) = \frac{D^*(s)G_1G_0^*(s)}{1+G_1H^*(s)+D^*(s)G_1G_0^*(s)}R^*(s)$$

$$\Rightarrow W(z) = \frac{D(z)G_1G_0(z)}{1+G_1H(z)+D(z)G_1G_0(z)}$$