Examples of Uncertainty Propagation

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This code was developed by Miodrag Bolic for the book PERVASIVE CARDIAC AND RESPIRATORY MONITORING DEVICES. The author thanks Joe Klebba for developing the toolbox for uncertainty propagation in Matlab.

Introduction

This notebook provides introduction to uncertainty propagation through several simple models. Uncertainty Quantification (UQ) aims at developing rigorous methods to characterize the impact of "limited knowledge" on quantities of interest.

Example 1 Uncertainty of corellated variables

Let us define a ratio R of AC currents obtained from a photodetector after emitting red and infrared light as:

$$R = I_R/I_{IR}$$

Assume the AC currents at the photodiode obtained after emitting red (I_R) and infrared (I_{IR}) light are measured with the standard deviation of 0.5 μA . The values of I_R and I_R are I_R = 10 μA and I_{IR} = 20 μA. Let us assume that the measurements are correlated, and that the correlation coefficient is ρ= 0.7. What is the uncertainty of the ratio R? Let us assume that the current follow normal distribution. Also, the distribution of R obtain after dividing the random variable that follow normal distribution will be also normal.

Solution:

Method 1 Perturbation

```
I_R=10; I_IR=20;
```

```
u_R=0.5; u_IR=0.5;
corrMatrix=[1 0.7; 0.7 1];
R = @(I_R,I_IR)I_R./I_IR;

[uncertCD,valCD]=propUncertCD(R,[I_R I_IR],[u_R u_IR],corrMatrix)
```

```
uncertCD = 0.0185
valCD = 0.5000
```

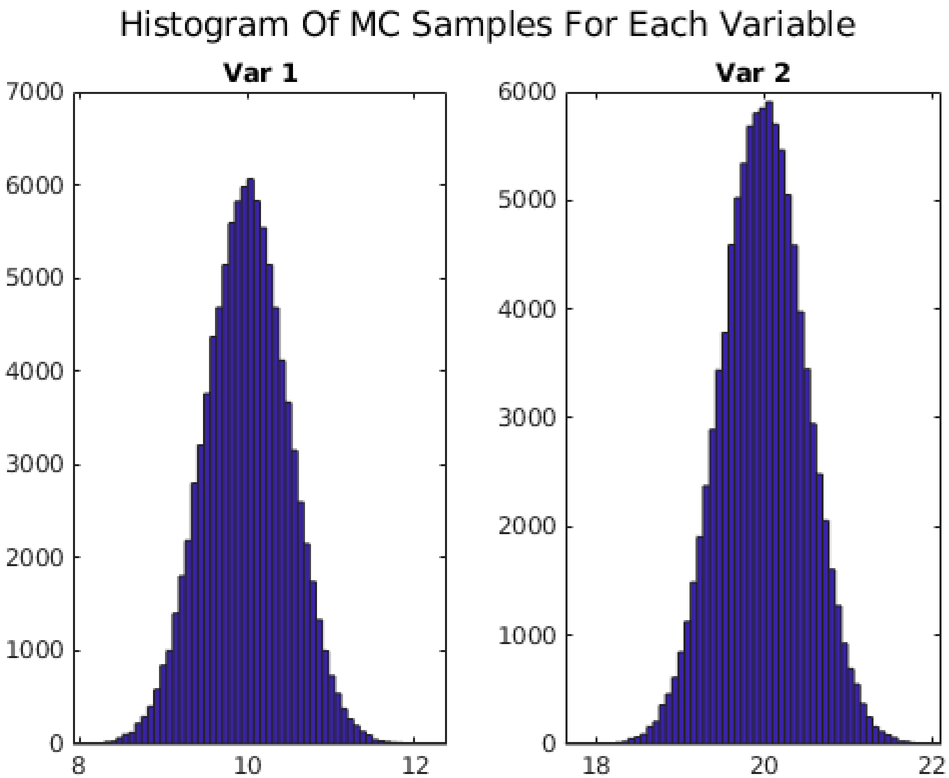
```
%Computation based on the formula from the text book
un_R=sqrt((u_R*1/I_IR)^2+(-u_IR*I_R/I_IR^2)^2-2*corrMatrix(1,2)*u_R*u_IR*I_R/I_IR^3)
```

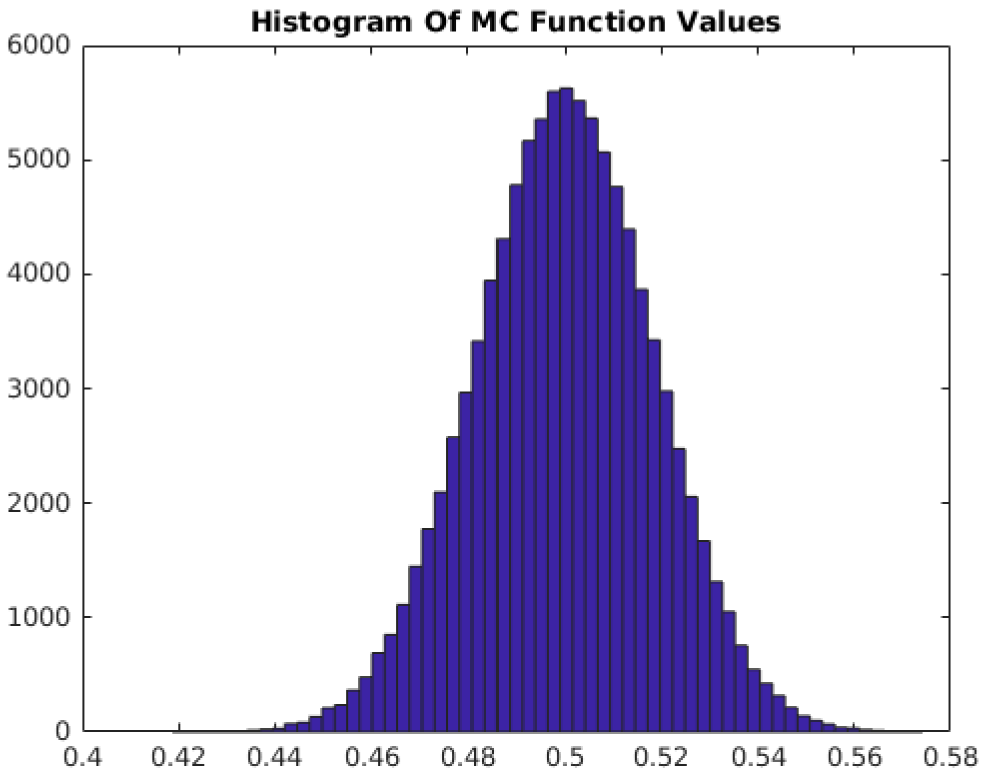
```
un_R = 0.0185
```

Method 2 Monte Carlo

The same result u_R=0.0185 can be obtained using Monte Carlo sampling. First, we need to sample from two-dimensional multivariate Gaussian distribution and obtain correlated samples $I_R^{(i)}$ and $I_{IR}^{(i)}$ for $i = 1,\ldots M$ where M is the number of Monte Carlo samples. We are using M=100,000 samples. Then, these samples are passed through the function $R^{(i)} = I_R^{(i)}/I_{IR}^{(i)}$. Values $R^{(i)}$ are fitted to a normal distribution and confidence intervals are computed. The solution in Matlab is shown in the book web page.

```
% Monte Carlo computation based on the method proposed by Joe Klebba
[CI, funcVal, MCfuncVals, MCsamples]=propUncertMC(R,{{'Corr',{{I_R u_R};{I_IR u_IR}},co
```






```
uncertMC = (CI(2)-CI(1))/2
```

```
uncertMC = 0.0184
```

```
valMC
```

```
valMC = 0.4999
```

```
% Monte Carlo computation based on the method from the text book
Cov=[u_R^2 corrMatrix(1,2)*u_R*u_IR; corrMatrix(1,2)*u_R*u_IR u_IR^2]
```

```
Cov = 2×2
    0.2500    0.1750
    0.1750    0.2500
```

```
rng('default')  % For reproducibility
MvRnd = mvnrnd([I_R I_IR],Cov,100000);

MCfuncVals1=R(MvRnd(:,1),MvRnd(:,2));
pd = fitdist(MCfuncVals1,'Normal');
ci = paramci(pd,'Alpha',.33);
ci(2,2)
```

```
ans = 0.0186
```

Example 2 Uncertainty based on GUM

a. For a transducer with the following calibration curve, y = bx, estimate the expanded uncertainty for x = 5.00 , if b =1 with Ub =0.01 and U_x =0.05 at 95% confidence. Assume that all variables follow normal distribution. Please note that we did not introduce units here and that we assume that all the values are relative.

b. Compute the expanded uncertainty at 99% confidence.

Solution:

```
clear all
x=5; u_x=0.05/2;
b=1; u_b=0.01/2;
y=@(x,b)x.*b;
[uncertCD,valCD]=propUncertCD(y,[x b],[u_x u_b])
```

```
uncertCD = 0.0354
valCD = 5
```

```
U_95=uncertCD*2
```

```
U_95 = 0.0707
```

```
U_99=uncertCD*3
```

```
U_99 = 0.1061
```

```
% Calculation from the textbook
u_y=sqrt((x*u_b)^2+(b*u_x)^2)
```

```
u_y = 0.0354
```

```

[CI,valMC]=propUncertMC(y,{{x u_x};{b u_b}},100000);
uncertMC = (CI(2)-CI(1))/2
```

```
uncertMC = 0.0351
```

```
valMC
```

```
valMC = 5.0012
```

```
% Uncertainty can be directly computed to the desired level
[CI,valMC]=propUncertMC(y,{{x u_x};{b u_b}},100000,'CI',0.99);
uncertMC = (CI(2)-CI(1))/2
```

```
uncertMC = 0.0910
```

```
valMC
```

```
valMC = 5.0015
```

Example 3 Combination of custom and normal distribution

Here, we will consider and example in which the data that is coming from a normal distribution and is quantized using ADC converter with the resulution of 8 bits and the range of 5 V.

```
n=100000;
%x_data = unifrnd(3,4,1,15);
x = normrnd(1,0.01,n,1);
ADC_res=5/2^8;
u_ADC=ADC_res/sqrt(12); % standard uncertainty for the "rounding" ADC
% Uncertainty in the measurements of x is Type A uncertainty while ADC
% converter uncertainty is type B. The total uncertainty based on GUM is:
u_total=sqrt(u_ADC^2+0.01^2)
```

```
u_total = 0.0115
```

```
f = @(x,u)x+u;
[CI,valMC]=propUncertMC(f,{{'Custom',x};{'uniform',0,ADC_res}},n);
uncertMC = (CI(2)-CI(1))/2
```

```
uncertMC = 0.0115
```

```
valMC
```

```
valMC = 1.0101
```