Confidence intervals

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This code was developed by Miodrag Bolic for the book PERVASIVE CARDIOVASCULAR AND RESPIRATORY MONITORING DEVICES

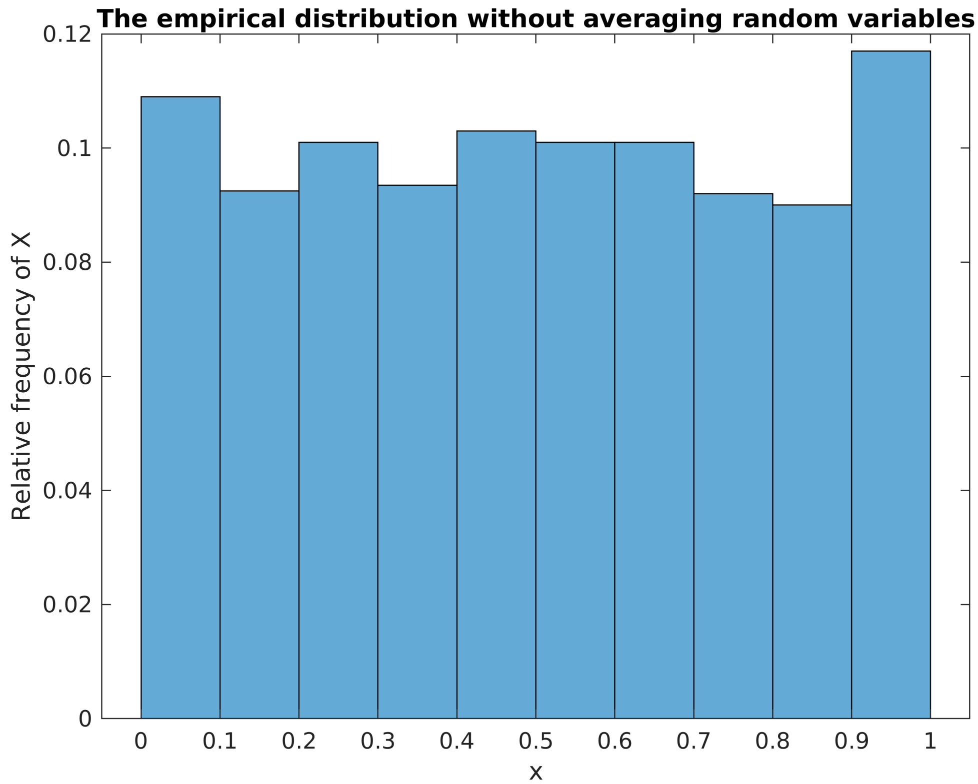
```
% Changing the path from main_folder to a particular chapter
main_path=fileparts(which('Main_Content.mlx'));
if ~isempty(main_path)
    %addpath(append(main_path,'/Chapter2'))
    cd (append(main_path,'/Chapter1/CLT/'))
    addpath(append(main_path,'/Service'))
end
SAVE_FLAG=0; % saving the figures in a file
```

Introduction

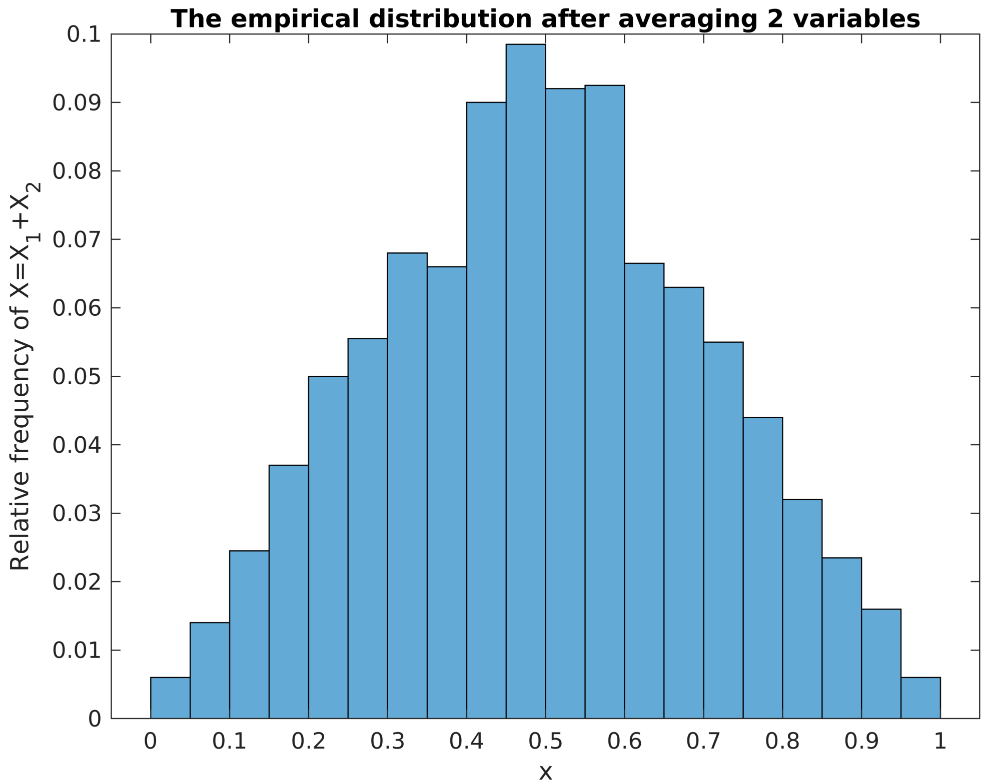
Example 1 Probability distribution of the mean of normal random variables

Plot the empirical distribution of the mean of a) n=1, b) n=2 and c) n=20 uniform random variables.

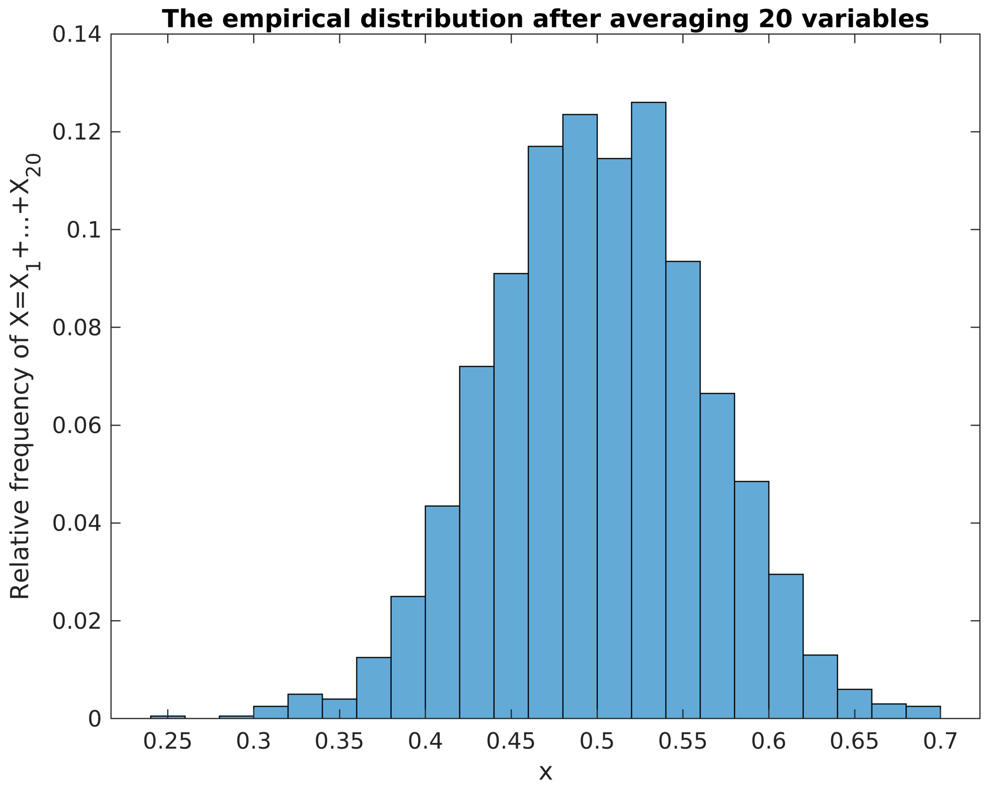
```
close all
clear_all_but('SAVE_FLAG');

% a) Plot hystogram for n=1
n1=rand(1,2000);
figure
histogram(n1, 'Normalization','probability')
ylabel('Relative frequency of X')
xlabel('x')
title('The empirical distribution without averaging random variables')
```




```
annonation_save('a)',"Fig1.1a.jpg", SAVE_FLAG);

%annotation('textbox',...
%    [0.0117142857142857 0.00952380952380952 0.0615 0.0547619047619048],...
%    'String',{'a)'},...
%    'FitBoxToText','off',...
%    'EdgeColor',[1 1 1]);


% b) Plot hystogram for n=2
n2=rand(2,2000);
m=mean(n2);
figure
histogram(m, 'Normalization','probability')
ylabel('Relative frequency of X=X_1+X_2')
xlabel('x')
title('The empirical distribution after averaging 2 variables')
```




```
annonation_save('b)',"Fig1.1b.jpg", SAVE_FLAG);

% c) Plot hystogram for n=20
n2=rand(20,2000);
m=mean(n2);
figure
histogram(m, 'Normalization','probability')
ylabel('Relative frequency of X=X_1+...+X_{20}')
xlabel('x')
title('The empirical distribution after averaging 20 variables')
```



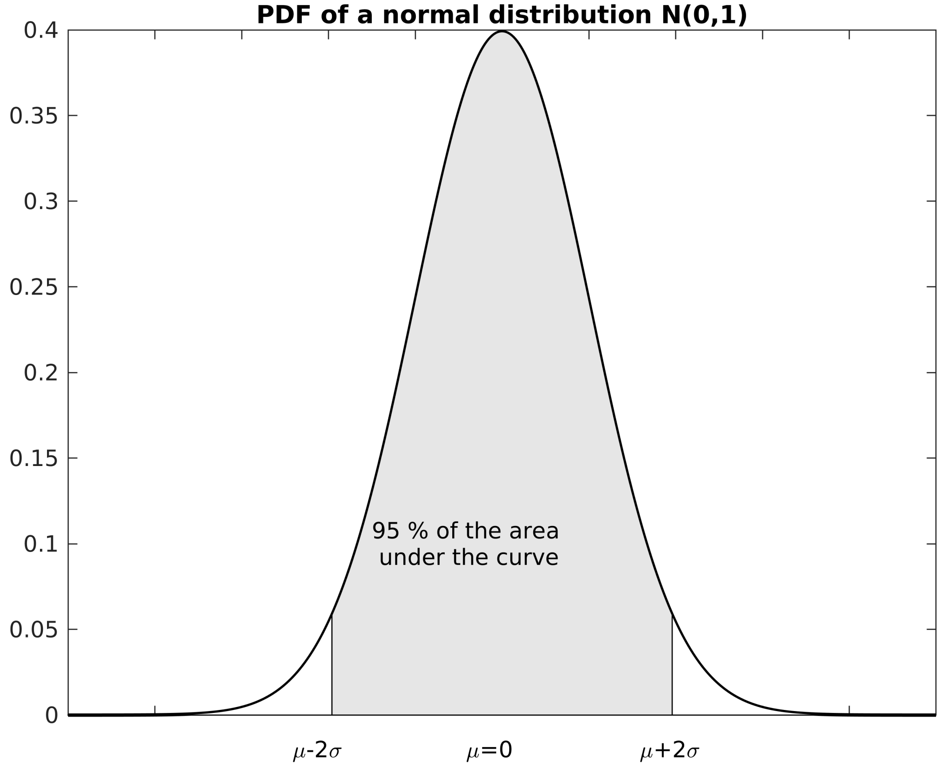

```
annonation_save('c)',"Fig1.1c.jpg", SAVE_FLAG);

%histfit(m)
```

Example 2 Plot area under normal distribution

This code is from: https://www.mathworks.com/matlabcentral/answers/128788-need-help-plotting-confidence-intervals

```
alpha = 0.05;           % significance level - change to 0.34 for 66% confidence
mu = 0;                % mean
sigma = 1;            % std
x = linspace(mu-5*sigma, mu+5*sigma, 500);

cutoff1 = norminv(alpha/2, mu, sigma);          % Lower 95% CI is p = 0.025
cutoff2 = norminv(1-alpha/2, mu, sigma);        % Upper 95% CI is p = 0.975
y = normpdf(x, mu, sigma);

xci = [linspace(mu-5*sigma, cutoff1); linspace(cutoff2, mu+5*sigma)];
yci = normpdf(xci, mu, sigma);

figure(1)
plot(x, y, '-k', 'LineWidth', 1.5)
patch(x, y, [0.9 0.9 0.9])
patch([xci(1,:) cutoff1], [yci(1,:) 0], [1 1 1])
patch([cutoff2 xci(2,:)], [0 yci(2,:)], [1 1 1])
xticklabels({})
```

```
text(-2.5,-0.02,{' \mu-2\sigma'})
text(-0.5,-0.02,{' \mu=0'})
text(1.5,-0.02,{' \mu+2\sigma'})
message = sprintf('95 %% of the area \n under the curve');
text(-1.5,0.1,message)
title('PDF of a normal distribution N(0,1)')
```



Example 3 Computing confidence intervals using Student t distribution

Generate a random sample of size 50 drawn from a normal population with mean 2V and standard deviation 0.5V. Compute confidence intervals at 95% confidence. This solution is from Mathworks example at https://www.mathworks.com/help/stats/tinv.html.

```
mu = 2;
sigma = 0.5;
n = 50;

x = normrnd(mu,sigma,n,1);
```

Compute the sample mean, standard error, and degrees of freedom.

```
xbar = mean(x);
se = std(x)/sqrt(n);
nu = n - 1;
```

Find the upper and lower confidence bounds for the 95% confidence interval.

```
conf = 0.95;
alpha = 1 - conf;
pLo = alpha/2;
pUp = 1 - alpha/2;
```

Compute the critical values for the confidence bounds.

```
crit = tinv([pLo pUp], nu);
```

Determine the confidence interval for the population mean.

```
ci = xbar + crit*se
```

```
ci = 1×2
    1.7935    2.0828
```

Uncertainty at 95% confidence is:

```
U_95=(ci(2)-xbar)/2
```

```
U_95 = 0.0723
```

If we replace the t-ditribution with the normal distribution at 95% confidence, we will obtain similar results:

```
k=2;
ci_normal = [xbar - k*se, xbar + k*se]
```

```
ci_normal = 1×2
    1.7942    2.0821
```

Uncertainty at 95% confidence is:

```
U_95_normal=(ci_normal(2)-xbar)/2
```

```
U_95_normal = 0.0720
```

Exersizes

Excersize 1: Repeat the analysis for Example 3 and comment on the difference in confidence intervals when a) n=5, n=15, n=200. Replace k=2 with k=1.96.

Excersize 2: Repeat the analysis for Example 3 but compute confidence intervals at 99% confidence.