Accuracy and precision of linear systems

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This code was developed by Miodrag Bolic for the book PERVASIVE CARDIOVASCULAR AND RESPIRATORY MONITORING DEVICES

Introduction

In this notebook, we will introduce two sensors models. They include:

1. The linear model that is used for generating data and comparing the output of paired measurements. The measurements are done with a sensor that has no bias and has small random error agains anouther sensor with a bias, scaling factor not equal 1 and larger measurement noise. The model is used to determine the **errors in measurements** and for **evaluating the agreement**.
2. The sensor model in Simulink that includes linear function as well as the gain, saturation block, bandlimiter and so on. This model allows for simulating different types of sensors.
3. The sensor model in Simulink that includes 3rd order polonomial nonlinear function as well as the gain, saturation block, bandlimiter and so on. This model allows for obtaining **linearity error**.

Linear model in Matlab

Evaluating error

In this example, we consider measurement where the "true" value of measurement is 1. The measurement is repeated N times. It is performed using 2 methods: 1 and 2. The measurement errors are modeled as random numbers coming from Normal distribution. Also, there could be some internal variability in x that is also modeled using the standard deviation sigma_x. If the variable Input_variance=0, then this internal variability is not taken into account.

Several combinations are considered:

1. Concordance coefficient slide

Input_variance=1;

sigma_e1=0.01;

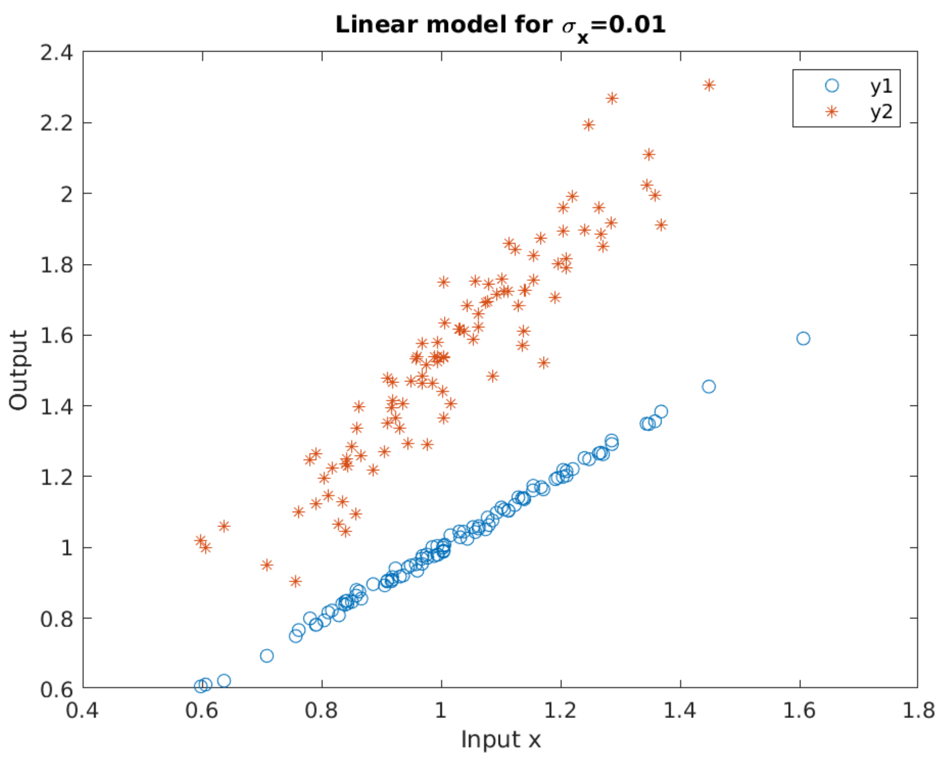
sigma_e2=0.1;

beta0=0;%0.01

beta1=1.5;%1.01;

sigma_x=0.2;

mu_x=1;



2. Bland Altman and scatter-plot slides - no biases

N=100;

sigma_e1=0.01;

sigma_e2=0.03;

beta0=0;%0.01

beta1=1;%1.01;

if Input_variance==0

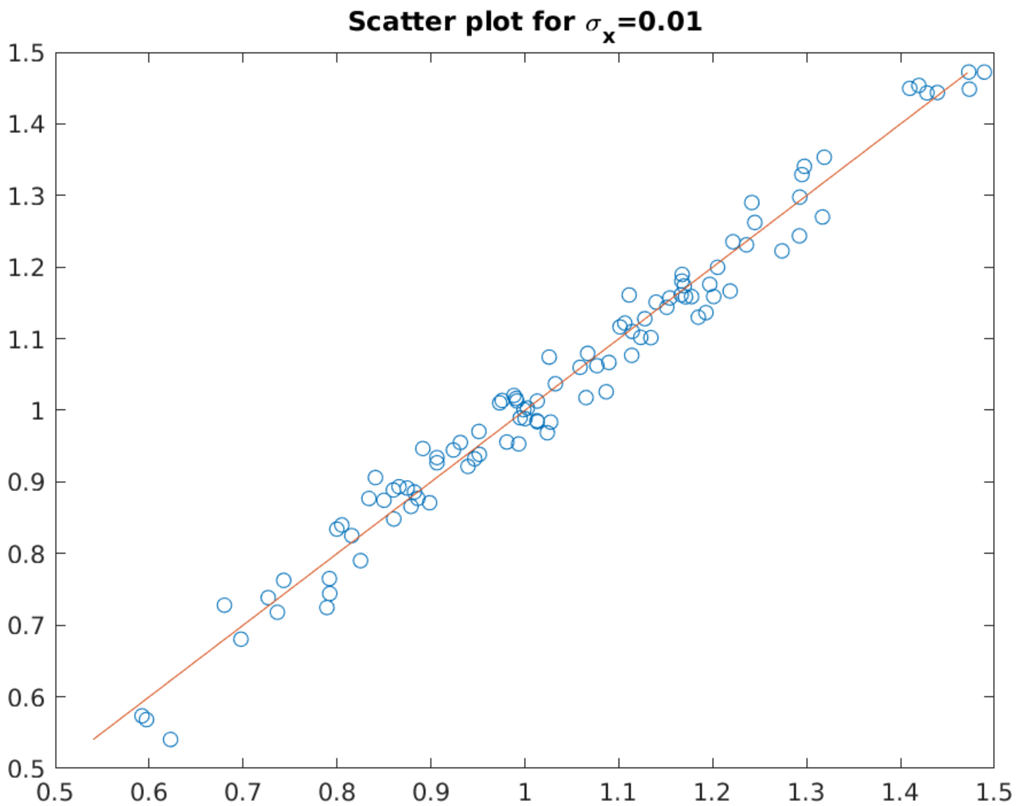
sigma_x=0;

else

sigma_x=0.2;

end

mu_x=1;



3. Bland Altman and scatter-plot slides - with biases

N=100;

sigma_e1=0.01;

sigma_e2=0.03;

beta0=0.02;%0.01

beta1=1.01;%1.01;

if Input_variance==0

sigma_x=0;

else

sigma_x=0.2;

end

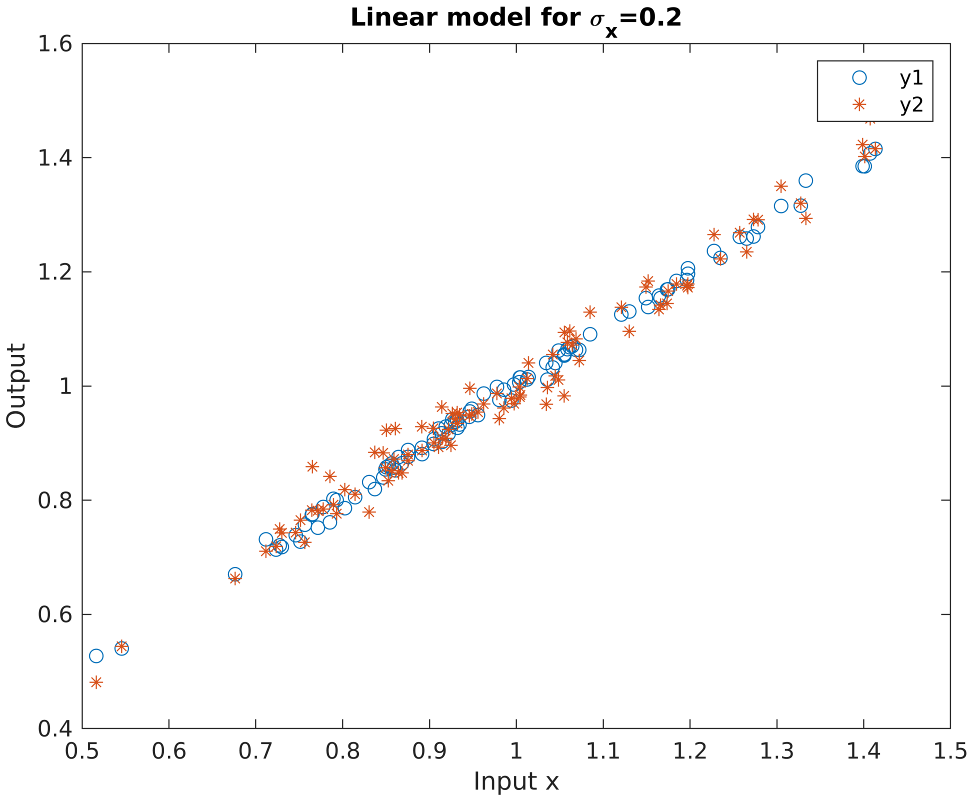
```
Input_variance=1; %or 1

N=100;
sigma_e1=0.01;
sigma_e2=0.03;
beta0=0;%0.01
beta1=1;%1.01;
if Input_variance==0
    sigma_x=0;
else
    sigma_x=0.2;
end
mu_x=1;

x=mu_x+sigma_x*randn(N,1);
e1=sigma_e1*randn(N,1);
e2=sigma_e2*randn(N,1);

y1=x+e1;
y2=beta0+beta1*x+e2;

D=y2-y1;

CCC = f_CCC([y1 y2],0.05);
Ch1 = ['Pearson coeff is: ',num2str(CCC{1, 1}.pearsonCorrCoeff)];
disp(Ch1)
```

```
Pearson coeff is: 0.9863
```

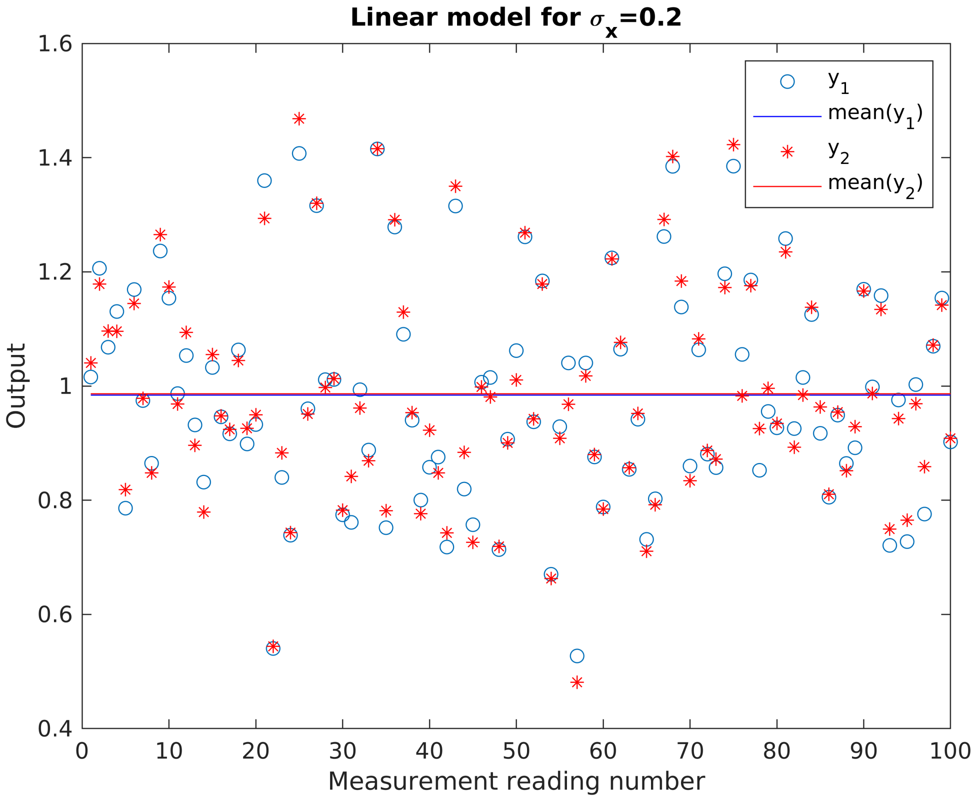
```
Ch2 = ['Concordance correlation coeff is: ',num2str(CCC{1, 1}.est)];
disp(Ch2)
```

```
Concordance correlation coeff is: 0.98622
```

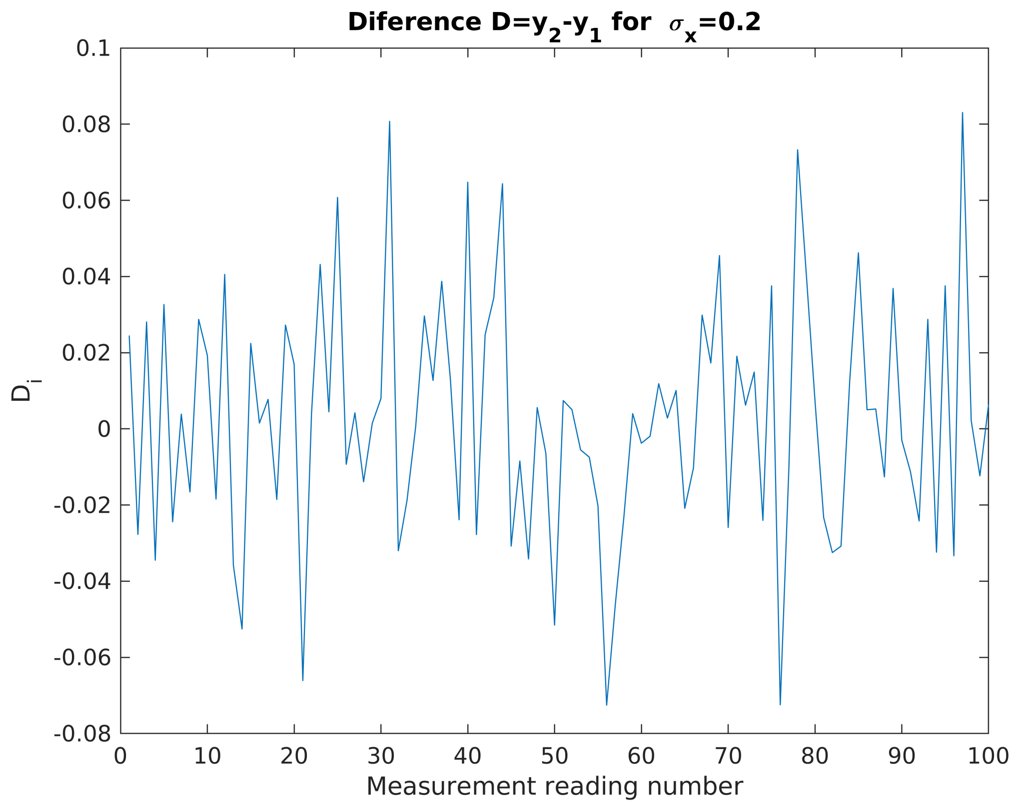
```

figure
plot(x,y1,'o')
hold on, plot(x,y2,'*')
xlabel('Input x')
ylabel('Output')
if Input_variance==0
    title('Linear model for \sigma_x=0')
else
    Ch3 = ['Linear model for \sigma_x=',num2str(sigma_x)];
    title(Ch3)
end
legend('y1','y2')
```




```
figure
plot(y1,'o')
hold on,
plot([1 length(y1)],[mean(y1) mean(y1)],'b')
plot(y2,'*r')
plot([1 length(y2)],[mean(y2) mean(y2)] ,'r')
xlabel('Measurement reading number')
ylabel('Output')
if Input_variance==0
    title('Linear model for \sigma_x=0')
else
    Ch3 = ['Linear model for \sigma_x=',num2str(sigma_x)];
    title(Ch3)
end
legend('y_1','mean(y_1)', 'y_2', 'mean(y_2)')
```

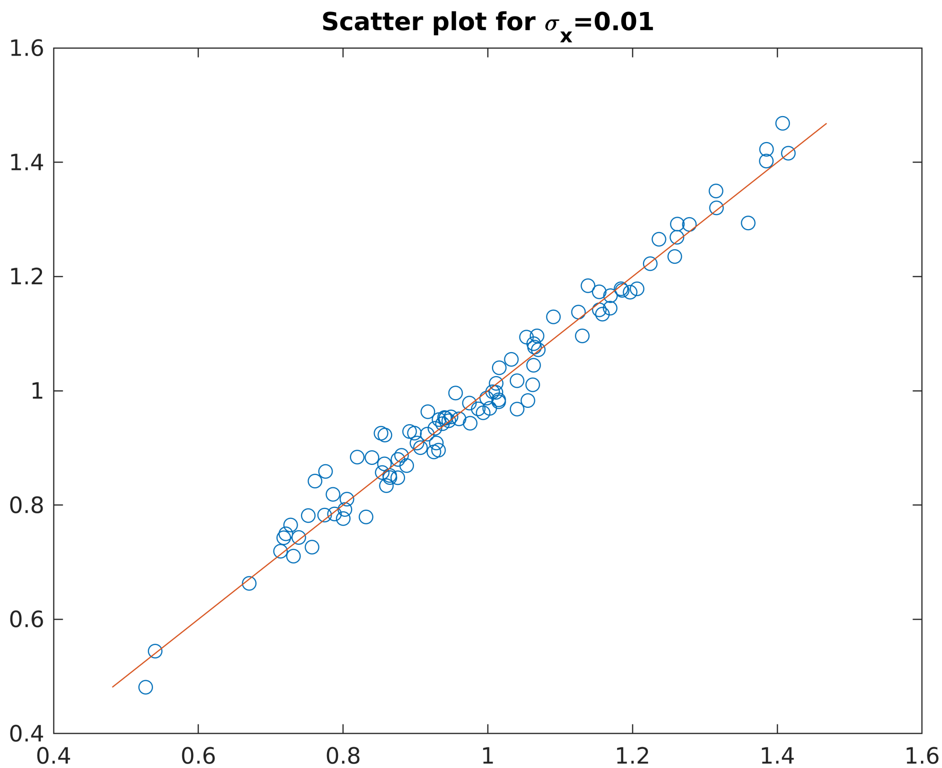



```
figure
plot(D)
xlabel('Measurement reading number')
ylabel('D_i')
if Input_variance==0
    title('Diference D=y_2-y_1 for \sigma_x=0')
else
    Ch3 = ['Diference D=y_2-y_1 for  \sigma_x=',num2str(sigma_x)];
    title(Ch3)
end
```

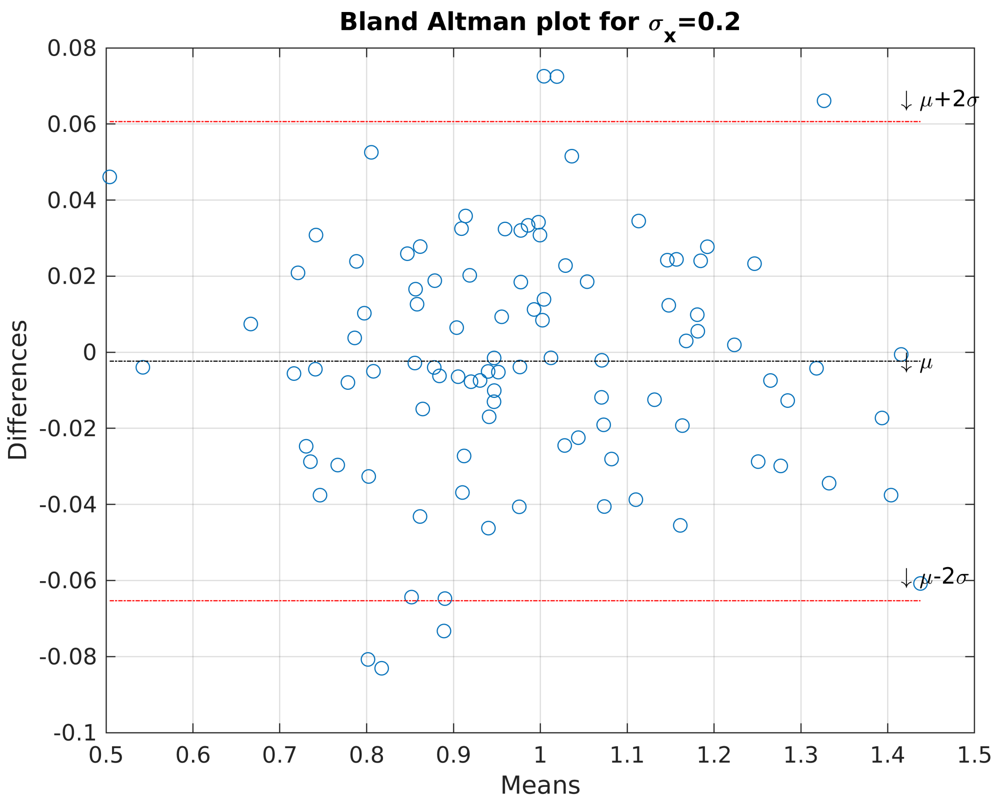


Evaluating agreement

```
% Scatter plot
x2 = min(y2):0.001:max(y2);
figure
plot(y1,y2,'o')
hold on
plot(x2,x2)
if Input_variance==0
    title('Scatter plot for \sigma_x=0')
else
    Ch3 = ['Scatter plot for \sigma_x=',num2str(sigma_x)];
    title(Ch3)
    title('Scatter plot for \sigma_x=0.01')
end
```

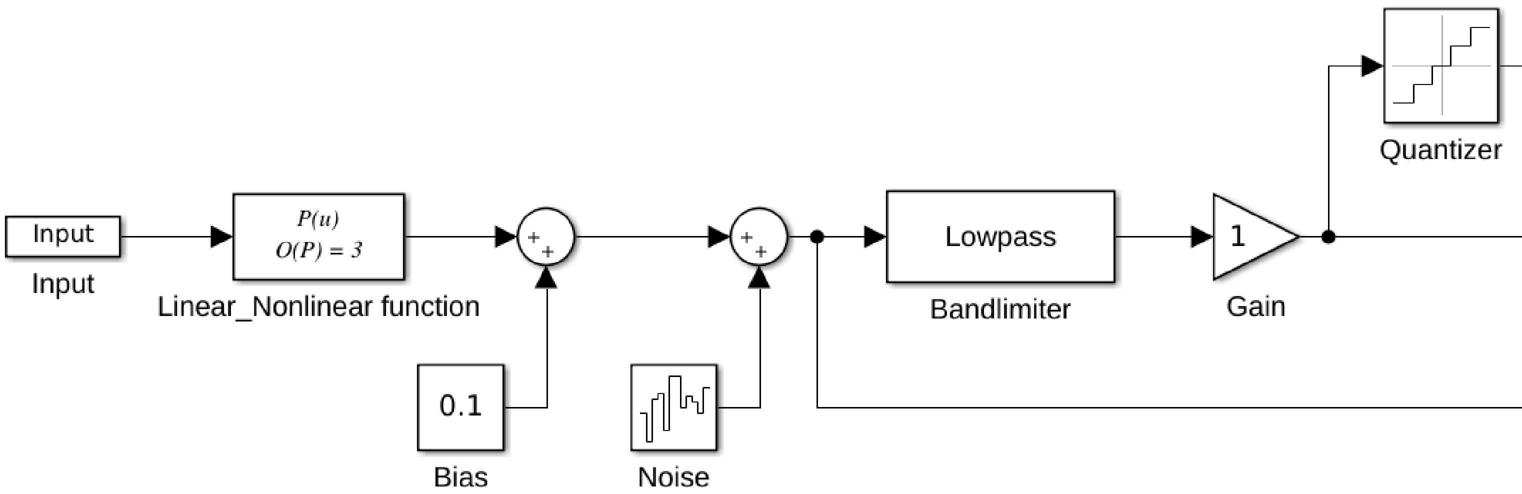



```
% Bland Altman plot
method1=y1';
method2=y2';
meanArray = mean([method1;method2]);
diffArray = method1-method2;
meanOfDiffs = mean(diffArray);
stdOfDiffs = std(diffArray);
confRange = [meanOfDiffs + 2.0 * stdOfDiffs, meanOfDiffs - 2.0 * stdOfDiffs];
figure
plot(meanArray,diffArray,'o');
hold on;
line([min(meanArray) max(meanArray)],[confRange(1) confRange(1)],'Color','red','LineSty
line([min(meanArray) max(meanArray)],[confRange(2) confRange(2)],'Color','red','LineSty
line([min(meanArray) max(meanArray)],[meanOfDiffs meanOfDiffs],'Color','black','LineSty
grid; ylabel('Differences'); xlabel('Means');
text(max(method1),1.1*confRange(1),{'\downarrow \mu+2\sigma'})
text(max(method1),0.9*meanOfDiffs,{'\downarrow \mu'})
text(max(method1),0.9*confRange(2),{'\downarrow \mu-2\sigma'})
%xlim([0.97 1.03])
if Input_variance==0
    title('Bland Altman plot for \sigma_x=0')
else
    Ch3 = ['Bland Altman plot for \sigma_x=',num2str(sigma_x)];
    title(Ch3)

end
hold off;
```



Sensor model

Here, we introduce sensor model that is shown in figure below. We first start the model with the linear function. We then plot different sensor outputs.

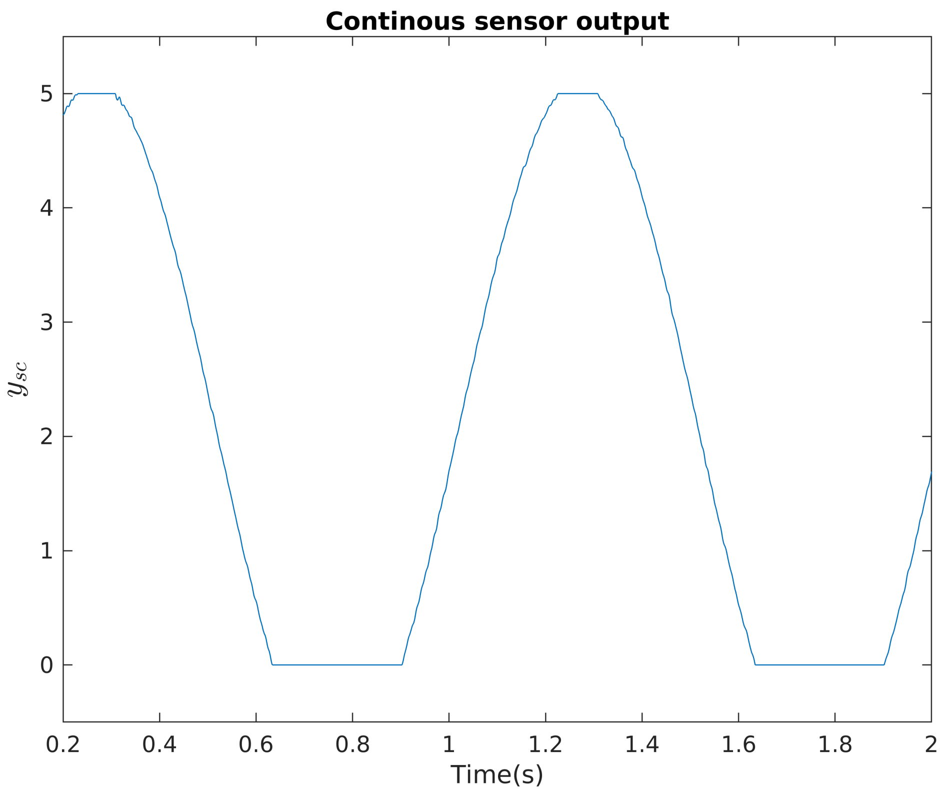


1. Figure 2.4 from chapter 2 was obtained by setting: Input(:,2) = mu_x+0.1*sin(2*pi*t*1);
2. Saturation in slides is modeled by Input(:,2) = mu_x+1.5*sin(2*pi*t*1);

```
clear all
mu_x=1;
fs=10000;
dur=5*fs; % 5 seconds
t=0:1/fs:dur/fs;
Input(:,1) = t;
Input(:,2) = mu_x+1.5*sin(2*pi*t*1); % mu_x+0.1*sin(2*pi*t*1);
model_name = 'SimpleSensorModel';
open_system(model_name);
% Linear model
set_param('SimpleSensorModel/Linear_Nonlinear function','Coefs','[1.01,0]') % Bias is s
set_param('SimpleSensorModel/Bias','Value','0.005')
set_param('SimpleSensorModel/Noise','Cov','[0.0000001]')
set_param('SimpleSensorModel/Gain','Gain','2')
out=sim(model_name);
```

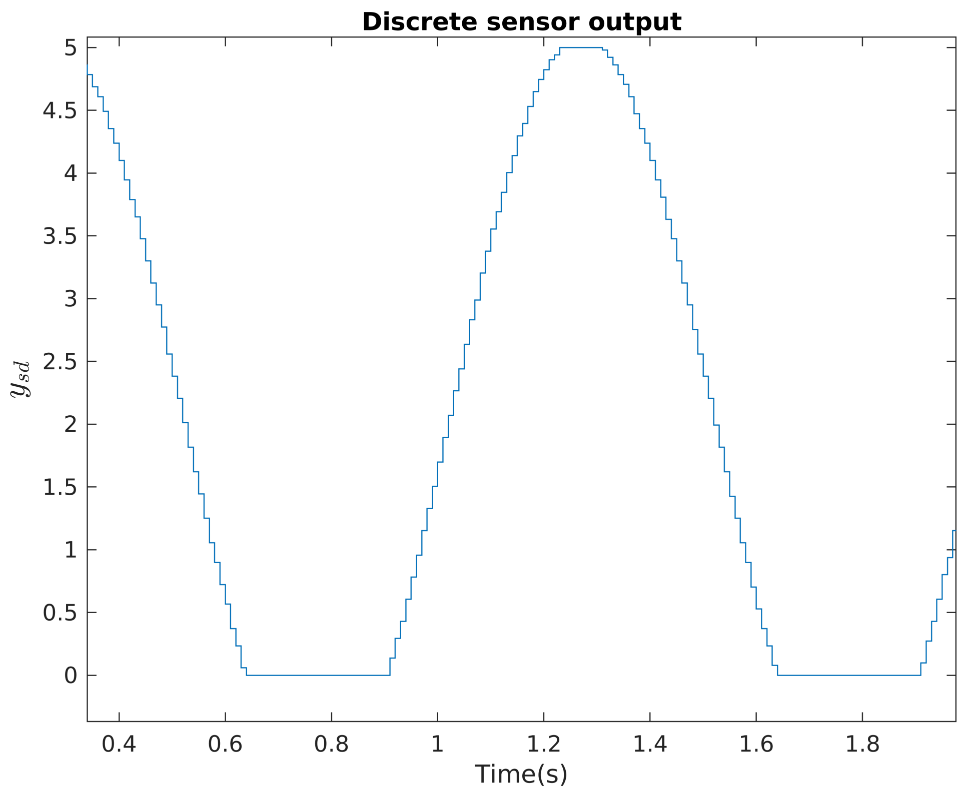
```
Warning: The file containing block diagram 'Lab_Simulation_1' has been changed on disk since it was
 loaded. You should close it, and Simulink will reload it if necessary.
```

```
figure, plot(out.y_sc)
xlim([0.2,2])
ylim([-0.5,5.5])
title('Continous sensor output')
L=ylabel('$y_{sc}$', 'FontSize',14);
set(L,'Interpreter','Latex');
xlabel('Time(s)')
```

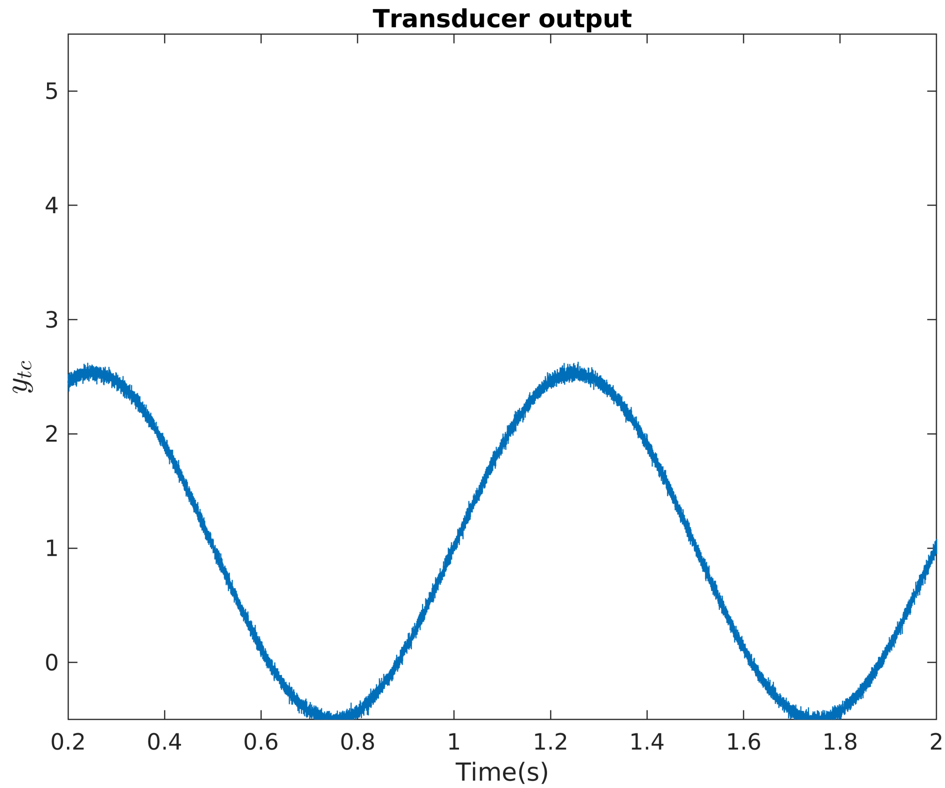



```
figure, plot(out.y_sd)
```

```
xlim([0.2,2])
ylim([-0.5,5.5])
title('Discrete sensor output')
L=ylabel('$y_{sd}$', 'FontSize',14);
set(L,'Interpreter','Latex');
xlabel('Time(s)')
```




```
figure, plot(out.y_tc)
xlim([0.2,2])
ylim([-0.5,5.5])
title('Transducer output')
L=ylabel('$y_{tc}$', 'FontSize',14);
set(L,'Interpreter','Latex');
xlabel('Time(s)')
```

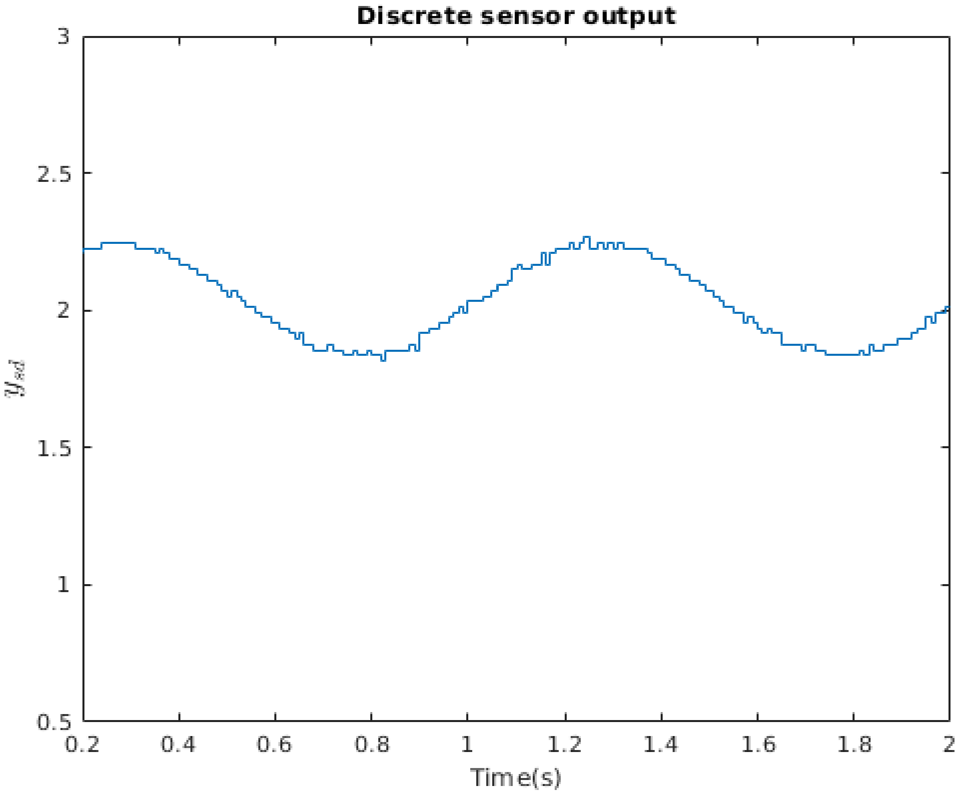


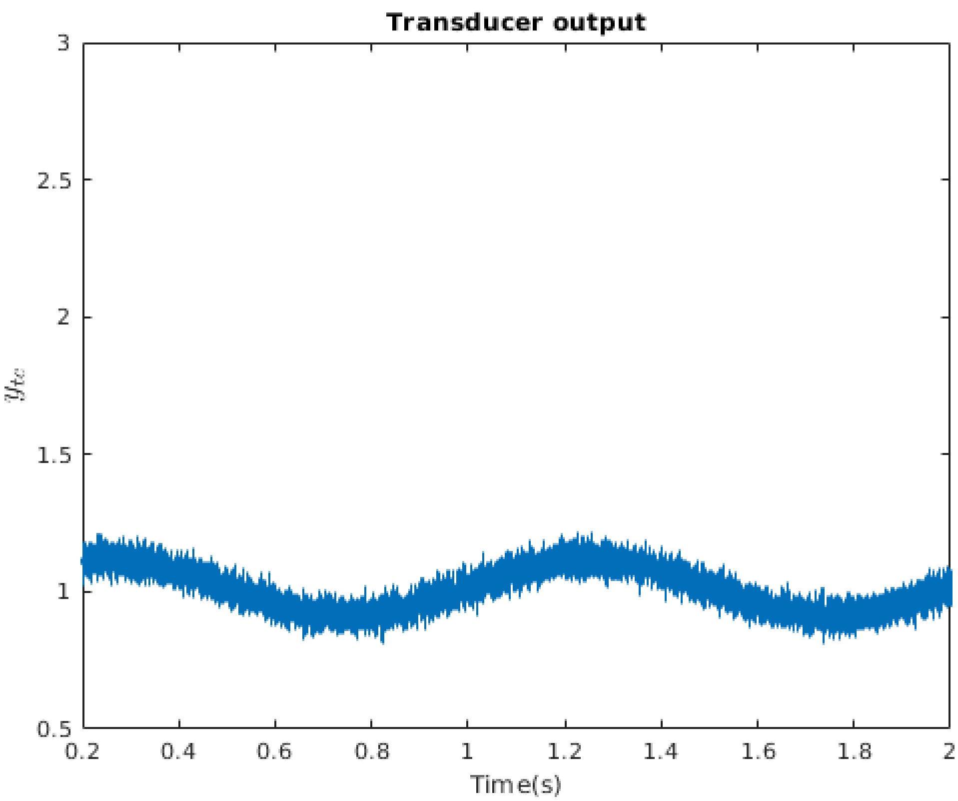
Evaluating nonlinearity

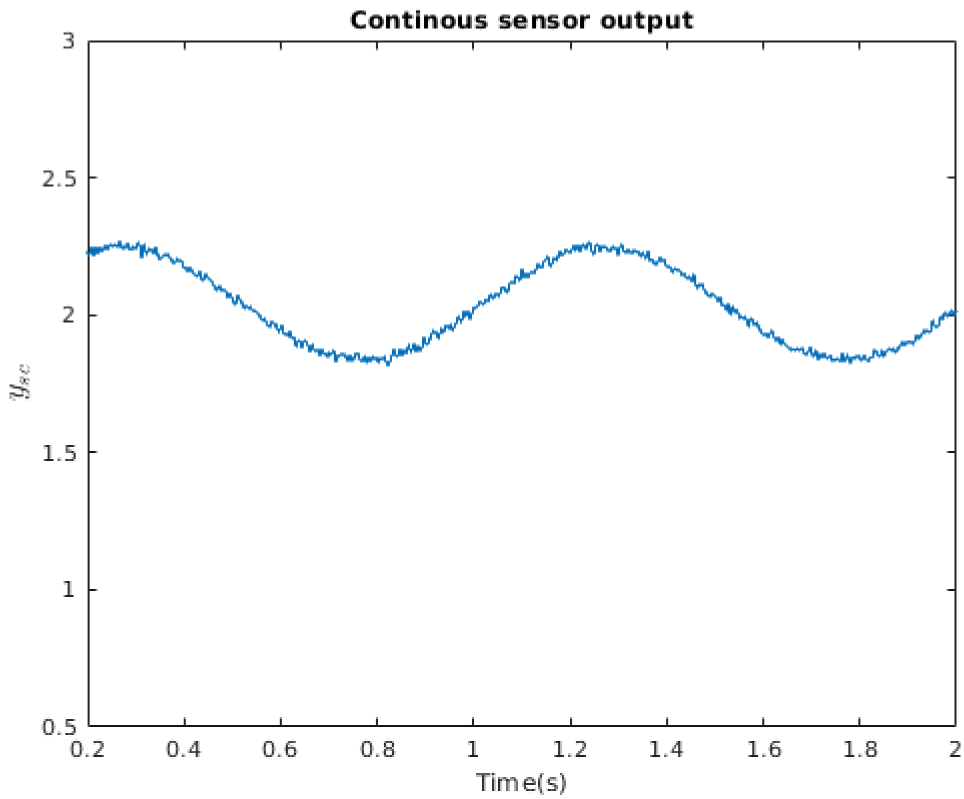
Here, the nonlinear model is introduced first. The model is simulated and the output is obtained from Simulink. Terminal and best fit error are calculated next.

```
model_name = 'SimpleSensorModel';
open_system(model_name);
Input(:,1) = t;
Input(:,2) = t; % input is modeled as an linear function of t
% NonLinear model
% yt=b1*x+b2*xt.^2+b3*xt.^3;
b0=0.1; b1=1; b2=0.08; b3=-0.015;
set_param('SimpleSensorModel/Linear_Nonlinear function','Coefs','[-0.015, 0.08,1,0]') %
set_param('SimpleSensorModel/Bias','Value','0.1')
set_param('SimpleSensorModel/Noise','Cov','0')
set_param('SimpleSensorModel/Gain','Gain','1')
out=sim(model_name);
```

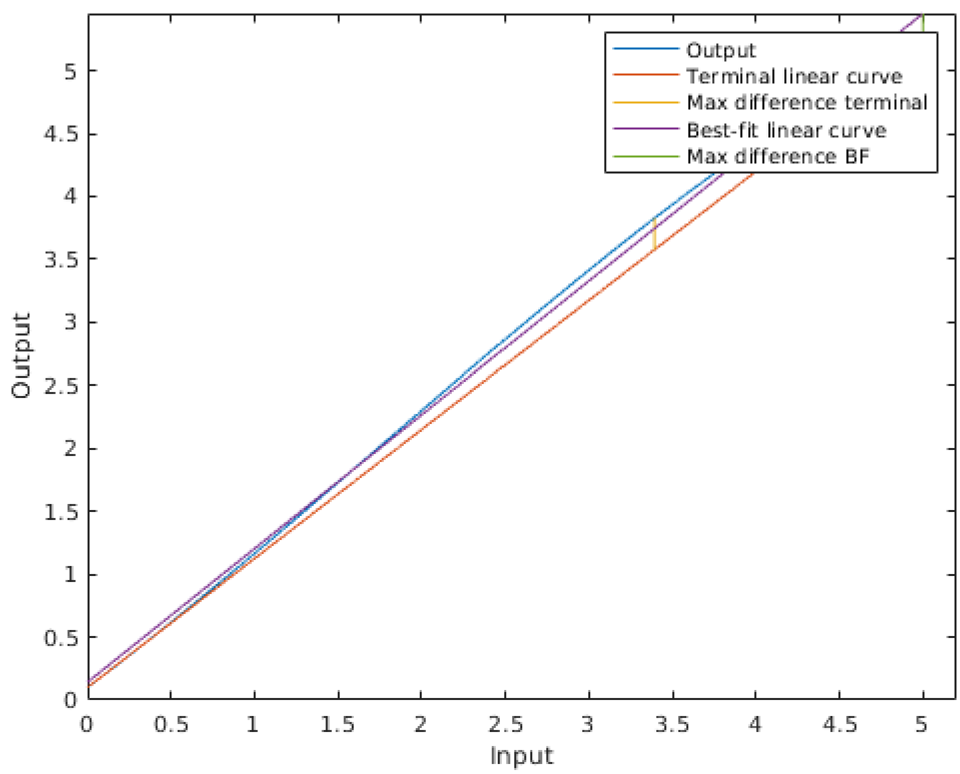
```
Warning: The file containing block diagram 'Lab_Simulation_1' has been changed on disk since it was
 loaded. You should close it, and Simulink will reload it if necessary.
```


Discrete sensor output
3
2.5
2
1.5
1
0.5
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time(s)
y_{sd}


Transducer output
3
2.5
2
1.5
1
0.5
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time(s)
y_{tc}




```
lin_error=PlotNonlinearity(fs, b0, t, Input(:,2), out.y_tc.Data);
```




```
fprintf('The terminal non-linearity error is %.1f %%.\n', 100*lin_error(1));
```

```
The terminal non-linearity error is 4.8 %.
```

```
fprintf('The best fit non-linearity error is %.1f %%.\n', 100*lin_error(2));
```

```
The best fit non-linearity error is 4.2 %.
```

```
function lin_error=PlotNonlinearity(fs, b0, t, xt, yt)

figure1 = figure;
axes1 = axes('Parent',figure1);
plot(xt, yt);
xlabel("Input")
ylabel("Output")
hold on

%% Terminal fit computation
b1=(yt(end)-yt(1))/(xt(end)-xt(1));
[m,i]=max(abs(yt-(b0 + b1*xt)));
lin_error(1)=m/max(b0 + b1*xt ); %terminal erorr
% Ploting terminal fit
plot(xt,b0 + b1*xt)
plot([xt(i),xt(i)],[b0 + b1*xt(i),yt(i)])

%% Best fit computation
P = polyfit(xt,yt,1);
y_BF = P(1)*xt+P(2); % best fit
[m,i]=max(abs(yt-y_BF));
lin_error(2)=m/max(y_BF);
% Ploting best fit
plot(xt,y_BF)
plot([xt(i),xt(i)],[y_BF(i),yt(i)])
ylim([0,max(y_BF)])
xlim([0,max(xt)+0.2])
legend("Output", "Terminal linear curve", "Max difference terminal", "Best-fit linear c
saveas(figure1,'Nonlinearity.jpg')

end
```

Exersizes

Excersize 1: Modify the gain to 15 of the simulation called Sensor model to generate the output signal that saturates.

Excersize 2: Modify the resolution of the quantizer from 8 to 16 bits and visualize the output.