Practical Assignment
Automated Reasoning 2IMF25

prof dr H. Zantema
MF room 6.078
email: `h.zantema@tue.nl`
November, 2015

The programs to be used

Recommended programs to be used:

- `Yices`: `http://yices.csl.sri.com/`.
- `Z3`: `http://z3.codeplex.com/`.

  Both `Yices` and `Z3` are programs for satisfiability modulo theories (SMT). They accept standard SMT format, in particular boolean SAT format.
- `NuSMV`: `http://nusmv.fbk.eu/`.

  This is a symbolic model checker based on BDDs
- `Prover9` and `Mace4`: `https://www.cs.unm.edu/~mccune/mace4/`

  These are tools for predicate and equational logic: `Prover9` for giving proofs based on resolution, and `Mace4` for finding counterexamples.

Each of the problems should be solved using one of these tools. The tools should do the job: manual modifications of the problems should be avoided.

For further information about the course we refer to
`http://www.win.tue.nl/~hzantema/ar.html`

The assignment

The practical assignment has to be executed by one or two persons. It consists of two parts.

The results of the assignment have to be described in two reports that should be handed in on paper. For the report on the first part the deadline is **December 7, 2015**, for the report on the second part the deadline is **January 11, 2016**.

For all used formulas an extensive documentation is required, explaining the approach and the overall structure. A generic approach is preferred, since this may result in clearer descriptions, increasing the confidence in the correctness of the results. Formulas of more than half a page should not be contained in the report, instead the structure of the formula should be explained. From the output of the programs relevant parts should be contained in the report, and observations on computation time should be reported. The answers on the problems should be motivated. Every report should contain name, student number and email address of each of the authors. In case of two authors each of them is considered to be responsible for the full text and all results.

Guidelines for grading:

- Clear and generic descriptions are appreciated, both of the formulas themselves and the way they were designed. An example of appreciated style is given at

  `http://www.win.tue.nl/~hzantema/prvb.pdf`.

- For both parts at least 3 out of the 4 solutions should be correct to obtain a 7.
- Not giving a solution at all for one problem is preferred over giving a wrong solution.
- Reasons for obtaining higher than a 7 may be:
  - all problems correctly solved,
  - remarkably clear and structured descriptions,
  - approaches allowing generalizations,
  - original approaches and solutions.

The problems for the first part

For the first part (deadline December 7, 2015) you have to find and describe solutions of the following 4 problems using the indicated programs.

1. Six trucks have to deliver pallets of obscure building blocks to a magic factory. Every truck has a capacity of 7800 kg and can carry at most eight pallets. In total, the following has to be delivered:

   - Four pallets of nuzzles, each of weight 700 kg.
   - A number of pallets of prittles, each of weight 800 kg.
   - Eight pallets of skipples, each of weight 1000 kg.
   - Ten pallets of crottles, each of weight 1500 kg.
   - Five pallets of dupples, each of weight 100 kg.

   Prittles and crottles are an explosive combination: they are not allowed to be put in the same truck.

   Skipples need to be cooled; only two of the six trucks have facility for cooling skipples.

   Dupples are very valuable; to distribute the risk of loss no two pallets of dupples may be in the same truck.

   Investigate what is the maximum number of pallets of prittles that can be delivered, and show how for that number all pallets may be divided over the six trucks.

2. Give a chip design containing three power components and eight regular components satisfying the following constraints:

- The width of the chip is 29 and the height is 22.
- All power components have width 4 and height 2.
- The sizes of the eight regular components are 9×7, 12×6, 10×7, 18×5, 20×4, 10×6, 8×6 and 10×8, respectively.
- All components may be turned 90°, but may not overlap.
- In order to get power, all regular components should directly be connected to a power component, that is, an edge of the component should have at least one point in common with an edge of the power component.
- Due to limits on heat production the power components should be not too close: their centers should differ at least 17 in either the x direction or the y direction (or both).

3. Twelve jobs numbered from 1 to 12 have to be executed satisfying the following requirements:

   - The running time of job i is i, for $i = 1, 2, \ldots, 12$.
   - All jobs run without interrupt.
   - Job 3 may only start if jobs 1 and 2 have been finished.
   - Job 5 may only start if jobs 3 and 4 have been finished.
   - Job 7 may only start if jobs 3, 4 and 6 have been finished.
   - Job 9 may only start if jobs 5 and 8 have been finished.
   - Job 11 may only start if job 10 has been finished.
   - Job 12 may only start if jobs 9 and 11 have been finished.
   - Jobs 5,7 en 10 require a special equipment of which only one copy is available, so no two of these jobs may run at the same time.

   Find a solution of this scheduling problem for which the total running time is minimal.

4. Seven integer variables $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are given, for which the initial value of a_i is i for $i = 1, \ldots, 7$. The following steps are defined: choose i with $1 < i < 7$ and execute

   $$a_i := a_{i-1} + a_{i+1},$$

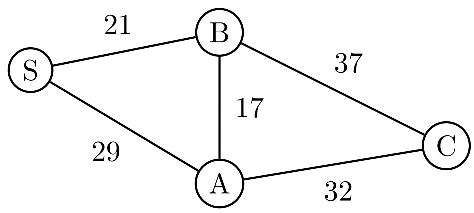
   that is, a_i gets the sum of the values of its neighbors and all other values remain unchanged. Show how it is possible that after a number of steps there is a number ≥ 50 that occurs at least twice in $a_1, a_2, a_3, a_4, a_5, a_6, a_7$.

The problems for the second part

For the second part (deadline January 11, 2016) you have to find and describe solutions of the following 4 problems using the indicated programs.

Warning: not all tools are suitable for all problems.

1. Three non-self-supporting villages A, B and C in the middle of nowhere consume one food package each per time unit. The required food packages are delivered by a truck, having a capacity of 300 food packages. The locations of the villages are given in the following picture, in which the numbers indicate the distance, more precisely, the number of time units the truck needs to travel from one village to another, including loading or delivering. The truck has to pick up its food packages at location S containing an unbounded supply. The villages only have a limited capacity to store food packages: for A and B this capacity is 120, for C it is 200. Initially, the truck is in S and is fully loaded, and in A, B and C there are 40, 30 and 145 food packages, respectively.



(a) Show that it is impossible to deliver food packages in such a way that each of the villages consumes one food package per time unit forever.

(b) Show that this is possible if the capacity of the truck is increased to 320 food packages. (Note that a finite graph contains an infinite path starting in a node v if and only if there is a path from v to a node w for which there is a non-empty path from w to itself.)

(c) Figure out whether it is possible if the capacity of the truck is set to 318.

2. Three bottles can hold 144, 72 and 16 units (say, centiliters), respectively. Initially the first one contains 3 units of water, the others are empty. The following actions may be performed any number of times:

- One of the bottles is fully filled, at some water tap.
- One of the bottles is emptied.
- The content of one bottle is poured into another one. If it fits, then the full content is poured, otherwise the pouring stops when the other bottle is full.

(a) Determine whether it is possible to arrive at a situation in which the first bottle contains 8 units and the second one contains 11 units. If so, give a scenario reaching this situation.

(b) Do the same for the variant in which the second bottle is replaced by a bottle that can hold 80 units, and all the rest remains the same.

(c) Do the same for the variant in which the third bottle is replaced by a bottle that can hold 28 units, and all the rest (including the capacity of 72 of the second bottle) remains the same.

3. The goal of this problem is to exploit the power of the recommended tools rather than elaborating the questions by hand.

   (a) In mathematics, a *group* is defined to be a set G with an element $I \in G$, a binary operator $*$ and a unary operator *inv* satisfying

$$x * (y * z) = (x * y) * z), \quad x * I = x \text{ and } x * inv(x) = I,$$

   for all $x, y, z \in G$. Determine whether in every group each of the four properties

$$I * x = x, \quad inv(inv(x)) = x, \quad inv(x) * x = I \text{ and } x * y = y * x$$

   holds for all $x, y \in G$. If a property does not hold, determine the size of the smallest finite group for which it does not hold.

   (b) A term rewrite system consists of the single rule

$$a(x, a(y, a(z, u))) \rightarrow a(y, a(z, a(x, u)))),$$

   in which a is a binary symbol and x, y, z, u are variables. Moreover, there are constants b, c, d, e, f, g. Determine whether c and d may swapped in $a(b, a(c, a(d, a(e, a(f, a(b, g))))))$ by rewriting, that is, $a(b, a(c, a(d, a(e, a(f, a(b, g))))))$ rewrites in a finite number of steps to $a(b, a(d, a(c, a(e, a(f, a(b, g))))))$.

4. Give a precise description of a non-trivial problem of your own choice, and encode this and solve it by one of the given programs.