# FIGURES

2.1	Various ways of interpreting the example ambiguous transformation: (a) change of coordinates; (b) transformation of plane onto itself;	
	and (c) transformation from one plane to another.	11
2.2	The solutions of the linear equation $3x_1 + 2x_2 = 6$ .	26
2.3	Three possible two-equation linear system solutions.	27
2.4	Schematic diagram of a function.	46
2.5	Composition of two functions.	47
2.6	One-to-one, onto, and isomorphic maps.	48
2.7	One-to-one and onto functions.	48
2.8	An invertible mapping.	49
2.9	Least squares example.	57
3.1	Vectors as directed line segments.	64
3.2	Two vectors.	65
3.3	Two vectors drawn head-to-tail.	65
3.4	Vector addition.	66
3.5	Vector addition chain.	66
3.6	Vector subtraction.	67
3.7	Vector multiplication.	67
3.8	Vector negation.	68
3.9	Commutativity of vector addition.	68
3.10	Associativity of vector addition.	68
3.11	Distributivity of addition over multiplication.	69
3.12	Distributivity of multiplication over addition.	69
3.13	The span of two vectors in 3D space is a plane.	70
3.14	A vector as the linear combination of basis vectors.	72
3.15	Angle between vectors.	74
3.16	The right-hand rule for orientation.	75
3.17	A vector as the linear combination of two different sets of basis	
	vectors.	76
3.18	The sine function.	77

xxiii

# **xxiv** Figures

3.19	Linear transformation "scale by two."	78
3.20	Nonuniform scale linear transformation.	79
3.21	Rotation transformation.	79
3.22	Shear transformation.	79
3.23	Definition of point subtraction.	81
3.24	The Head-to-Tail axiom.	81
3.25	Affine combination of two points.	83
3.26	Affine combination of several points.	85
3.27	Angle between vectors and vector length.	86
3.28	Parallelogram rule for vector addition.	87
3.29	Vector projection.	87
3.30	$\cos \theta$ negative.	89
3.31	The vector product.	92
3.32	The right-hand rule.	93
3.33	Parallelepiped defined by three vectors.	94
3.34	The scalar triple product.	95
3.35	Coordinates of an affine point, relative to an arbitrary frame.	98
3.36	Affine maps preserve relative ratios.	99
3.37	Vector sum.	101
3.38	Vector scale.	102
3.39	Sum of point and vector.	102
3.40	Composition of affine maps (rotation).	104
3.41	Affine (a) and barycentric (b) coordinates.	105
3.42	The first three simplexes: a line (a), a triangle (b), and a	
	tetrahedron (c).	107
4.1	$P = p_1 \vec{i} + p_2 \vec{j} + p_3 \vec{k} + \mathcal{O} = [p_1  p_2  p_3  1].$	114
4.2	$\vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k} = [v_1  v_2  v_3  0].$	114
4.3	The "perp" operator.	122
4.4	The perp dot product reflects the signed angle between vectors.	124
4.5	The perp dot product is related to the signed area of the triangle	
	formed by two vectors.	125
4.6	Representing $P$ in $\mathcal{A}$ and $\mathcal{B}$ .	129
4.7	Representing $\mathcal{O}$ in $\mathcal{G}$ .	130
4.8	Representing $\vec{v}_i$ in $\mathcal{B}$ .	131
4.9	Translation.	134
4.10	Translation of a frame.	136
4.11	Simple rotation of a frame.	137
4.12	General rotation.	140

4.13	General rotation shown in the plane $\mathcal{A}$ perpendicular to $\hat{u}$ and	
	containing <i>P</i> .	141
4.14	Scaling a frame.	143
4.15	Uniform scale.	145
4.16	Nonuniform scale.	146
4.17	Mirror image.	148
4.18	Simple reflection in 2D.	150
4.19	Simple reflection in 3D.	151
4.20	General reflection in 3D.	152
4.21	Mirror image in 2D.	153
4.22	Mirror image in 3D.	154
4.23	Shearing in 2D.	155
4.24	$T_{xy,\theta}$ .	155
4.25	General shear specification.	157
4.26	Orthographic (orthogonal) projection.	160
4.27	Oblique projection.	161
4.28	Edge-on view of oblique projection.	162
4.29	Perspective projection.	163
4.30	Cross-ratio.	164
4.31	Perspective map for vectors.	165
4.32	The plane $x + y = k$ .	166
4.33	Incorrectly transformed normal.	167
4.34	Normal as cross product of surface tangents.	167
5.1	Examples of (a) a line, (b) a ray, and (c) a segment.	172
5.2	Implicit definition of a line.	173
5.3	The two possible orderings for a triangle.	175
5.4	The domain and range of the parametric form of a triangle.	176
5.5	The domain and range of the barycentric form of a triangle.	176
5.6	The domain and range for the parametric form of a rectangle.	177
5.7	The symmetric form of a rectangle.	178
5.8	A typical polyline.	178
5.9	Examples of (a) a simple concave polygon and (b) a simple convex polygon.	179
5.10	Examples of nonsimple polygons. (a) The intersection is not a vertex. (b) The intersection is a vertex. The polygon is a polysolid.	180
5.11	Examples of polygonal chains: (a) strictly monotonic; (b) monotonic, but not strict.	180

5.12	A monotone polygon. The squares are the vertices of one chain. The triangles are the vertices of the other chain. The circles are those vertices on both chains.	18]
5.13	Solutions to the quadratic equation depending on the values for $d_0 \neq 0$ , $d_1 \neq 0$ , and $r$ .	183
5.14	Solutions to the quadratic equation depending on the values for $d_0 \neq 0$ , $d_1 = 0$ , $e_1$ , and $r$ .	184
5.15	Circles defined in distance (implicit) and parametric forms.	184
5.16	Definition of an ellipse.	185
5.17	A cubic Bézier curve.	186
5.18	A cubic B-spline curve.	187
6.1	Closest point $X(\bar{t})$ on a line to a specified point $Y$ .	190
6.2	Closest point on a ray to a given point: (a) $X(\bar{t})$ closest to $Y$ ; (b) $P$ closest to $Y$ .	192
6.3	Closest point on a segment to a given point: (a) $X(\bar{t})$ closest to $Y$ ; (b) $P_0$ closest to $Y$ ; (c) $P_1$ closest to $Y$ .	192
6.4	The segment $S_0$ generated the current minimum distance $\mu$ between the polyline and $Y$ . $S_1$ and $S_2$ cannot cause $\mu$ to be updated because they are outside the circle of radius $\mu$ centered at $Y$ . Segment $S_3$ does cause an update since it intersects the circle. The infinite-strip test does not reject $S_1$ and $S_3$ since they lie partly in both infinite strips, but $S_2$ is rejected since it is outside the vertical strip. The rectangle test rejects $S_1$ and $S_2$ since both are outside the rectangle containing the circle, but does not reject $S_3$ .	195
6.5	Closest point on a triangle to a given point: (a) $\operatorname{Dist}(Y, \mathcal{T}) = 0$ ; (b) $\operatorname{Dist}(Y, \mathcal{T}) = \operatorname{Dist}(Y, < P_0, P_1 >)$ ; (c) $\operatorname{Dist}(Y, \mathcal{T}) = \operatorname{Dist}(Y, P_2)$ ; (d) $\operatorname{Dist}(Y, \mathcal{T}) = \operatorname{Dist}(Y, < P_1, P_2 >)$ .	197
6.6	Partitioning of the parameter plane into seven regions.	199
6.7	Contact points of level curves of $F(t_0, t_1)$ with the triangle: (a) contact	
	with an edge; (b) contact with a vertex.	199
6.8	Contact points of level curves of $F(t_0, t_1)$ with the triangle: (a) contact with an edge; (b) contact with another edge; (c) contact with a vertex.	200
6.9	A triangle, a bounding box of the triangle, and the regions of points closest to vertices and to edges.	207
6.10	Partitioning of the plane by a rectangle.	212
6.11	(a) An example of a single cone. (b) A frustum of a cone. (c) An orthogonal frustum.	214
6.12	Portion of frustum in first quadrant.	215
6.13	Only those edges visible to the test point must be searched for the closest point to the test point. The three visible edges are dotted. The	

	with vertex at the test point and whose sides are tangent to the convex polygon.	217
6.14	Closest point on a quadratic curve to a given point.	218
6.15	Closest point on a polynomial curve to a given point.	220
6.16	Various line-line configurations: (a) zero distance; (b) positive distance.	222
6.17	Various line-ray configurations: (a) zero distance; (b) positive distance.	223
6.18	Various line-segment configurations: (a) zero distance; (b) positive distance.	224
6.19	Various nonparallel ray-ray configurations: (a) zero distance; (b) positive distance from end point to interior point; (c) positive distance from end point to end point.	224
6.20	Relationship of level curves of $F$ to boundary minimum at $(\hat{t}_0, 0)$ or $(0, 0)$ .	226
6.21	Various parallel ray-ray configurations: (a) rays pointing in the same direction; (b) rays pointing in opposite directions with overlap; (c) rays pointing in opposite directions with no overlap.	227
6.22	The configuration for the segment $S$ attaining current minimum distance $\mu$ that is the analogy of Figure 6.4 for the point $Y$ attaining current minimum distance.	230
6.23	Segment connecting closest points is perpendicular to both objects.	232
6.24	(a) Triangles $A$ and $B$ ; (b) set $-B$ ; (c) set $A + B$ ; (d) set $A - B$ , where the gray point is the closest point in $A - B$ to the origin. The black dots are the origin $(0,0)$ .	235
6.25	The first iteration in the GJK algorithm.	237
6.26	The second iteration in the GJK algorithm.	237
6.27	The third iteration in the GJK algorithm.	238
6.28	The fourth iteration in the GJK algorithm.	238
6.29	(a) Construction of $V_{k+1}$ in the convex hull of $S_k \cup \{W_k\}$ . (b) The new simplex $\bar{S}_{k+1}$ generated from $M = \{W_0, W_2, W_3\}$ .	239
7.1	An arc of a circle spanned counterclockwise from $A$ to $B$ . The line containing $A$ and $B$ separates the circle into the arc itself and the remainder of the circle. Point $P$ is on the arc since it is on the same side of the line as the arc. Point $Q$ is not on the arc since it is on the approximation of the line.	249
7.2	opposite side of the line.  Intersection of a line and a cubic curve.	250
7.2	Line-curve intersection testing using a hierarchy of bounding boxes.	252
1.5	the curve intersection testing using a meratery of bounding boxes.	232

## xxviii Figures

7.4	A line and a curve rasterized on a grid that is initially zero. The line is rasterized by or-ing the grid with the mask 1 (light gray). The curve is rasterized by or-ing the grid with the mask 2 (dark gray). Grid cells that contain both the line and the curve have a value 3 (dotted).	254
7.5	Intersections of an ellipse and a parabola.	256
7.6	Relationship of two circles, $\vec{u} = C_1 - C_0$ : (a) $\ \vec{u}\  =  r_0 + r_1 $ ; (b) $\ \vec{u}\  =  r_0 - r_1 $ ; (c) $ r_0 - r_1  < \ \vec{u}\  <  r_0 + r_1 $ .	258
7.7	$\mathcal{E}_1$ is contained in $\mathcal{E}_0$ . The maximum $\mathcal{E}_0$ level curve value $\lambda_1$ for $\mathcal{E}_1$ is negative.	259
7.8	$\mathcal{E}_1$ transversely intersects $\mathcal{E}_0$ . The minimum $\mathcal{E}_0$ level curve value $\lambda_0$ for $\mathcal{E}_1$ is negative; the maximum value $\lambda_1$ is positive.	260
7.9	$\mathcal{E}_1$ is separated from $\mathcal{E}_0$ . The minimum $\mathcal{E}_0$ level curve value $\lambda_0$ for $\mathcal{E}_1$ is positive.	261
7.10	Intersection of two ellipses.	262
7.11	Two curves rasterized on a grid that is initially zero. The first curve is rasterized by or-ing the grid with the mask 1 (light gray). The second curve is rasterized by or-ing the grid with the mask 2 (dark gray). Grid cells that contain both curves have a value 3 (dotted).	264
7.12	Nonintersecting convex objects and a separating line for them.	266
7.13	(a) Nonintersecting convex polygons. (b) Intersecting convex	
	polygons.	267
7.14	(a) Edge-edge contact, (b) vertex-edge contact, and (c) vertex-vertex contact.	267
7.15	The edge normals closest to a non-edge-normal separation direction: (a) from the same triangle and (b) from different triangles.	268
7.16	Two polygons separated by an edge-normal direction of the first polygon.	270
7.17	(a) Edge-edge intersection predicted. (b) Vertex-vertex intersection predicted. (c) No intersection predicted.	276
7.18	Edge-edge contact for two moving triangles.	282
8.1	Circle through three points.	286
8.2	Circle tangent to three lines.	286
8.3	Line tangent to a circle at a given point.	287
8.4	Line through point, tangent to a circle.	288
8.5	In general, there are two tangents, but there may be one or none.	288
8.6	Line tangent to two circles.	291
8.7	Depending on the relative sizes and positions of the circles, the number of tangents between them will vary.	292
8.8	Circle through two points with a given radius.	297
8.9	Both possible circles through two points with a given radius.	297
	1 1 1	'

8.10	Insight for computing circle of given radius through two points.	298
8.11	Circle through a point and tangent to a line with a given radius.	299
8.12	In general, there are two distinct circles through the given point.	299
8.13	If <i>P</i> lies on the line, the circles are mirrored across the line; if <i>P</i> is further from the line than the diameter of the circle, there are no	
	solutions.	300
8.14	Circles tangent to two lines with a given radius.	303
8.15	In general, there are four circles of a given radius tangent to two lines.	303
8.16	Constructive approach for circle tangent to two lines.	304
8.17	Circles through a point and tangent to a circle with a given radius.	305
8.18	Depending on the relative positions and radii of the circle, there may be four, two, or no solutions.	306
8.19	Insight for solving problem.	307
8.20	Constructive approach to solving problem.	308
8.21	Special case for constructive approach.	309
8.22	Circles tangent to a line and a circle with a given radius.	309
8.23	The number of distinct solutions varies depending on the relative	
	positions of the line and circle, and the circle's radius.	310
8.24	No solutions if given radius is too small.	311
8.25	Insight for finding circle of given radius.	311
8.26	Schematic for the solution.	312
8.27	Circles tangent to two circles with a given radius.	314
8.28	In general there are two solutions, but the number of distinct solutions varies with the relative sizes and positions of the given	
	circles.	315
8.29	Construction for a circle tangent to two circles.	316
8.30	Line normal to a given line and through a given point.	317
8.31	Line between and equidistant to two points.	318
8.32	Line parallel to a given line at a distance $d$ .	319
8.33	Line parallel to a given line at a vertical or horizontal distance $d$ .	321
8.34	Lines tangent to a given circle and normal to a given line.	322
9.1	A plane is defined as the set of all points <i>X</i> satisfying $\vec{n} \cdot (X - P) = 0$ .	327
9.2	Geometric interpretation of plane equation coefficients.	328
9.3	The parametric representation of a plane.	329
9.4	The parametric representation of a circle in 3D.	332
9.5	A convex polygon and its decomposition into a triangle fan.	334
9.6	A nonconvex polygon and its decomposition into triangles.	335
9.7	A triangle mesh.	335

9.8	Vertices, edges, and triangles are not a mesh since a vertex is isolated.	336
9.9	Vertices, edges, and triangles are not a mesh since an edge is isolated.	336
9.10	Vertices, edges, and triangles are not a mesh since two triangles	
	interpenetrate.	337
9.11	A polyhedron that consists of a tetrahedron, but an additional vertex	
	was added to form a depression in the centered face.	338
9.12	A polymesh that is not a polyhedron since it is not connected.	
	The fact that the tetrahedron and rectangle mesh share a common vertex does not make them connected in the sense of edge-triangle	
	connectivity.	338
9.13	A polymesh that is not a polyhedron since an edge is shared by three	
	faces.	339
9.14	A polytope, a regular dodecahedron.	339
9.15	The four possible configurations for ordering of two adjacent	
	triangles.	343
9.16	A rectangle has two parallel edges joined together forming (a) a	
	cylindrical strip (orientable) or (b) a Möbius strip (nonorientable).	344
9.17	The five Platonic solids. Left to right: tetrahedron, hexahedron,	246
0.10	octahedron, dodecahedron, icosahedron.	346
9.18	Quadrics having three nonzero eigenvalues.	353
9.19	Quadrics having two nonzero eigenvalues.	354
9.20	Quadrics having one nonzero eigenvalue.	355
9.21	A standard "ring" torus.	355
9.22	A cubic Bézier curve.	358
9.23	A cubic B-spline curve.	359
9.24	A bicubic Bézier surface.	361
9.25	A cubic triangular Bézier surface.	362
9.26	A uniform bicubic B-spline surface.	363
10.1	Distance between a line and a point.	366
10.2	The projection of $Q$ on $\mathcal{L}$ .	367
10.3	Distance between a line segment and a point.	368
10.4	Utilizing half-spaces to speed up point/polyline distance tests.	370
10.5	Rejection example for point/polyline distance.	371
10.6	Distance between a point and a plane.	374
10.7	Edge-on view of plane $\mathcal{P}$ .	375
10.8	Distance between a point and a triangle. The closest point may be on the interior of the triangle (a), on an edge (b), or be one of the	
	vertices.	376
10.9	Partition of the $st$ -plane by triangle domain $D$ .	378
	1 / 0	

	Figures	XXXI
10.10	Various level curves $Q(s,t) = V$ .	379
10.11	Alternative definition of a rectangle.	382
10.12	Distance between a point and a rectangle.	383
10.13	Partition of the plane by a rectangle.	383
10.14	Distance from a point to a polygon.	385
10.15	Solving the 3D point-polygon distance test by projecting to 2D.	386
10.16	Typical case, closest point to circle.	389
10.17	Closest point is circle center.	390
10.18	Closest point when <i>P</i> projects inside the disk.	390
10.19	Closest point when <i>P</i> projects outside the disk.	391
10.20	Distance from a point to a polyhedron (tetrahedron).	392
10.21	Distance from a point to an oriented bounding box.	394
10.22	Computing the distance between a point and an OBB.	395
10.23	The portion of the frustum in the first octant.	398
10.24	Six possible "closest points" on an ellipsoid's surface.	404
10.25	Distance from an arbitrary point to a parametric curve.	405
10.26	Distance from an arbitrary point to a parametric surface.	407
10.27	Distance between two lines.	410
10.28	Domains for each possible combination of linear component distance calculation.	413
10.29		413
	Definition of visibility of domain boundaries.	
10.30 10.31	Cases for the four edges of the domain.	416
	Distance between two line segments.	416
10.32 10.33	Distance between a line and a line accoment	419 420
10.33	Distance between a line and a line segment.  Distance between two rays.	420
10.34	Distance between a ray and a line segment.	424
10.35	Partitioning of the <i>st</i> -plane by the unit square.	424
10.37	Various level curves $Q(s,t) = V$ .	427
10.37	Distance between a line and a triangle.	434
10.39	Parametric representation of a triangle.	435
10.39	Possible partitionings of the solution space for the linear	433
10.40	component/triangle distance problem.	436
10.41	Boundary strip and planes for region 3.	440
10.42	Distance between a line and a rectangle.	442
10.43	The partitioning of the solution domain for a line segment and	

444

447

rectangle.

10.44 Distance between a line and a tetrahedron.

#### xxxii Figures

10.45	Projecting a tetrahedron (a) onto a plane perpendicular to $\hat{d}$ and then (b) into 2D.	448
10.46	Distance between a line and an oriented bounding box.	451
10.47	Schematic for line-OBB distance algorithm.	452
10.48	Case of two zero-components.	453
10.49	Case of one zero-component.	456
10.50	Determining where to look for the closest point on the box.	457
10.51	Determining whether the line intersects the box.	458
10.52	Each "positive" face of the OBB has two edges and three vertices that may be closest to the line.	460
11.1	Intersection of a line and a plane.	483
11.2	Intersection of a line and a triangle.	486
11.3	Intersection of a ray and a polygon.	489
11.4	Intersection of a linear component and a disk.	492
11.5	Intersection of a ray and a polyhedron (octahedron).	494
11.6	Intersection of a line segment and a polygonal (triangle) mesh.	494
11.7	The logical intersection of half-lines defines the intersection of a line with a polyhedron.	496
11.8	The logical intersection of half-lines fails to exist if the line does not intersect the polyhedron.	498
11.9	Possible ray-sphere intersections.	503
11.10	Intersection of a linear component and an ellipsoid.	505
11.11	Parameterized standard cylinder representation.	508
11.12	General cylinder representation.	509
11.13	Parameterized standard cone representation.	513
11.14	General cone representation.	514
11.15	An acute cone. The inside region is shaded.	514
11.16	An acute double cone. The inside region is shaded.	515
11.17	Case $c_2 = 0$ . (a) $c_0 \neq 0$ ; (b) $c_0 = 0$ .	516
11.18	Intersection of a ray with a NURBS surface.	520
11.19	Failed intersection calculation due to insufficient surface tessellation (shown in cross section for clarity).	522
11.20	A ray represented as the intersection of two planes.	524
11.21	Leaf-node bounding boxes are constructed from the Bézier polygon between each pair of refined vertices.	528
11.22	Adjacent bounding boxes are coalesced into a single box at the next level in the hierarchy.	529
11.23	Intersection of two planes.	530

11.24	Possible configurations for three planes described in Table 11.1.	532
11.25	Plane-triangle intersection.	535
11.26	Plane-triangle intersection configurations.	536
11.27	Triangle-triangle intersection configurations: (a) $\mathcal{P}_0 \  \mathcal{P}_1$ , but $\mathcal{P}_0 \neq \mathcal{P}_1$ ; (b) $\mathcal{P}_0 = \mathcal{P}_1$ ; (c) $\mathcal{T}_0$ intersects $\mathcal{T}_1$ ; (d) $\mathcal{T}_0$ does not intersect $\mathcal{T}_1$ .	540
11.28	Triangle-triangle interval overlap configurations: (a) intersection; (b) no intersection; (c)?.	541
11.29	Intersection of a trimesh and a plane.	544
11.30	Intersection of a polygon and a plane.	545
11.31	Intersection of a polygon and a triangle.	546
11.32	Intersection of a plane and a sphere.	548
11.33	Cross-sectional view of sphere-plane intersection.	550
11.34	Intersection of a plane and a cylinder.	551
11.35	Some of the ways a plane and a cylinder can intersect.	552
11.36	Edge-on view of plane-cylinder intersection.	554
11.37	Ellipse in 3D.	556
11.38	Circle in 3D.	556
11.39	Dandelin's construction.	557
11.40	Cross section of a plane intersecting a cylinder, with the two spheres used to define the intersecting ellipse. After Miller and Goldman (1992).	558
11.41	The intersection of a plane and a cylinder is a circle if the plane's normal is parallel to the cylinder's axis.	561
11.42	Intersection of a plane and a cone.	564
11.43	Some of the ways a plane and a cone can intersect.	565
11.44	Intersection test for a plane and an infinite cone.	566
11.45	Edge-on view of plane-cone intersection.	568
11.46	Infinite cone definition.	569
11.47	Geometric definitions for hyperbola and parabola.	570
11.48	Parabolic curve intersection of plane and cone. After Miller and Goldman (1992).	572
11.49	Circular curve intersection of plane and cone. After Miller and Goldman (1992).	576
11.50	Ellipse intersection of plane and cone. After Miller and Goldman (1992).	577
11.51	Hyperbola intersection of plane and cone. After Miller and Goldman (1992).	578

#### xxxiv Figures

11.52	Degenerate intersections of a plane and a cone. After Miller and Goldman (1992).	581
11.53	Intersection of a plane and a parametric surface.	587
11.54	Hermite basis functions (cubic).	589
11.55	Cubic Hermite curve, specified by end points and tangents.	590
11.56	A subpatch in parameter space maps to a topologically rectangular	370
	region on the patch. After Lee and Fredricks (1984).	591
11.57	3-space intersection curve $R(t)$ .	592
11.58	Parametric space intersection curve $p(t)$ .	593
11.59	Intersection of two B-spline surfaces.	609
11.60	Intersection curves in one surface's parameter space.	610
11.61	Intersection of a linear component with an axis-aligned bounding box.	627
11.62	Axis-aligned box as the intersection of three "slabs."	628
11.63	Clipping a line against a slab.	629
11.64	How slab clipping correctly computes ray-AABB intersection.	631
11.65	Specifying an oriented bounding box.	631
11.66	Clipping against an "oriented slab."	632
11.67	Intersection of a plane and an axis-aligned bounding box.	636
11.68	We only need to check the corners at the end of the diagonal most closely aligned with the normal to the plane.	636
11.69	The intersection of a plane and an oriented bounding box.	637
11.70	Projecting the diagonal of an OBB onto the plane normal.	638
11.71	Intersection of two axis-aligned bounding boxes.	639
11.72	2D schematic for OBB intersection detection. After Gottschalk, Lin, and Manocha (1996).	640
11.73	Intersection of an axis-aligned bounding box and a sphere.	644
11.74	Intersection of a linear component and a torus.	659
11.75	Computing the normal of a torus at a point (of intersection).	660
11.76	The <i>u</i> parameter of a point on a torus.	661
11.77	The $v$ parameter of a point on a torus.	662
12.1	Projection of a point onto a plane.	664
12.2	Projection of a vector onto a plane.	665
12.3	Projection of one vector onto another.	666
12.4	Angle between a line and a plane.	666
12.5	Angle between two planes.	667
12.6	Plane normal to a line through a point.	668
12.7	Computing the distance coefficient for the plane.	669

12.8	Three points defining a plane.	670
12.9	Angle between two lines in 3D.	671
12.10	Angle between two lines in 3D, with one line reversed.	671
13.1	BSP tree partitioning of the plane.	674
13.2	A partitioning line for which two coincident edges have opposite	
	direction normals.	676
13.3	A sample polygon for construction of a BSP tree.	678
13.4	Current state after processing edge $(9,0)$ .	678
13.5	Current state after processing edge $(0, 1)$ .	679
13.6	Current state after processing edge $\langle 1, 2 \rangle$ . This edge forces a split of $\langle 4, 5 \rangle$ to $\langle 4, 10 \rangle$ and $\langle 10, 5 \rangle$ . It also forces a split of $\langle 8, 9 \rangle$ to $\langle 8, 11 \rangle$ and $\langle 11, 9 \rangle$ .	679
13.7	Current state after processing edge $\langle 10, 5 \rangle$ . This edge forces a split of $\langle 7, 8 \rangle$ to $\langle 7, 12 \rangle$ and $\langle 12, 8 \rangle$ .	680
13.8	Current state after processing edge $\langle 5, 6 \rangle$ .	680
13.9	Final state after processing edge $\langle 13, 9 \rangle$ .	681
13.10	Partition for a convex polygon and the corresponding BSP tree.	682
13.11	Partition for a convex polygon and the corresponding balanced BSP tree.	683
13.12	Partition of a line segment.	687
13.13	Point-in-convex-polygon test by determining two edges intersected by the vertical line through the test point. $P$ is inside the polygon. $Q$ is outside the polygon.	700
13.14	Point-in-polygon test by counting intersections of ray with polygon. The ray for point $P_0$ only crosses edges transversely. The number of crossings is odd (5), so the point is inside the polygon. The ray for point $P_1$ is more complex to analyze.	701
13.15	Points $P$ on the "left" edges of the polygon are classified as inside. Points $Q$ on the "right" edges of the polygon are classified as outside.	704
13.16	Point tags for the horizontal line containing $P_1$ in Figure 13.14.	705
13.17	Interval tags for the horizontal line containing $P_1$ in Figure 13.14.	706
13.18	Two configurations for when the test ray $P + t\hat{d}$ intersects a shared edge $\vec{e}$ at an interior edge point. (a) The faces are on the same side of the plane formed by the edge and the ray. Parity is not changed. (b) The faces are on opposite sides. Parity is toggled.	712
13.19	The spherical polygon implied by the edges sharing a vertex $V$ that the test ray intersects. If the point $A$ corresponds to the ray direction, the ray interpenetrates the polyhedron. If the point $B$ corresponds to the ray direction, the ray does not interpenetrate the polyhedron.	713

13.20	Bounded and unbounded polygons that partition the plane into inside and outside regions. The inside region is gray. The unbounded polygon on the right is a half-space with a single line as the boundary of the region.	714
13.21	A polygon and its negation. The inside regions are gray. The edges are shown with the appropriate directions so that the inside is always to the left.	715
13.22	Two polygons whose inside regions are bounded.	716
13.23	The intersection of two polygons shown in gray.	716
13.24	The union of two polygons shown in gray.	717
13.25	The difference of two polygons: (a) The inverted L-shaped polygon minus the pentagon. (b) The pentagon minus the inverted L-shaped polygon.	718
13.26	The exclusive-or of two polygons shown in gray. This polygon is the union of the two differences shown in Figure 13.25.	719
13.27	Intersection of two triangles: (a) The two triangles, $A$ and $B$ . (b) Edges of $A$ intersected with inside of $B$ . (c) Edges of $B$ intersected with inside of $A$ . (d) $A \cap B$ as the collection of all intersected edges.	721
13.28	(a) Two polygons that are reported not to intersect by the pseudocode. (b) The actual set intersection, a line segment.	722
13.29	(a) Two polygons and (b) their true set of intersection.	723
13.30	<ul><li>(a) Polygon with a hole requiring two lists of vertices/edges.</li><li>(b) Keyhole version to allow a single list of vertices/edges.</li></ul>	723
13.31	Intersection of a rectangle and a keyhole polygon.	723
13.32	(a) Convex. (b) Not convex, since the line segment connecting <i>P</i> and <i>Q</i> is not entirely inside the original set.	730
13.33	A point set and its convex hull. The points are in dark gray, except for those points that became hull vertices, marked in black. The hull is shown in light gray.	730
13.34	A convex hull $H$ , a point $V$ outside $H$ , and the two tangents from $V$ to the hull. The upper and lower tangent points are labeled as $P_U$ and $P_L$ , respectively.	731
13.35	The five possibilities for the relationship of $P$ to a line segment with end points $Q_0$ and $Q_1$ : $P$ is (a) to the left of the segment, (b) to the right of the segment, (c) on the line to the left of the segment, (d) on the line to the right of the segment, or (e) on the line and contained by the segment.	734
13.36	Two convex hulls $H_L$ and $H_R$ and their upper and lower tangents.	740
13.37	Two convex hulls $H_L$ and $H_R$ and the incremental search for the	, 10
10.07	lower tangent.	741

13.38	The extreme points used to initialize tangent search are on the same vertical line. The initial visibility tests both do not yield a NEGATIVE test, yet the initial segment connecting the extremes is not a tangent to the hulls. The current candidate for the tangent is shown as a dotted line.	745
13.39	The current hull and point to be merged. The visible faces are drawn in light gray. The hidden faces are drawn in dark gray. The polyline separating the two sets is dashed. The other edges of the visibility cone are dotted.	746
13.40	(a) Two icosahedrons. (b) The merged hull. The dashed lines indicate those edges that are part of faces of the original hulls. The dotted lines indicate those edges that are part of the newly added faces.	749
13.41	(a) A side view of the pyramid and line segment. (b) A view from behind the line segment. The line segment $\langle 0, a \rangle$ can only see triangle $\langle 2, 3, 6 \rangle$ and quadrilateral $\langle 3, 4, 5, 6 \rangle$ . The line segment $\langle a, b \rangle$ can only see the quadrilateral. The line segment $\langle b, 1 \rangle$ can only see triangle $\langle 2, 4, 5 \rangle$ and the quadrilateral. The faces that are hidden in all cases are the triangles $\langle 2, 3, 4 \rangle$ and $\langle 2, 5, 6 \rangle$ . The terminator consists of the boundaries of these triangles, a sequence of line segments forming two cycles, not a simple cycle.	750
13.42	Triangulations of finite point sets: (a) with optional requirements; (b) without.	756
13.43	The two triangulations for a convex quadrilateral. The angle $\alpha \doteq 0.46$ radians and the angle $\beta \doteq 1.11$ radians. (a) The minimum angle of the top triangle is $\alpha$ (smaller than $\beta$ ). (b) The minimum angle is $2\alpha$ radians (smaller than $\beta$ ); the triangles maximize the minimum angle.	757
13.44	Two circumcircles for the triangles of Figure 13.43.	757
13.45	(a) The newly inserted point $P$ , shown as an unlabeled black dot, is interior to a triangle, in which case the triangle is split into three subtriangles, or (b) it is on an edge of a triangle, in which case each triangle sharing the edge (if any) is split into two subtriangles.	760
13.46	A triangle pair $\langle T,A\rangle$ that needs an edge swap. The index tracking is necessary so that the correct objects in the vertex-edge-triangle table of the mesh are manipulated. After the edge swap, up to two new pairs of triangles occur, $\langle N_0, B_0 \rangle$ and $\langle N_1, B_1 \rangle$ , each pair possibly needing an edge swap. These are pushed onto the stack of pairs that	
	need to be processed.	762
13.47	Supertriangle of the input point set.	763
13.48	Circumcircles containing the next point to be inserted.	764
13.49	The insertion polygon for the next point to be inserted.	765
13.50	The modified insertion polygon that restores the empty circumcircle condition for the total mesh.	765

## xxxviii Figures

13.51	The final mesh triangles are dark gray. The removed triangles are shown in light gray.	766
13.52	(a) Convex hull of 2D points lifted onto a paraboloid in 3D. (b) The corresponding Delaunay triangulation, the projection of the lower hull onto the <i>xy</i> -plane.	767
13.53	(a) Two vertices that are visible to each other. The diagonal connecting them is shown. (b) Two vertices that are not visible to each other, blocked by a vertex between them. (c) Two vertices that are not visible to each other, blocked by a single edge. (d) Two vertices that are not visible to each other, blocked by a region outside the polygon.	768
13.54	Illustration of why lack of visibility between $V_0$ and $V_2$ is equivalent to triangle $\langle V_0, V_1, V_2 \rangle$ containing a reflex vertex $R$ .	769
13.55	Cone containment (a) for a convex vertex and (b) for a reflex vertex.	770
13.56	A simple polygon that is used to illustrate the horizontal decomposition into trapezoids. The edges are labeled randomly and are processed in that order in the figures that follow.	776
13.57	The entire plane is a single trapezoid.	777
13.58	Split by $s_1.y_0$ .	777
13.59	Split by $s_1, y_1$ .	778
13.60	Insert $s_1$ .	778
13.61	Split by $s_2.y_1$ .	779
13.62	Split by $s_2$ , $y_0$ .	779
13.63	Insert $s_2$ .	779
13.64	Split by $s_3.y_1$ .	780
13.65	Insert $s_3$ .	780
13.66	Insert $s_9$ .	781
13.67	The plane after trapezoids are merged into maximally sized ones.	783
13.68	The sample polygon after trapezoids are merged into maximally sized ones.	784
13.69	The sample polygon as a union of monotone polygons. The two polygons are drawn in light gray and dark gray. The horizontal line segments from the trapezoidal decomposition are still shown.	784
13.70	If the triangle at an extreme vertex is an ear, removing the ear yields another monotone polygon.	785
13.71	Failure of triangle $\langle V_0, V_{\min}, V_1 \rangle$ to be an ear.	786
13.72	(a) Not all reflex chain vertices are visible to $W$ . (b) Removal of the triangles leads to $W$ being the next vertex to be added to the reflex	
	chain.	787

13.73	(a) $W$ occurs above the current strip, $V_0$ is visible to all reflex chain vertices. (b) Removal of the triangles leads to a reduced monotone polygon, so the process can be repeated.	789
13.74	(a) Partition using only vertices. (b) Partition using an additional	70)
13.71	point interior to the polygon.	790
13.75	Vertex $V_0$ is reflex. The diagonal $\langle V_0, V_1 \rangle$ is inessential. The diagonal $\langle V_0, V_2 \rangle$ is essential for $V_0$ .	791
13.76	Original polygon (upper left) and 11 minimum convex decompositions, with the narrowest pairs shaded in gray. A dotted line indicates that the edge of the polygon is treated instead as a diagonal.	793
13.77	Canonical triangulations of the convex polygons in the minimum convex decomposition of a polygon. The original polygon has edges shown in a heavy line. The diagonals used in the decomposition are dotted. The diagonals used in the triangle fans for the canonical triangulations are shown in a light line.	794
13.78	Circumscribed and inscribed circles for a triangle.	799
13.79	Purported minimum-area rectangle that has no coincident polygon edges.	804
13.80	(a) Current bounding circle and a point that is outside the circle, causing the circle to grow. (b) The new bounding circle, but a point inside the old circle is now outside the new circle, causing a restart of the algorithm.	809
13.81	Points $U_1$ and $U_2$ chosen for computing Equation 13.2. Only one edge of the triangle is visible to the first point. Two edges of the triangle are visible to the second point.	819
A.1	The top sequence shows a nonuniform scale $(x, y) \rightarrow (2x, y)$ applied first, a counterclockwise rotation by $\pi/4$ second. The bottom sequence shows a rotation by any angle (the circle is invariant under rotations), but clearly there is no nonuniform scaling along coordinate axes that can force the circle to become the ellipse of the	
	top sequence.	856
A.2	Intersection of a function $f(x, y) = z$ and plane $z = 0.8$ yields a level curve, shown projected on the $xy$ -plane.	895
A.3	Level curves for $f(x, y) = \frac{2x^2}{3} + y^2$ .	896
A.4	Level curves for $f(x, y) = \frac{2x^2}{3} + y^2$ , projected onto the xy-plane.	897
A.5	Two functions lacking either a minimum or maximum value.	898
A.6	Two functions that violate the assumptions of the Extreme Value	
	Theorem.	899
A.7	A variety of functions, showing critical points—(a) and (b) are stationary points; (c) and (d) are inflection points.	900

A.8	The maximum of a function may occur at the boundary of an interval or within the interval.	901
A.9	$f(x) = x^3 + 6x^2 - 7x + 19, \forall x \in [-8, 3].$	902
A.10	$f'(x) = 3x^2 + 12x - 7, \forall x \in [-8, 3].$	902
A.11	Relative extrema.	903
A.12	Relative extrema of $f(x) = 3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$ .	904
A.13	Relative extrema of a function of two variables are the hills and valleys of its graph.	906
A.14	The relative maximum of a function $z = f(x, y)$ . After Anton (1980).	907
A.15	A "saddle function"—the point (0, 0) is not an extremum, in spite of the first partial derivatives being zero.	909
A.16	Graph of $2y^2x - yx^2 + 4xy$ , showing saddle points and relative	
	minimum.	911
A.17	Contour plot of $2y^2x - yx^2 + 4xy$ .	912
A.18	Plot of the ellipse $17x^2 + 8y^2 + 12xy = 100$ .	915
A.19	Plot of the function $x^2 + y^2$ .	915
A.20	Level curves for $x^2 + y^2$ .	916
A.21	The constraint curve and the ellipse are tangent at the minima.	916
A.22	The closest and farthest point to $(1, 2, 2)$ on the sphere $x^2 + y^2 + z^2 = 36$ .	919
A.23	Constraint equations $g_1(x, y, z) = x - y + z = 1$ and $g_2(x, y, z) = x^2 + y^2 = 1$ .	921
A.24	Extrema of $f$ shown as points on the constraint curve determined by the intersection of implicit surfaces defined by $g_1 = 0$ and $g_2 = 0$ , and the level sets of $f$ at those extrema.	922
B.1	Standard terminology for angles.	922
B.2	Definition of arc length.	925
B.3	Definition of radians.	925
B.4	The ratios of sides of a right triangle can be used to define trig	923
	functions.	927
B.5	Generalized definition for trigonometric functions.	928
B.6	Geometrical interpretation of trigonometric functions.	930
B.7	Graphs of the fundamental trigonometric functions.	932
B.8	The law of sines.	937
B.9	Proof of the law of sines.	938
B.10	Graphs of the fundamental inverse trigonometric functions.	947