# Distance Between Point and Line, Ray, or Line Segment

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The following construction applies in any dimension, not just in 3D. Let the test point be $\vec{P}$. A line is parameterized as $\vec{L}(t) = \vec{B} + t\vec{M}$ where $\vec{B}$ is a point on the line, $\vec{M}$ is the line direction, and $t \in \mathbb{R}$. A *ray* is of the same form but with restriction $t \geq 0$. A *line segment* is restricted even further with $t \in [0, 1]$. The end points of the line segment are $\vec{B}$ and $\vec{B} + \vec{M}$.

The closest point on the line to $\vec{P}$ is the projection of $\vec{P}$ onto the line, $\vec{Q} = \vec{B} + t_0\vec{M}$, where

$$t_0 = \frac{\vec{M} \cdot (\vec{P} - \vec{B})}{\vec{M} \cdot \vec{M}}.$$

The distance from $\vec{P}$ to the line is

$$D = |\vec{P} - (\vec{B} + t_0\vec{M})|.$$

If $t_0 \leq 0$, then the closest point on the ray to $\vec{P}$ is $\vec{B}$. For $t_0 > 0$, the projection $\vec{B} + t_0\vec{M}$ is the closest point. The distance from $\vec{P}$ to the ray is

$$D = \left\{ \begin{array}{ll} |\vec{P} - \vec{B}|, & t_0 \leq 0 \\ |\vec{P} - (\vec{B} + t_0\vec{M})|, & t_0 > 0 \end{array} \right\}.$$

Finally, if $t_0 > 1$, then the closest point on the line segment to $\vec{P}$ is $\vec{B} + \vec{M}$. The distance from $\vec{P}$ to the line segment is

$$D = \left\{ \begin{array}{ll} |\vec{P} - \vec{B}|, & t_0 \leq 0 \\ |\vec{P} - (\vec{B} + t_0\vec{M})|, & 0 < t_0 < 1 \\ |\vec{P} - (\vec{B} + \vec{M})|, & t_0 \geq 1 \end{array} \right\}.$$

The division by $\vec{M} \cdot \vec{M}$ is the most expensive algebraic operation. The implementation should defer the division as late as possible. The code in file `pt3lin3.cpp` does defer the division.