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ECS 020 A03

Homework 3

Exercise 1: .

Show that this implication is a tautology, by using a truth table:

$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$

p	q	r	$p \lor q$	$p \to r$	$q \to r$	$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Exercise 2: .

Show that $(p \lor q) \land (\neg p \lor r) \to (q \lor r)$ is a tautology.

p	q	r	$p \lor q$	$\neg p \lor r$	$q \lor r$	$(p \lor q) \land (\neg p \lor r) \to (q \lor r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	F	T	T	T
F	F	F	F	T	F	T

Exercise 3: .

Determine whether these are valid arguments:

(a) “If x^2 is irrational, then x is irrational. Therefore, if x is irrational, it follows that $x^2$is irrational.“

if $x = \sqrt{2}, then x^2 = 2$

$\because \sqrt{2}$ is irrational, and 2 is rational

$\therefore$ this statement is not valid.

(b) “if x^2 is irrational, then x is irrational. The number $y = \pi^2$ is irrational. Therefore, the number $x = \pi$ is irrational.

$\forall x \quad x^2 \in$ irrational number,$x \in$ irrational number

$\because \pi^2 \in$ irrational number

$\therefore \pi \in$ irrational number

Exercise 4: .

Prove that a square of an integer ends with a 0, 1, 4, 5 6 or 9. (Hint:let $n = 10k + \ell$, where $\ell = 0$, 1, ...,9)

let $n = 10k + \ell, n \in \mathbb{Z}, k \in \mathbb{Z}$

$n^2 = 100k^2 + 20k\ell + \ell^2$

$\therefore n^2 \ mod\ 10 = \ell^2 \ mod\ 10$

ℓ	$\ell^2 \ mod\ 10$	l	$\ell^2 \ mod\ 10$
0	0	5	5
1	1	6	6
2	4	7	9
3	9	8	4
4	6	9	1

according to the table above, we can know that a square of an integer only can ends with a 0, 1, 4, 5 6 or 9.

Exercise 5: .

Prove that if n is a positive integer, then n is even if and only if 7n+4 is even.

p : "n is even" q : "7n+4 is even"

we want to proof that $p \leftrightarrow q$

1) $p \rightarrow q$

if n is even, we can assume that $n = 2k, k \in \mathbb{Z}$

$\therefore 7n + 4 = 14k + 4$

$\because 14k + 4 mod 2 \equiv 0$

$\therefore q$ is true

$p \rightarrow q$ is true

2) $q \rightarrow p$

we can proof it by contrapositive, so we should prove that $\neg p \rightarrow \neg q$ is true.

if n is odd, we can assume that $n = 2k + 1, k \in \mathbb{Z}$

$\therefore 7n + 4 = 14k + 11$

$\because 14k + 4 mod 2 \equiv 1$

$\therefore \neg q$ is true

$\therefore \neg p \rightarrow \neg q$ is true

$\therefore q \rightarrow p$ is true

in the case that $(q \rightarrow p) \wedge (p \rightarrow q)$ is true, we can conclude that $p \leftrightarrow q$ is true

so n is even if and only if 7n+4 is even.

Exercise 6: .

Prove that either $2.10^{500} + 15$ or $2.10^{500} + 16$ is not a perfect square. Is your proof constructive, or non-constructive?

first of all, I want to prove that the difference between two perfect square cannot be 1.

suppose we have two integers, m and n ($m < n$).

$n^2 - m^2 = (n - m)(n + m)$

$\because n - m \geqslant 1, n + m > 1$

$\therefore n^2 - m^2 > 1$

so the difference between two perfect square cannot be 1.

$\because 2.10^{500} + 16 - 2.10^{500} + 15 = 1$

$\therefore$ there are at least 1 number is not perfect sauare number

so we can conclude that either $2.10^{500} + 15$ or $2.10^{500} + 16$ is not a perfect square.

because I do not proof that either $2.10^{500} + 15$ or $2.10^{500} + 16$ is not a perfect square directly, this proof is constructive.

Exercise 7: .

Prove or disprove that if a and b are rational numbers, then a^b is also rational.

if a = 2 and b = 0.5. The number, $2^{0.5}$ will be a irrational number. So this statement is false.

Exercise 8: .

Prove that at least one of the real numbers $a_1, a_2, ..., a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use?

if all of the number is less than the average, we can say that $a_i = \bar{a} - k_i,\ k_i \in \mathbb{Z}^+$

$\bar{a} = \frac{1}{n} \sum_{i=1}^{n} \bar{a} - k_i = \bar{a} - \sum_{i=1}^{n} k_i$

$\because k_i > 0$

$\therefore$ we get that $\bar{a} < \bar{a}$, which is imposable.

So my hypothesis is false, which means that there are at least one of the real numbers $a_1, a_2, ..., a_n$ is greater than or equal to the average of these numbers.

I proof is by contrapositive.

Exercise 9: .

The proof below has been scrambled. Please put it back in the correct order.

claim: For all $n \geqslant 9$, if n is a perfect square, then n-1 is not prime.

1: Since (n-1) is the product of 2 integers greater than 1, we know (n-1) is not prime

2: Since $m \geqslant 3$, it follows that $m-1 \geqslant 2$ and $m+1 \geqslant 4$

3: Let n be a perfect square such that $n \geqslant 9$

4: This means that $n-1 = m^2 - 1 = (m-1)(m+1)$

5: There is an integer $m \geqslant 3$ such that $n = m^2$

Answer:

$3 \to 5 \to 4 \to 2 \to 1$

Exercise 10: .

Prove that these four statements are equivalent: (i) $n^2$is odd, (ii) 1-n is even, (iii) n^3 is odd, (iv) n^2+1 is even.

if n is odd, we can assume that $n = 2k+1$

$n^2 = 4k^2 + 4k + 1$

$\therefore n^2$ is odd

$1 - n = -2k$

$\therefore 1-n$ is even

$n^3 = 8k^3 + 4k^2 + 2k + 1$

$\therefore n^3$ is odd

$n^2 + 1 = 4k^2 + 4k + 2$

$\therefore n^2 + 1$ is even

if n is even, we can assume that $n = 2k$

$n^2 = 4k^2$

$\therefore n^2$ is even

$1 - n = 1 - 2k$

$\therefore 1 - n$ is odd

$n^3 = 8k^3$

$\therefore n^3$ is even

$n^2 + 1 = 4k^2 + 1$

$\therefore n^2 + 1$ is odd

when n is odd, all of the statements are true. When n is even, all of the statements are false. So all of the statements are equivalent.

Exercise 11: .

Use Exercise 8 to show that if the first 10 strictly positive integers are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.

let $A_1 = 1+2+3,\ A_2 = 2+3+4\ ...\ A_{10} = 10+1+2$

$\therefore \sum_{i=1}^{10} A_i = 55 * 3 = 165$

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$\therefore$ the average number of A_i is 16.5.

$\because$ all of the numbers are integers

$\therefore$ there must have a sum which is greater than or equal to 17.