

Homework 8

Exercise 1: .

How many strings are there of four lower case letters that have the letter x in them?

Number of strings of length 4 = 26^4

Number of strings of length 4 other than $x = 25^4$

Thus, the answer is $26^4 - 25^4 = 66351$ strings.

Exercise 2: .

How many strings of four decimal digits

a)do not contains the same digit twice?

$$10 * 9 * 8 * 7 = 5040$$

b)End with an even digit?

$$10^3 * 5 = 5000$$

c)Have exactly three digits that are 9s?

$$9 * 4 = 36$$

Exercise 3: .

How many license plates can be made using either three digits followed by three letters, or three letters followed by three digits?

$$9^3 * 26^3 + 26^3 * 9^3 = 2(26^3 * 9^3) = 25625808$$

Exercise 4: .

How many subsets of a set with 100 elements have more than one element?

All of the subset = 2^{100}

The sub set has less than one element = $100 + 1$

Thus, the answer is $2^{100} - 100 - 1$.

Exercise 5: .

How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?

let's consider 5 consecutive 1s first:

if it start at position 1, remaining digits can be both 1 and 0: $2^5 = 32$

if it start at position $k = 2, 3, 4, 5, 6$, the number at position $k - 1$ must be 0, or it will be in previous cases: $5 * 2^4$

So the total number is $2^5 + 5 * 2^4 = 112$

5 consecutive 0s is similar as 5 consecutive 1s, so we get another 112.

because we count 0000011111 and 1111100000 twice, we should subtract 2 from the answer.

Thus the final answer is $112 * 2 - 2 = 222$

