Anze Wang ID: 912777492
ECS 020 A04

Homework 7

Exercise 1: .

e)-2002 is divided by 89?

$\because -2002 \ mod \ 89 = 45$

$\therefore$ -2002 is not divided by 89

f) 0 is divided by 19?

$\because 0 \ mod \ 19 = 0$

$\therefore$ 0 is divided by 19

g) 1,234,567 is divided by 101?

$\because 1234567 \ mod \ 19 = 14$

$\therefore$ 1,234,567 is not divided by 19

h)-100 is divided by 103?

$\because -100 \ mod \ 103 = 3$

$\therefore$ -100 is not divided by 103

Exercise 2: .

a) Let a be a positive integer. Show that $gcd(a, a-1) = 1$

$$\begin{aligned} &gcd(a, a-1) \\ =&gcd(a-1, 1) \\ =&1 \end{aligned}$$

b) Use the result of part a) to solve the Diophantin eequation

$$a + 2b = 2ab$$

where (a, b) are positive integers

$$\begin{aligned} &a + 2b = 2ab \\ &a = 2(a-1)b \\ &b = \frac{a}{2(a-1)} \end{aligned}$$

Because both a and b are positive integer, we can only get one set of solution, a = 2, b = 1.

Exercise 3: Extra Credit

Let a and b be two strictly positive integers. Solve $gcd(a, b) + lcm(a, b) = b + 9$

Let $a = pd$, $b = qd$,which p, q, d are positive integers

$$\begin{aligned} &gcd(a, b) + lcm(a, b) = b + 9 \\ &pd + \frac{pqd^2}{d} = qd + 9 \\ &pd + pqd - qd = 9 \\ &d(p - q + pq) = 9 \end{aligned}$$

Because p ,q ,d are positive integers, we can get these solutions:

$$\begin{cases} d = 9 \\ p = 1 \\ q = 1 \end{cases} \qquad \begin{cases} d = 3 \\ p = 2 \\ q = 1 \end{cases}$$

So we can know that $a = 9, b = 9$ and $a = 6, b = 3$