Anze Wang ID: 912777492
ECS 020 A03

Homework 4

Exercise 1: .

Give a direct proof, an indirect proof, and a proof by contradiction of the statement: if n is even, then n+4 is even.

p: n is even; q: n+4 is even

direct proof: $p \to q$

Because n is even, let n = 2k. then n + 4 = 2k +4, which is even.

indirect proof: $\neg q \to \neg p$

Because n + 4 is odd, n+4 = 2k +1. Then n = 2k - 3, which is odd.

proof by contradiction: $\neg p \to q$

Because n is odd, n = 2k + 1. Then n + 4 = 2k + 5, which is odd. This is contradict with the statement n +4 is even. So if n is even, n+4 is even.

Exercise 2: .

Let A, B and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$

$$\begin{aligned}
&(A - C) - (B - C) \\
&= \{x|(x \in A \wedge x \notin C) \wedge \neg(x \in B \wedge x \notin C)\} \\
&= \{x|(x \in A \wedge x \notin C) \wedge (x \notin B \vee x \in C)\} \\
&= \{x|x \in A \wedge (x \notin C \wedge (x \notin B \vee x \in C))\} \\
&= \{x|x \in A \wedge ((x \notin C \wedge x \notin B) \vee (x \notin C \wedge x \in C))\} \\
&= \{x|x \in A \wedge (x \notin C \wedge x \notin B)\} \\
&= \{x|(x \in A \wedge x \notin B) \wedge x \notin C\} \\
&= (A - B) - C
\end{aligned}$$

Exercise 3: .

Show that $A \oplus B = (A - B) \cup (B - A)$

According to the definition, we know that

$$\begin{aligned}
&A \oplus B \\
&= \{x|(x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B)\} \\
&= \{x|(x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)\} \\
&= \{x|(x \in A \wedge (x \notin A \vee x \notin B)) \vee (x \in B \wedge (x \notin A \vee x \notin B))\} \\
&= \{x|(x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\} \\
&= (A - B) \cup (B - A)
\end{aligned}$$

Exercise 4: .

(a) show that $A \oplus B = B \oplus A$

$$\begin{aligned}
& A \oplus B \\
&= \{x|(x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B)\} \\
&= \{x|(x \in B \vee x \in A) \wedge \neg(x \in B \wedge x \in A)\} \\
&= B \oplus A
\end{aligned}$$

(b) show that $(A \oplus B) \oplus B = A$

$$\begin{aligned}
& (A \oplus B) \oplus B \\
&= \{x|(x \in A \oplus x \in B) \oplus x \in B\} \\
&= \{x|x \in A \oplus (x \in B \oplus x \in B)\} \\
&= \{x|x \in A\} \\
&= A
\end{aligned}$$

(c) show that $A \neq A \oplus A$ if A is a non empty set.

$$\begin{aligned}
& A \oplus A \\
&= \{x|x \in A \oplus x \in A\} \\
&= \varnothing
\end{aligned}$$

$\therefore$ if A is not empty set, A cannot equal to $A \oplus A$

Exercise 5: .

Can you conclude that A = B if A, B, and C are sets such that:

(a) $A \cup C = B \cup C$

if $A = \{1\}$, $B = \{2\}$, $C = \{1,\ 2,\ 3\}$

then,$A \cup C = B \cup C = \{1,\ 2,\ 3\}$

However, $A \neq B$

So I cannot conclude that A = B if $A \cup C = B \cup C$

(b) $A \cap C = B \cap C$

if $A = \{1.\ 3,\ 4,\ 5\}$, $B = \{2,\ 3,\ 4,\ 5\}$, $C = \{3,\ 4,\ 5\}$

then,$A \cap C = B \cap C = \{3,\ 4,\ 5\}$

However, $A \neq B$

So I cannot conclude that A = B if $A \cap C = B \cap C$

Exercise 6: .

Show that if A, B, and C are sets then

$$|A \cup B \cup C| \;=\; |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$\begin{aligned}
&|A \cup B \cup C| \\
&= |A \cup (B \cup C)| \\
&= |A| + |B \cup C| - |A \cap (B \cup C)| \\
&= |A| + |B \cup C| - |(A \cap B) \cup (A \cap C)| \\
&= |A| + |B \cup C| - |A \cap B| - |A \cap C| + |A \cap B \cap C| \\
&= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|
\end{aligned}$$

Exercise 7: .

Let A and B be subsets of the finite universal set U. Show that:

$$|\overline{A} \cap \overline{B}| = |U| - |A| - |B| + |A \cap B|$$

$$\begin{aligned}
&|\overline{A} \cap \overline{B}| \\
&= |\overline{A \cup B}| \\
&= |U| - |A \cup B| \\
&= |U| - |A| - |B| + |A \cap B|
\end{aligned}$$

Exercise 8: .

let $A_i = \{..., \; -2, \; -1, \; 0, \; 1, \; 2, ..., \; i\}$. Find:

a) $\bigcup_{i=1}^{n} A_i = A_n = \{..., \; -2, \; -1, \; 0, \; 1, \; 2, ..., \; n\}$

b) $\bigcap_{i=1}^{n} A_i = A_1 = \{..., \; -2, \; -1, \; 0, \; 1\}$

Exercise 9: .

Let A and B be two sets. Show that if $A \cup B = B$ then $A \cap B = A$

$$\begin{aligned}
&\because A \cup B = B \\
&\therefore A \subseteq B \\
&\therefore A \cap B = A
\end{aligned}$$

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Exercise 10: .

Let A and B be two sets. Show that if $A \cap B = A$ then $B \cap (\overline{B \cap \overline{A}}) = A$

$$\begin{aligned}
&B \cap (\overline{B \cap \overline{A}}) \\
&= B \cap (\overline{B} \cup A) \\
&= (B \cap \overline{B}) \cup (B \cap A) \\
&= \phi \cup (B \cap A) \\
&= B \cap A \\
&= A
\end{aligned}$$

Exercise 11: Extra credit:

A 4x4 magic square is an arrangement of the number from 1 to 16 in a 4x4 table, such that the sum of the elements on any rows, columns and main diagonals is the same. Show that the square shown below cannot be completed to be a magic square:

1	2	3	?
?	4	5	6
7	?	8	?
?	9	?	10

Because $1 + 4 + 8 + 10 = 23$, we can know that the sum of each line is 23.

Therefore, The question mark in the first line must be 23 - 1 - 2 -3 = 17, which is bigger than 16.

So this square cannot be completed.