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ECS 020 A03

Homework two

Exercise 1: .

Construct a truth table for each of these compound propositions

a) $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

b) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	T	T	T

c) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Exercise 2: .

Construct a truth table for each of these compound propositions

a) $(p \oplus q) \vee (p \oplus \neg q)$

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

b) $(p \oplus q) \wedge (p \oplus \neg q)$

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

Exercise 3: .

Is the assertion “This statement is false“ a proposition?

According to the textbook, a proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both. The assertion “This statement is false“ is neither true nor false. It is a paradox. So it is not a proposition

Exercise 4: .

A contestant in a TV game show is presented with three boxes, A, B, and C. He is told that one of the boxes contains a prize, while the two others are empty. Each box has a statement written on it:
Box A: The prize is in this box
Box B: The prize is not in box A
Box C: The prize is not in this box
The host of the show tells the contestant that only one of the statements is true. Can the contestant find logically which box contains the prize? Justify your answer.

p: the prize is in box A

q: the prize is in box B

r: the prize is in box C

so the statement of box A is p; the statement of box B is $\neg p$; the statement of box C is $\neg r$

$\because$ there are only one price

$\therefore p \wedge q \equiv F$

$\therefore$ the truth table is:

p	r	p	$\neg p$	$\neg r$
T	F	T	F	T
F	T	F	T	F
F	F	F	T	T

According to the truth table, we can know that only when p is false and the r is true, there is only one true statement on the box. So the price is in box C.

Exercise 5: .

This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?

A says The two of us are both knights, and B says A is a knave.

p: A is Knight, $\neg p$: A is knave

q: B is Knight, $\neg q$: B is knave

$\because$ knights always tell the truth and knaves always lie

$\therefore$ the value of the type of person, p, must equal to the value of his of her statement, r.

According to the truth table below, we can know that the $\neg(p \oplus r)$ is true, if and only if when the knave lie or the knight tell truth.

p	r	$\neg(p \oplus r)$
T	T	T
T	F	F
F	T	F
F	F	T

A : $p \wedge q$

B : $\neg p$

So we can know that :

$\neg(p \oplus (p \wedge q))$: A's statement is true when he is a knight, and is false when he is a knave

$\neg(q \oplus \neg p)$: B's statement is true when he is a knight, and is false when he is a knave

p	q	$\neg(p \oplus (p \wedge q))$	$\neg(q \oplus \neg p)$
T	T	T	F
T	F	F	T
F	T	T	T
F	F	T	F

According to the truth table, we can know that that A is a knave, and B is a knight.

Exercise 6: .

Use truth tables to verify the associative laws:

a) $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

b) $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Exercise 7: .

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$are equivalent.

$$\begin{aligned}
LHS &= p \leftrightarrow q \\
&= (p \rightarrow q) \wedge (q \rightarrow p) \\
&= (\neg p \wedge q) \wedge (\neg q \wedge p) \\
&= (\neg p \vee q) \wedge (\neg q \vee p) \\
&= ((\neg p \vee q) \wedge \neg q)) \vee ((\neg p \vee q) \wedge p) \\
&= ((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee (\neg p \wedge p) \vee (q \wedge q)) \\
&= (\neg p \wedge \neg q) \vee (q \wedge q)
\end{aligned}$$

$= (p \wedge q) \vee (\neg p \wedge \neg q)$

$RHS = (p \wedge q) \vee (\neg p \wedge \neg q)$

$\therefore p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$are equivalent.

Exercise 8: Extra Credit

We are back on the island of knights and knaves (see exercise 5 above). John and Bill are residents.
John: if Bill is a knave, then I am a knight
Bill: we are different
Who is who?

p: Bill is a knight.

q: John is a knight.

$\therefore$ Bill's statement is $p \oplus q$

John's statement is $\neg p \rightarrow q$

p	q	$\neg(p \oplus (p \oplus q))$	$\neg(q \oplus (\neg p \rightarrow q))$
T	T	F	T
T	F	T	F
F	T	F	T
F	F	T	T

According to the truth table, we can know that both of them are knave.