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ECS 020 A04

Homework 1

Exercise 1: .

A ball and a bat cost \$1.10 (total). The bat costs \$1.0 more than the ball. How much does the ball cost?

let the ball costs \$x, $x \in \mathbb{R}$

$(x + 1.0) + x = 1.1$

$x = 0.05$

So, the ball costs \$0.05

Exercise 2: .

Prove the following statements:

a) The sum of any three consecutive even numbers is always a multiple of 6

$\forall n \in \mathbb{Z}$, we have $2 * n,\ 2 * n + 2$ and $2 * n + 4$, which are consecutive even numbers.

$2 * n + (2 * n + 2) + (2 * n + 4) = 6 * n + 6$

$\because 6 * n + 6 \equiv 0 \pmod 6$

$\therefore 2 * n + (2 * n + 2) + (2 * n + 4) \equiv 0 \pmod 6$

So the sum of any three consecutive even numbers is always a multiple of 6

b) The product of any three consecutive even numbers is always a multiple of 8

$\forall n \in \mathbb{Z}$, we have $2 * n,\ 2 * n + 2$ and $2 * n + 4$, which are consecutive even numbers.

$(2 * n) * (2 * n + 2) * (2 * n + 4) = 8 * n * (n + 1) * (n + 2)$

$\because 8 * n * (n + a1) * (n + 2) \equiv 0 \pmod 8$

$\therefore (2 * n) * (2 * n + 2) * (2 * n + 4) \equiv 0 \pmod 8$

So the product of any three consecutive even numbers is always a multiple of 8

c) Prove that if you add the squares of three consecutive integer numbers and then subtract two, you always get a multiple of 3.

$\forall n \in \mathbb{Z}$, we have $n,\ n + 1$ and $n + 2$, which are consecutive integer numbers.

$n^2 + (n + 1)^2 + (n + 2)^2 - 2$

$= n^2 + n^2 + 2 * n + 1 + n^2 + 4 * n + 4$

$= 3 * n^2 + 6 * n + 3$

$= 3 * (n^2 + 2 * n + 1)$

$\because 3 * (n^2 + 2 * n + 1) \equiv 0 \pmod 3$

$\therefore n^2 + (n + 1)^2 + (n + 2)^2 - 2 \equiv 0 \pmod 3$

We can conclude that if I add the squares of three consecutive integer numbers and then subtract two, I always get a multiple of 3.

Exercise 3: .

Roger is an amateur magician. In one of his tricks he invites people in the audience to think of a number (integer). He then asks them to carry out the following simple instructions:

	double your number
then	add 5
then	multiply the number you now have by itself
then	subtract 25
then	divide by 4
then	divide by your original number

Based on the final number obtained, Roger can then guess the initial number.
Show that there is no magic in this. Justify your answer.

Let the audience chooses n, $n \in \mathbb{N}$

$N_0 = 2 * n$

$N_1 = N_0 + 5 = 2 * n + 5$

$N_2 = N_1 * N_1 = (2 * n + 5)^2$

$N_3 = N_2 - 25 = 4 * n^2 + 20 * n$

$N_4 = N_3/4 = n^2 + 5 * n$

$N_5 = n_4/n = n + 5$

The final number we get is N_5, which is equal to $n + 5$. If we subtract 5 from the final number, we can get the initial number. So there is no magic in this.

Exercise 4: . Prove the following identities, where p, q, x, m, and n are real numbers:

a)$8(p - q) + 3(p + q) = 2(p + 2q) + 9(p - q)$

$LHS = 8 * p - 8 * q + 3 * p + 3 * q = 11 * p - 5 * q$

$RHS = 2 * p + 4 * q + 9 * p - 9 * q = 11 * p - 5 * q$

$\because LHS = RHS$

$\therefore$ This identity is true.

b)$x(m + n) + y(n - m) = m(x - y) + n(x + y)$

$LHS = x * m + x * n + y * n - y * m$

$RHS = x * m - y * m + x * n + y * n = x * m + x * n + y * n - y * m$

$\because LHS = RHS$

$\therefore$ This identity is true.

c)$(x + 2)(x + 10) - (x - 5)(x - 4) = 21 * x$

$LHS = x^2 + 12 * x + 20 - x^2 + 9 * x - 20 = 21 * x$

$RHS = 21 * x$

$\because LHS = RHS$

$\therefore$ This identity is true.

d)$m^4 - 1 = (m^2 + 1)(m^2 - 1)$

$LHS = m^4 - 1$

$RHS = m^4 + m^2 - m^2 - 1 = m^4 - 1$

$\because LHS = RHS$

$\therefore$ This identity is true.

Exercise 5: Extra Credit

Four persons need to cross a bridge to get back to their camp at night. Unfortunately, they only have one flashlight and it only has enough light left for seventeen minutes. The bridge is too dangerous to cross without a flashlight, and it is strong enough to support only two persons at any given time. The flashlight cannot be thrown from one side of the bridge to the other. Each of the campers walks at a different speed. One can cross the bridge in 1 minute, another in 2 minutes, the third in 5 minutes, and the last one takes 10 minutes to cross.

Tell these people how they can make it across in 17 minutes.

according to the question, we can know that the first people, I will call him 'A', can cross the bridge in 1 minute; the second, called 'B', in 2 minutes, the third, called 'C', in 5 minutes, and the last one , called 'D', takes 10 minutes to cross.

1st step: let A and B cross the bridge, which will spend 2 min

2nd step: let A go back with the light, which will spend 1 min

3rd step: let C and D cross the bridge, which will spend 10 min

4th step: let B go back with the light, which will spend 2 min

5th step: let A and B cross the bridge, which will spend 2 min

After 5 steps, all of the people crossed the bridge. Because $2 + 1 + 10 + 2 + 2 = 17$, they can cross the bridge in 17 minute.