2. (10 marks) **IMLD vs. MED**
Consider the binary code $C = \{c_1 = 0000, c_2 = 1110, c_3 = 0111\}$. Suppose that $P(c_1) = 0.1$, $P(c_2) = 0.2$, and $P(c_3) = 0.7$, where $P(c_i)$ denotes the probability that $c_i$ is sent. Suppose that a binary symmetric channel with symbol error probability $p$ is being used, and $r = 1001$ is received.

**(a) What is the distance of $C$?**

We have $\binom{M}{2} = \binom{3}{2} = 3$ comparisons.

1. $d(c_1, c_2) = 3$
2. $d(c_1, c_3) = 3$
3. $d(c_2, c_3) = 2$

We take the minimum value, and get $d(C) = 2$.

**(b) Suppose that $p = 0.1$. Decode $r$ using IMLD.**

IMLD decodes to the minimum Hamming Distance.

1. $d(r, c_1) = 2$
2. $d(r, c_2) = 3$
3. $d(r, c_3) = 3$

Thus, $r = 1001$ decodes to $r = 0000$.

**(c) Suppose that $p = 0.1$. Decode $r$ using MED.**

$P(r \mid c_1) = (1-p)^{n-d} \left(\frac{p}{q-1}\right)^d = (1-0.1)^{4-2} \left(\frac{0.1}{2-1}\right)^2 = 0.0081$

$P(r \mid c_2) = (1-p)^{n-d} \left(\frac{p}{q-1}\right)^d = (1-0.1)^{4-3} \left(\frac{0.1}{2-1}\right)^3 = 0.0009$

$P(r \mid c_3) = 0.0009$

$P(c_1 \mid r) = \frac{P(r|c_1)P(c_1)}{P(r)} = \frac{(0.0081)(0.1)}{P(r)} = \frac{0.00081}{P(r)}$

$P(c_2 \mid r) = \frac{P(r|c_2)P(c_1)}{P(r)} = \frac{(0.0009)(0.2)}{P(r)} = \frac{0.00018}{P(r)}$

$P(c_3 \mid r) = \frac{P(r|c_3)P(c_1)}{P(r)} = \frac{(0.0009)(0.7)}{P(r)} = \frac{0.00063}{P(r)}$

Since we want to maximize $P(c \mid r)$, we pick $c_1$. Thus, $r = 1001$ decodes to $r = 0000$.

**(d) Suppose that $p = 0.4$. Decode $r$ using IMLD.**

Since the probability does not matter for IMLD, the answer will be the same as (b). Thus, $r = 1001$ decodes to $r = 0000$.

**(e) Suppose that $p = 0.4$. Decode $r$ using MED.**

$P(r \mid c_1) = (1-p)^{n-d} \left(\frac{p}{q-1}\right)^d = (1-0.4)^{4-2} \left(\frac{0.4}{2-1}\right)^2 = 0.0576$

$P(r \mid c_2) = (1-p)^{n-d} \left(\frac{p}{q-1}\right)^d = (1-0.4)^{4-3} \left(\frac{0.4}{2-1}\right)^3 = 0.0384$

$P(r \mid c_3) = 0.0384$

$P(c_1 \mid r) = \frac{P(r|c_1)P(c_1)}{P(r)} = \frac{(0.0576)(0.1)}{P(r)} = \frac{0.00576}{P(r)}$

$P(c_2 \mid r) = \frac{P(r|c_2)P(c_1)}{P(r)} = \frac{(0.0384)(0.2)}{P(r)} = \frac{0.00768}{P(r)}$

$P(c_3 \mid r) = \frac{P(r|c_3)P(c_1)}{P(r)} = \frac{(0.0384)(0.7)}{P(r)} = \frac{0.02688}{P(r)}$

Since we want to maximize $P(c \mid r)$, we pick $c_3$. Thus, $r = 1001$ decodes to $r = 0111$.