CO 351 - Network Flow Theory

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Introduction.

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Definitions:

DEFINITION 1.0.1: Graph, Vertices, Edges

A **graph** $G = (V, E)$ consists of a set of **vertices** V, and a set of **edges** E that are unordered pairs of vertices.

DEFINITION 1.0.2: Degree

The degree of a vertex v is the number of edges incident with v, denoted $d_G(v)$ or $d(v)$.

DEFINITION 1.0.3: Walk

A **walk** is a sequence of vertices $v_1, v_2, \ldots, v_k$ where $v_i v_{i+1}$ are edges.

DEFINITION 1.0.4: Path

A **path** is a walk where all vertices are distinct.

DEFINITION 1.0.5: Cycle

A **cycle** is a walk $v_1, v_2, \ldots, v_k, v_1$ where $v_1, \ldots, v_k$ are distinct and $k \geqslant 3$.

Connectivity and cuts:

DEFINITION 1.0.6: Connected, Disconnected

A graph is **connected** if there is a path between every pair of vertices. Otherwise it is **disconnected.**

DEFINITION 1.0.7: Cut

For $S \subseteq V$, the cut induced by S is the set of all edges with exactly one end in S.

$$\delta(S) = \{uv \in E : u \in S, v \notin S\}$$

DEFINITION 1.0.8: s, t-cut

If $s, t \in V$ where $s \in S$ and $t \notin S$, then $\delta(S)$ is an ***s*,*t*-cut**.

PROPOSITION 1.0.9

There exists an s, t-path if and only of every s, t-cut is non-empty.

Trees:

DEFINITION 1.0.10: Tree

A **tree** is a connected graph with no cycles.

PROPOSITION 1.0.11

A tree with n vertices has $n - 1$ edges.

DEFINITION 1.0.12: Spanning tree

A **spanning tree** of a graph G is a subgraph that is a tree which uses all vertices of G.

PROPOSITION 1.0.13

G has a spanning tree if and only if G is connected.

PROPOSITION 1.0.14

If T is a tree and u, v are not adjacent, then $T + uv$ has exactly one cycle C. If xy is an edge of C, then $T + uv - xy$ is another tree.

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