STAT 231

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Roadmap:

(i) Recap and the relationship between Confidence and Hypothesis

(ii) Example: Bias Testing

(iii) Testing for variance (Normal)

(iv) What if we don't know how to construct a Test-Statistic?

EXAMPLE 0.1.1. $Y_1, \ldots Y_n$ iid $N(\mu, \sigma^2)$

- $\sigma^2 =$ known
- $\mu =$ unknown
- Sample: $\{y_1, \ldots, y_n\}$
- $\overline{y} =$ sample mean
- H_0: $\mu = \mu_0$
- H_1: $\mu \neq \mu_0$

$$D = \left| \frac{\overline{Y} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \right| \qquad \rightarrow \quad \text{Test-Statistic (r.v.)}$$

$$d = \left| \frac{\overline{y} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \right| \qquad \rightarrow \quad \text{Value of the Test-Statistic}$$

$$\begin{aligned} p\text{-value} &= P(D \geqslant d) && \text{assuming } H_0 \text{ is true} \\ &= P(|Z| \geqslant d) && Z \sim N(0,1) \end{aligned}$$

Question: Suppose the p-value for the test > 0.05 if and only if μ_0 belongs in the 95% confidence interval for μ?

YES.

Suppose μ_0 is in the 95% confidence interval for μ, i.e.

$$\overline{y} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$\mu_0 \leqslant \overline{y} + 1.96 \frac{\sigma}{\sqrt{n}}$$
$$\mu_0 \geqslant \overline{y} - 1.96 \frac{\sigma}{\sqrt{n}}$$

These two equations yield

$$d = \left| \frac{\overline{y} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \right| \leqslant 1.96$$
$$P(|Z| \geqslant d) > 0.05$$

General result (assuming same pivot)

p-value of a test H_0: $\theta = \theta_0$ vs H_1: $\theta \neq \theta_0$ is more than $q\%$, then θ_0 belongs to the $100(1-q)\%$ confidence interval and vice versa.

EXAMPLE 0.1.2 (Bias). A 10 kg weighted 20 times $(y_1, \ldots, y_n)$

- H_0: The scale is unbiased
- H_1: The scale is biased

If the scale was unbiased,

$$Y_1, \ldots, Y_n \sim N(10, \sigma^2)$$

If the scale was biased,

$$Y_1, \ldots, Y_n \sim N(10 + \delta, \sigma^2)$$

- H_0: $\delta = 0$ (unbiased)
- H_1: $\delta \neq 0$ (biased)

is equivalent to

- H_0: $\mu = 10$
- H_1: $\mu \neq 10$

Test-statistic:

$$D = \left| \frac{\overline{Y} - 10}{\frac{s}{\sqrt{n}}} \right|$$

Compute d.

$$d = \left| \frac{\overline{y} - 10}{\frac{s}{\sqrt{n}}} \right|$$

$$\begin{aligned} p\text{-value} &= P(D \geqslant d) \\ &= P(|T_{19}| \geqslant d) \end{aligned}$$

EXAMPLE 0.1.3 (Draw Conclusions). $Y_1, \ldots, Y_n =$ co-op salaries. $Y_1, \ldots, Y_n \sim N(\mu, \sigma^2)$

- H_0: $\mu = 3000$
- H_1: $\mu < 3000$ ($\mu \neq 3000$)

$$D = \left| \frac{\overline{Y} - \mu_0}{\frac{s}{\sqrt{n}}} \right|$$

$$D = \begin{cases} 0 & \overline{Y} > \mu_0 \\ \frac{\overline{Y} - \mu_0}{\frac{s}{\sqrt{n}}} & \overline{Y} < \mu_0 \end{cases}$$

If n is large, then

$$Y_1, \ldots, Y_n \sim f(y_i; \theta)$$

- H_0: $\theta = \theta_0$
- H_1: $\theta \neq \theta_0$

$$\Lambda(\theta) = -2 \ln \left[\frac{L(\theta_0)}{L(\tilde{\theta})} \right]$$

where Λ satisfies all the properties of D. Also,

$$\lambda(\theta) = -2 \ln \left[\frac{L(\theta_0)}{L(\hat{\theta})} \right]$$

and

$$p\text{-value} = P(\Lambda \geqslant \lambda) = P(Z^2 \geqslant \lambda)$$

<u>Roadmap</u>:

(i) General info

(ii) Testing for variance for Normal

(iii) An example

The general problem: $Y_1, \ldots, Y_n \sim N(\mu, \sigma^2)$ iid where μ and σ^2 are both unknown. H_0: $\sigma^2 = \sigma_0^2$ vs two sided alternative.

(i) Test statistic? Problem

(ii) Convention?

The pivot is:

$$U = \frac{(n-1)s^2}{\sigma_0^2} \sim \chi^2_{n-1}$$

can we use this as our test statistic?

EXAMPLE 0.1.4.

- Normal population: $\{y_1, \ldots, y_n\}$
- $n = 20$, $\sum y_i = 888.1$, $\sum y_i^2 = 39545.03$
- H_0: $\sigma = 2$
- H_1: $\sigma \neq 2$

What is the p-value? We know

$$s^2 = \frac{1}{n-1}\left[\sum y_i^2 - n\overline{y}^2\right] = 5.7342$$

Compute U:

$$U = \frac{(n-1)s^2}{\sigma_0^2} = 27.24$$

χ^2_{19}

$$\begin{aligned} p\text{-value} &= 2P(U \geqslant 27.24) \\ &= 2P(\chi^2_{19} \geqslant 27.24) \\ &= 10\% \text{ and } 20\% \end{aligned}$$

so, $p > 0.1$ means there is no evidence against null-hypothesis.