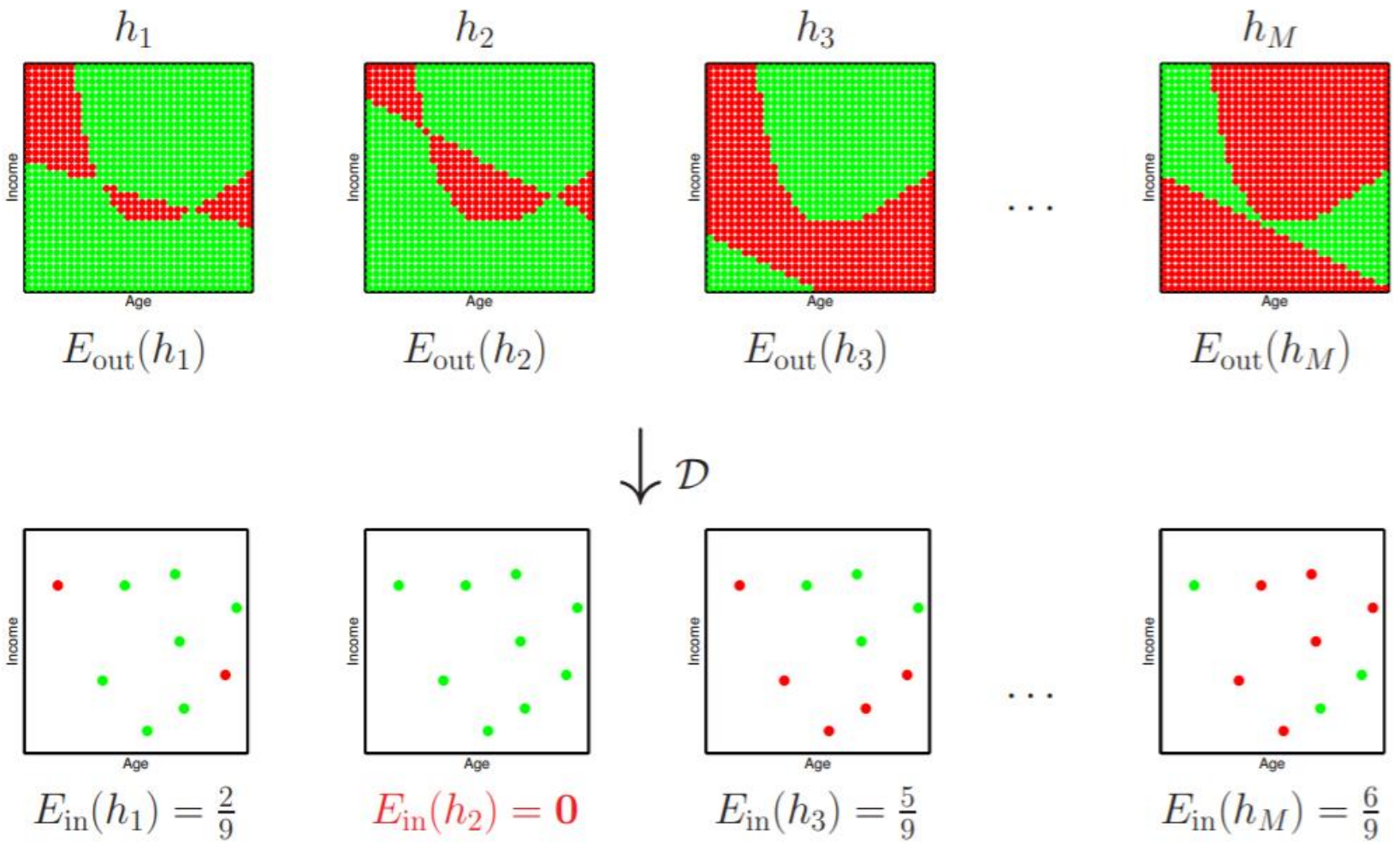
Machine Learning from Data

Lecture 4: Spring 2021

Today's Lecture

- Feasibility of Learning
- Two Step Solution to Learning
- Error and Noise



Pick the hypothesis with minimum E_{in}; will E_{out} be small?

Verification and Selection Bias

- If we pick the hypothesis with minimum in-sample error, it does not approximate out-of-sample error.
- Search Causes Selection Bias
- In Real Learning in-sample error cannot reach out to out-of-sample error.

Using Hoeffding’s Inequality in Learning

- Definition - “Hoeffding's inequality provides an upper bound on the probability that the sum of bounded independent random variables deviates from its expected value by more than a certain amount.”

Updating Hoeffding's Bound

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Feasibility of Learning

- Two Questions to answer:
  - Can we make sure that $E_{out}(g)$ is close enough to $E_{in}(g)$
  - Can we make $E_{in}(g)$ small enough

Complexity of *H* and *f*

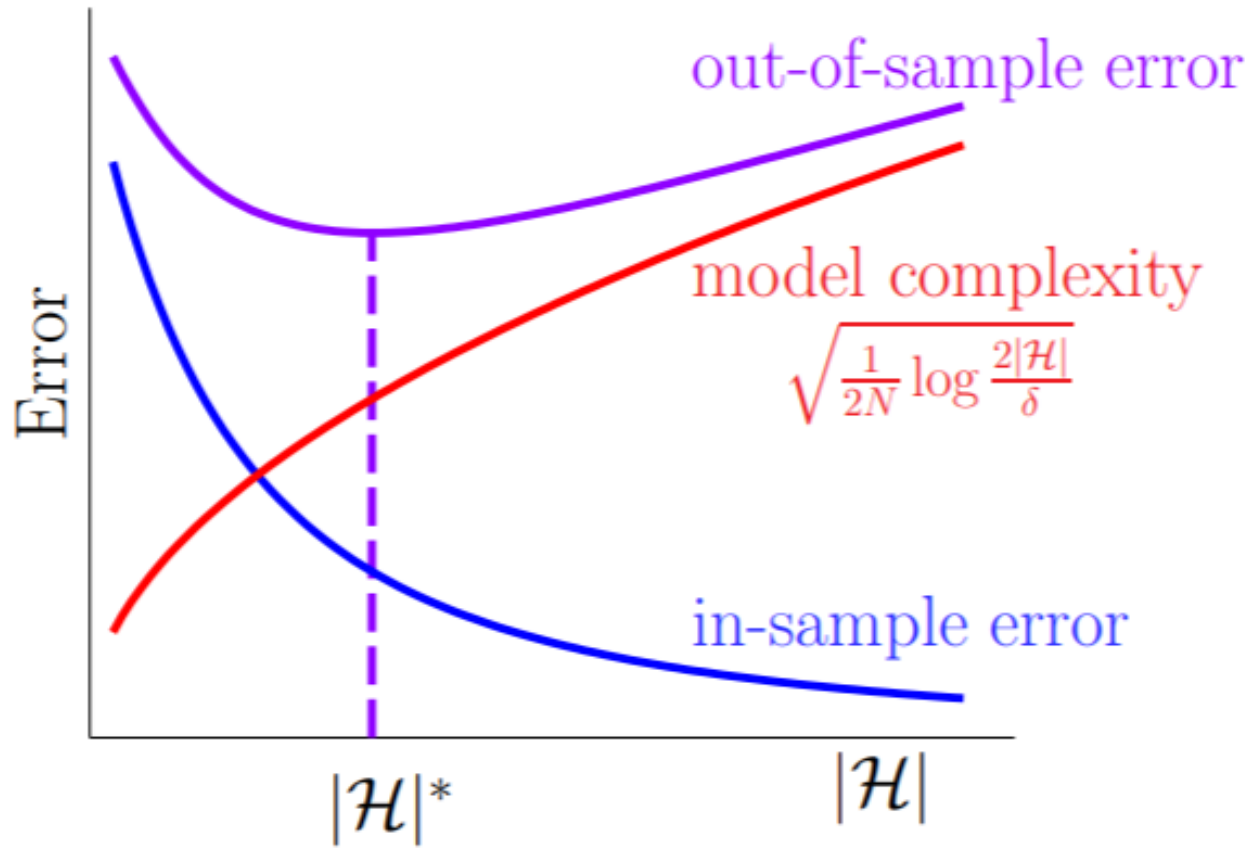
Interpreting the Hoeffding's Bound

$$\mathbb{P}\left[|\boldsymbol{E}_{\text{in}}(\boldsymbol{g}) - \boldsymbol{E}_{\text{out}}(\boldsymbol{g})| > \epsilon\right] \leq 2|\mathcal{H}|e^{-2\epsilon^2 N}, \qquad \text{for any } \epsilon > 0.$$

$$\mathbb{P}\left[|\boldsymbol{E}_{\text{in}}(\boldsymbol{g}) - \boldsymbol{E}_{\text{out}}(\boldsymbol{g})| \leq \epsilon\right] \geq 1 - 2|\mathcal{H}|e^{-2\epsilon^2 N}, \qquad \text{for any } \epsilon > 0.$$

E_{in} *reaches out to* E_{out} when H is small

$$E_{\text{out}}(g) \leq E_{\text{in}}(g) + \sqrt{\frac{1}{2N}\log\frac{2|\mathcal{H}|}{\delta}}.$$


out-of-sample error
model complexity
$\sqrt{\frac{1}{2N}\log\frac{2|\mathcal{H}|}{\delta}}$
in-sample error
Error
$|\mathcal{H}|^*$
$|\mathcal{H}|$

2 Step Approach

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Summarize

- Is Learning Feasible?

Our Learning Approach is General

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Target Function

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Noisy target

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Error

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Interpretation of Error

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Pointwise Errors

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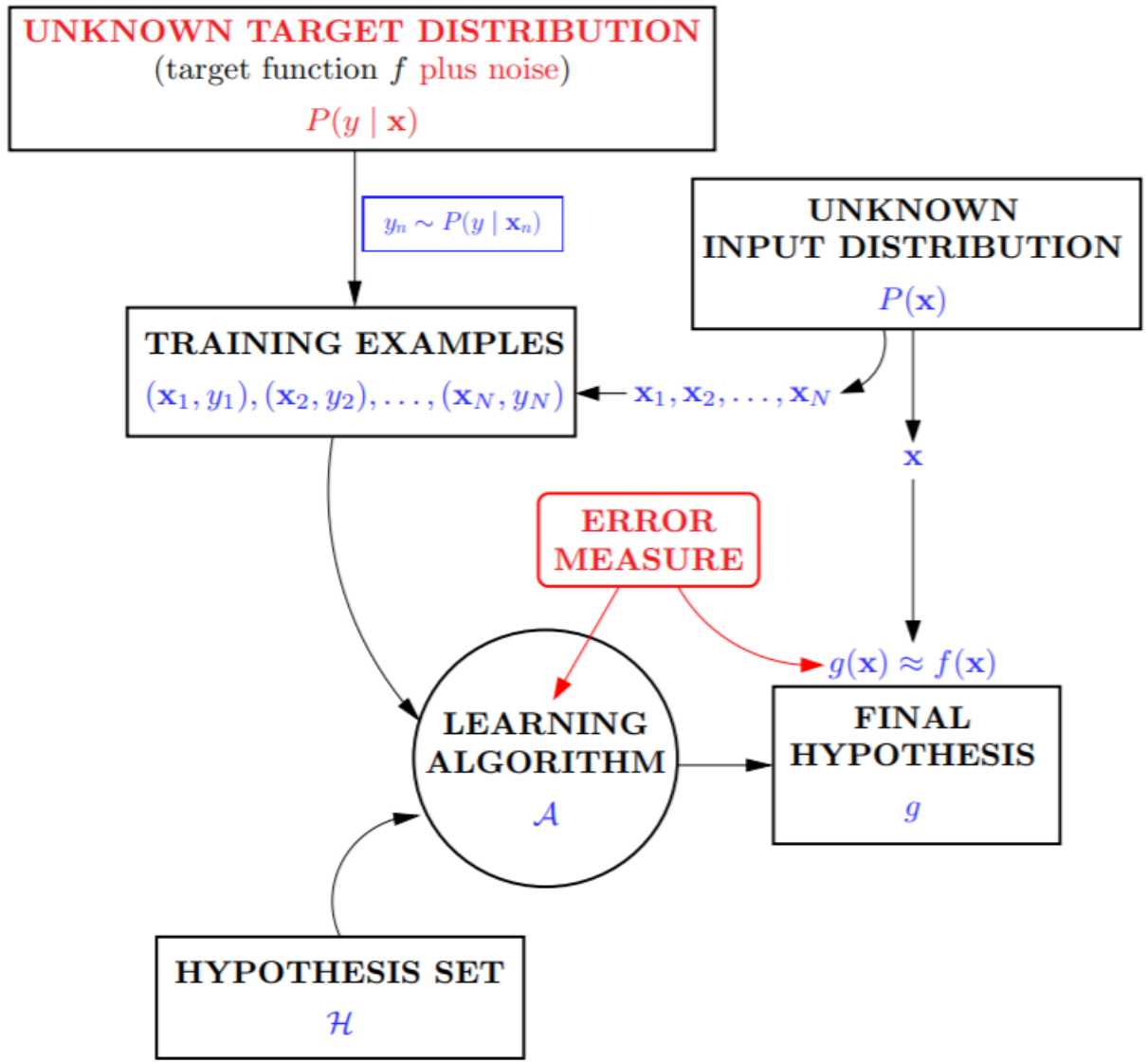
UNKNOWN TARGET DISTRIBUTION
(target function f plus noise)
$P(y \mid \mathbf{x})$
$y_n \sim P(y \mid \mathbf{x}_n)$
UNKNOWN INPUT DISTRIBUTION
$P(\mathbf{x})$
TRAINING EXAMPLES
$(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_N, y_N)$
$\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$
$\mathbf{x}$
ERROR MEASURE
$g(\mathbf{x}) \approx f(\mathbf{x})$
LEARNING ALGORITHM
$\mathcal{A}$
FINAL HYPOTHESIS
g
HYPOTHESIS SET
$\mathcal{H}$