NOTE ON TYPE THEORIES

KEISUKE HOSHINO

[Cis19]

Definition 0.1. A ***clan*** $\mathbf{C} = (\mathbf{C}, \mathsf{Fib})$ is a pair of a category $\mathbf{C}$ and a class Fib of morphisms satisfying the following conditions. Arrows in Fib are called ***fibrations*** of $\mathbf{C}$.

- Fib contains all isomorphisms.
- $\mathbf{C}$ has a terminal object.
- Let $h\colon A \longrightarrow B$ and $f\colon X \twoheadrightarrow B$ be morphisms in $\mathbf{C}$ such that f is a fibration. Then there is a pullback square

$$
\begin{array}{ccc}
\cdot & \longrightarrow & X \\
\downarrow & \lrcorner & \downarrow f \\
A & \underset{h}{\longrightarrow} & B
\end{array}
$$

  in $\mathbf{C}$ such that the left side is also a fibration.
- For each object $A \in \mathbf{C}$, the unique morphism $A \longrightarrow 1$ towards the terminal object is a fibraion.
- Fib is closed under composition. ∎

REFERENCES

[Cis19] D.-C. Cisinski. *Higher Categories and Homotopical Algebra*, volume 180 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2019. doi:10.1017/9781108588737.

Email address: hoshinok@kurims.kyoto-u.ac.jp

RESEARCH INSTITUTE OF MATHEMATICAL SCIENCE, KYOTO UNIVERSITY