NOTE ON TYPE THEORIES

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[Cis19]

Notation 0.1. We employ the following notations.

- Possibly large (2,1)-categories are denoted by bf symbols: $\mathbf{C}, \mathbf{A}, \mathbf{E}, \dots$
- Small categories are denoted by bb symbols: $\mathbb{I}, \mathbb{D}, \mathbb{A}, \dots$
- **Set** is the category of small sets.
- $\mathfrak{C}at$ is the 2-category of small categories.
- **Cat** is the (2,1)-category of small categories.
- Δ^1 is the 1-simplex seen as a category. $\mathbf{C}^{\Delta^1}$ is the arrow category of $\mathbf{C}$.
- $\mathbf{C}_{/A}$ and $\mathbf{C}_{A/}$ are over and under categories respectively.
- cod and dom means codomain and domain respectively. They often have as their type $\mathbf{C}^{\Delta^1} \longrightarrow \mathbf{C}$, $\mathbf{C}_{/A} \longrightarrow \mathbf{C}$, or $\mathbf{C}_{A/} \longrightarrow \mathbf{C}$.
- By a ***replete class of morphisms*** of $\mathbf{C}$, we mean a replete subcategory of $\mathbf{C}^{\Delta^1}$ that is a groupoid.
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■

Definition 0.2.

$$\begin{tikzcd} \mathbf{StrCat} \arrow[r] \arrow[d] & \mathbf{Cat}^{\Delta^1} \arrow[d, "\mathsf{cod}"] \\ \mathbf{Set} \arrow[r] & \mathbf{Cat} \end{tikzcd}$$

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Definition 0.3. A ***clan*** $\mathbf{C} = (\mathbf{C}, \mathsf{Fib})$ is a pair of a category $\mathbf{C}$ and a replete class Fib of morphisms satisfying the following conditions. Arrows in Fib are called ***fibrations*** of $\mathbf{C}$.

- $\mathbf{C}$ has a terminal object.
- Let $h\colon A \longrightarrow B$ and $f\colon X \twoheadrightarrow B$ be morphisms in $\mathbf{C}$ such that f is a fibration. Then there is a pullback square

$$\begin{tikzcd} \cdot \arrow[r] \arrow[d, two heads] \arrow[dr, phantom, "\lrcorner" very near start] & X \arrow[d, two heads, "f"] \\ A \arrow[r, "h"'] & B \end{tikzcd}$$

  in $\mathbf{C}$ such that the left side is also a fibration.
- For each object $A \in \mathbf{C}$, the unique morphism $A \longrightarrow 1$ towards the terminal object is a fibraion.
- Fib is closed under composition. ■

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Theorem 0.4. shoumei no aidani claim ireru yatu ♦

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Theorem 0.5. Saigo ni claim kuru taipu. ♦

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References

[Cis19] D.-C. Cisinski. *Higher Categories and Homotopical Algebra*, volume 180 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2019. doi:10.1017/9781108588737.

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