NOTE ON TYPE THEORIES

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[Cis19]

Notation 0.1. We employ the following notations.

- (2,1)-categories are denoted by bf symbols: $\mathbb{C}, \mathbb{A}, \mathbb{E}, \ldots$ or bb symbols: $\mathbb{I}, \mathbb{D}, \mathbb{A}, \ldots$
- **Set** is the category of sets.
- Cat is the large (2,1)-category of categories.
- There is a fully faithful (2,1)-functor disc: Set \hookrightarrow Cat.
- Cat is the large 2-category of categories.
- Δ^1 is the 1-simplex seen as a category. \mathbf{C}^{Δ^1} is the arrow category of \mathbf{C} .
- \bullet $\mathbf{C}_{/A}$ and $\mathbf{C}_{A/}$ are over and under categories respectively.
- cod and dom mean codomain and domain respectively. They often have as their type $\mathbf{C}^{\Delta^1} \longrightarrow \mathbf{C}$, $\mathbf{C}_{/A} \longrightarrow \mathbf{C}$, or $\mathbf{C}_{A/} \longrightarrow \mathbf{C}$.
- By a *replete class of morphisms* of C, we mean a replete full subcategory of C^{Δ^1} .
- **WSub** is the replete full sub (2,1)-category of \mathbf{Cat}^{Δ^1} consisting of wide subcategory inclusions; i.e., faithful functors that induce equivalences on core groupoids.

Definition 0.2. A *display map category* C = (C, Dis) is a pair of a category C and a replete class Dis of morphisms satisfying the following conditions. Arrows in Dis are called *display maps* of C.

- C has a terminal object.
- Let $h: A \longrightarrow B$ and $f: X \longrightarrow B$ be morphisms in ${\bf C}$ such that f is a display map. Then there is a pullback square

$$\begin{array}{ccc}
\cdot & \longrightarrow & X \\
\downarrow & & \downarrow f \\
A & \longrightarrow & B
\end{array}$$

in C such that the left side is also a display map.

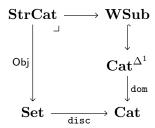
Definition 0.3. A $clan\ \mathbf{C} = (\mathbf{C}, \mathsf{Fib})$ is a display map category satisfying the following conditions. Display maps (i.e., arrows in Fib) are called *fibrations* of \mathbf{C} .

- For each object $A \in \mathbb{C}$, the unique morphism $A \longrightarrow 1$ towards the terminal object is a fibraion.
- Fib is closed under composition.

Definition 0.4. A representable map category $\mathbf{R} = (\mathbf{R}, \mathsf{Rep})$ is a display map category satisfying the following conditions. Display maps (i.e., arrows in Rep) are called representable maps of \mathbf{R} .

- **R** is finitely complete.
- For each representable map $f: X \longrightarrow Y$, the pullback functor $f^*: \mathbf{R}_{/Y} \longrightarrow \mathbf{R}_{/X}$ has a right adjoint f_* .

Definition 0.5. A *strict category* is a category equipped with its wide subcategory that is a set. In other words, the (2,1)-category **StrCat** of strict categories is defined by the following pullback square.



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The functor on the left is locally fully faithful and surjective on objects. In particular, StrCat is a category.

Definition 0.6. A *contextual category* $C = (C, Dis, \langle \rangle, \ell)$ consists of the following data.

- A strict category **C**.
- A structure Dis of a display map category for the underlying category of C.
- An object $\langle \rangle$ of **C** which is a terminal object in the underlying category of **C**.
- A function $\ell \colon \mathsf{Obj}(\mathbf{C}) \longrightarrow \mathbb{N}$ satisfying the following conditions.

 - $-\ell^{-1}(0) = \{\langle\rangle\}.$ The canonical function

$$(\Gamma,A,f)\mapsto A\colon \coprod_{\Gamma\in\ell^{-1}(n)}\{(A,f)\,|\,f\colon A \twoheadrightarrow \Gamma\}\longrightarrow \mathsf{Obj}(\mathbf{C})$$

is a monomorphism and $\ell^{-1}(n+1)$ coincides with its image.

Theorem 0.7. shoumei no aidani claim ireru yatu

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References

[Cis19] D.-C. Cisinski. Higher Categories and Homotopical Algebra, volume 180 of Cambridge Studies in Advanced Mathematics. Cambridge University Press, Cambridge, 2019. doi:10.1017/9781108588737.

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