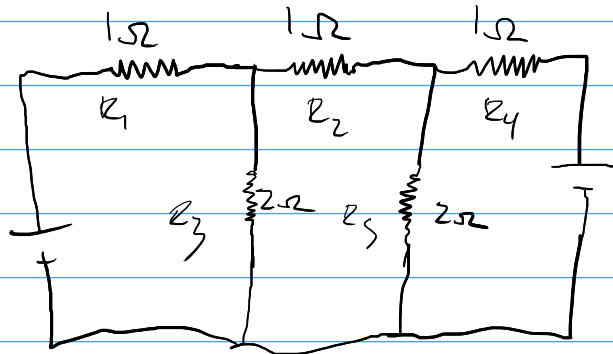


Nodal Voltage Analysis

Example: find all currents and voltage in the circuit



$$V_A - V_{R_1} + 10$$

$$V_{R_1} = V_A - 10$$

$$I_1 = \frac{V_A - 10}{1\Omega}$$

$$V_A - V_B = V_{R_2}$$

$$V_A = V_{R_2} + V_{R_4} + 20$$

$$V_B = V_{R_4} + 20$$

$$V_A - V_B = V_{R_2} + V_{R_4} + 20 - V_{R_4} - 20$$

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_A - V_B}{1\Omega}$$

$$I_3 = \frac{V_A - 0}{2\Omega}$$

$$I_1 + I_4 + I_3 = 0$$

$$\frac{V_A - 10}{1} + \frac{V_A - V_B}{1} + \frac{V_A}{2} = 0$$

At node B

$$\frac{V_B - V_A}{1} + \frac{V_B - 0}{2} + \frac{V_B - 20}{1} = 0$$

$$2 \left(\frac{V_A - 10}{1} + \frac{V_A - V_B}{1} + \frac{V_A}{2} \right) = 0$$

$$2 \left(\frac{V_A - V_A}{1} + \frac{V_B - 0}{2} + \frac{V_B - 20}{1} \right) = 0$$

$$2(V_A - 10) + 2(V_A - V_B) + V_A = 0$$

$$2(V_B - V_A) + V_B + 2(V_B - 20) = 0$$

$$2V_A - 20 + 2V_A - 2V_B + V_A = 0$$

$$5V_A - 2V_B = 20$$

$$2V_B - 2V_A + V_B + 2V_B - 40 = 0$$

$$-2V_A + 5V_B = 40$$

$$2\left(\frac{V_A - 10}{1} + \frac{V_A - V_B}{1} + \frac{V_A}{2}\right) = 0$$

$$2(V_A - 10) + 2(V_A - V_B) + V_A = 0$$

$$2\left(\frac{V_A - V_A}{1} + \frac{V_B - 0}{2} + \frac{V_B - 20}{1}\right) = 0$$

$$2(V_B - V_A) + V_B + 2(V_B - 20) = 0$$

$$10V_A - 4V_B = 40$$

$$-10V_A + 25V_B = 200$$

$$25V_B = 240$$

$$V_B = \frac{240}{21} = 11.4V$$

$$5V_A - 2(11.4) = 20$$

$$V_A = 8.56V$$

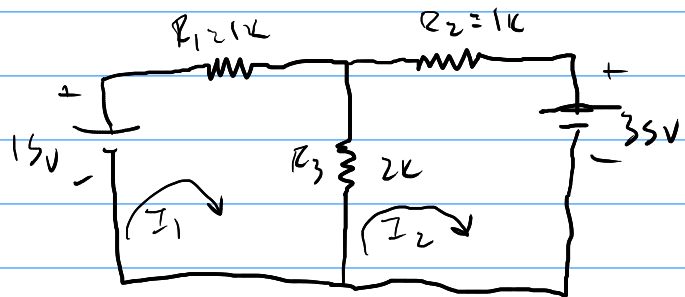
$$5V_A = 20 + 2(11.4)$$

$$I_{R1} = I_1 = \frac{V_A - 10}{R_1} = \frac{8.56 - 10}{1} = -1.44$$

$$I_{R2} = \frac{V_A - V_B}{R_2} = \frac{8.56 - 11.4}{1} = -2.84A$$

Mesh Current Method Analysis

assign a current for
each loop



$$-15 + I_1 R_1 + R_3 (I_1 - I_2) = 0$$

$$I_2 R_2 + 35 + R_3 (I_2 - I_1) = 0$$

$$-15 + (1k) I_1 + 2k (I_1 - I_2) = 0$$

$$(1k) I_2 + 35 + 2k (I_2 - I_1) = 0$$

$$3k I_1 - 2k I_2 = 15$$

$$-2k I_1 + 3k I_2 = -35$$

$$2x \left\{ \begin{array}{l} 3I_1 - 2I_2 = 15 \\ -2I_1 + 3I_2 = -35 \end{array} \right. \quad I_1, I_2 \text{ mA}$$

$$6I_1 - 4I_2 = 30$$

$$-6I_1 + 9I_2 = -105$$

$$5I_2 = -75$$

$$I_2 = \frac{-75}{5} = -15 \text{ mA}$$

$$3k I_1 - 2k (-15 \text{ mA}) = 15$$

$$3k I_1 + 30 = 15$$

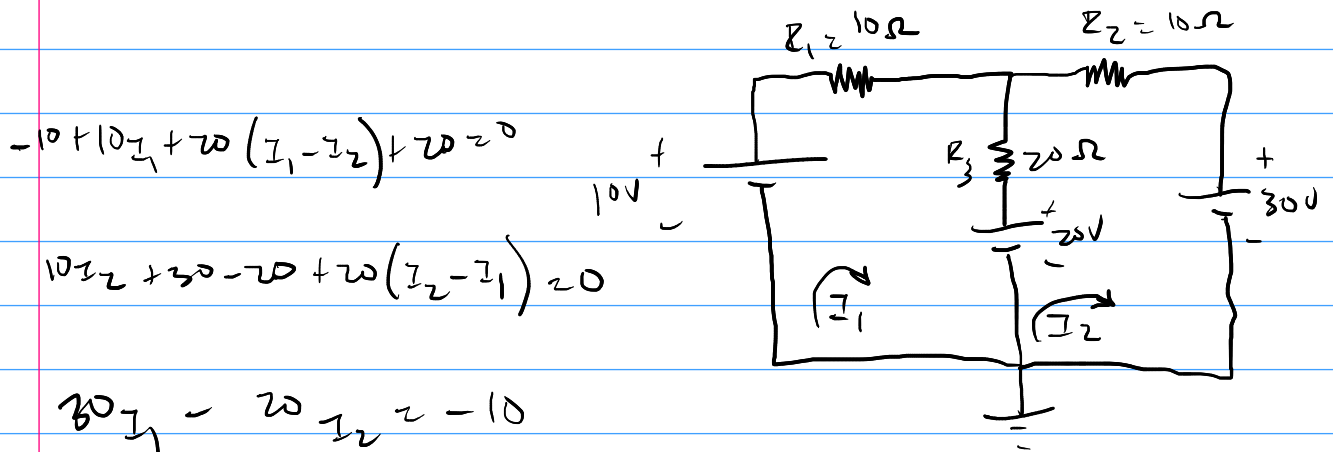
$$I_1 = \frac{-15}{3k}$$

$$= -5 \text{ mA}$$

$$\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 15 \\ -35 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 15 & -2 \\ 35 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -2 & 3 \end{vmatrix}} = \frac{(15)(3) - (-2)(35)}{(3)(3) - (-2)(-2)}$$

Example: Use Mesh Current method



$$-10 + 10I_1 + 20(I_1 - I_2) + 20 = 0$$

$$10I_2 + 30 - 20 + 20(I_2 - I_1) = 0$$

$$30I_1 - 20I_2 = -10$$

$$-20I_1 + 30I_2 = -10$$

$$3I_1 - 2I_2 = -1$$

$$-2I_1 + 3I_2 = -1$$

$$60I_1 - 210I_2 = -20$$

$$-60I_1 + 90I_2 = -30$$

$$50I_2 = -50$$

$$I_2 = \frac{-50}{50} = -1 \text{ A}$$

$$30I_1 - 20(-1) = -10$$

$$V_{R_3} = 20(I_1 - I_2) = 20(-1 - (-1)) = 0$$