Logic simplification Using Boolean Algebra

Example: Using Boolean Algebra techniques, simplify this expression:

AB + A(B + C) + B(B + C) =>

AB + AB + AC + BB + BC => Distributive Law

AB + AB + AC + B + BC => Rule 7 BB = B

AB + AC + B + BC => Rule 5 (AB+AB=AB)

AB + AC + B => Rule 10 (B+BC = B)

B + AC => Rule 10 (AB + B = B)

$$\left[A\bar{B}\left(C+BD\right) +\bar{A}\ \bar{B} \right] C$$

$$= \left(A\bar{B}C + A\bar{B}BD + \bar{A}\bar{B} \right) C$$

$$\left(A\bar{B}C + A0D + \bar{A}\bar{B} \right) C$$

$$\left(A\bar{B}C + 0 + \bar{A}\bar{B} \right) C$$

$$\left(A\bar{B}C + \bar{A}\bar{B} \right) C$$

$$= A\bar{B}CC + \bar{A}\bar{B}C$$

$$= A\bar{B}C + \bar{A}\bar{B}C$$

$$= \bar{B}C\left(A + \bar{A} \right)$$

$$= \bar{B}C \cdot 1$$

$$= \bar{B}C$$

$$\overline{\overline{AB} + \overline{AC}} + \overline{A} \cdot \overline{BC}$$

$$\overline{A(B+C)} + \overline{A} \cdot \overline{BC}$$

$$\overline{AB} \cdot \overline{AC} + \overline{A} \; \overline{BC} \qquad \text{Demorgan theorem}$$

$$\left(\overline{A} + \overline{B}\right)\left(\overline{A} + \overline{C}\right) + \overline{A} \; \overline{B} \; C$$

$$\overline{A}\overline{A} + \overline{A}\overline{C} + \overline{B}\overline{A} + \overline{B}\overline{C} + \overline{A}\overline{B}C$$

$$\overline{A} + \overline{A}\overline{C} + \overline{B}\overline{A} + \overline{B}\overline{C} + \overline{A}\overline{B}C$$

$$\overline{A} + \overline{A}\overline{C} + \overline{A}\overline{B}(1+C) + \overline{B}\overline{C}$$

$$\overline{A} + \overline{A}\overline{C} + \overline{A}\overline{B}(1) + \overline{B}\overline{C}$$

$$\overline{A} + \overline{A}\overline{C} + \overline{A}\overline{B} + \overline{B}\overline{C}$$

$$\overline{A}(1+C) + \overline{A}\;\overline{B} + \overline{B}\overline{C}$$

$$\overline{A} + \overline{A}\;\overline{B} + \overline{B}\overline{C}$$

$$\overline{A}(1+\overline{B}) + \overline{B}\overline{C}$$

$$\overline{A} + \overline{B}\overline{C}$$

Standard Forms of Boolean Expressions

Sum of Products (SOP) Form: When two or more product terms are summed by boolean addition, the resulting expression is a sum of products (SOP).

AB + ABC
AB + CDE + $\overline{B}\overline{C}\overline{D}$

Domain of a Boolean Expression: The domain of a general Boolean expression is the set of variables contained in the expression in either complemented or uncomplemented form.

AB + ABC

$AB\overline{C} + \overline{C}\overline{D}E + \overline{B}\overline{C}\overline{D}$

Conversion of a General Expression to SOP form:

Any logic expression can be changed into SOP form by applying Boolean algebra techniques.

Example: Convert each of the following Boolean expressions to SOP form:

a.) AB + B(CD + EF)
b.) (A+B)(B+C+D)

a: AB + B(CD + EF)
AB + BCD + BEF

b: (A + B)(B + C + D)
AB + AC + AD + BB + BC + BD