

11. Sketch an angle  $\theta$  in standard position such that  $\theta$  has the least positive measure, and the given point is on the terminal side of  $\theta$ . Then find the values of the six trigonometric functions for each angle. Rationalize denominators when applicable.

$$(5, -12)$$

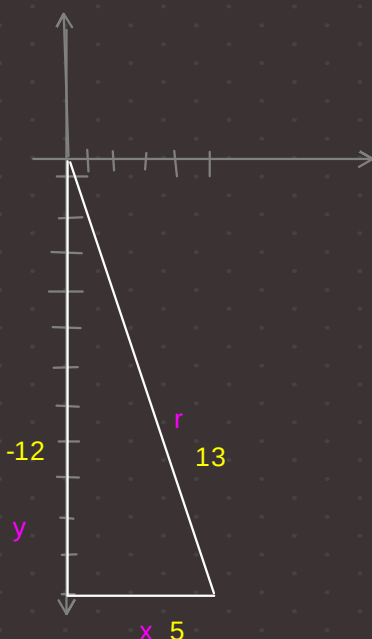
$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{5^2 + 12^2}$$

$$r = \sqrt{25 + 144}$$

$$r = \sqrt{169}$$

$$r = 13$$



$$\sin \theta = \frac{y}{r} = \frac{-12}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{5}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{-12}{5}$$

$$\csc \theta = \frac{r}{y} = \frac{13}{-12}$$

$$\sec \theta = \frac{r}{x} = \frac{13}{5}$$

$$\cot \theta = \frac{x}{y} = \frac{5}{-12}$$

12.

$$(-12, -5)$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-12)^2 + (-5)^2}$$

$$r = \sqrt{144 + 25}$$

$$r = \sqrt{169}$$

$$r = 13$$

$$\sin \theta = y/r = -5/13$$

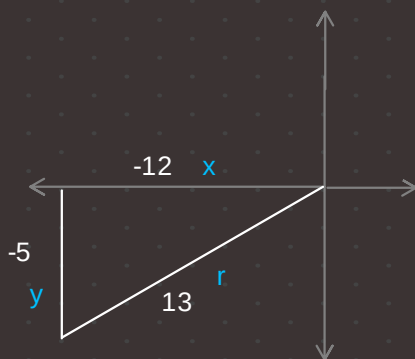
$$\cos \theta = x/r = -12/13$$

$$\tan \theta = y/x = -5/-12 = 5/12$$

$$\csc \theta = r/y = 13/-5$$

$$\sec \theta = r/x = 13/-12$$

$$\cot \theta = x/y = -12/-5 = 12/5$$



19.

$$(0, 2)$$



$$x = 0$$

$$y = 2$$

$$r = 2$$

$$\sin \theta = y/r = 2/2 = 1$$

$$\cos \theta = x/r = 0/2 = 0$$

$$\tan \theta = y/x = 2/0 = \text{undef}$$

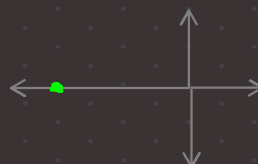
$$\csc \theta = r/y = 2/2 = 1$$

$$\sec \theta = r/x = 2/0 = \text{undef}$$

$$\cot \theta = x/y = 0/2 = 0$$

21.

$$(-4, 0)$$



$$x = -4$$

$$y = 0$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-4)^2 + 0^2}$$

$$r = \sqrt{16 + 0}$$

$$r = \sqrt{16}$$

$$r = 4$$

$$\sin \theta = y/r = 0/4 = 0$$

$$\cos \theta = x/r = -4/4 = -1$$

$$\tan \theta = y/x = 0/-4 = 0$$

$$\csc \theta = r/y = 4/0 = \text{undef}$$

$$\sec \theta = r/x = 4/-4 = -1$$

$$\cot \theta = x/y = -4/0 = \text{undef}$$

13.

$$(3, 4)$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{3^2 + 4^2}$$

$$r = \sqrt{9 + 16}$$

$$r = \sqrt{25}$$

$$r = 5$$

$$\sin \theta = y/r = 4/5$$

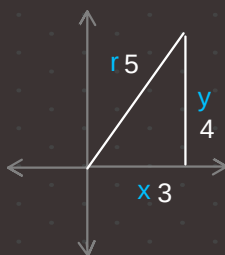
$$\cos \theta = x/r = 3/5$$

$$\tan \theta = y/x = 4/3$$

$$\csc \theta = r/y = 5/4$$

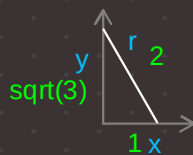
$$\sec \theta = r/x = 5/3$$

$$\cot \theta = x/y = 3/4$$



25.

(1,  $\sqrt{3}$ )



$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ r &= \sqrt{1^2 + (\sqrt{3})^2} \\ r &= \sqrt{1+3} \\ r &= \sqrt{4} \\ r &= 2 \end{aligned}$$

$$\begin{aligned} \sin \theta &= y/r = \sqrt{3}/2 \\ \cos \theta &= x/r = 1/2 \\ \tan \theta &= y/x = \sqrt{3}/1 = \sqrt{3} \\ \csc \theta &= r/y = 2/\sqrt{3} \\ \sec \theta &= r/x = 2/1 = 2 \\ \cot \theta &= x/y = 1/\sqrt{3} \end{aligned}$$

31. Suppose that the point (x, y) is in the indicated quadrant. Determine whether the given ratio is positive or negative. Recall that  $r = \sqrt{x^2 + y^2}$ .

Quadrant II,  $x/r$ , negative

32.

Quadrant III,  $y/r$ , negative

33.

Quadrant IV,  $y/x$ , negative

34.

Quadrant IV,  $x/y$ , negative

35.

Quadrant II,  $y/r$ , positive

36.

Quadrant III,  $x/r$ , negative

37.

Quadrant IV,  $x/r$ , positive

38.

Quadrant IV,  $y/r$ , negative

39.

Quadrant II,  $x/y$ , negative

40.

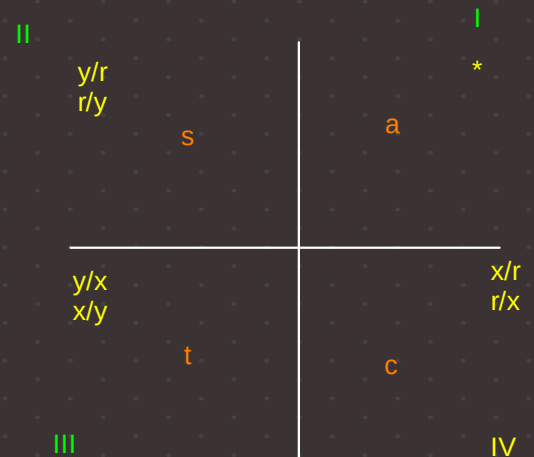
Quadrant II,  $y/x$ , negative

41.

Quadrant III,  $y/x$ , positive

42.

Quadrant III,  $x/y$ , positive



51.

An equation of the terminal side of an angle  $\theta$  in standard position is given with a restriction on x. Sketch the least positive such angle  $\theta$ , and find the values of the six trigonometric functions of  $\theta$ .

$$2x + y = 0, x \geq 0$$

$$-2x \quad -2x$$

$$y = 2x$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ r &= \sqrt{2^2 + (-4)^2} \\ r &= \sqrt{4+16} \\ r &= \sqrt{20} \end{aligned}$$

$$\begin{aligned} \sin \theta &= y/r = -4/\sqrt{20} \\ \cos \theta &= x/r = 2/\sqrt{20} \\ \tan \theta &= y/x = -4/2 = -2/1 = -2 \\ \csc \theta &= r/y = \sqrt{20}/(-4) \\ \sec \theta &= r/x = \sqrt{20}/2 \\ \cot \theta &= x/y = 2/(-4) = -1/2 \end{aligned}$$

