

- 11.) Sketch an angle θ in standard position such that θ has the least positive measure, and the given point is on the terminal side of θ . Then find the values of the six trigonometric functions for each angle. Rationalize denominators when applicable.

$(5, -12)$

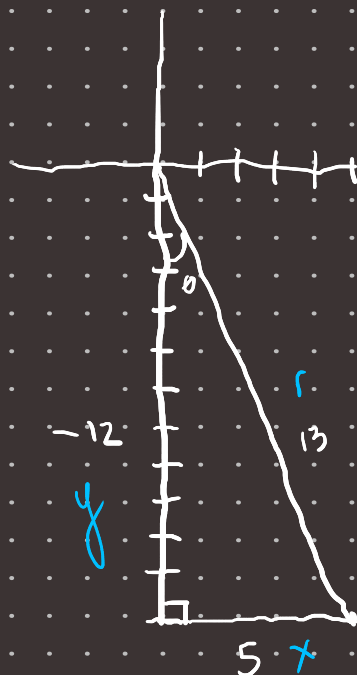
$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{5^2 + (-12)^2}$$

$$r = \sqrt{25 + 144}$$

$$r = \sqrt{169}$$

$$r = 13$$



$$\sin \theta = \frac{y}{r} = \frac{-12}{13}$$

$$\csc \theta = \frac{r}{y} = \frac{13}{-12}$$

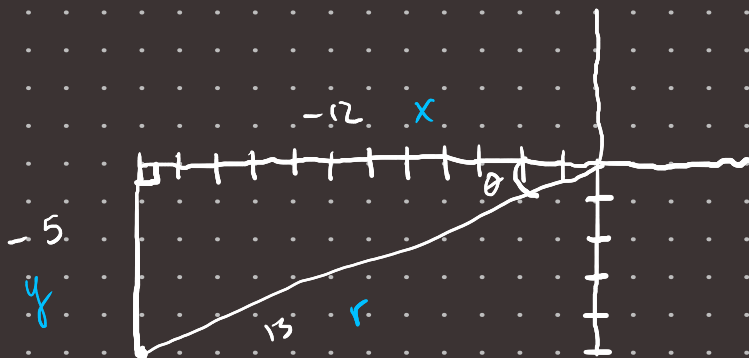
$$\cos \theta = \frac{x}{r} = \frac{5}{13}$$

$$\sec \theta = \frac{r}{x} = \frac{13}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-12}{5}$$

$$\cot \theta = \frac{x}{y} = \frac{5}{-12}$$

12.) $(-12, -5)$



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-12)^2 + (-5)^2}$$

$$r = \sqrt{144 + 25}$$

$$r = \sqrt{169}$$

$$r = 13$$

$$\sin \theta = \frac{y}{r} = \frac{-5}{13}$$

$$\csc \theta = \frac{r}{y} = \frac{13}{-5}$$

$$\cos \theta = \frac{x}{r} = \frac{-12}{13}$$

$$\sec \theta = \frac{r}{x} = \frac{13}{-12}$$

$$\tan \theta = \frac{y}{x} = \frac{-5}{-12}$$

$$\cot \theta = \frac{x}{y} = \frac{-12}{-5}$$

$$\frac{5}{12} = \swarrow$$

$$\frac{12}{5} = \swarrow$$

13.) (3, 4)

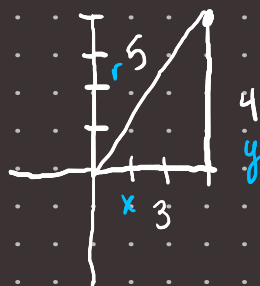
$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{3^2 + 4^2}$$

$$r = \sqrt{9 + 16}$$

$$r = \sqrt{25}$$

$$r = 5$$



$$\sin \theta = \frac{y}{r} = \frac{4}{5}$$

$$\csc \theta = \frac{r}{y} = \frac{5}{4}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{5}$$

$$\sec \theta = \frac{r}{x} = \frac{5}{3}$$

$$\tan \theta = \frac{y}{x} = \frac{4}{3}$$

$$\cot \theta = \frac{x}{y} = \frac{3}{4}$$

19.) (0, 2)



$$x = 0$$

$$y = 2$$

$$r = 2$$

$$\sin \theta = \frac{y}{r} = \frac{2}{2} = 1$$

$$\csc \theta = \frac{r}{y} = \frac{2}{2} = 1$$

$$\cos \theta = \frac{x}{r} = \frac{0}{2} = 0$$

$$\sec \theta = \frac{r}{x} = \frac{2}{0} = \text{undefined}$$

$$\tan \theta = \frac{y}{x} = \frac{2}{0} = \text{undefined}$$

$$\cot \theta = \frac{x}{y} = \frac{0}{2} = 0$$

21.) (-4, 0)



$$x = -4$$

$$y = 0$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-4)^2 + 0^2}$$

$$r = \sqrt{16 + 0}$$

$$r = \sqrt{16}$$

$$r = 4$$

$$\sin \theta = \frac{y}{r} = \frac{0}{4} = 0$$

$$\csc \theta = \frac{r}{y} = \frac{4}{0} = \text{undefined}$$

$$\cos \theta = \frac{x}{r} = \frac{-4}{4} = -1$$

$$\sec \theta = \frac{r}{x} = \frac{4}{-4} = -1$$

$$\tan \theta = \frac{y}{x} = \frac{0}{-4} = 0$$

$$\cot \theta = \frac{x}{y} = \frac{-4}{0} = \text{undefined}$$

25.) $(1, \sqrt{3})$



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{1^2 + (\sqrt{3})^2}$$

$$r = \sqrt{1 + 3}$$

$$r = \sqrt{4}$$

$$r = 2$$

$$\sin \theta = \frac{y}{r} = \frac{\sqrt{3}}{2} \quad \csc \theta = \frac{r}{y} = \frac{2}{\sqrt{3}}$$

$$\cos \theta = \frac{x}{r} = \frac{1}{2} \quad \sec \theta = \frac{r}{x} = \frac{2}{1} = 2$$

$$\tan \theta = \frac{y}{x} = \frac{\sqrt{3}}{1} = \sqrt{3} \quad \cot \theta = \frac{x}{y} = \frac{1}{\sqrt{3}}$$

31.) Suppose that the point (x, y) is in the indicated quadrant. Determine whether the given ratio is positive or negative. Recall that $r = \sqrt{x^2 + y^2}$.

Quadrant II, $\frac{x}{r}$, Negative

32.) Quadrant III, $\frac{y}{r}$, Negative

33.) Quadrant IV, $\frac{y}{x}$, Negative

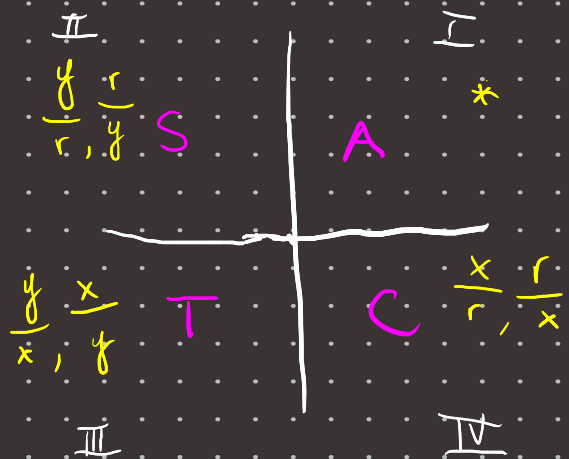
34.) Quadrant II, $\frac{x}{y}$, Negative

35.) Quadrant II, $\frac{y}{r}$, Positive

36.) Quadrant III, $\frac{x}{r}$, Negative

37.) Quadrant IV, $\frac{x}{r}$, Positive

38.) Quadrant II, $\frac{y}{r}$, Negative



39.) Quadrant II, $\frac{x}{y}$, Negative

40.) Quadrant II, $\frac{y}{x}$, Negative

41.) Quadrant III, $\frac{y}{x}$, Positive

42.) Quadrant III, $\frac{x}{y}$, Positive

51.)

An equation of the terminal side of an angle θ in standard position is given with a restriction on x . Sketch the least positive such angle θ , and find the values of the six trigonometric functions of θ .

$$2x + y = 0, \quad x \geq 0$$

$$-2x$$

$$-2x$$

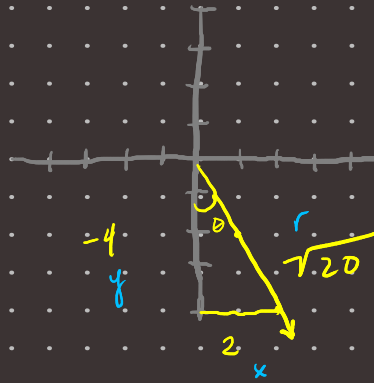
$$y = -2x$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{2^2 + (-4)^2}$$

$$r = \sqrt{4 + 16}$$

$$r = \sqrt{20}$$



$$\sin \theta = \frac{y}{r} = \frac{-4}{\sqrt{20}}$$

$$\csc \theta = \frac{r}{y} = \frac{\sqrt{20}}{-4}$$

$$\cos \theta = \frac{x}{r} = \frac{2}{\sqrt{20}}$$

$$\sec \theta = \frac{r}{x} = \frac{\sqrt{20}}{2}$$

$$\tan \theta = \frac{y}{x} = \frac{-4}{2} = -2 \quad \cot \theta = \frac{x}{y} = \frac{2}{-4} = -\frac{1}{2}$$

66.)

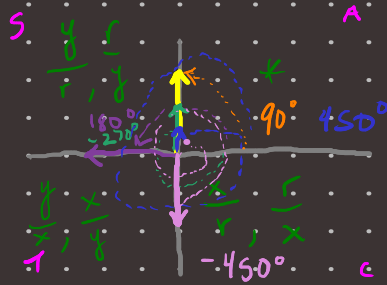
Find the indicated function value. If it is undefined, say so.

$$\cot 90^\circ$$

$$x = 0$$

$$y = 1$$

$$\cot 90^\circ = \frac{x}{y} = \frac{0}{1} = 0$$



$$x = 0$$

$$y = 1$$

$$x = 0$$

$$y = 1$$

$$x = -1$$

$$y = 0$$

$$x = 0$$

$$y = 1$$

$$x = 0$$

$$y = -1$$

67.) $\sec 180^\circ$

$$x = -1$$

$$y = 0$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-1)^2 + 0^2}$$

$$r = \sqrt{1 + 0}$$

$$r = \sqrt{1}$$

$$r = 1$$

$$\sec 180^\circ = \frac{r}{x} = \frac{1}{-1} = -1$$

69.) $\sin(-270^\circ)$

$$x = 0$$

$$y = 1$$

$$\sin(-270^\circ) = \frac{y}{r} = \frac{1}{1} = 1$$

72.) $\tan 450^\circ$
 $x=0$
 $y=-1$
 $\tan 450^\circ = \frac{y}{x} = \frac{-1}{0} = \text{undefined}$

73.) $\csc(-450^\circ)$
 $x=0$
 $y=-1$
 $\csc(-450^\circ) = \frac{r}{y} = \frac{1}{-1} = -1$

79.) $\sec 1800^\circ$
 $x=1$
 $y=0$
 $\sec 1800^\circ = \frac{x}{y} = \frac{1}{0} = \text{undefined}$