 tan(number • pi / 180)

Example: Use your calculator to compute the following (Make sure that your calculator is in degree mode!!!!)

a. $\tan(68°43')$

2.56

b. $\cos(193.622°)$

-0.97

c. $\csc(35.8471°)$ $\frac{1}{\sin(35.8471°)}$

1.70

d. $\sec(-287°)$ $\frac{1}{\cos(-287°)}$

3.44

Example: Use your calculator to compute the following (Make sure that you calculator is in degree mode!!!!)

a. $sin^{-1}(.9677091705)$

~75.4

b. $cos^{-1}(.9677091705)$

~14.6

c. $tan^{-1}(.9677091705)$

~44.06

Example: Use the calculator to find an acute angle that satisfies:

a. $\cos\theta = .9211854056$

$\sim 22.9°$

b. $\cot\theta = 1.4466474$ $\tan^{-1}\left(\frac{1}{1.4466474}\right)$

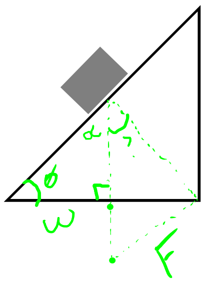
$\sim 34.65°$

Example: Use a calculator to evaluate $\cos 100° \sin 80° - \cos 80° \sin 100°$.

~ -0.342

Grade Resistance is given by $F = W \sin\theta$ where W is the weight of the object and θ is the angle of the incline.

You must be wondering from where that formula comes?



$\varnothing + \alpha = 90^\circ$

$\alpha + ? = 90^\circ$

$\varnothing + \alpha = \alpha + ?$

$\varnothing = ?$

$2\varnothing = 90^\circ$

$\frac{90}{2} = \varnothing$

$\varnothing = 45^\circ$

$\sin\varnothing = \frac{O}{H}$

$\sin\varnothing = \frac{F}{W} \Rightarrow W(\sin\varnothing) = F$

Example:

a. Find F for a 5500-lb. car traveling an uphill grade with $\theta = 3.9°$.

$F = W\sin\varnothing$

$= 5{,}500(\sin 3.9^\circ)$

≈ 373.9 lbs

grade = slope
= Tangent value
NOT
Angle

b. Find F for a 2800-lb. car traveling a downhill grade with $\theta = -4.8°$.

$F = W\sin\varnothing$

$= 2{,}800(\sin -4.8)$

≈ -234.298

c. A 2400 lb. car traveling uphill has a grade resistance of 288 lbs. What is the angle of the grade?

$F = W\sin\varnothing$

$288 = 2400\sin\varnothing$

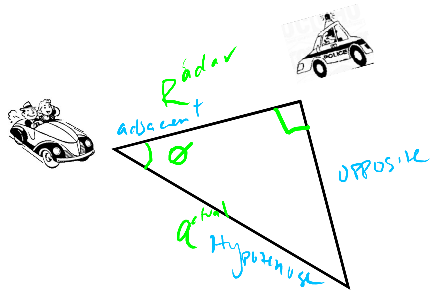
$\frac{288}{2400} = \sin^{-1}(\sin\varnothing)$

$= 0.12 = \sin^{-1}(\sin\varnothing)$

$\sin\varnothing = \sim 6.89^\circ$

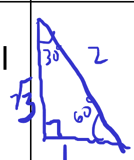
$radar\ speed = (actual\ speed)\cos\theta$, where θ is the angle between radar gun and the vehicle.

You must be wondering from where that formula comes?

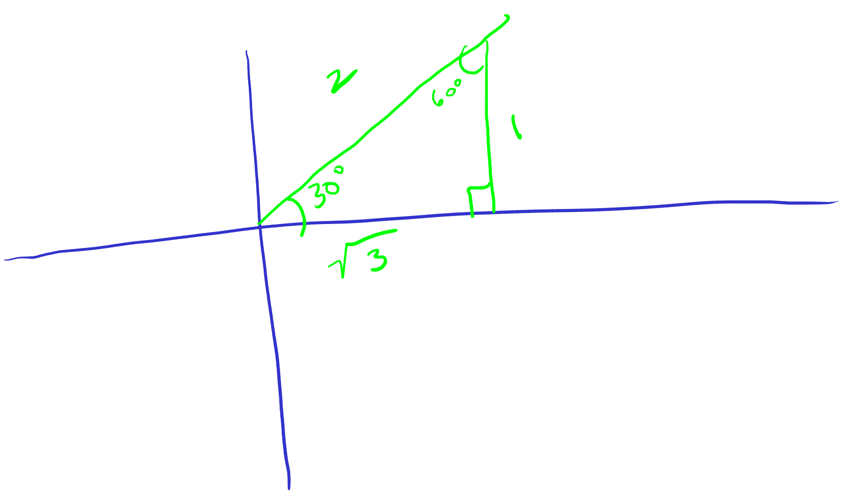


$\cos\phi = \frac{a}{h}$
$= \frac{R}{a}$
$a\cos\phi = r$

Quiz Prep:

Question	Answer
1. Draw a $30° - 60° - 90°$ triangle. Label angle measures, label side lengths.	 $1<\sqrt{3}<2$ $30°<60°<90°$
2. Draw a $45° - 45° - 90°$ triangle. Label angle measures, label side lengths.	

Find the equation of the line that makes a $30°$ angle with the x-axis. A $60°$ angle? A $225°$ angle? Any other (standard) angle?



$y = mx + b$

$b = 0$

$m(x) = \frac{1}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}x$

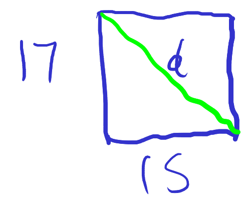
Accuracy

Measurement to the nearest foot is 15 $14.5 \leq$ actual measurement < 15.5

Measurement to the nearest tenth of a foot is 17.2 $17.15 \leq$ actual measurement < 17.25

The results of a calculation can be no more accurate than the least accurate number in the calculation.

So one would estimate the diagonal of a rectangular room that measures 15 ft. by 17 ft. as ______ft. See the computations below.



$$d^2 = 15^2 + 17^2$$
$$d^2 = 514$$
$$d = \sqrt{514} \approx 22.6715681$$

Significant digits are the digits obtained by actual measurement.
Note that 0 is not significant when used only to locate the decimal place.

For .0025 0 is not significant

For 2500 0 is assumed to not be significant

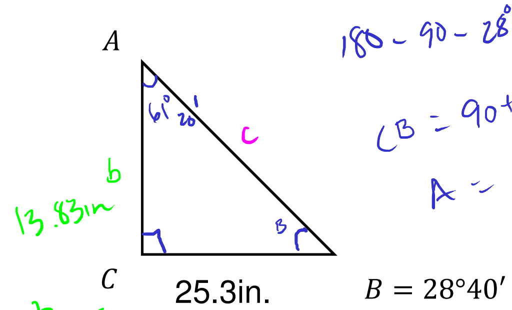
For .2500 0 is significant

$4.08 \cdot 10^3$

Example	Number of Significant Digits
408	3
6.700	4
.0025	2
7300	2
73°	2
73.5°	3
73°30'	3
73°15'	4
73.25°	4

To solve a triangle means to find the measures of each of the interior angles and each of the sides.

Example: Solve the triangle below.



$180 - 90 - 28°40' = A$

$CB = 90 + 28°40' = 118°40'$

$A = 180 - 118°40'$

$A = 61°20'$

$\tan B = \frac{b}{25.3}$

$\tan 28°40' = \frac{b}{25.3}$

$25.3(\tan 28°40') = b$

$b = \sim 13.83\text{ in}$

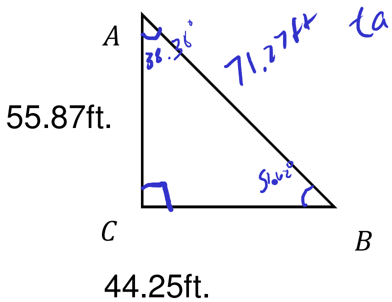
$\cos 28°40' = \frac{25.3}{c}$

$c(\cos 28°40') = 25.3$

$c = \frac{25.3}{\cos 28°40'}$

$c = \sim 28.8\text{ in}$

Example: Solve the triangle below.



$\tan A = \frac{44.25\text{ft}}{55.87\text{ft}}$

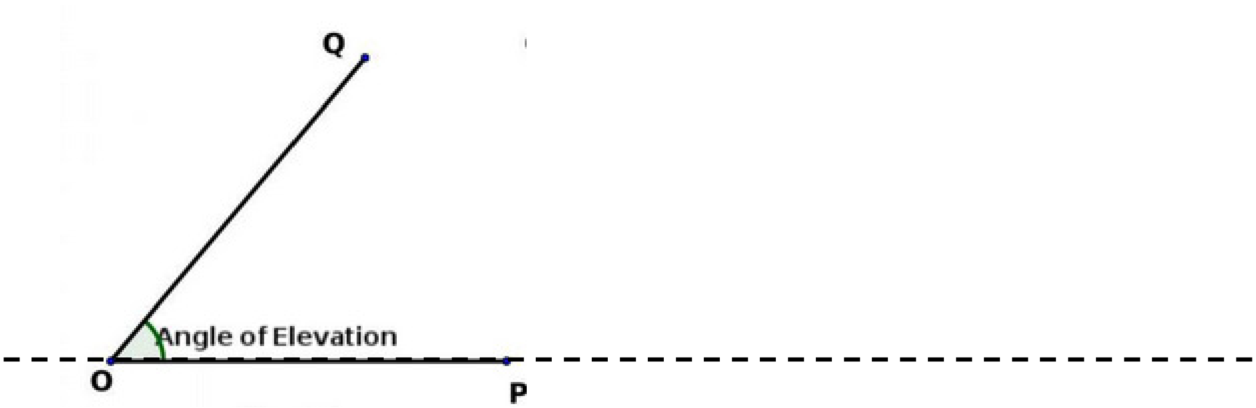
$\tan^{-1}\left(\frac{44.25}{55.87}\right) = A = \sim 38.38°$

$90° - 38.38° = 51.62°$

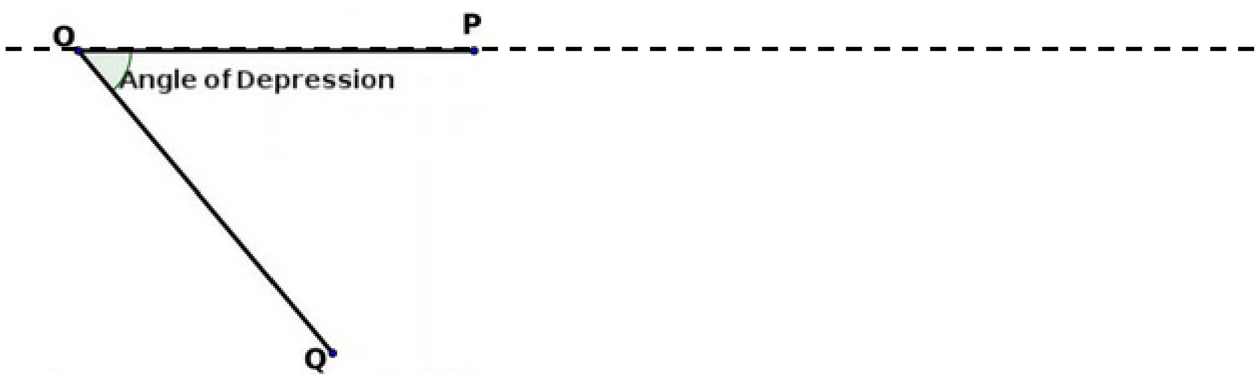
$a^2 + b^2 = c^2$

$\sqrt{44.25^2 + 55.87^2} = c = 71.27\text{ft}$

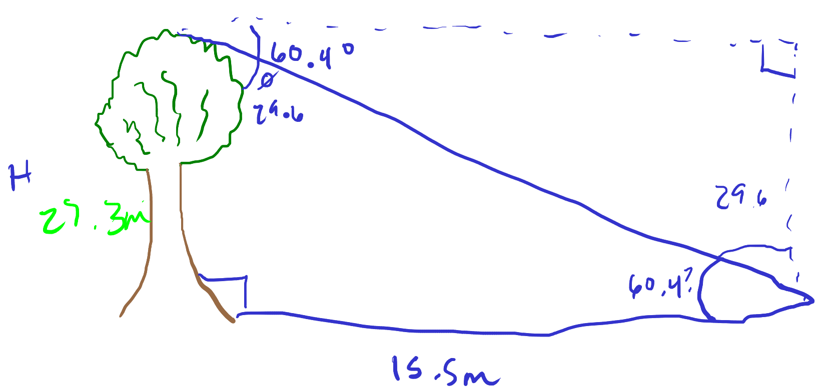
The Angle of Elevation is the angle formed by the ray $\overrightarrow{OQ}$ and a horizontal ray with end point O.



The Angle of Depression is the angle formed by the ray $\overrightarrow{OQ}$ and a horizontal ray with end point O.



Example: The angle of depression from the top of a tree to a point on the ground 15.5 m. from the base of the tree is $60.4°$. Find the height of the tree.



$60.4 + \emptyset = 90$

$? + \emptyset = 90°$

$? = 60.4°$

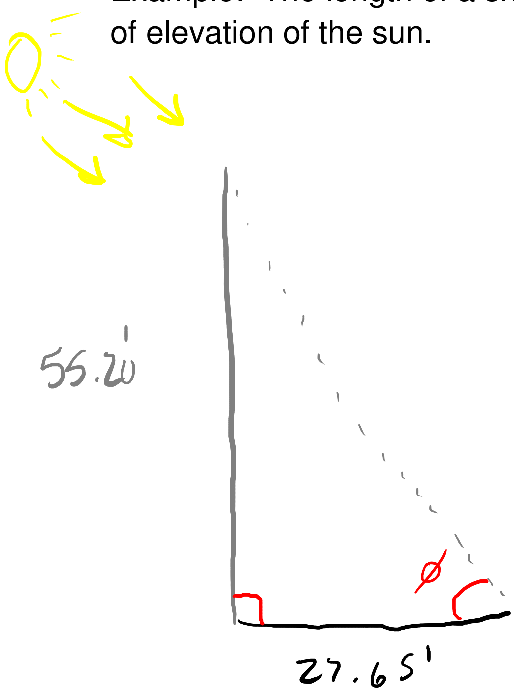
$\emptyset = 29.6°$

$\tan 60.4 = \frac{H}{15.5}$

$H = 15.5(\tan 60.4 \cdot \pi/180) =$

$27.3m$

Example: The length of a shadow of a flagpole that is 55.20 ft. tall is 27.65 ft. Find the angle of elevation of the sun.



$$\tan \emptyset = \frac{55.20'}{27.65}$$

$$\tan^{-1} \tan \emptyset = \tan^{-1}\left(\frac{55.20}{27.65}\right)$$

$$\emptyset = 63.39^\circ$$

Navigation Method 1: Given one angle θ, θ is measured in clockwise direction from due North.

Example: Illustrate

a. A bearing of $53°$

b. A bearing of $323°$

Navigation Method 2: A north or south direction is indicated along with an acute angle and an east or west direction.

Example: Illustrate

a. A bearing of N$64°$W

b. A bearing of N$26°$E

c. A bearing of S$82°$W

Example: Consider an observer standing at the origin of a coordinate system. Find the bearing of a boat located at each of the following points.

a. (3,0)

b. (0,-2)

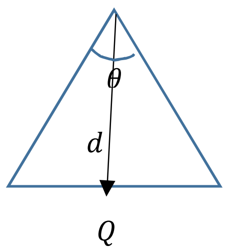
c. (2,-2)

Example Radar stations A and B are on an east-west line 8.6 km apart. Station A detects a plane at C on a bearing of $53°$. Station B simultaneously detects the same plane on a bearing of 323°. Find the distance from B to C.

Example The bearing from A to C is N64°W. The bearing from A to B is S82°W. The bearing from B to C is N26°E. A plane flying 350 mph takes 1.8 hours to go from A to B. Find the distance from B to C.

Subtense Bar Method

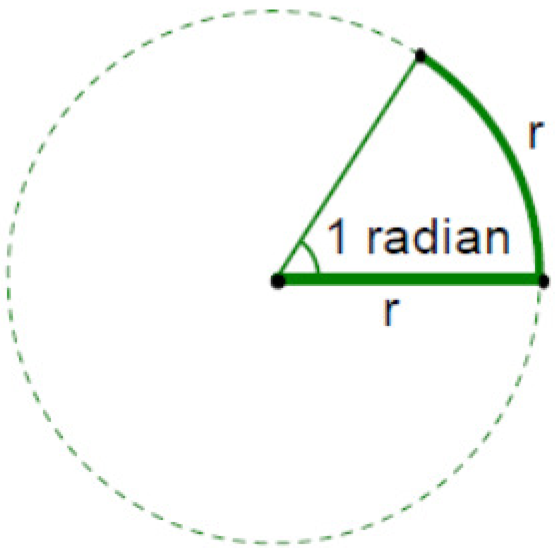
A method used by surveyors to determine a small distance between two points P and Q. The subtense bar of length b is centered at Q and situated perpendicular to the line of sight between P and Q. Find



a. d when θ= $2°41'38"$ and $b = 3.5000$ cm

b. d when θ= $2°41'39"$ and $b = 3.5000$ cm

Radian Measure: An angle with its vertex at the center of a circle that intercepts an arc on the circle equal in length to the radius of the circle has a measure of 1 radian.



- as one sweeps out an arc equal in length to the radius,
  one sweeps out an angle that has measure 1 radian

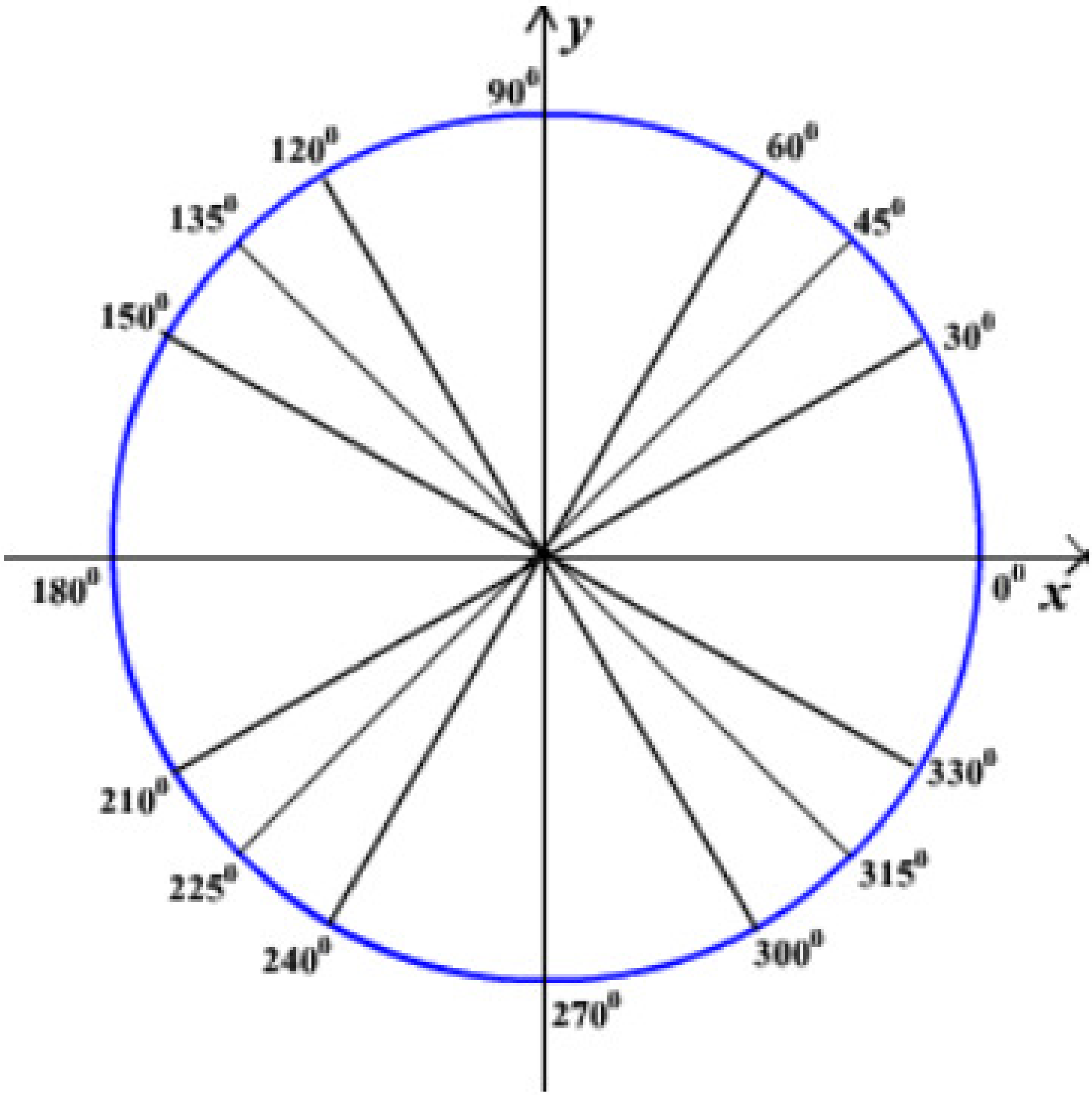
- as one sweeps out $360°$,
  one sweeps out 2π arcs each of length equal to the radius, thus
  one sweeps out 2π angles of measure 1 radian

So $360° = 2\pi$ radians

$180° = \pi$ radians

$\frac{180°}{\pi\ radians} = 1$ and $1 = \frac{\pi\ radians}{180°}$

Identify the radian measure of each of the angles:



Example: When converting degree measure to radian measure which ratio should be used?
$\frac{180°}{\pi\ radians} = 1$ or $1 = \frac{\pi\ radians}{180°}$?

Convert the following angles in degrees to radians:

a. $108°$

b. $-135°$

c. $325.7°$

Example: When converting radian measure to degree measure, which ratio should be used?
$\frac{180°}{\pi\ radians} = 1$ or $1 = \frac{\pi\ radians}{180°}$?

a. $\frac{11\pi}{12}$ radians

b. $\frac{-7\pi}{6}$ radians

c. -2.92 radians

Note that if the unit of the angle isn't given, one should assume that the angle is given in radians.

Example: Find each value

a. $\cos\frac{5\pi}{6}$

b. $\cot\frac{3\pi}{4}$

c. $\sec\frac{5\pi}{3}$

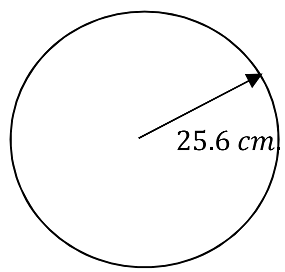
Arclength: The length of an arc that corresponds to an angle θ
Geometrical Fact: The length of an arc is proportional to the measure of its central angle.

$$\frac{r}{1\ radian} = \frac{s}{\theta\ in\ radians}$$

$$s = r\theta$$

Where θ is in radians and the arclength s will be in the units of r.

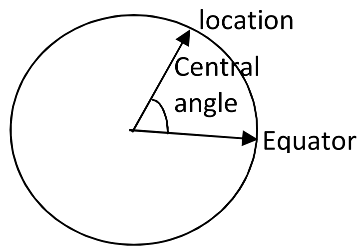
Example: For the circle below, find the length of the arc intercepted by a central angle with measure



a. $\frac{7\pi}{8}$ radians

b. $54°$

Latitude- the latitude of a location gives the measure of a central angle through the equator (the initial side starts at the center of the earth and goes through the equator) whose terminal side goes through the location. (See illustration below.)

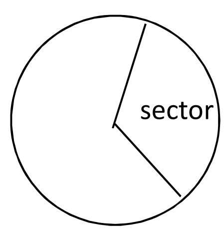


Example: Erie, Pennsylvania is due north of Columbia, South Carolina. The latitude of Erie is $42°$ north, while that of Columbia is $34°$ north. Find the north-south distance between the two cities.

Example: A rope is being wound around a drum with radius .327 m. How much rope will be wound around the drum if the drum is rotated through an angle of $132.6°$?

Example: Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of $150°$, through how many degrees will the larger gear rotate? The larger gear has radius 5.4 inches, and the smaller has radius 3.6 inches.

Sector of a circle- A sector of a circle is the portion of the interior of a circle intercepted by a central angle. See illustration below. (One can think of a sector of a circle as a piece of the pie.)



Area of a sector of a circle- is $\frac{1}{2}r^2\theta$ where r is the length of the radius and θ is the central angle.

But where did that formula come from you ask? Good question.

Geometrical Fact: The area of the sector of a circle is proportional to the measure of the corresponding central angle. That means that the ratio $\frac{area\ of\ a\ sector\ of\ a\ circle}{measure\ of\ the\ central\ angle}$ is constant.

So for any sector of a circle

- With radius r
- With central angle we will call θ

$$\frac{area(sector)}{\theta} = \frac{\pi r^2}{2\pi}$$

$$area(sector) = \frac{1}{2}r^2\theta$$

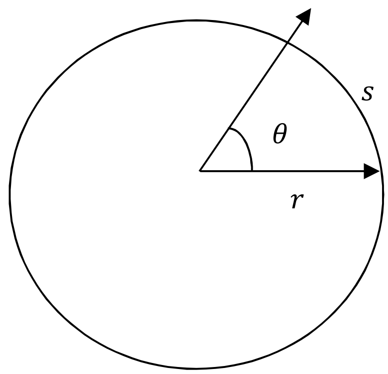
Example: On the Model A Ford, the windshield wiper arm is 10 inches long, the actual blade is 7 inches long, and the arm sweeps out a $95°$ angle. What is the area of the portion of the windshield that the wiper cleans?

Example: If the area of the sector of a circle is 217.8 ft^2 and its radius is 15.20 ft., then what is the central angle?

Recall: The length of an arc that corresponds to a central angle θ is

$$s = r\theta$$

Where θ is in radians and the arclength, s, will be in the units of the radius, r.



Question: On the unit circle, what is the length of the radius? Or another way to pose the same question is, what is the value of r?

Since

- **it doesn't matter which point we pick on the terminal side of an angle to evaluate the trigonometric functions, and**

- **on the unit circle $r = 1$**

we can define the trigonometric functions for the radian measures of an angle as follows:

Definition: For an angle in standard position with
- radian measure s
- point (x, y) as the point of intersection of the terminal side and the unit circle

$\sin(s) = sin\theta = \frac{y}{r} = \frac{y}{1} = y$

$\cos(s) = cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\tan(s) = tan\theta = \frac{y}{x}$

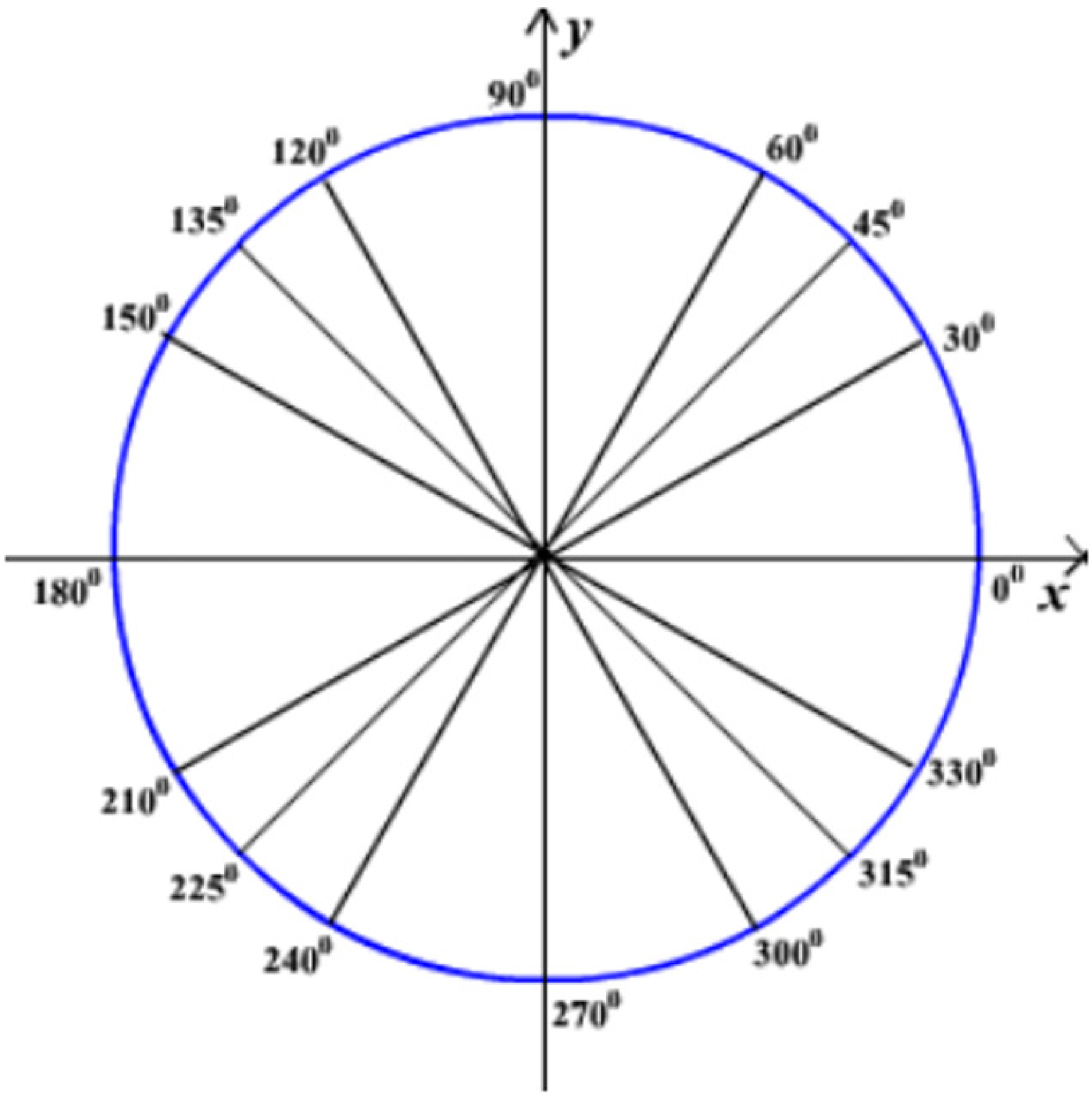
$\csc(s) = csc\theta = \frac{r}{y} = \frac{1}{y}$

$\sec(s) = sec\theta = \frac{r}{x} = \frac{1}{x}$

$\cot(s) = cot\theta = \frac{x}{y}$

Assume that the given circle is the unit circle. That means that the radius is 1.

(1) Identify the radian measure of each of the angles.
(2) Label the x, y- coordinates of the point of intersection of the terminal side of each angle with the unit circle.



Example: Find the trigonometric function values for 0 radians.

$\sin(0) =$ $\csc(0) =$

$\cos(0) =$ $\sec(0) =$

$\tan(0) =$ $\cot(0) =$

Example: Find the exact value of the following.

$\sin(-\pi) =$

$\cos(-\pi) =$

$\tan(-\pi) =$

Example: Use the unit circle to find each of the following.

1. $sin\left(\frac{4\pi}{3}\right)$

2. $tan\left(\frac{-9\pi}{4}\right)$

3. $sec\left(\frac{11\pi}{6}\right)$

Example: Find the calculator approximation of

1. $\sin(3.42)$

2. $\tan(.8234)$

3. $\sec(5.6041)$

4. $\csc(-2.7335)$

Example:

1. Solve $\sin(s) = .3210$ for an s in the interval $\left[0, \frac{\pi}{2}\right]$.

2. Solve $\tan(s) = \frac{-\sqrt{3}}{3}$ for an s in $\left[\frac{3\pi}{2}, 2\pi\right]$.

From the unit circle, let's determine the domain of all of the trigonometric functions.

Function	Domain
$y = sinx$	
$y = cosx$	
$y = tanx$	
$y = cscx$	
$y = secx$	
$y = cotx$	

Now let's find the range of the sine and cosine functions.

Function	Range
$y = sinx$	
$y = cosx$	

Linear speed- is the measure of how fast the position is changing.

Linear speed = $\frac{change\ in\ distance}{change\ in\ time}$ linear speed in a circular path = $\frac{change\ in\ arc\ length}{change\ in\ time}$

The book uses the following notation: $v = \frac{s}{t}$

Angular speed –is the measure of how fast the angle is changing.

The book uses the following notation to denote this: $\omega = \frac{\theta}{t}$

Here is something really cool, speed:

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

Example: Suppose P is on a circle of radius 15 inches, and ray OP is rotating with angular speed $\frac{\pi}{12}$ radians per second.

a. Find the angle generated by P in 10 seconds.
b. Find the distance traveled by P along the circle in 10 seconds.
c. Find the linear speed of P in inches per second.

Example: A belt runs a pulley of radius 5 inches at 120 revolutions per minute.

a. Find the angular speed of the pulley in radians per second.
b. Find the linear speed of the belt in inchers per second.

Example: A satellite traveling in a circular orbit approximately 1800 km above the surface of earth takes 2.5 hours to make an orbit. (The radius of Earth is 6,400 km.)

a. Approximate the linear speed of the satellite in kilometers per hour.
b. Approximate the distance the satellite travels in 3.5 hours.