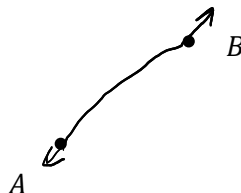


Math 1230 Trigonometry
Section 1.1

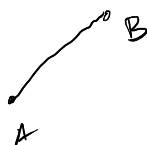
Name: Derek White

Lines:

- 2 points determine a line: How many lines can you draw between points A and B below?



- line segment between points A and B



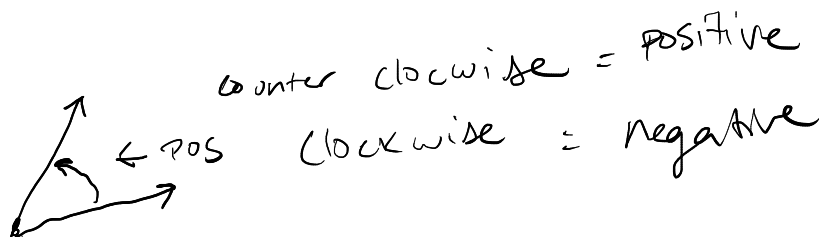
- ray starting a point A that goes through point B



Angle:

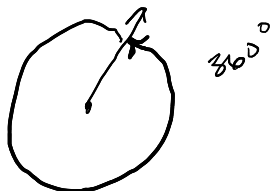
Two line segments or two rays with a common end point.

- Vertex- The common end point of the lines/rays
- Initial side
- Terminal side
- Positive angle- rotation is counter-clockwise
- Negative angle- clockwise
- Measure of an angle is in degrees. The symbol is $^\circ$.

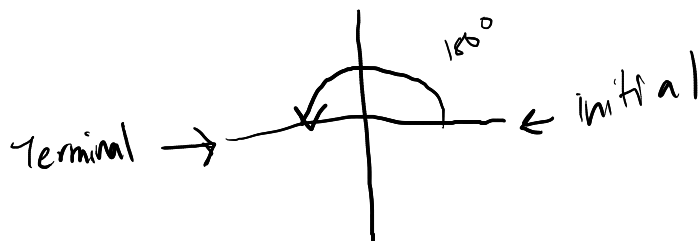


Measure of an angle:

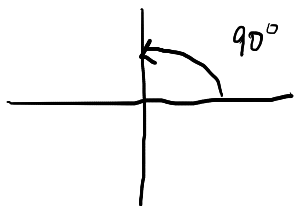
360° : one complete rotation in the counter-clockwise direction



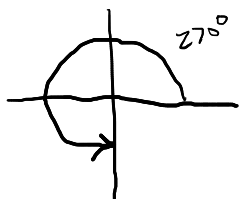
180° : half of a counter-clockwise rotation



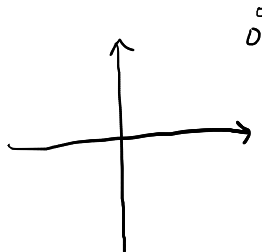
90° : half of a 180° , i.e. $\frac{1}{4}$ of a counterclockwise rotation



270° : $\frac{3}{4}$ of a counterclockwise rotation



0° : no rotation (the initial and terminal sides correspond)

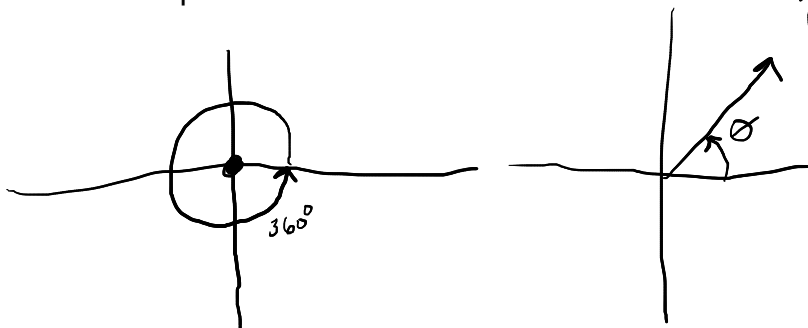


initial side = starting point
terminal side = ending point

Angle in standard position:

- the vertex is at the origin
- the initial side lies on the positive x -axis

θ (theta)
is standardized notation
for an angle variable

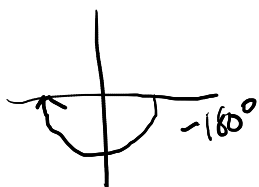


Example: Draw the following angles in standard position. Give the quadrant of each angle, if possible.

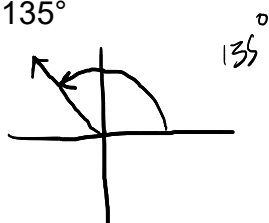
1. 90°



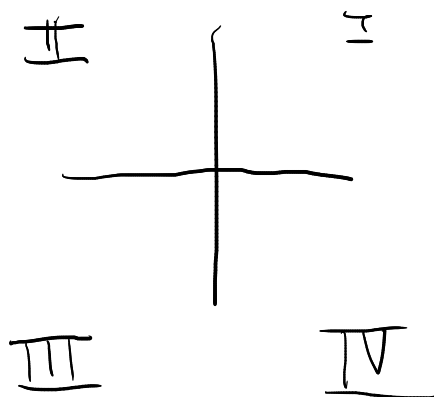
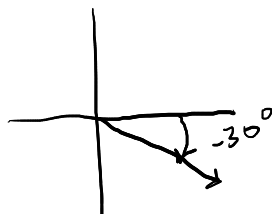
2. -180°



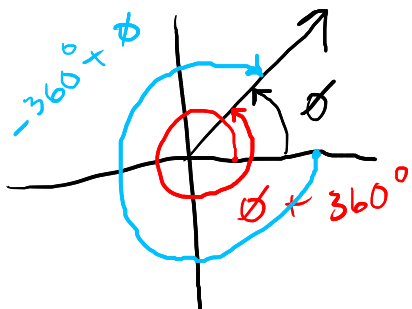
3. 135°



4. -30°



Coterminal Angles: Angles are coterminal if their measures differ by a multiple of 360°



Example:

- a. List three positive angles that are coterminal with 45° .

$$\begin{aligned} 45^\circ + 360^\circ \\ 45^\circ + 720^\circ \\ 45^\circ + 1,080^\circ \end{aligned}$$

- b. List three negative angles that are coterminal with 45° .

$$\begin{aligned} 45^\circ - 360^\circ &= -315^\circ \\ 45^\circ - 720^\circ &= -675^\circ \\ 45^\circ - 1,080^\circ &= -1,035^\circ \end{aligned}$$

Example: List two positive angles and two negative angles that are coterminal with each of the following angles.

- a. 1106°

$$\begin{aligned} 1106^\circ - 360^\circ &= 746^\circ \\ 1106^\circ - 360^\circ - 360^\circ &= 386^\circ \end{aligned}$$

$$1106 - 360 - 360 - 360 - 360 = -334^\circ$$

- b. -150°

$$1106 - 360 - 360 - 360 - 360 - 360 = -694^\circ$$

- c. -603°

Types of angles:

- θ is an acute angle if: $0 \leq \text{measure of } \theta < 90^\circ$
- θ is a right angle if: $\text{measure of } \theta = 90^\circ$
- θ is a straight angle if: $\text{measure of } \theta = 180^\circ$
- θ is an obtuse angle if: $90^\circ < \text{measure of } \theta < 180^\circ$

Complementary angles: α and β are complementary angles provided $\alpha + \beta = 90^\circ$

$45^\circ, 45^\circ$
 $30^\circ, 60^\circ$
 $89.5^\circ, .5^\circ$

Supplementary angles: α and β are supplementary angles provided $\alpha + \beta = 180^\circ$

$90^\circ, 90^\circ$
 $30^\circ, 150^\circ$

Measure of angles:

- Just like a dollar is equivalent to 4 quarters, and each quarter is equivalent to 25 pennies.
- Just like an hour is equivalent to 60 minutes, and each minute is equivalent to 60 seconds.
- The smaller pieces of an angle are also called minutes and seconds.

Minutes $1^\circ = 60 \text{ minutes (denoted by } 60')$

Seconds $1 \text{ minute} = 60 \text{ seconds (denoted by } 60'')$

Example: Do the following computations.

1. $60^\circ 45' + 13^\circ 20'$

$$\begin{array}{r} 45' \\ + 20' \\ \hline 1^\circ 5' \end{array}$$

$$\begin{array}{r} 60 \\ 13 \\ 1 \\ \hline \end{array}$$

$74^\circ 5'$

2. $31^\circ 10'18'' + 20^\circ 15'45''$

$$\begin{array}{r} 18'' \\ + 45'' \\ \hline 1' 3'' \end{array}$$

$$\begin{array}{r} 10' \\ 15' \\ + 1' \\ \hline 26' \end{array}$$

$$\begin{array}{r} 31^\circ \\ + 20^\circ \\ \hline 51^\circ \end{array}$$

$51^\circ 26' 3''$

Example: Find the supplement of the following angles.

1. 90°

2. $45^\circ 3'$

Example: Find the complement of $60^\circ 59'11''$.

$$\begin{array}{r} 89^\circ 59' 60'' \\ - 60^\circ 59' 11'' \\ \hline 29^\circ 0' 49'' \end{array}$$

$$1^{\circ} = 60' = 3600''$$

$$30' \frac{1^{\circ}}{60'} = \frac{30}{60}^{\circ}$$

Example:

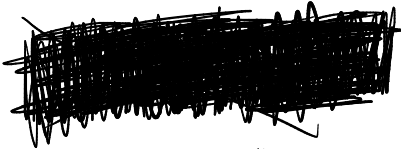
a. Convert $30'$ to degrees.

b. Convert $45^{\circ} 30'$ to degrees.

Example:

a. Convert $25'$ to degrees.

b. Convert $17''$ to degrees.



$$17'' \frac{1'}{60''} = \frac{17'}{60}$$

$$\frac{17}{60} \frac{1^{\circ}}{60} = \frac{17}{3600}^{\circ} \approx .0047^{\circ}$$

Example:

a. Convert $.25^{\circ}$ to minutes.

b. Convert 16.25° to degrees, minutes, and seconds as is appropriate.

Example:

a. Convert $.27^\circ$ to minutes.

b. Convert $.27^\circ$ to minutes and seconds.

c. Convert 16.27° to minutes and seconds as is appropriate.

Example application: A wheel makes 270 revolutions per minute. Through how many degrees will a point on the edge of the wheel move in 5 seconds?

$$\frac{360}{60}$$

$$\begin{array}{l} \frac{270 \text{ rev}}{\text{min}} \quad 1 \text{ rev} = 360^\circ \\ \frac{360^\circ}{1 \text{ rev}} \\ \left(\frac{270 (360)}{60 \text{ sec}} \right) 5 \text{ sec} \quad \frac{1 \text{ min}}{60 \text{ sec}} \\ = 8100^\circ \end{array}$$

Activity:

- 1) Cut a triangle out of a piece of paper.
- 2) Label the angles A, B, and C. For each vertex of the triangle, draw an arrow pointing at the vertex.
- 3) With two cuts, separate the triangle into three pieces. Make sure not to cut thru any of the vertices.
- 4) Line up the angles A, B, and C side by side.
- 5) To what value do the angles sum?

Vertical Angles

Example: Identify the pairs of angles below that are vertical angles.

Fact: Vertical angles are equal in measure.

Why are vertical angles equal in measure?

Transversal: Any line intersecting two parallel lines.

Parallel Postulate: If a line cuts two lines and the interior angles on the same side sum to less than 180° , then the lines intersect.

Alternate Interior Angles

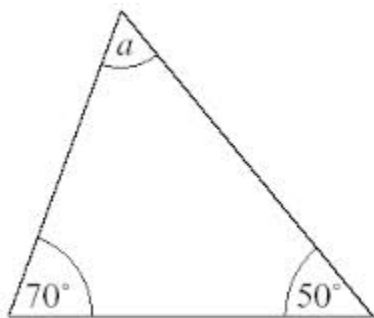
Example: Find the pairs of alternate interior angles below.

Fact: Alternate interior angles are equal in measure.
Why are alternate interior angles equal in measure?

Fact: The interior angles of any triangle sum to 180° .
Why do the interior angles of any triangle sum to 180° ?

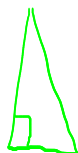
Example: What is the measure of a ?

$$180 - 70 - 50 = 60^\circ$$

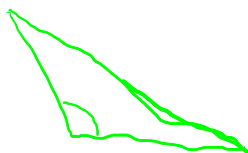


Types of triangles based on interior angles

- Acute triangle: all interior angles are acute
- Right triangle: one interior angle is a 90° angle
- Obtuse triangle: one interior angle is obtuse



- Obtuse triangle: one interior angle is obtuse



Types of triangles based on the lengths of the sides

- Equilateral triangle: all sides are of equal length



- Isosceles triangle: two sides are of equal length



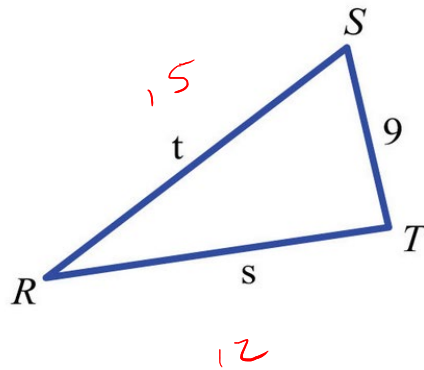
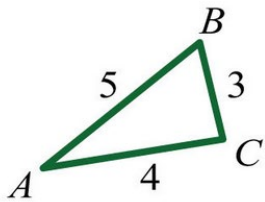
- Scalene triangle: no sides are of equal length



Similar Triangles: Two triangles are similar provided

- Corresponding angles have equal measure.
- Corresponding sides are proportional (the ratio of the lengths of corresponding sides are the same).

Example: Assume the two triangles are similar. Solve for s and t .



$$\frac{9}{3} = 3$$

$$5 \cdot 3 = 15 = t$$

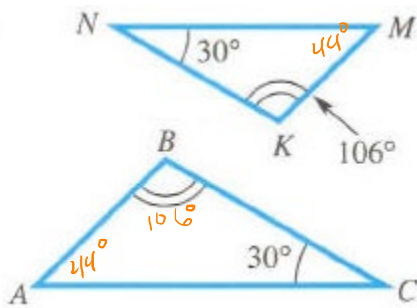
$$4 \cdot 3 = 12 = s$$

$$\frac{5}{3} = \frac{t}{9}$$

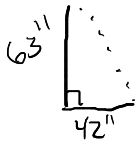
$$9\left(\frac{5}{3}\right) = t$$

$$9\left(\frac{3}{5}\right) = s$$

Example: Solve for A , B , and M .



Application: Nina wants to know the height of a tree in a park near her home. The tree casts a 38 ft shadow the same time that Nina casts a 42 inch shadow. Nina is 63 inches tall. What is the height of the tree?

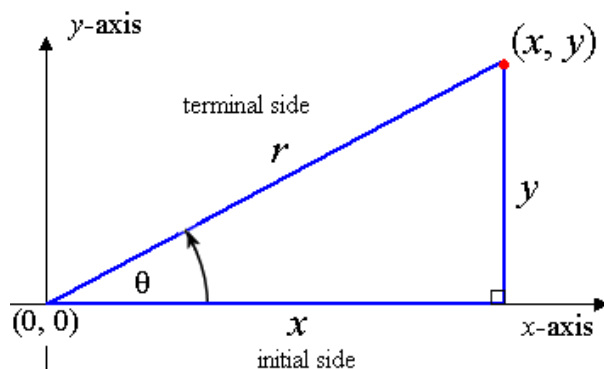


$$\frac{63}{42} = \frac{X}{38}$$

$$X = \frac{63}{42} (38) = 57"$$

$$\begin{array}{r} 63 \cancel{1} \quad \cancel{1} \\ \underline{42 \quad 12} \\ 12 \end{array}$$

The Definition of the Trigonometric Functions: Let (x, y) be a point on the terminal side of an angle θ



- θ in standard position
- $(x, y) \neq (0, 0)$
- Set $r = \sqrt{x^2 + y^2}$

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

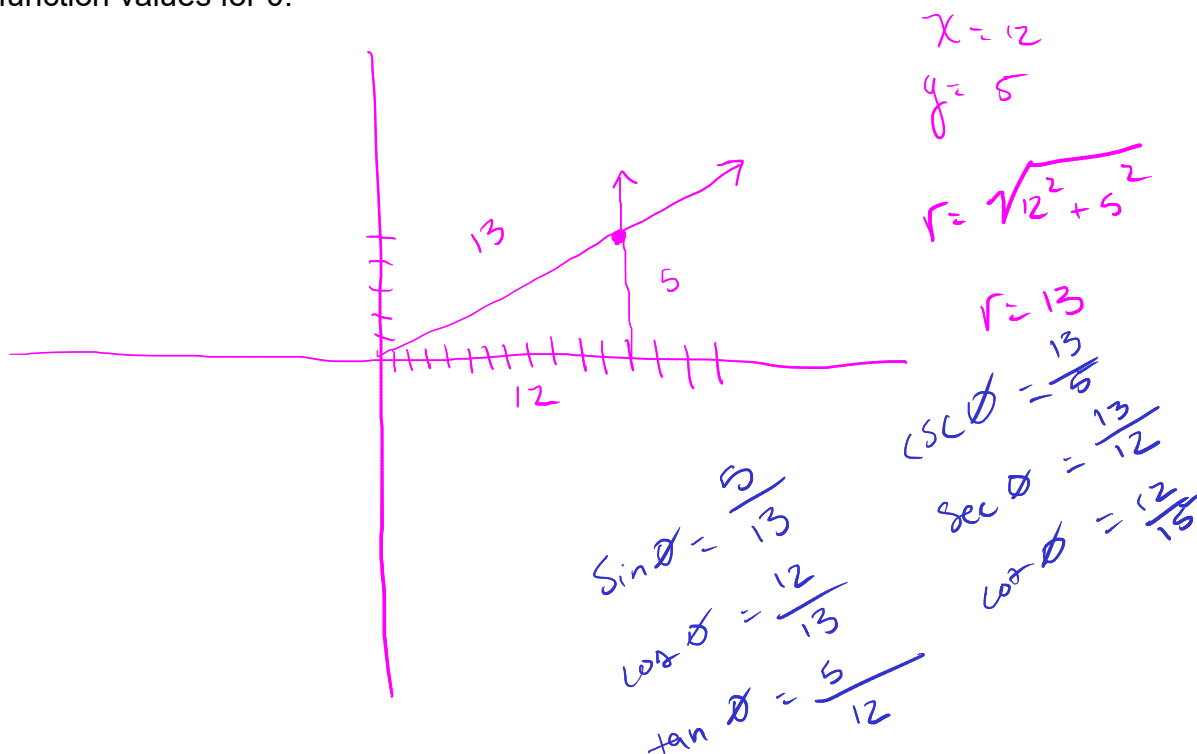
$$\sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{x}{y}$$

cosecant

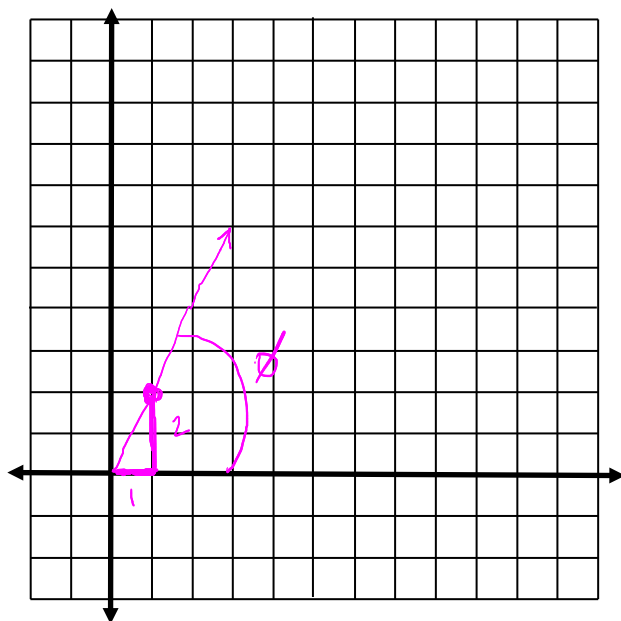
secant

Example: For an angle θ with terminal side passing thru $(12, 5)$, find the six trigonometric function values for θ .



Important Observation:

- Draw the ray with endpoint (0,0) that goes thru (1,2). Let this ray be the terminal side of a positive angle θ in standard position.



$x=1$
 $y=2$
 $r = \sqrt{x^2 + y^2}$

$\sqrt{ab} = \sqrt{a} \sqrt{b}$

Find $\sin \theta = \frac{y}{r}$
 $= \frac{2}{\sqrt{5}}$

$\cos \theta = \frac{x}{r}$
 $= \frac{1}{\sqrt{5}}$

$\tan \theta = \frac{y}{x}$
 $= \frac{2}{1} = 2$

Note that the terminal side of θ also goes thru (2,4). Use this point to find

$r = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

$\sin \theta = \frac{y}{r}$
 $\frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$

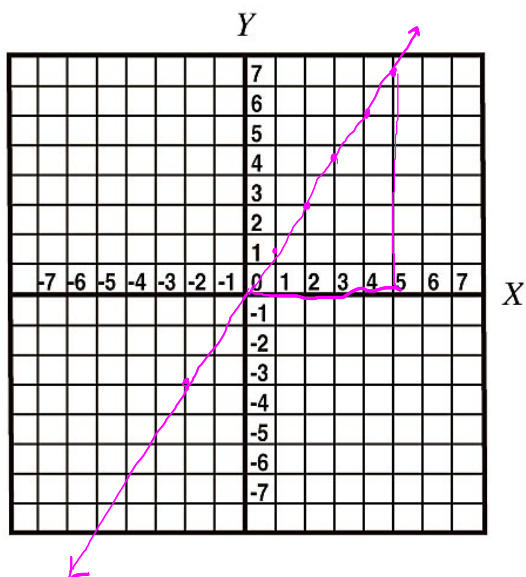
$\cos \theta = \frac{x}{r}$
 $\frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\tan \theta = \frac{y}{x}$
 $\frac{4}{2} = 2$

- How does using different points on the terminal side of an angle affect the value of the trigonometric function values?

Example:

- a. Graph $3x - 2y = 0$.



$$\frac{-2y}{-2} = \frac{-3x}{-2}$$

$$y = 1.5x$$

$$\begin{array}{c|c} x & y = \frac{3}{2}x \\ \hline 0 & 0 \\ 2 & \frac{3}{2} \cdot 2 = 3 \end{array}$$

- b. Now graph the portion of $3x - 2y = 0$ where $x \leq 0$.
- c. To find the 6 trigonometric function values for an angle θ whose terminal side coincides with $3x - 2y = 0$ where $x \leq 0$, recall that you need to know the (x, y) coordinate pair of a point that lies on the terminal side of θ .

$$x = -2$$

$$y = -3$$

$$r = \sqrt{13}$$

- d. Find

$$\sin \theta = \frac{y}{r} = \frac{-3}{\sqrt{13}}$$

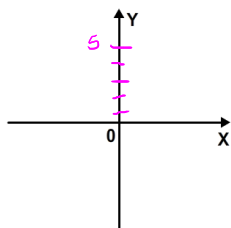
$$\cos \theta = \frac{x}{r} = \frac{-2}{\sqrt{13}}$$

$$\tan \theta = \frac{y}{x} = \frac{-3}{-2} = \frac{3}{2}$$

Quadrantal Angles: To find the trigonometric function values for quadrantal angles, just as you would have to do for any other angle, you need to find the (x, y) coordinate pair of a point that lies on the terminal side of the quadrantal angle.

0°
90°
180°
270°

Example: Let's find the trigonometric function values for 90°.



$$\begin{aligned}x &= 0 \\y &= 5 \\r &= 5\end{aligned}$$

$$\sin \theta = \frac{y}{r} = \frac{5}{5} = 1$$

$$\csc \theta = \frac{r}{y} = \frac{5}{5} = 1$$

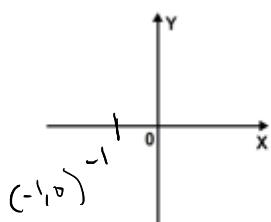
$$\cos \theta = \frac{x}{r} = \frac{0}{5} = 0$$

$$\sec \theta = \frac{r}{x} = \frac{5}{0} = \text{undefined}$$

$$\tan \theta = \frac{y}{x} = \frac{5}{0} = \text{undefined}$$

$$\cot \theta = \frac{x}{y} = \frac{0}{5} = 0$$

Example: Find the six trigonometric function values for 180° .



$$x = -1$$

$$y = 0$$

$$r = 1$$

$$\sin \theta = \frac{y}{r} = \frac{0}{1} = 0$$

$$\cos \theta = \frac{x}{r} = \frac{-1}{1} = -1$$

$$\tan \theta = \frac{y}{x} = \frac{0}{-1} = 0$$

$$\csc \theta = \frac{r}{y} = \frac{1}{0} = \text{undefined}$$

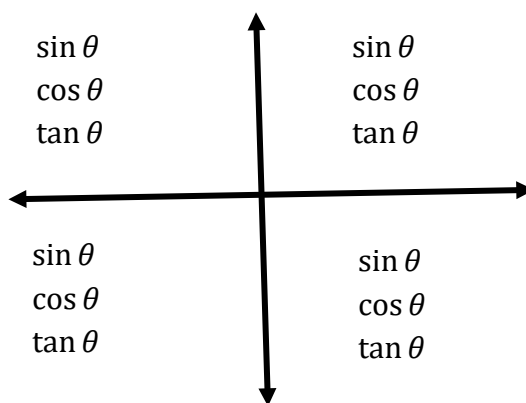
$$\sec \theta = \frac{r}{x} = \frac{1}{-1} = -1$$

$$\cot \theta = \frac{x}{y} = \frac{-1}{0} = \text{undefined}$$

The signs of the coordinates in the different quadrants:

Quadrant	x –coordinate	y –coordinate
I		
II		
III		
IV		

Signs of the Trigonometric Functions in the different coordinates:



Reciprocal Identities:

$\sin \theta =$

$\csc \theta =$

$\cos \theta =$

$\sec \theta =$

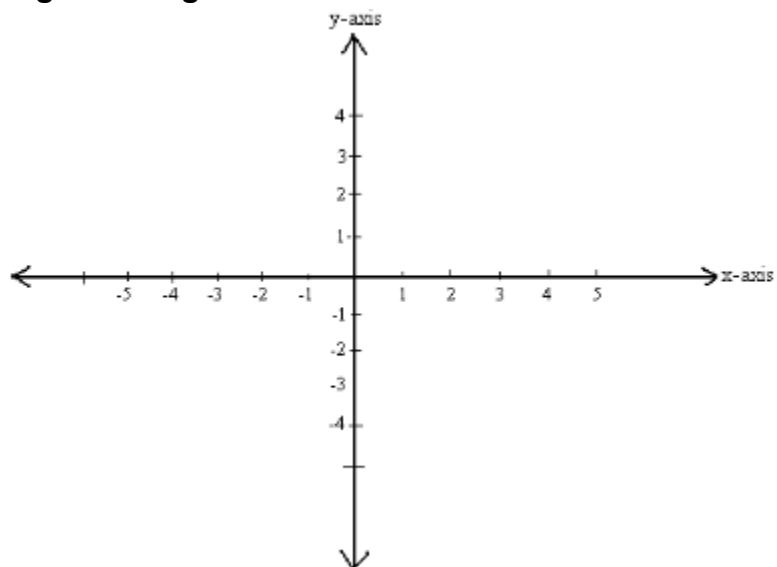
$\tan \theta =$

$\cot \theta =$

Example: Find each of the following values.

a. If $\tan \theta = \frac{1}{4}$ then $\cot \theta =$.

b. If $\cos \theta = \frac{-2}{\sqrt{20}}$ then $\sec \theta =$.

Signs of Trig Functions

Example: Find the signs of the trigonometric functions for each of the following angles.

a. 54°

b. 260°

c. -60°

Example: Find the quadrant of the terminal side of the angle θ that satisfies the following conditions.

a. $\tan \theta > 0$ and $\csc \theta < 0$

b. $\sin \theta > 0$ and $\csc \theta > 0$

Example: Given θ is in Quadrant III and $\tan \theta = \frac{8}{5}$, find

a. $\sin \theta =$

b. $\cos \theta =$

Derive the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$.

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1.$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Quotient Identities

$$\tan\theta =$$

$$\cot\theta =$$

Example: Given $\sin \theta = \frac{-\sqrt{2}}{3}$ and $\cos \theta > 0$, find

a. $\cos \theta$

b. $\tan \theta$

Example: Given $\cos \theta = \frac{-7}{25}$ and θ is in Quadrant II, find

a. $\cot \theta =$

b. $\csc \theta =$

Range of Trigonometric Functions:

$\sin \theta$:

Function	Range
$\sin \theta$	
$\csc \theta$	
$\cos \theta$	
$\sec \theta$	
$\tan \theta$	
$\cot \theta$	

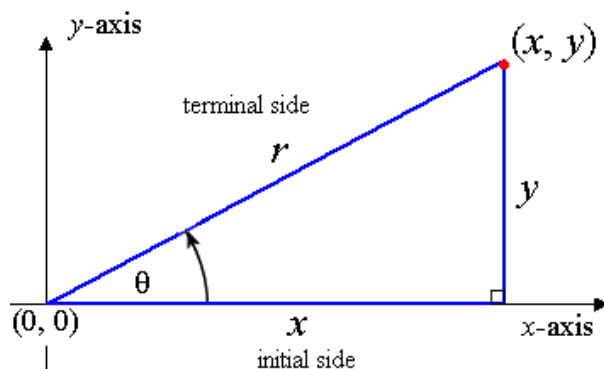
Example: Possible or not possible?

a. $\cot \theta = -.999$

b. $\cos \theta = -1.7$

c. $\csc \theta = 0$

The Definition of the Trigonometric Functions for acute angles: For any angle θ in standard position



- θ acute in standard position
- $(x, y) \neq (0, 0)$
- $r = \sqrt{x^2 + y^2}$ from the distance formula

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

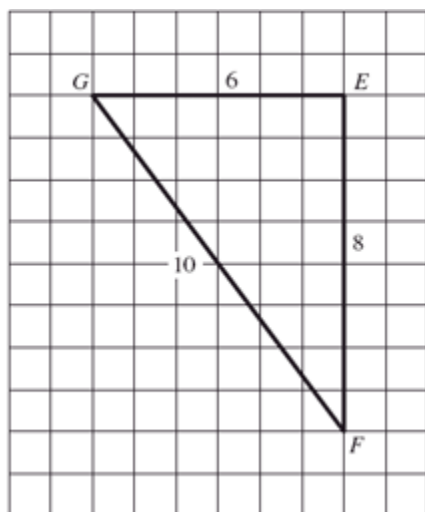
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite side}}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

Example: Find the following trigonometric values for the following interior angles of the triangle below.



$$\sin G =$$

$$\sin F =$$

$$\cos G =$$

$$\cos F =$$

$$\csc G =$$

$$\csc F =$$

$$\sec G =$$

$$\sec F =$$

$$\tan G =$$

$$\tan F =$$

$$\cot G =$$

$$\cot F =$$

Note:

Angle F and angle G are complementary angles because

$$(\text{measure of } G) + (\text{measure of } F) = \underline{\hspace{2cm}}$$

Which of their trigonometric function values are equivalent?

$$\sin G =$$

$$\cos G =$$

$$\csc G =$$

$$\sec G =$$

$$\tan G =$$

$$\cot G =$$

Cofunction Identities for Acute Angles

$$\sin G = \cos(90^\circ - G)$$

$$\cos G = \sin(90^\circ - G)$$

$$\csc G = \sec(90^\circ - G)$$

$$\sec G = \csc(90^\circ - G)$$

$$\tan G = \cot(90^\circ - G)$$

$$\cot G = \tan(90^\circ - G)$$

Example: Use the cofunction identities to fill in the blanks.

a. $\sin 9^\circ =$ _____

b. $\cot 76^\circ =$ _____

c. $\csc 45^\circ =$ _____

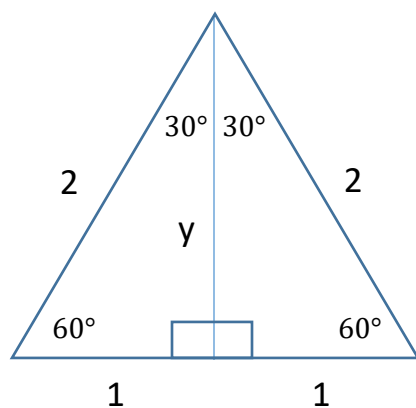
Example: Use the cofunction identities to solve for θ .

a. $\cot(\theta - 8^\circ) = \tan(4\theta + 13^\circ)$

b. $\sec(5\theta + 14^\circ) = \csc(2\theta - 8^\circ)$

Trigonometric Function Values 30° and 60° :

1. Solve for the value of y for the triangle below.



2. Use the definition of the trigonometric functions given at the beginning of this section, and the Cofunction Identities to find the following trigonometric function values.

$$\sin 30^\circ =$$

$$\csc 30^\circ =$$

$$\cos 30^\circ =$$

$$\sec 30^\circ =$$

$$\tan 30^\circ =$$

$$\cot 30^\circ =$$

$$\sin 60^\circ =$$

$$\csc 60^\circ =$$

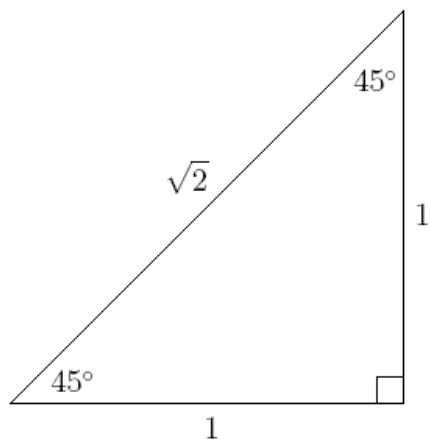
$$\cos 60^\circ =$$

$$\sec 60^\circ =$$

$$\tan 60^\circ =$$

$$\cot 60^\circ =$$

The Trigonometric Function Values for 45° - 45° - 90° triangle



a. Why would a triangle with sides of length 1 unit have a hypotenuse of length $\sqrt{2}$?

b. Use the triangle above to find the trigonometric function values for 45° .

$$\sin 45^\circ =$$

$$\csc 45^\circ =$$

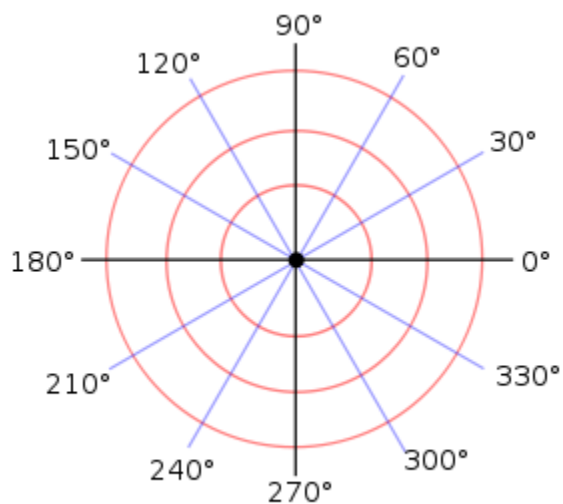
$$\cos 45^\circ =$$

$$\sec 45^\circ =$$

$$\tan 45^\circ =$$

$$\cot 45^\circ =$$

Behavior of the trigonometric functions as θ goes from 0° to 90° :



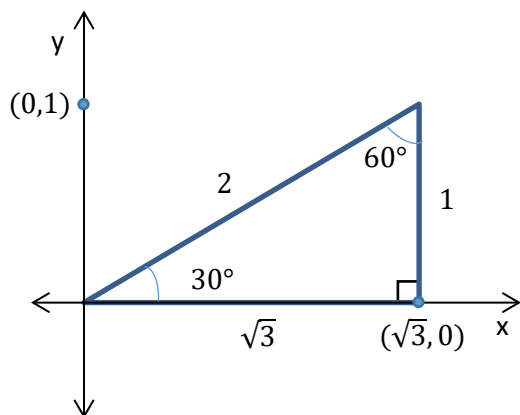
Function	Behavior of the numerator	Behavior of the denominator	Overall Behavior
$\sin \theta = \frac{y}{r}$			
$\cos \theta = \frac{x}{r}$			
$\tan \theta = \frac{y}{x}$			
$\csc \theta = \frac{r}{y}$			
$\sec \theta = \frac{r}{x}$			
$\cot \theta = \frac{x}{y}$			

Example: Indicate whether the following statements are true or false. Explain.

a. $\tan 25^\circ < \tan 23^\circ$

b. $\csc 44^\circ < \csc 40^\circ$

Example: Find the equation of the line that is collinear with the terminal side of a 30° angle.



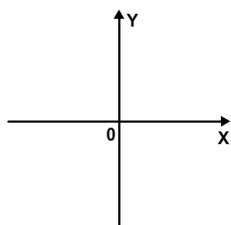
Reference Triangles

Question	Answer
1. Draw a $30^\circ - 60^\circ - 90^\circ$ triangle. Label angle measures, label side lengths.	
2. Draw a $45^\circ - 45^\circ - 90^\circ$ triangle. Label angle measures, label side lengths.	

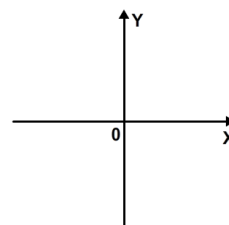
The reference angle for an angle in standard position: The reference angle for an angle θ in standard position is the acute angle that **the terminal side** makes with the **x -axis**.

Draw each of these angles and determine its reference angle:

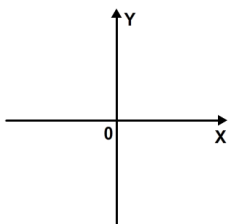
a. 30°



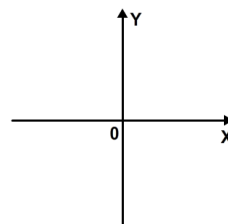
-30°



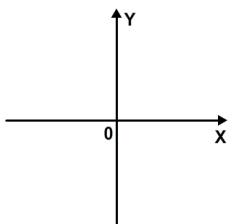
b. 150°



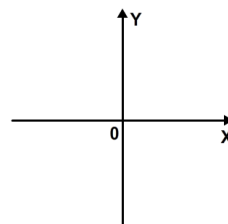
-150°



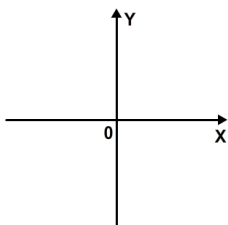
c. 210°



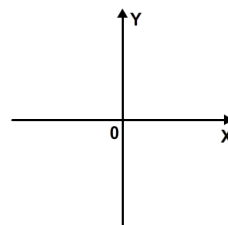
-210°



d. 330°

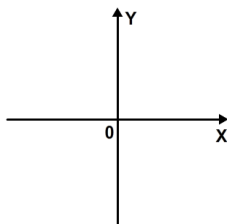


-330°

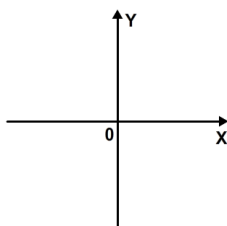


Example: Find the reference angle for

a. 294°

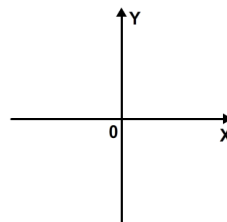


b. -883°

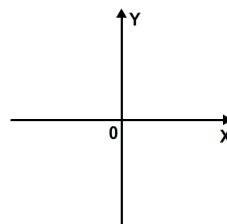


Example:

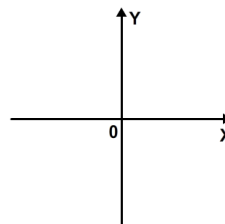
a. Draw the angle 135° .



b. Find and label the reference angle for 135° .



- c. Draw and label the reference triangle for 135° .



- d. Find the six trigonometric function values for 135° .

$$\sin 135^\circ =$$

$$\csc 135^\circ =$$

$$\cos 135^\circ =$$

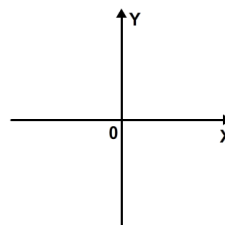
$$\sec 135^\circ =$$

$$\tan 135^\circ =$$

$$\cot 135^\circ =$$

Example:

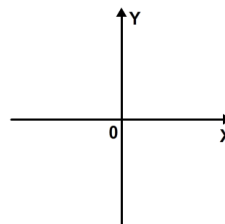
- a. Draw and label the reference triangle for -150° .



- b. Find $\sin(-150^\circ) =$

Example:

- a. Draw and label the reference triangle for 780° .



- b. Find $\cot(780^\circ) =$

Example: Recall the order of operations and note that $\sin^2\theta = (\sin\theta)^2$.
Evaluate $\sin^2(45^\circ) + 3\cos^2(135^\circ) - 2\tan(225^\circ) =$

Example: Find all values of θ in $[0, 360^\circ)$ that satisfy $\sin \theta = -\frac{\sqrt{3}}{2}$.

