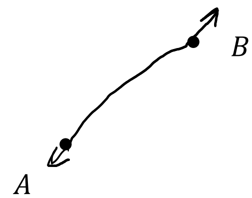
Math 1230 Trigonometry **Name:**____________________________

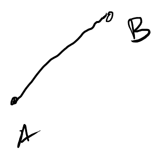
Section 1.1

Lines:

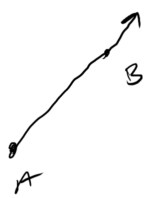
- 2 points determine a line: How many lines can you draw between points A and B below?



- line segment between points A and B



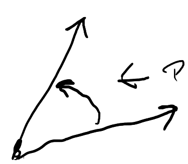
- ray starting a point A that goes through point B



Angle: Two line segments or two rays with a common end point.

- Vertex- The common end point of the lines/rays
- Initial side
- Terminal side
- Positive angle- rotation is counter-clockwise
- Negative angle- clockwise
- Measure of an angle is in degrees. The symbol is °.

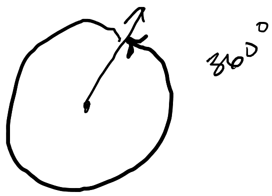
counter clockwise = positive



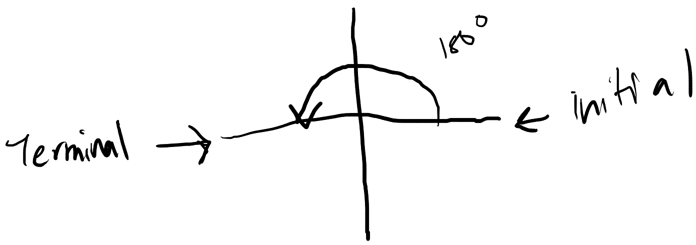
← POS

Clockwise = negative

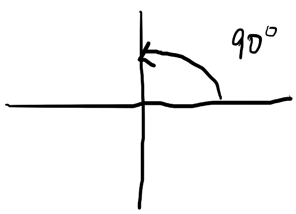
Measure of an angle:
360°: one complete rotation in the counter-clockwise direction



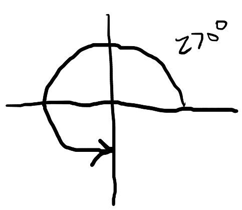
180°: half of a counter-clockwise rotation



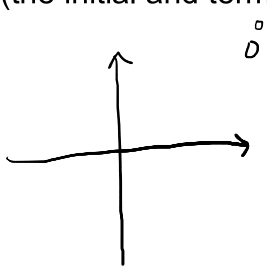
90°: half of a 180°, i.e. ¼ of a counterclockwise rotation



270°: ¾ of a counterclockwise rotation



0°: no rotation (the initial and terminal sides correspond)

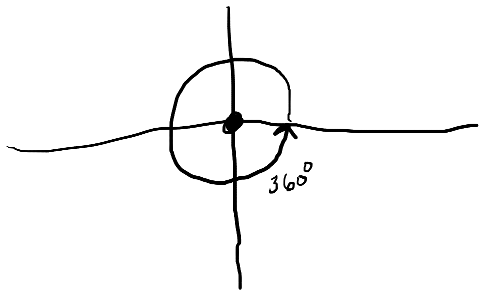


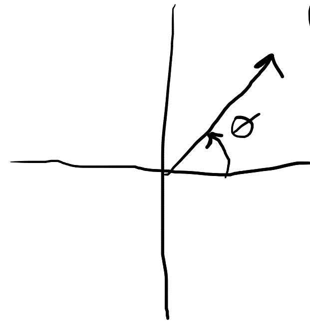
initial side = Starting Point
Terminal Side = ending Point

Angle in standard position:

- the vertex is at the origin
- the initial side lies on the positive x-axis

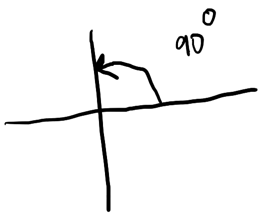
θ (theta) is standardized notation for an angle variable



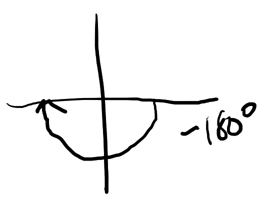


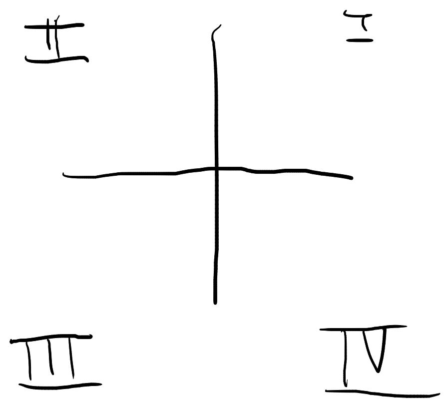
Example: Draw the following angles in standard position. Give the quadrant of each angle, if possible.

1. 90°

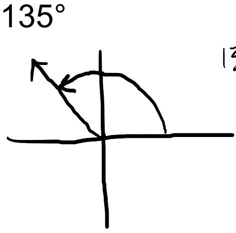


2. $-180°$

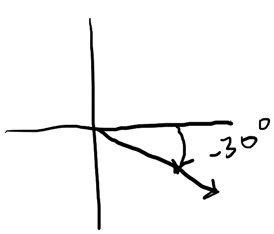




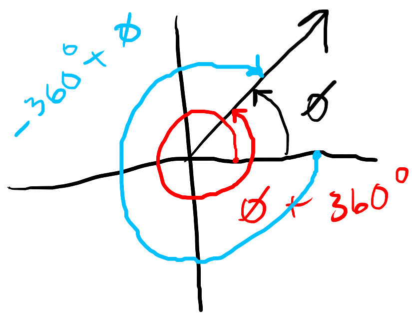
3. 135°



4. $-30°$



Coterminal Angles: Angles are coterminal if their measures differ by a multiple of 360°



Example:

a. List three positive angles that are coterminal with 45°.

$45° + 360°$
$45° + 720°$
$45° + 1{,}080°$

b. List three negative angles that are coterminal with 45°.

$45° - 360° = -315°$
$45° - 720° = -675°$
$45° - 1{,}080° = -1{,}035°$

Example: List two positive angles and two negative angles that are coterminal with each of the following angles.

a. 1106°

$1106° - 360 = 746°$
$1106 - 360 - 360 = 386°$

$1106 - 360 - 360 - 360 - 360 = -334°$
$1106 - 360 - 360 - 360 - 360 - 360 = -694°$

b. -150°

$-150° + 360° = 210°$
$-150° + 360° + 360° = 570°$

$-150 - 360 = -510°$
$-150 - 360 - 360 = -870°$

c. -603°

$-603° + 360° + 360° = 117°$
$-603° + 360° + 360° + 360° = 477°$

$-603° + 360° = -243°$
$-603° - 360° = -963°$

Types of angles:

- θ is an acute angle if : $0 \le measure\ of\ \theta < 90°$
- θ is a right angle if: $measure\ of\ \theta = 90°$
- θ is a straight angle if: $measure\ of\ \theta = 180°$
- θ is an obtuse angle if: $90° < measure\ of\ \theta < 180°$

Complementary angles: α and β are complementary angles provided $\alpha + \beta = 90°$

45°, 45°

30°, 60°

89.5°, .5°

Supplementary angles: α and β are supplementary angles provided $\alpha + \beta = 180°$

90°, 90°

30°, 150°

Measure of angles:

- Just like a dollar is equivalent to 4 quarters, and each quarter is equivalent to 25 pennies.
- Just like an hour is equivalent to 60 minutes, and each minute is equivalent to 60 seconds.
- The smaller pieces of an angle are also called minutes and seconds.

Minutes $1° = 60\ minutes\ (denoted\ by\ 60')$
Seconds $1\ minute = 60\ seconds\ (denoted\ by\ 60'')$

Example: Do the following computations.

1. $60°\,45' + 13°\,20'$

$$\begin{array}{r} 45' \\ +\ 20' \\ \hline 1^{\circ}\ \ 5' \end{array} \qquad \begin{array}{r} 60 \\ 13 \\ 1 \\ + \\ \hline 74^{\circ}\ 5' \end{array}$$

2. $31°\,10'18'' + 20°\,15'45''$

$$\begin{array}{r} 18'' \\ +\ 45'' \\ \hline 1'\ \ 3'' \end{array} \qquad \begin{array}{r} 10' \\ 15' \\ +\ 1' \\ \hline 26' \end{array} \qquad \begin{array}{r} 31^{\circ} \\ +\ 20^{\circ} \\ \hline 51^{\circ} \end{array}$$

$51^{\circ}\ 26'\ 3''$

Example: Find the supplement of the following angles.

1. $90°$

$$\begin{array}{r} 180 \\ -\ 90 \\ \hline \end{array} = 90^{\circ}$$

2. $45°\,3'$

$$\begin{array}{r} 180 \\ -\ 45\ \ 3' \\ \hline \cancel{1}35 \end{array} \qquad 134 \qquad \begin{array}{r} 60' \\ -\ 3 \\ \hline 57' \end{array}$$

$134\ \ 57'$

Example: Find the complement of $60°\,59'11''$.

$$\begin{array}{rrr} \overset{89^{\circ}}{\cancel{9}0^{\circ}} & \overset{59'}{\cancel{6}0'} & 60'' \\ -\ 60^{\circ} & 59' & 11'' \\ \hline 29^{\circ} & 0' & 49'' \end{array}$$

Example:

a. Convert $30'$ to degrees.

$$\frac{30}{60}^{\circ} = \frac{1}{2}^{\circ} = 0.5^{\circ}$$

b. Convert $45° \, 30'$ to degrees.

$$45 \; \frac{30}{60}^{\circ} = 45\tfrac{1}{2}^{\circ} = 45.5^{\circ}$$

Example:

a. Convert $25'$ to degrees.

$$\frac{25}{60}^{\circ} = \sim 0.41\overline{6}^{\circ}$$

b. Convert $17''$ to degrees.

$$\frac{17}{60}^{\circ} = \sim 0.28\overline{3}^{\circ}$$

Example:

a. Convert $.25°$ to minutes.

$$.25^{\circ}(60') = 15'$$

b. Convert $16.25°$ to degrees, minutes, and seconds as is appropriate.

$$.25^{\circ}(60') = 15'$$

$$16^{\circ} \, 15'$$

Example:

a. Convert $.27°$ to minutes.

$.27°(60') = 16.2'$

b. Convert $.27°$ to minutes and seconds.

$.27°(60') = 16.2'$

$.2'(60'') = 12''$

$16'\ 12''$

c. Convert $16.27°$ to minutes and seconds as is appropriate.

$16.27°$

$.27°(60') = 16.2'$

$.2'(60'') = 12''$

$16°\ 16'\ 12''$

Example application: A wheel makes 270 revolutions per minute. Through how many degrees will a point on the edge of the wheel move in 5 seconds?

$270'(60) = 16200''$

$\frac{16200''}{5''} = 3240''$

$\frac{3240''}{60''} = \frac{54'}{60'} = 0.9°$

Activity:

1) Cut a triangle out of a piece of paper.
2) Label the angles A, B, and C. For each vertex of the triangle, draw an arrow pointing at the vertex.
3) With two cuts, separate the triangle into three pieces. Make sure not to cut thru any of the vertices.
4) Line up the angles A, B, and C side by side.
5) To what value do the angles sum?

Vertical Angles

Example: Identify the pairs of angles below that are vertical angles.

Fact: Vertical angles are equal in measure.
Why are vertical angles equal in measure?

Transversal: Any line intersecting two parallel lines.

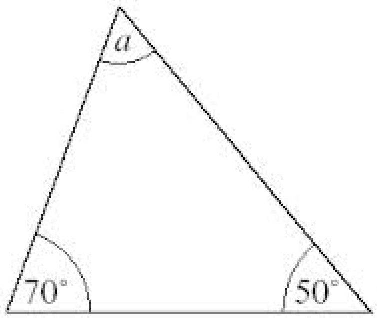
Parallel Postulate: If a line cuts two lines and the interior angles on the same side sum to less than $180°$, then the lines intersect.

Alternate Interior Angles
Example: Find the pairs of alternate interior angles below.

Fact: Alternate interior angles are equal in measure.
Why are alternate interior angles equal in measure?

Fact: The interior angles of any triangle sum to 180°.
Why do the interior angles of any triangle sum to 180°?

Example: What is the measure of a?



Types of triangles based on interior angles

- Acute triangle: all interior angles are acute

- Right triangle: one interior angle is a 90° angle

- Obtuse triangle: one interior angle is obtuse

Types of triangles based on the lengths of the sides

- Equilateral triangle: all sides are of equal length

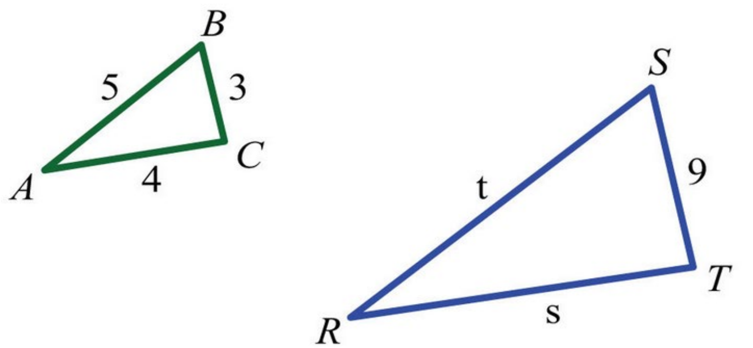
- Isosceles triangle: two sides are of equal length

- Scalene triangle: no sides are of equal length

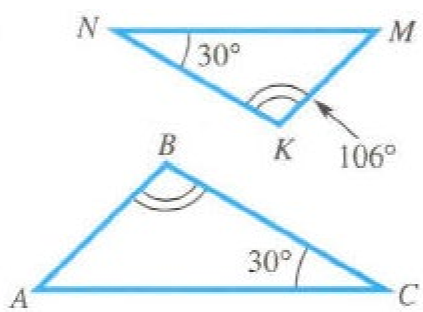
Similar Triangles: Two triangles are similar provided

- Corresponding angles have equal measure.
- Corresponding sides are proportional (the ratio of the lengths of corresponding sides are the same).

Example: Assume the two triangles are similar. Solve for s and t.

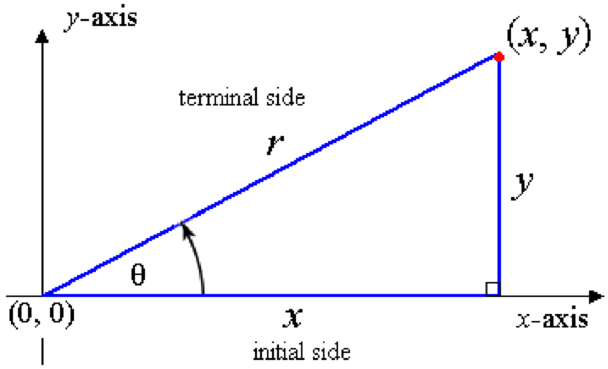


Example: Solve for A, B, and M.



Application: Nina wants to know the height of a tree in a park near her home. The tree casts a 38 ft shadow the same time that Nina casts a 42 inch shadow. Nina is 63 inches tall. What is the height of the tree?

The Definition of the Trigonometric Functions: Let (x, y) be a point on the terminal side of an angle θ



- θ in standard position
- $(\boldsymbol{x}, \boldsymbol{y}) \neq (\mathbf{0}, \mathbf{0})$
- Set $r = \sqrt{x^2 + y^2}$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

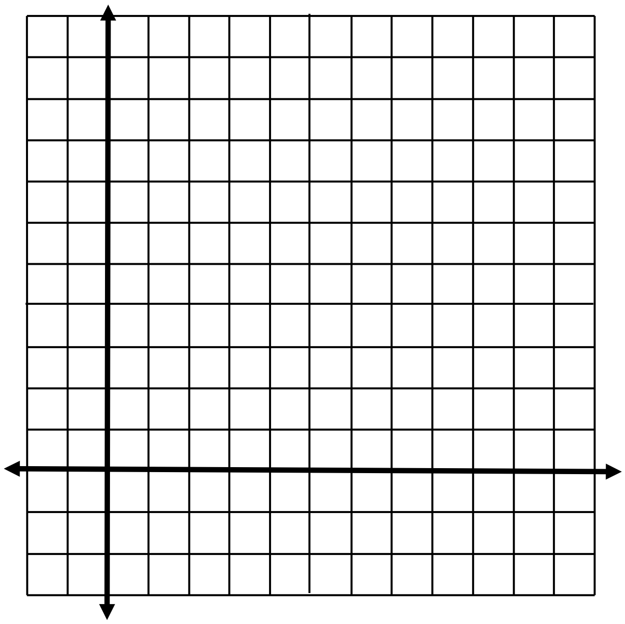
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Example: For an angle θ with terminal side passing thru $(12,5)$, find the six trigonometric function values for θ.

Important Observation:

- Draw the ray with endpoint $(0,0)$ that goes thru $(1,2)$. Let this ray be the terminal side of a positive angle θ in standard position.



- Find $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

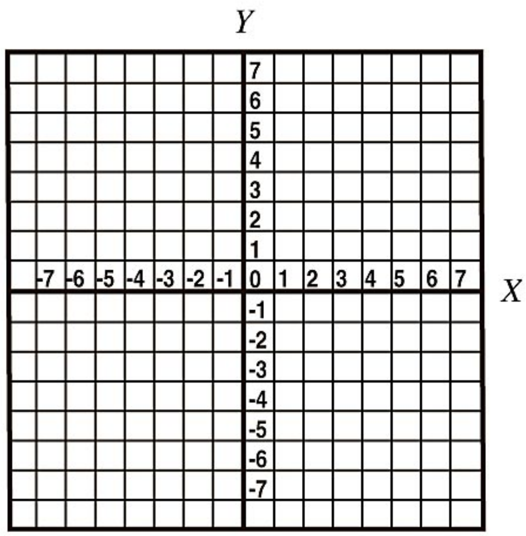
Note that the terminal side of θ also goes thru $(2,4)$. Use this point to find

- $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

- How does using different points on the terminal side of an angle affect the value of the trigonometric function values?

Example:

a. Graph $3x - 2y = 0$.



b. Now graph the portion of $3x - 2y = 0$ where $x \leq 0$.

c. To find the 6 trigonometric function values for an angle θ whose terminal side coincides with $3x - 2y = 0$ where $x \leq 0$, recall that you need to know the (x, y) coordinate pair of a point that lies on the terminal side of θ.

$x =$

$y =$

$r =$

d. Find

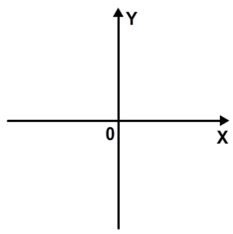
$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

Quadrantal Angles: To find the trigonometric function values for quadrantal angles, just as you would have to do for any other angle, you need to find the (x, y) coordinate pair of a point that lies on the terminal side of the quadrantal angle.

Example: Let's find the trigonometric function values for $90°$.



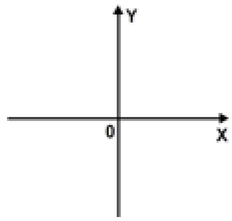
$x =$
$y =$
$r =$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

Example: Find the six trigonometric function values for $180°$.



$x =$
$y =$
$r =$

$\sin\theta =$ $\csc\theta =$

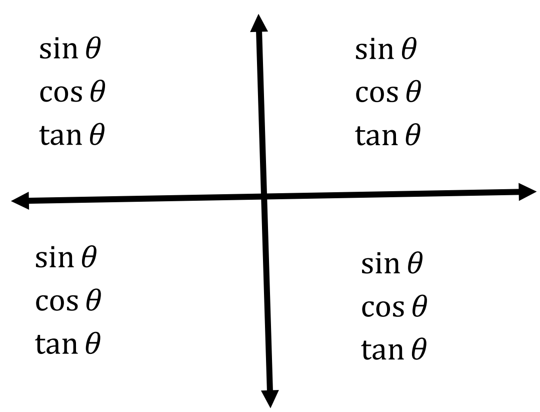
$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

The signs of the coordinates in the different quadrants:

Quadrant	x −coordinate	y −coordinate
I		
II		
III		
IV		

Signs of the Trigonometric Functions in the different coordinates:



Reciprocal Identities:

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

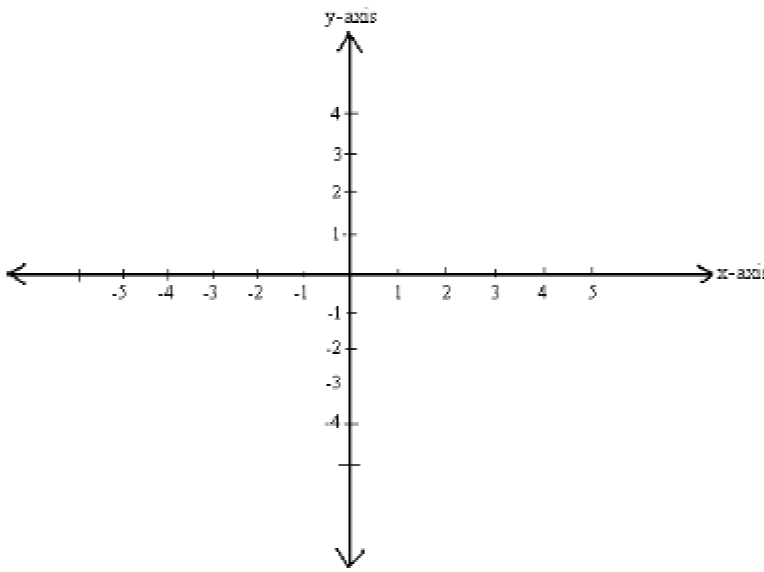
$\tan\theta =$ $\cot\theta =$

Example: Find each of the following values.

a. If $\tan\theta = \frac{1}{4}$ then $\cot\theta =$.

b. If $\cos\theta = \frac{-2}{\sqrt{20}}$ then $\sec\theta =$.

Signs of Trig Functions



Example: Find the signs of the trigonometric functions for each of the following angles.

a. $54°$

b. $260°$

c. $-60°$

Example: Find the quadrant of the terminal side of the angle θ that satisfies the following conditions.

a. $\tan\theta > 0$ and $\csc\theta < 0$
b. $\sin\theta > 0$ and $\csc\theta > 0$

Example: Given θ is in Quadrant III and $\tan\theta = \frac{8}{5}$, find

a. $\sin\theta =$
b. $\cos\theta =$

Derive the Pythagorean Identity $sin^2\theta + cos^2\theta = 1$.

Pythagorean Identities

$$sin^2\theta + cos^2\theta = 1.$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + cot^2\theta = csc^2\theta$$

Quotient Identities

$\tan\theta =$

$\cot\theta =$

Example: Given $\sin\theta = \frac{-\sqrt{2}}{3}$ and $\cos\theta > 0$, find

a. $\cos\theta$

b. $\tan\theta$

Example: Given $\cos\theta = \frac{-7}{25}$ and θ is in Quadrant II, find

a. $\cot\theta =$

b. $\csc\theta =$

Range of Trigonometric Functions:

$\sin\theta$:

Function	Range
$\sin\theta$	
$\csc\theta$	
$\cos\theta$	
$\sec\theta$	
$\tan\theta$	
$\cot\theta$	

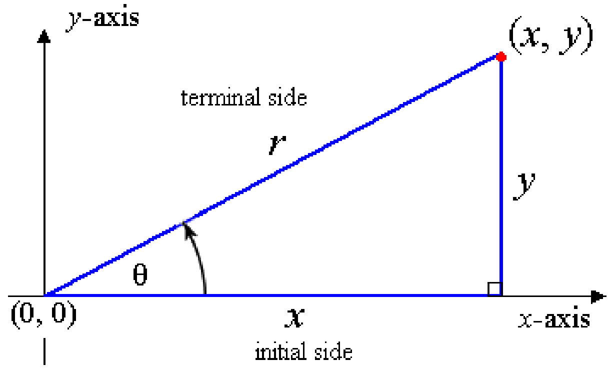
Example: Possible or not possible?

a. $\cot\theta = -.999$

b. $\cos\theta = -1.7$

c. $\csc\theta = 0$

The Definition of the Trigonometric Functions for acute angles: For any angle θ in standard position



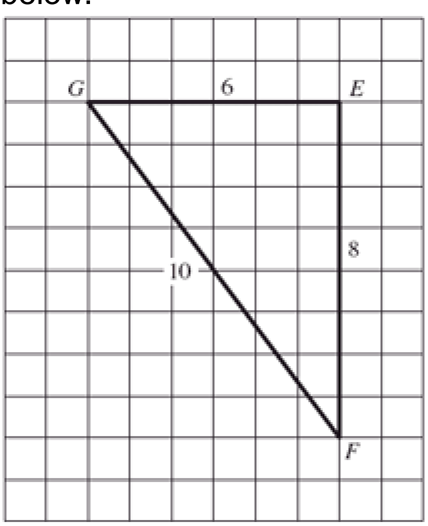
- Θ acute in standard position
- $(x, y) \neq (0, 0)$
- $r = \sqrt{x^2 + y^2}$ from the distance formula

$$\sin\theta = \frac{opposite\ side}{hypotenuse} \qquad \csc\theta = \frac{hypotenuse}{opposite\ side}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad \sec\theta = \frac{hypotenuse}{adjacent}$$

$$\tan\theta = \frac{opposite}{adjacent} \qquad \cot\theta = \frac{adjacent}{opposite}$$

Example: Find the following trigonometric values for the following interior angles of the triangle below.



$\sin G =$	$\sin F =$
$\cos G =$	$\cos F =$
$\csc G =$	$\csc F =$
$\sec G =$	$\sec F =$
$\tan G =$	$\tan F =$
$\cot G =$	$\cot F =$

Note:

Angle F and angle G are complementary angles because

$$(measure\ of\ G) + (measure\ of\ F) = \_\_\_\_\_\_\_\_$$

Which of their trigonometric function values are equivalent?

$\sin G =$

$\cos G =$

$\csc G =$

$\sec G =$

$\tan G =$

$\cot G =$

Cofunction Identities for Acute Angles

$\sin G = \cos(90° - G)$

$\cos G = \sin(90° - G)$

$\csc G = \sec(90° - G)$

$\sec G = \csc(90° - G)$

$\tan G = \cot(90° - G)$

$\cot G = \tan(90° - G)$

Example: Use the cofunction identities to fill in the blanks.

a. $\sin 9° =$ ____________

b. $\cot 76° =$ ___________

c. $\csc 45° =$ ___________

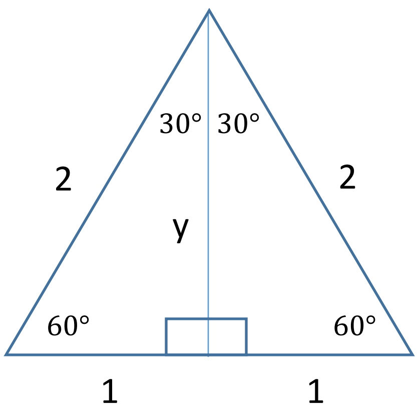
Example: Use the cofunction identities to solve for θ.

a. $\cot(\theta - 8°) = \tan(4\theta + 13°)$

b. $\sec(5\theta + 14°) = \csc(2\theta - 8°)$

Trigonometric Function Values $30°$ and $60°$:

1. Solve for the value of y for the triangle below.



2. Use the definition of the trigonometric functions given at the beginning of this section, and the Cofunction Identities to find the following trigonometric function values.

$\sin 30° =$ $\csc 30° =$

$\cos 30° =$ $\sec 30° =$

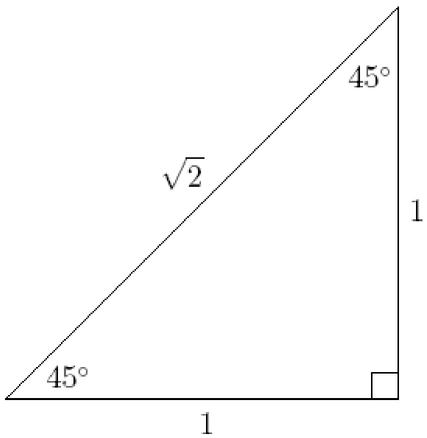
$\tan 30° =$ $\cot 30° =$

$\sin 60° =$ $\csc 60° =$

$\cos 60° =$ $\sec 60° =$

$\tan 60° =$ $\cot 60° =$

The Trigonometric Function Values for $45°$ - $45°$ - $90°$ triangle



a. Why would a triangle with sides of length 1 unit have a hypotenuse of length $\sqrt{2}$?

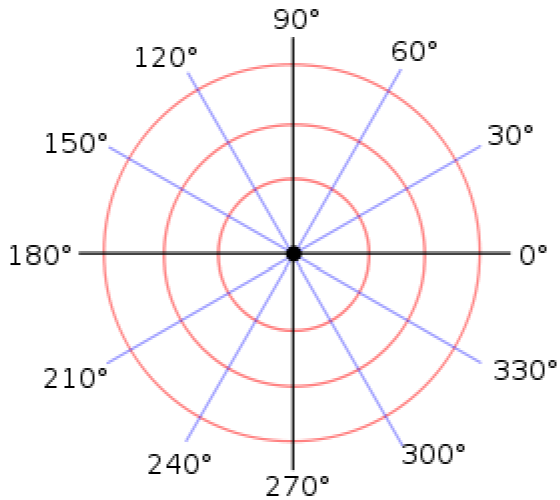
b. Use the triangle above to find the trigonometric function values for $45°$.

$\mathbf{\sin 45° =}$ $\mathbf{\csc 45° =}$

$\mathbf{\cos 45° =}$ $\mathbf{\sec 45° =}$

$\mathbf{\tan 45° =}$ $\mathbf{\cot 45° =}$

Behavior of the trigonometric functions as θ goes from $0°$ to $90°$:



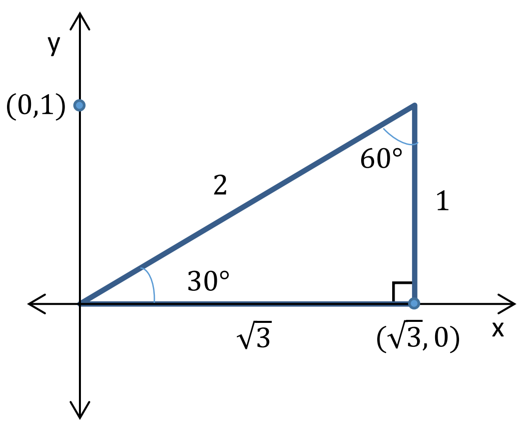
Function	Behavior of the numerator	Behavior of the denominator	Overall Behavior
$\sin\theta = \frac{y}{r}$			
$\cos\theta = \frac{x}{r}$			
$\tan\theta = \frac{y}{x}$			
$\csc\theta = \frac{r}{y}$			
$\sec\theta = \frac{r}{x}$			
$\cot\theta = \frac{x}{y}$			

Example: Indicate whether the following statements are true or false. Explain.

a. $\tan 25° < \tan 23°$

b. $\csc 44° < \csc 40°$

Example: Find the equation of the line that is collinear with the terminal side of a $30°$ angle.



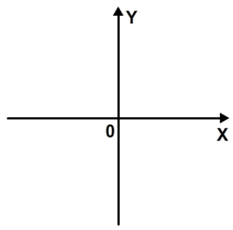
Reference Triangles

Question	Answer
1. Draw a $30° - 60° - 90°$ triangle. Label angle measures, label side lengths.	
2. Draw a $45° - 45° - 90°$ triangle. Label angle measures, label side lengths.	

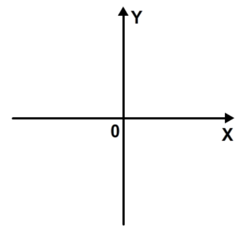
The reference angle for an angle in standard position: The reference angle for an angle θ in standard position is the **acute** angle that **the terminal side** makes with the **x-axis**.

Draw each of these angles and determine its reference angle:

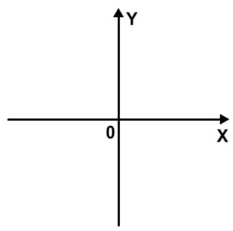
a. $30°$



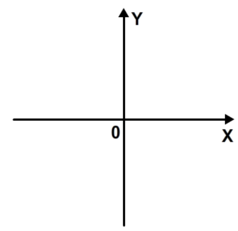
$-30°$



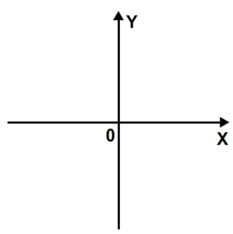
b. $150°$



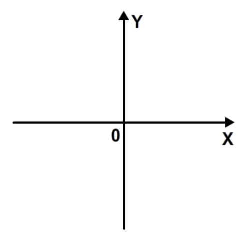
$-150°$



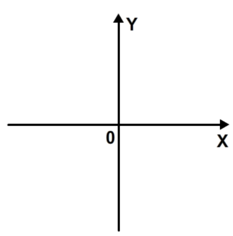
c. $210°$



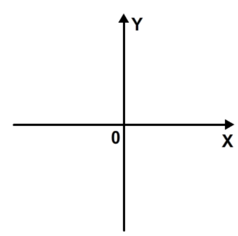
$-210°$



d. $330°$

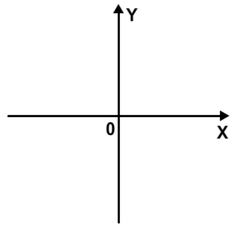


$-330°$

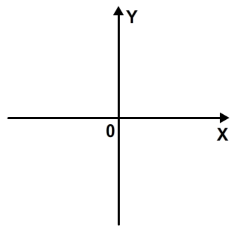


Example: Find the reference angle for

a. $294°$

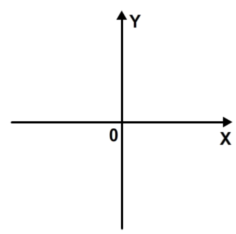


b. $-883°$

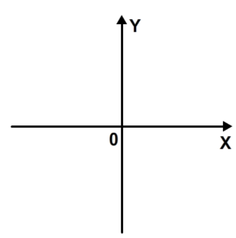


Example:

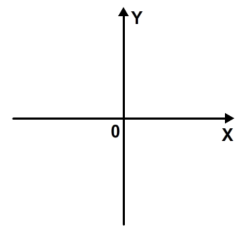
a. Draw the angle $135°$.



b. Find and label the reference angle for $135°$.



c. Draw and label the reference triangle for $135°$.



d. Find the six trigonometric function values for $135°$.

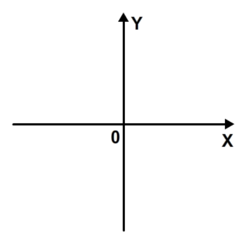
$\mathbf{sin\ 135°} =$ $\mathbf{csc}\ 135° =$

$\mathbf{cos}\ 135° =$ $\mathbf{sec}\ 135° =$

$\mathbf{tan}\ 135° =$ $\mathbf{cot}\ 135° =$

Example:

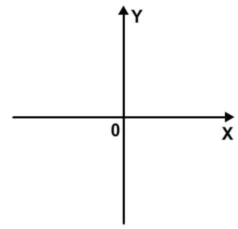
a. Draw and label the reference triangle for $-150°$.



b. Find $\sin(-150°) =$

Example:

a. Draw and label the reference triangle for $780°$.



b. Find $\cot(780°) =$

Example: Recall the order of operations and note that $sin^2\theta = (sin\theta)^2$.
Evaluate $sin^2(45°) + 3cos^2(135°) - 2\tan(225°) =$

Example: Find all values of θ in $[0,360°)$ that satisfy $\sin\theta = -\frac{\sqrt{3}}{2}$.