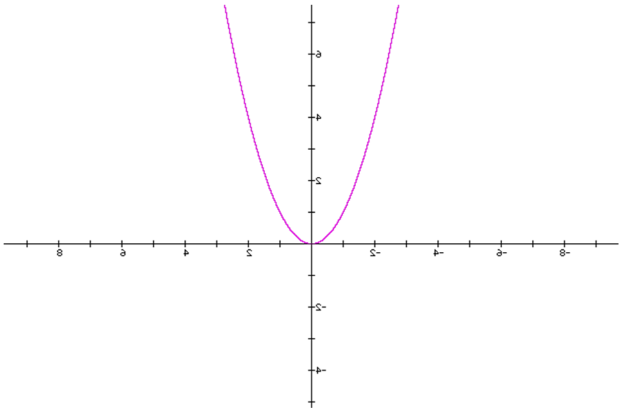
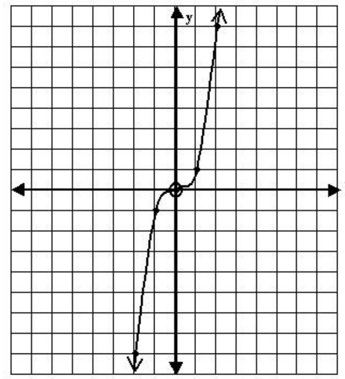
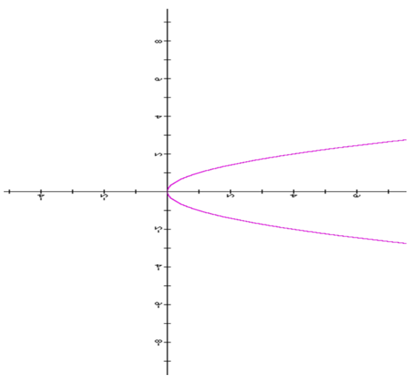
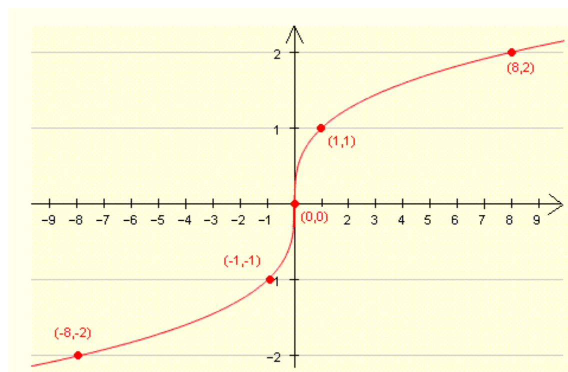
Recall the Vertical Line Test: If a vertical line crosses a graph more than once, then the graph is <u>not</u> the graph of a function.

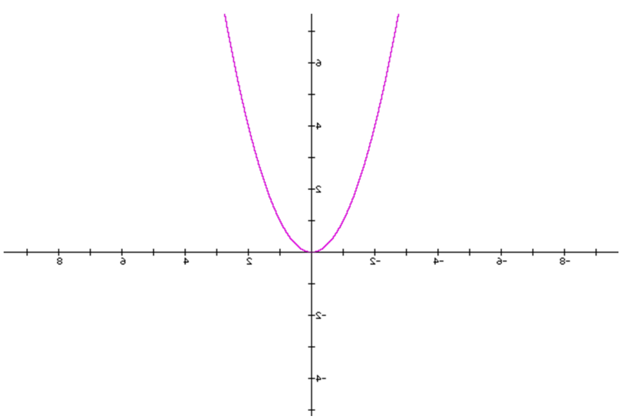


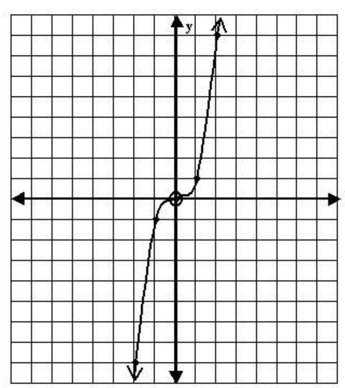






Recall the Horizontal Line Test: If a horizontal line crosses a graph of a function more than once, then the function <u>does not</u> have an inverse function.

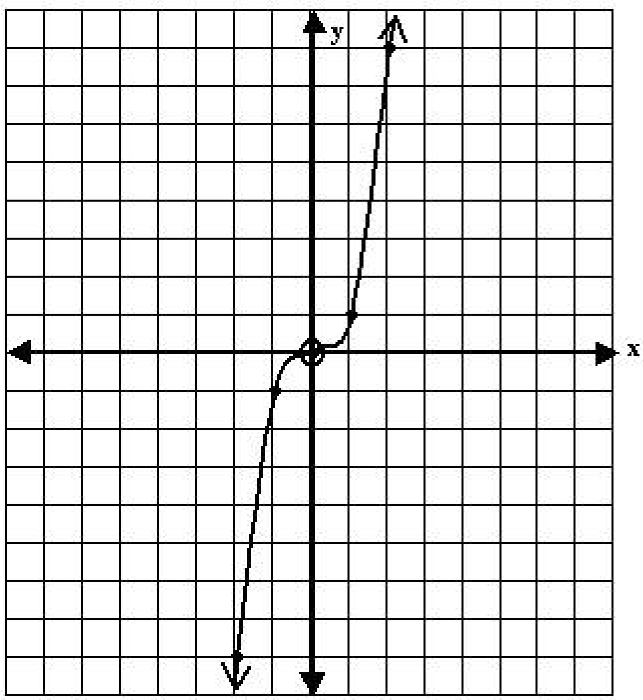


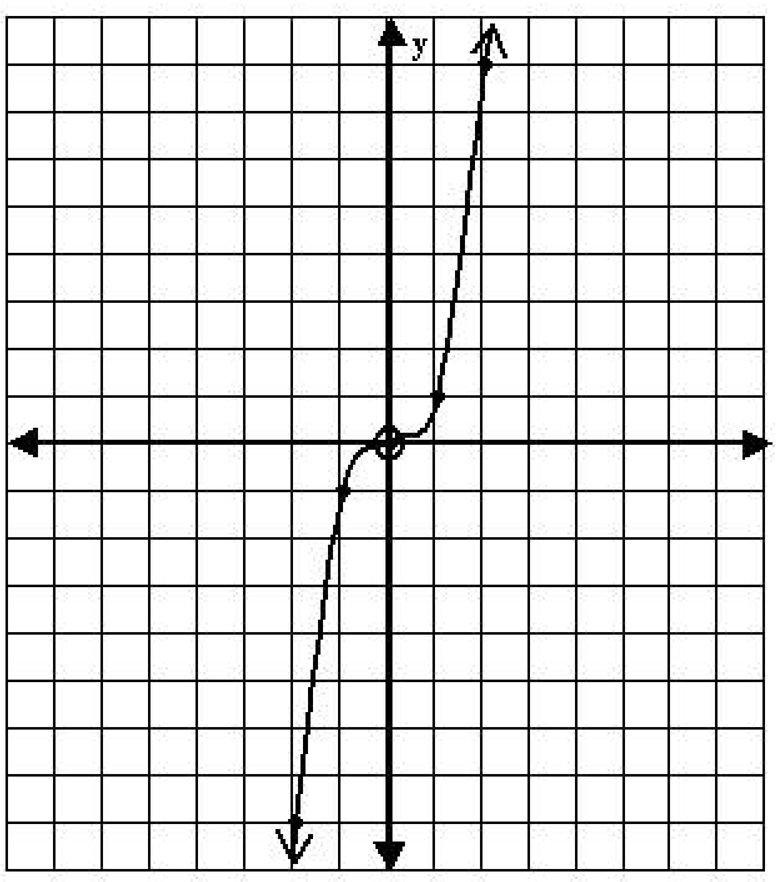


How do we get the graph of the inverse function from the graph of a function that passes the horizontal line test?

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

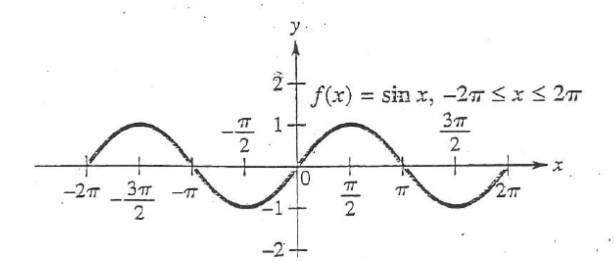


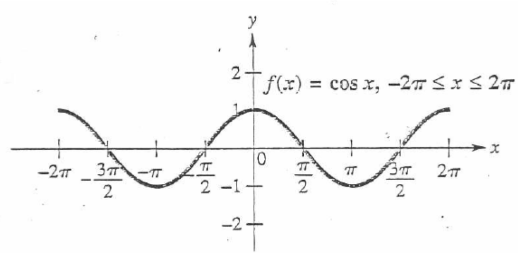


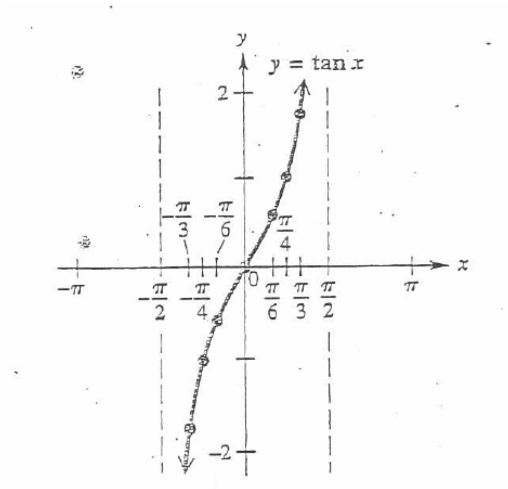
Notation for the Inverse Functions of the Trigonometric Functions:

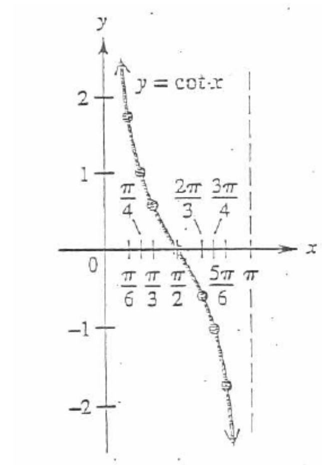
Function	Inverse Function
$y = \sin x$	$y = sin^{-1}(x)$ or $y = \arcsin(x)$
$y = \cos x$	$y = cos^{-1}(x)$ or $y = \arccos(x)$
$y = \tan x$	$y = tan^{-1}(x)$ or $y = \arctan(x)$
$y = cscx$	$y = csc^{-1}(x)$ or $y = \mathrm{arccsc}(x)$
$y = \sec x$	$y = sec^{-1}(x)$ or $y = \mathrm{arcsec}(x)$
$y = \cot x$	$y = cot^{-1}(x)$ or $y = \mathrm{arccot}(x)$

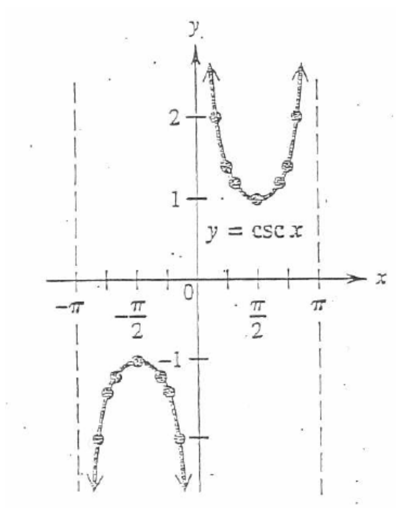
Let's find the graphs of the inverse trigonometric functions.

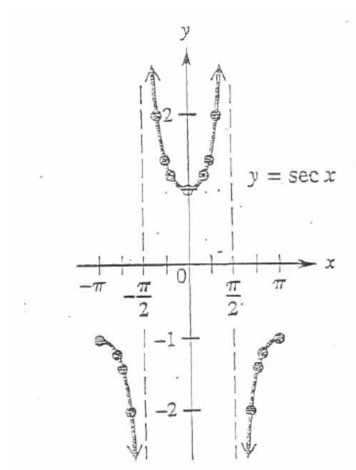






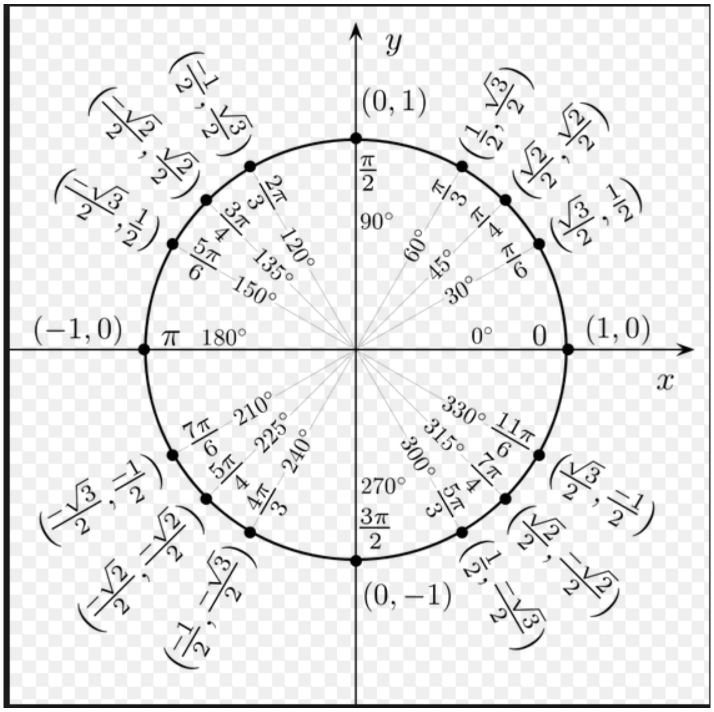






Trig Functions Chart

Degrees	Radians	Sin	Cosine	Tangent	Cotangent	Secant	Cosecant
0	0	0	1	0	undefined	1	undefined
30	π/6	1/2	√3/2	√3/3	√3	2√3/3	2
45	π/4	√2/2	√2/2	1	1	√2	√2
60	π/3	√3/2	1/2	√3	√3/3	2	2√3/3
90	π/2	1	0	Undefined	0	undefined	1
120	2π/3	√3/2	-1/2	-√3	-√3/3	-2	2√3/3
135	3π/4	√2/2	-√2/2	-1	-1	- √2	√2
150	5π/6	1/2	-√3/2	-√3/3	-√3	-2√3/3	2
180	π	0	-1	0	undefined	-1	undefined
210	7π/6	-1/2	-√3/2	√3/3	√3	-2√3/3	-2
225	5π/4	-√2/2	-√2/2	1	1	-√2	-√2
240	4π/3	-√3/2	-1/2	√3	√3/3	-2	-2√3/3
270	3π/2	-1	0	undefined	0	undefined	-1
300	5π/3	-√3/2	1/2	-√3	-√3/3	2	2√3/3
315	7π/4	-√2/2	√2/2	-1	-1	√2	-√2
330	11π/6	-1/2	√3/2	-√3/3	-√3	2√3/3	-2
360	2π	0	1	0	undefined	1	undefined



Example: Find y

a. $arcsin\frac{\sqrt{3}}{2} = y$

b. $sin^{-1}\left(\frac{-1}{2}\right) = y$

c. $sin^{-1}\left(\sqrt{2}\right) = y$

d. $arccos0 = y$

e. $cos^{-1}\left(\frac{1}{2}\right) = y$

Example: Find the degree measure:

a. $\arctan\sqrt{3} = \theta$

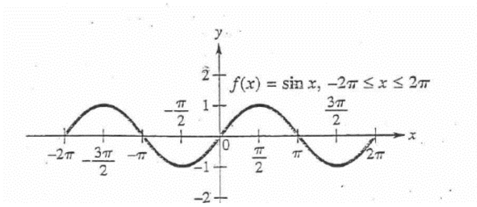
b. $csc^{-1}(-\sqrt{2}) = \theta$

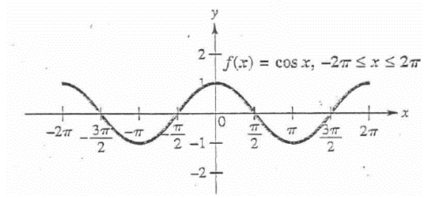
Example: Use the calculator to find

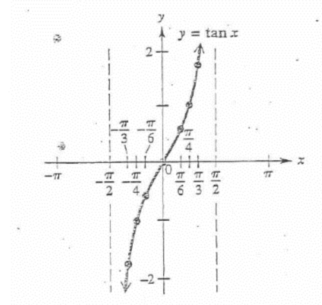
a. θ in degrees, if $\theta = \text{arccot}(-.2528)$. Be careful.

b. y if $y = sec^{-1}(-4)$

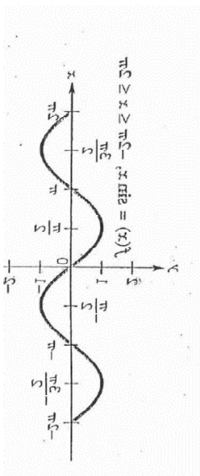
Functions

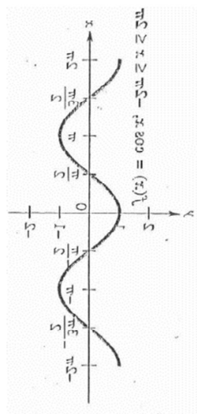


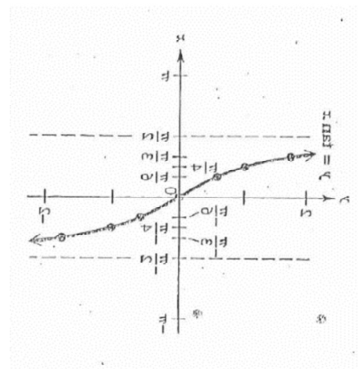




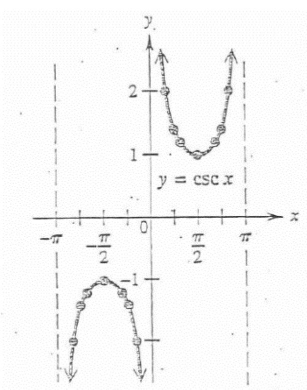
Inverses

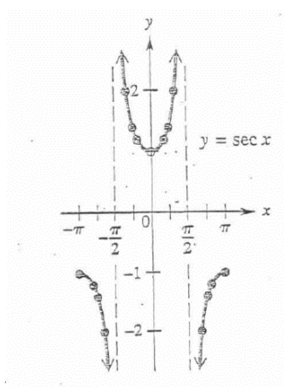


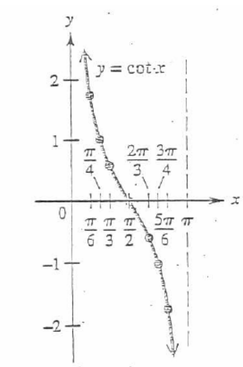




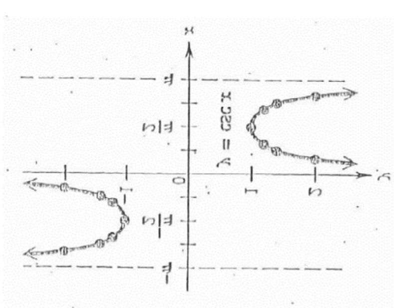
Functions

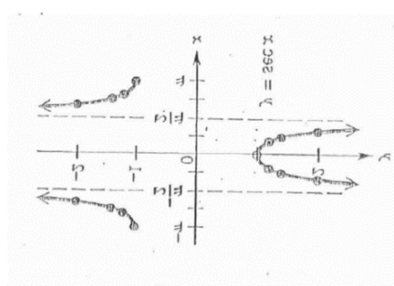


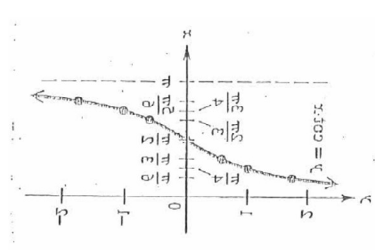




Inverses







Inverse Function	**Domain**	**Range**	**Quadrants**
$y = sin^{-1}(x)$			
$y = cos^{-1}(x)$			
$y = tan^{-1}(x)$			
$y = csc^{-1}(x)$			
$y = sec^{-1}(x)$			
$y = cot^{-1}(x)$			

Example: Evaluate each expression without a calculator.

a. $\cos\left(sin^{-1}\left(\frac{1}{3}\right)\right)$

b. $\sec\left(cot^{-1}\left(\frac{-15}{8}\right)\right)$

Example: Evaluate each expression without a calculator.

a. $\sin\left(\arctan\left(\frac{4}{3}\right) - \arccos\left(\frac{12}{13}\right)\right)$

b. $\sin(2\,\text{arccot}(-5))$

Example: Write each trigonometric expression as an algebraic expression in u.

a. $\cot(sec^{-1}u)$

b. $sin(2cos^{-1}u)$

Solving Trig Equations

Solve $3u - \sqrt{3} = 0$

Example 1: Solve $3tan\theta - \sqrt{3} = 0$, on the interval $[0°, 360°)$, by hand.

Example 2: Solve $3tan\theta - \sqrt{3} = 0$, on the interval $[0°, 360°)$, using the calculator.

Solve $uv + u = 0$

Example 3: Solve $cos\theta cot\theta = -cos\theta$, on the interval $[0°, 360°)$.

Solve $u(u+2)=1$

Example 4: Solve $cosx(cosx+2)=1$, on the interval $[0,2\pi)$.

Solve $2u^2 - u = 1$

Example 5: Solve $2cos^2x - cosx = 1$, on the interval $[0, 2\pi)$.

Example 6: Solve $\tan\theta(\tan\theta - 2) = 5$, on the interval $[0°, 360°)$.

Example 7: Solve $\cot\theta - \sqrt{3} = \csc\theta$, on the interval $[0, 2\pi)$.

For homework: If the instructions say to find solutions for an interval greater than $[0, 2\pi)$ or $[0, 360°)$, adjust it to those intervals.

1. Solve $2\cos\frac{x}{2} - \sqrt{2} = 0$ for the interval $[0, 2\pi)$.

2. Solve $\cos 2x = \sin x$ on the interval $[0, 2\pi)$.

3. Solve $2cos^2\theta - 2sin^2\theta + 1 = 0$ on the interval $[0, 360°)$.

4. Solve $sin\frac{\theta}{2} = \csc\frac{\theta}{2}$ on the interval $[0, 360°)$.

5. Solve $\sqrt{2}sin3x - 1 = 0$ on the interval $[0, 2\pi)$.

Review:

Solve $\theta = \ cot^{-1}(-.608)$ for θ.

Solve equations with Inverse Trig Functions

1. Solve $y = 4\tan 3x$ for x, where $-\frac{\pi}{6} < x < \frac{\pi}{6}$

2. Solve $3\arctan x = \pi$.

3. Solve $sec^{-1}x = csc^{-1}2$

4. Solve $\arccos x - \arcsin x = \pi$. (Extra space on following page, if needed.)

5. Solve $cot^{-1}x = \ tan^{-1}\frac{4}{3}$

7.1 Oblique Triangles and the Law of Sines

Congruence Axioms:	
Side-Angle-Side (SAS)	If two sides of the included angle of one triangle are equal, respectively, to two sides and the included angle of a second triangle, then the triangles are congruent.
Angle-Side-Angle (ASA)	If two angles and the included side of one triangle are equal, respectively, to two angles and the included side of a second triangle, then the triangles are congruent.
Side-Side-Side (SSS)	If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.

Remember that (AAA) guarantees similarity, but not congruence.

If you have two angles, the third is easily found by $A + B + C = 180°$.

Data Required for Solving Oblique Triangles*		**Strategy**
Case 1	One side and two angles are known (**SAA** or **ASA**). Find third angle with $A + B + C = 180°$.	Law of Sines (7.1)
Case 2	Two sides and one angle not included between the two sides are known (**SSA**). This case may lead to more than one triangle.	Ambiguous cases, but carefully use Law of Sines (7.2)
Case 3	Two sides and the angle included between the two sides are known (**SAS**).	Law of Cosines (7.3)
Case 4	Three sides are known (**SSS**).	Law of Cosines (7.3)
*Triangles that are not right triangles are oblique triangles.		

Law of Sines*

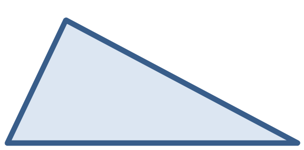
In any triangle ABC, with sides a, b, and c,

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \qquad \frac{a}{\sin A} = \frac{c}{\sin C}, \qquad \frac{b}{\sin B} = \frac{c}{\sin C}.$$

This can be written more compactly as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

*Useful if two angles and one side are known.

Labeling:



Solve triangles.

1. Solve the triangle ABC if $A = 28.8°$, $C = 102.6°$, and $c = 25.3$ inches.

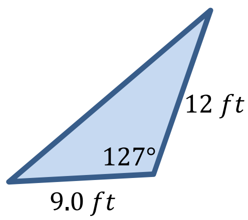
Area of a Triangle*

In any triangle ABC, the area, $\mathcal{A}$, is given by these formulas:

$$\mathcal{A} = \frac{1}{2}bc\sin A, \quad \mathcal{A} = \frac{1}{2}ab\sin C, \quad \mathcal{A} = \frac{1}{2}ac\sin B$$

*Useful if two sides and an included angle are known.

Find the area of triangle DEF in the figure.



7.2 Ambiguous Cases

Data Required for Solving Oblique Triangles*		Strategy
Case 1	One side and two angles are known (**SAA** or **ASA**). Find third angle with $A + B + C = 180°$.	Law of Sines (7.1)
Case 2	Two sides and one angle not included between the two sides are known (**SSA**). This case may lead to more than one triangle.	Ambiguous cases, but carefully use Law of Sines (7.2)
Case 3	Two sides and the angle included between the two sides are known (**SAS**).	Law of Cosines (7.3)
Case 4	Three sides are known (**SSS**).	Law of Cosines (7.3)
*Triangles that are not right triangles are oblique triangles.		

Case 2 (SSA) can result in multiple triangles. Be cautious when using the Law of Sines with these triangles.

Example 1: A segment of length L is to be drawn from the given point to the positive x-axis to form a triangle. For what values of L will you have:

One triangle?

Two triangles?

No triangles?

Example 2: How many triangles are possible?

$$B = 149.93°, c = 8.9\ m, b = 14.1\ m$$

Example 3: How many triangles are possible?

$$A = 36.4°, a = 3.2\ ft, c = 10.3\ ft$$

Example 4: How many triangles are possible?

$$A = 51.2°, c = 7986\ cm, a = 7208\ cm$$

7.3 Law of Cosines

Data Required for Solving Oblique Triangles*		**Strategy**
Case 1	One side and two angles are known (**SAA** or **ASA**). Find third angle with $A + B + C = 180°$.	Law of Sines (7.1)
Case 2	Two sides and one angle not included between the two sides are known (**SSA**). This case may lead to more than one triangle.	Ambiguous cases, but carefully use Law of Sines (7.2)
Case 3	Two sides and the angle included between the two sides are known (**SAS**).	Law of Cosines (7.3)
Case 4	Three sides are known (**SSS**).	Law of Cosines (7.3)
*Triangles that are not right triangles are oblique triangles.		

Triangle Side Length Restriction: In any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side.

Law of Cosines*

In any triangle ABC, with sides a, b, and c, the following hold:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

*Useful if two sides and one angle (SAS) are known or if three sides (SSS) are known.

Solve triangle ABC where $B = 73.5°$, $a = 28.2$ feet, and $c = 46.7$ feet.

Solve triangle ABC where $a = 25.4$ cm, $b = 42.8$ cm, and $c = 59.3$ cm.

Heron's Area Formula*

In any triangle ABC, with sides a, b, and c, the semi-perimeter is:

$$S = \frac{1}{2}(a + b + c)$$

The area is:

$$\mathcal{A} = \sqrt{S(s-a)(s-b)(s-c)}$$

*Useful when three sides (SSS) are known.

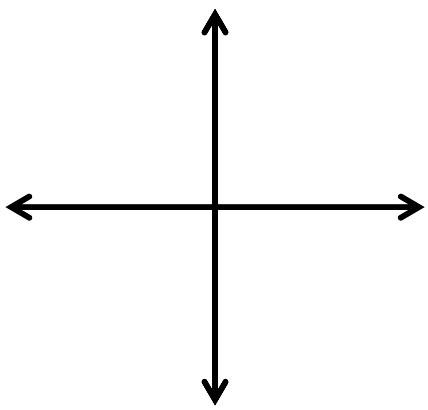
Find the area of a triangle with sides 3, 5, and 7 feet.

Put these complex numbers in order:

$2 - 4i$, $3 - 6i$, $8 - 3i$

Fact:

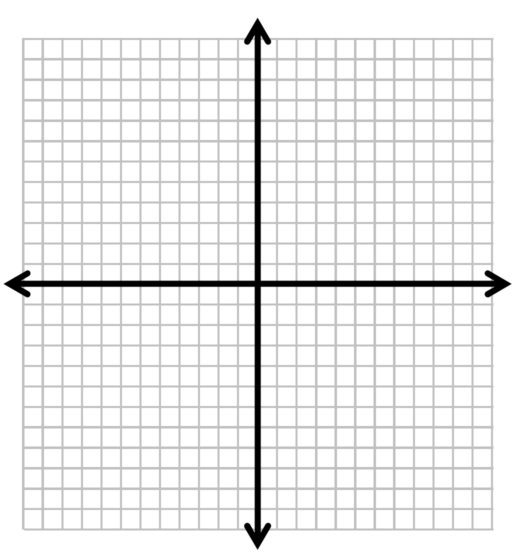
Visualize $2 + 3i$ on the rectangular coordinate system:



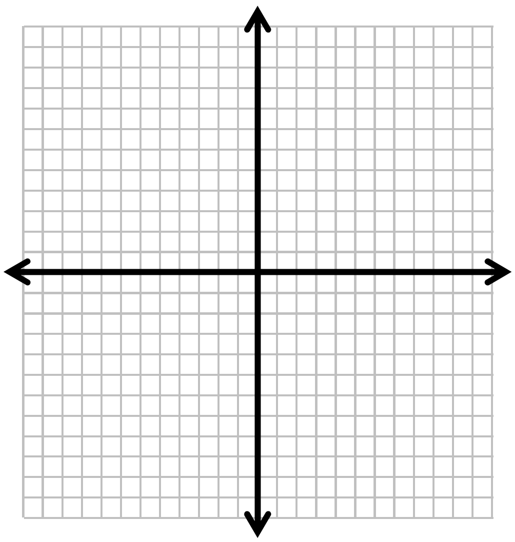
Each complex number, $a + bi$, determines a unique position vector with an initial point $(0,0)$ and terminal point (a, b).

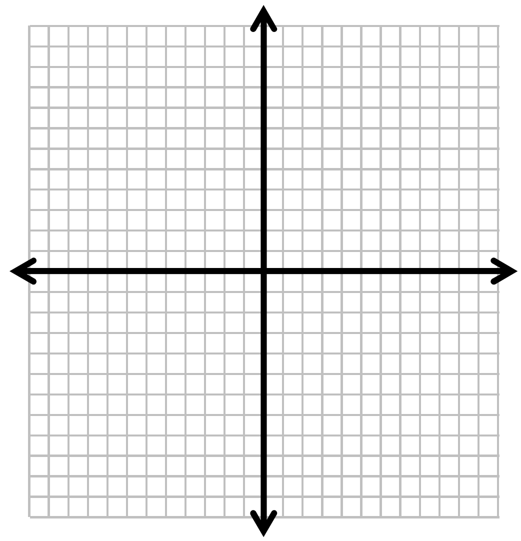
What about the sum of two complex numbers?

$(2 + i) + (3 + 4i)$



$(6-2i) + (-4-3i)$





How is this related to Trigonometry?

$x =$

$y =$

Rectangular form		Polar form
$x+yi$	$=$	$r\cos\theta + i\, r\sin\theta$
	$=$	$r(\cos\theta + i\sin\theta)$
	$=$	$r\ cis\ \theta$

Where r is

Where θ is

Express $10(\cos 135° + i \sin 135°)$ in its rectangular form.

Write in polar (i.e. trigonometric) form.

a.) $8 - 8i$

b.) -15