<u>Reciprocal Identities:</u> Fill in the blanks.

$cot\theta = \frac{1}{tan\theta}$ $tan\theta =$ ________

$sec\theta =$ ______ $cos\theta =$ _______

$csc\theta =$ ______ $sin\theta =$ _____

<u>Pythagorean Identities</u>: Fill in the blanks.

$$sin^2\theta + \_\_\_\_\_\_\_\_ = 1$$

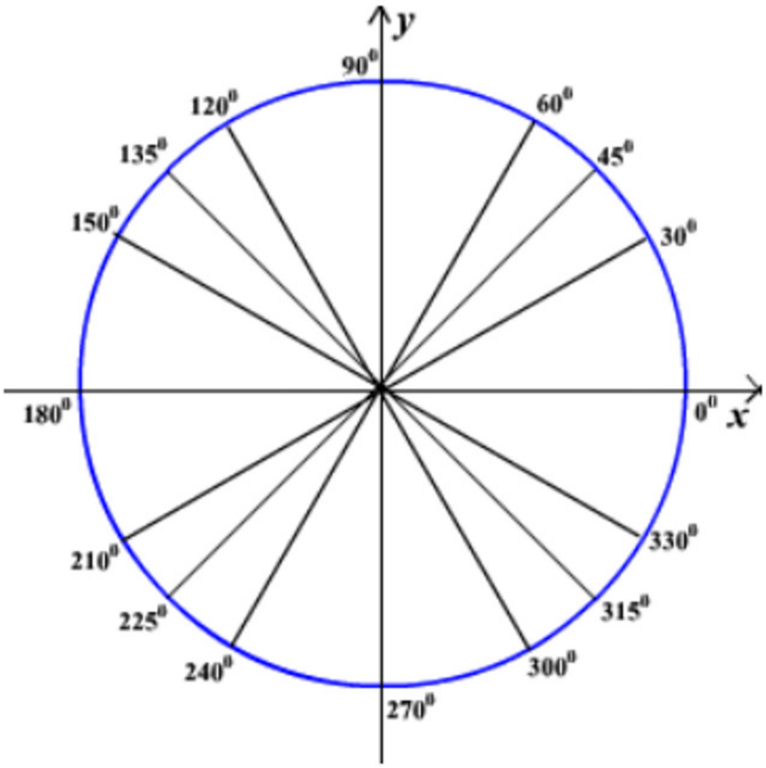
$$1 + \_\_\_\_\_\_\_\_ = \sec^2\theta$$

$$1 + \_\_\_\_\_\_\_\_ = csc^2\theta$$

<u>Quotient Identities</u>: Fill in the blanks.

$tan\theta =$ ____________ $cot\theta =$ ______________

In order to derive the negative identities, consider the unit circle.



<u>Negative Identities</u>:

$\sin(-\theta) =$ $\csc(-\theta) =$

$\cos(-\theta) =$ $\sec(-\theta) =$

$\tan(-\theta) =$ $\cot(-\theta) =$

Example:

If $cos\theta = \frac{5}{8}$ and θ is in Quadrant IV, find each value

a. $sin\theta$
b. $tan\theta$
c. $\sec(-\theta)$

Example: Write $tan\theta$ in terms of $cos\theta$.

Example: Write $sec\theta + csc\theta$ in terms of $sin\theta$ and $cos\theta$, then simplify.

<u>Reciprocal Identities:</u> Fill in the blanks.

$cot\theta = \frac{1}{tan\theta}$ $tan\theta =$ ________

$sec\theta =$ ______ $cos\theta =$ _______

$csc\theta =$ ______ $sin\theta =$ _____

<u>Pythagorean Identities</u>: Fill in the blanks.

$$sin^2\theta + \_\_\_\_\_\_\_\_ = 1$$

$$1 + \_\_\_\_\_\_\_\_ = \sec^2\theta$$

$$1 + \_\_\_\_\_\_\_\_ = csc^2\theta$$

<u>Negative Identities</u>:

$\sin(-\theta) =$ $\csc(-\theta) =$

$\cos(-\theta) =$ $\sec(-\theta) =$

$\tan(-\theta) =$ $\cot(-\theta) =$

<u>Quotient Identities</u>: Fill in the blanks.

$tan\theta = \frac{\quad}{\quad}$ $cot\theta = \frac{\quad}{\quad}$

Write $secx$ in terms of $sinx$.

Example: Write $sec\theta + csc\theta$ in terms of $sin\theta$ and $cos\theta$, then simplify.

Write each of the following in terms of sine and cosine as the first step and then simplify so that no quotients appear (using any trig functions necessary).

1. $(1 - cos\theta)(1 + sec\theta)$

2. $\frac{cos^2\,\theta - sin^2\theta}{sin\theta cos\theta}$

Write in terms of sine and cosine and simplify so that no quotients appear.

3. $\frac{1-sin^2\theta}{1+cot^2\theta}$

Reciprocal Identities: Fill in the blanks.

$cot\theta = \frac{1}{tan\theta}$

$tan\theta =$ ________

$sec\theta =$ ______

$cos\theta =$ _______

$csc\theta =$ ______

$sin\theta =$ _____

Pythagorean Identities: Fill in the blanks.

$sin^2\theta +$ ________ $= 1$

$1 +$ ________ $= \sec^2\theta$

$1 +$ ________ $= csc^2\theta$

Negative Identities:

$\sin(-\theta) =$

$\csc(-\theta) =$

$\cos(-\theta) =$

$\sec(-\theta) =$

$\tan(-\theta) =$

$\cot(-\theta) =$

Quotient Identities:

$\tan(\theta) =$

$\cot(\theta) =$

<u>Things to Try When Verifying Identities;</u>

1. Learn the Fundamental Identities.
2. Try to rewrite the more complicated side.
3. Try expressing all trigonometric functions in the equation in terms of sine and cosine.
4. Perform indicated factoring or algebraic operations.
5. Remember the formula for which you are aiming.
6. If an expression contains $1 + sinx$, multiply both numerator and denominator by $1 - \sin x$.

Verify that the following equations are an identity:

1. $\cot x \sec x \sin x = 1$

2. $cot^2\theta(tan^2\theta + 1) = csc^2\theta$

3. $\frac{tan^2 s}{sec^2 s} \quad = (1 + \cos(s))(1 - \cos(s))$

4. $\frac{\sec s+\tan s}{\sin s}=\frac{\csc s}{\sec s-\tan s}$

5. $\frac{\cot\theta-\csc\theta}{\cot\theta+\csc\theta}=\frac{1-2\cos\theta+cos^2\,\theta}{-sin^2\,\theta}$

6. $\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta} = 2\,sec^2\,\theta$

7. $\dfrac{1+\cos x}{1-\cos x}-\dfrac{1-\cos x}{1+\cos x} = 4\cot x\csc x$

8. $\frac{sin^4 \alpha - cos^4 \alpha}{sin^2 \alpha - cos^2 \alpha} = 1$

Show that $\sqrt{cos^2 \theta} = \cos \theta$ is not an identity.

<u>Reciprocal Identities:</u> Fill in the blanks.

$cot\theta = \frac{1}{tan\theta}$ $tan\theta =$ ________

$sec\theta =$ ______ $cos\theta =$ _______

$csc\theta =$ ______ $sin\theta =$ _____

<u>Pythagorean Identities</u>: Fill in the blanks

$sin^2\theta +$ ________ $= 1$

$1 +$ ________ $= \sec^2\theta$

$1 +$ ________ $= csc^2\theta$

<u>Negative Identities</u>:

$\sin(-\theta) =$ $\csc(-\theta) =$

$\cos(-\theta) =$ $\sec(-\theta) =$

$\tan(-\theta) =$ $\cot(-\theta) =$

<u>Quotient Identities</u>:

$\tan\theta =$ $cot\,\theta =$

Recall:

Given the points $(-1,-2)$ and $(-3,5)$, find the distance between them.

Label the points as follows:

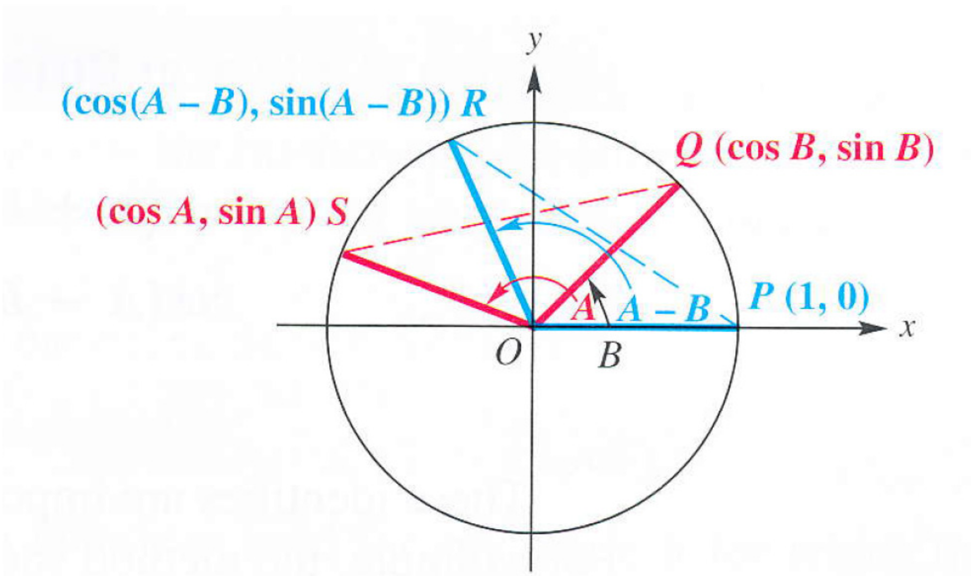
$(x_1, y_1) = (-1,-2)$ and $(x_2, y_2) = (-3,5)$.

So, $x_1 = -1$, $y_1 = -2$, and $x_2 = -3$, $y_2 = 5$.

To find the distance between the two points, use the distance formula:

$$
\begin{aligned}
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad &= \sqrt{(-3-(-1))^2 + (5-(-2))^2} \\
&= \sqrt{(-3+1)^2 + (5+2)^2} \\
&= \sqrt{(-2)^2 + (7)^2} \\
&= \sqrt{4+49} \\
&= \sqrt{53}
\end{aligned}
$$

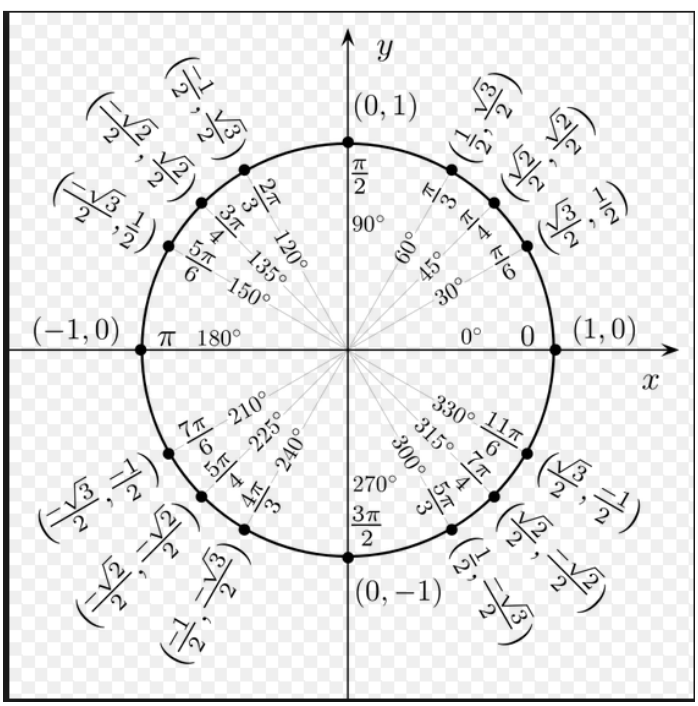
So the distance between $(-1,-2)$ and $(-3,5)$ is $\sqrt{53}$.



From the graph, we can derive the identity $\cos(A-B) = cosAcosB + sinAsinB$

Trig Functions Chart

Degrees	Radians	Sin	Cosine	Tangent	Cotangent	Secant	Cosecant
0	0	0	1	0	undefined	1	undefined
30	π/6	1/2	√3/2	√3/3	√3	2√3/3	2
45	π/4	√2/2	√2/2	1	1	√2	√2
60	π/3	√3/2	1/2	√3	√3/3	2	2√3/3
90	π/2	1	0	Undefined	0	undefined	1
120	2π/3	√3/2	-1/2	-√3	-√3/3	-2	2√3/3
135	3π/4	√2/2	-√2/2	-1	-1	- √2	√2
150	5π/6	1/2	-√3/2	-√3/3	-√3	-2√3/3	2
180	π	0	-1	0	undefined	-1	undefined
210	7π/6	-1/2	-√3/2	√3/3	√3	-2√3/3	-2
225	5π/4	-√2/2	-√2/2	1	1	-√2	-√2
240	4π/3	-√3/2	-1/2	√3	√3/3	-2	-2√3/3
270	3π/2	-1	0	undefined	0	undefined	-1
300	5π/3	-√3/2	1/2	-√3	-√3/3	2	2√3/3
315	7π/4	-√2/2	√2/2	-1	-1	√2	-√2
330	11π/6	-1/2	√3/2	-√3/3	-√3	2√3/3	-2
360	2π	0	1	0	undefined	1	undefined



From $\cos(A - B) = cosAcosB + sinAsinB$, we can derive

$$\cos(A + B) = cosAcosB - sinAsinB$$

Example: Find the exact values for each expression:

1. $\cos(-75°)$

2. $\cos\left(\frac{17\pi}{12}\right)$

3. $cos173°cos83° + sin173°sin83°$

Cofunction Identities:

1. From $\cos(A - B) = cosAcosB + sinAsinB$, we can derive the identity $\cos(90° - \theta) = sin\theta$

2. From $\cos(90° - \theta) = sin\theta$, we can derive the identity $\sin(90° - \theta) = cos\theta$

3. From $\cos(90° - \theta) = sin\theta$ and $\sin(90° - \theta) = cos\theta$, we can derive the rest of the cofunction identities:

$$\tan(90° - \theta) = cot\theta$$
$$\cot(90° - \theta) = tan\theta$$
$$\sec(90° - \theta) = csc\theta$$
$$\csc(90° - \theta) = sec\theta$$

Example: Find an angle θ that satisfies each of the following:

1. $sec\theta = csc62°$

2. $cos\theta = sin\left(\frac{7\pi}{6}\right)$

Ex: Use $\cos(A + B) = cosAcosB - sinAsinB$ to rewrite $\cos(90° + \theta)$ in its simplest form.

Ex: Suppose $cosu = \frac{15}{17}$, $sinv = \frac{-24}{25}$, and u and v are in Quadrant IV. Find $\cos(u - v)$.

Reciprocal Identities: **Fill in the blanks.**

$cot\theta = \frac{1}{tan\theta}$ $tan\theta =$ ________

$sec\theta =$ ______ $cos\theta =$ _______

$csc\theta =$ _______ $sin\theta =$ _____

Pythagorean Identities: **Fill in the blanks**

$$sin^2\theta + \text{\_\_\_\_\_\_\_\_} = 1$$

$$1 + \text{\_\_\_\_\_\_\_\_} = \sec^2\theta$$

$$1 + \text{\_\_\_\_\_\_\_\_} = csc^2\theta$$

Negative Identities:

$\sin(-\theta) =$ $\csc(-\theta) =$

$\cos(-\theta) =$ $\sec(-\theta) =$

$\tan(-\theta) =$ $\cot(-\theta) =$

Quotient Identities:

$\tan\theta =$ $cot\,\theta =$

Formula for $\sin(A + B)$

Now we will derive identities for sine: Recall

$\cos(A - B) = cosAcosB + sinAsinB$, and $sin\theta = \cos(90° - \theta)$

Combining the two for $\theta = (A + B)$, we get

$$\sin(A + B) = \cos(90° - (A + B))$$

Formula for $\sin(A - B)$

Now we can derive a formula for $\sin(A + B)$ from the identity

$\sin(A + B) = sinAcosB + sinBcosA$

$\sin(A - B) =$

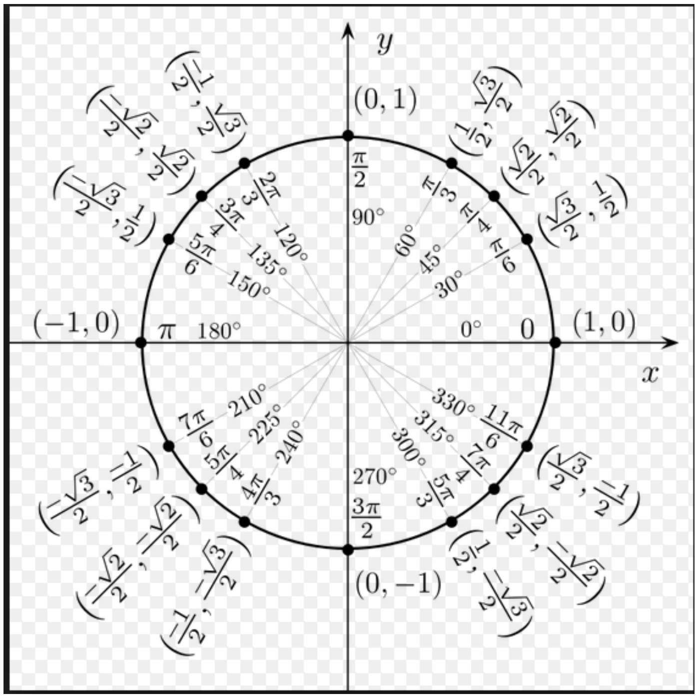
Formula for $\tan(A + B)$

Then combining $\sin(A+B) = sinAcosB + sinBcosA$
and $\cos(A+B) = cosAcosB - sinAsinB$ we get:

$\tan(A+B) =$

<u>Formula for $\tan(A-B)$</u>
Use $\tan(A+B)$=$\frac{tanA+tanB}{1-tanAtanB}$

$\tan(A-B) =$



Example: Find the exact values of

1. $\sin(-15°)$

2. $tan\left(\frac{13\pi}{12}\right)$

3. $\frac{tan100° - tan70°}{1 + tan100° tan70°}$

Example: Write each function as an expression involving functions of θ.

1. $\sin(\theta - 270°)$

2. $\tan(\theta + 3\pi)$

Example: Suppose that A and B are angles in standard position with $cosA = \frac{-7}{25}$ for $\pi < A < \frac{3\pi}{2}$ and $sinB = \frac{-3}{5}$ where $\frac{3\pi}{2} < B < 2\pi$. Find the following

1. $\sin(A - B)$
2. $\tan(A - B)$
3. The quadrant of $(A - B)$.

Time permitting:

Example: Verify that $\tan\left(\frac{\pi}{4}+t\right)+tan\left(\frac{\pi}{4}-t\right)=\frac{2sec^2t}{1-tan^2t}$

Reciprocal Identities: **Fill in the blanks.**

$cot\theta = \frac{1}{tan\theta}$ $tan\theta =$ ________

$sec\theta =$ ______ $cos\theta =$ _______

$csc\theta =$ ______ $sin\theta =$ _____

Pythagorean Identities: **Fill in the blanks**

$sin^2\theta +$ ________ $= 1$

$1 +$ ________ $= \sec^2\theta$

$1 +$ ________ $= csc^2\theta$

Negative Identities:

$\sin(-\theta) =$ $\csc(-\theta) =$

$\cos(-\theta) =$ $\sec(-\theta) =$

$\tan(-\theta) =$ $\cot(-\theta) =$

Quotient Identities:

$\tan\theta =$ $cot\,\theta =$

Formula $\cos(2A) = cos^2A - sin^2A$

Recall: $\cos(A + B) = cosAcosB - sinAsinB$

$\cos(2A) =$

Example:

$\cos(60°) =$

Note: $\boldsymbol{cos(2A) \neq 2cosA}$

Formula for $\cos(2A) = 1 - 2sin^2A$

Recall: $\cos(2A) = cos^2A - sin^2A$ and $cos^2A + sin^2A = 1$.

$\cos(2A)$

Example:

$\cos(60°) =$

Formula $\cos(2A) = 2cos^2A - 1$
Recall: $\cos(2A) = cos^2A - sin^2A$ and $cos^2A + sin^2A = 1$.

Example:
$\cos(60°) =$

Formula $\sin(2A) = 2sinAcosA$
Recall: $\sin(A + B) = sinAcosB + sinBcosA$.

$\sin(2A) =$

Example:
$\sin(60°) =$

Formula $\tan(2A) = \frac{2tanA}{1-tan^2A}$

Recall: $\tan(A+B)=\frac{tanA+tanB}{1-tanAtanB}$

$\tan(2A) =$

Example:
$\tan(60°) =$

Example: Given $sin\theta = \frac{8}{17}$ and $cos\theta < 0$. Find the values for

1. $\sin(2\theta)$

2. $cos(2\theta)$

3. $tan(2\theta)$

Example: Find the value of the six trigonometric functions for θ, if $cos2\theta = \frac{4}{5}$ and $90° < \theta < 180°$

Example: Verify that $cos^4\beta - sin^4\beta = cos2\beta$.

Example: Simplify each expression

1. $2cos^2 5x - 1 =$

2. $sin165° cos165° =$

Example: Write $cos3x$ in terms of $cosx$.

Formula for $\boldsymbol{sin}\left(\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)$

$\cos(2A) = 1 - 2sin^2A$

Formula for $\boldsymbol{cos}\left(\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)$

Recall $\cos(2A) = 2cos^2A - 1$

Formulas for $\boldsymbol{tan}\left(\frac{\boldsymbol{x}}{\boldsymbol{2}}\right)$

$$\boldsymbol{sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - cosx}{2}}}$$

$$\boldsymbol{cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + cosx}{2}}}$$

$$\boldsymbol{tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - cosx}{1 + cosx}}}$$

$$\boldsymbol{tan\left(\frac{x}{2}\right) = \frac{sinx}{cosx + 1}}$$

$$\boldsymbol{tan\left(\frac{x}{2}\right) = \frac{1 - cosx}{sinx}}$$

Use half angle identities to find the exact value of

1. $sin22.5°$

2. $tan75°$

$$\boldsymbol{sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{2}}}$$

$$\boldsymbol{cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+cosx}{2}}}$$

$$\boldsymbol{tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{1+cosx}} = \frac{sinx}{cosx+1} = \frac{1-cosx}{sinx}}$$

Example: Given $\cos s = \frac{-3}{7}$ with $\pi < s < \frac{3\pi}{2}$, find

1. $\sin\frac{s}{2}$

2. $\cos\frac{s}{2}$

$$\boldsymbol{sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{2}}} \qquad \boldsymbol{cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+cosx}{2}}}$$

$$\boldsymbol{tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-cosx}{1+cosx}} = \frac{sinx}{cosx+1} = \frac{1-cosx}{sinx}}$$

3. $\tan\frac{s}{2}$