Boolean Algebra and DeMorgan's Theorems
1. Apply DeMorgan's Theorems to each Expression:

a.) $\overline{A + \overline{B}}$

$= \overline{A}\,\overline{\overline{B}}$

$= \overline{A}B$

b) $\overline{(A + \overline{B})(\overline{C} + D)}$

$= (\overline{A}\,\overline{\overline{B}}) + (\overline{\overline{C}}\,\overline{D})$

$= \overline{A}B + C\overline{D}$

c. $\overline{A\overline{B}(C + \overline{D})}$

$= \overline{A\overline{B}C + A\overline{B}\,\overline{D}}$

$= \overline{A}B\overline{C} + \overline{A}BD$

Logic Simplification using Boolean Algebra
Using Boolean algebra, simplify each expression:

a. $A(A + \overline{A}B)$

$= \cancel{AA} + \cancel{A\overline{A}}B$

$= A + AB$

$= A$

b) $(A+\bar{B})(A+C)$

$= AA + AC + \bar{B}A + \bar{B}C$

$= A + AC + \bar{B}A + \bar{B}C$

$= A + \bar{B}A + \bar{B}C$

$= A + \bar{B}C$

c.) $AB + (\bar{A} + \bar{B})C + AB$

$= AB + \bar{A}C + \bar{B}C + AB$

$= AB + \bar{A}C + \bar{B}C$

D. $\bar{A}\bar{B}C + \overline{(A+B+\bar{C})} + \bar{A}\bar{B}\bar{C}D$

$= \bar{A}\bar{B}C + \overline{AB\bar{C}} + \bar{A}\bar{B}\bar{C}D$

$= \bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D$

$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D$

$= \bar{A}\bar{B}(\bar{C}D + C)$

$= \bar{A}\bar{B}(C + D)$

$= \bar{A}\bar{B}C + \bar{A}\bar{B}D$

Karnaugh Map and K-Map SOP Minimization
Use a Karnaugh map to simplify each expression to a minimum SOP form.

a) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C$

$\quad 000 \qquad 001 \qquad 101$

AB \ C	0	1
00	1	1
01		
11		
10		1

$\bar{A}\bar{B} + \bar{B}C$

b.) $AC(\bar{B} + C)$

$= A\bar{B}C + ACC$

$= A\bar{B}C + AC$

$\quad 1\ 0\ 1 \qquad \cancel{1\ 0\ 1}$

$\qquad\qquad\quad 1\ 1\ 1$

AB \ C	0	1
00		
01		
11		1
10		1

AC

c.) $\bar{A}(BC + B\bar{C}) + A(BC + B\bar{C})$

$= \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

$\quad 0\ 1\ 1 \quad 0\ 1\ 0 \quad 1\ 1\ 1 \quad 1\ 1\ 0$

AB \ C	0	1
00		
01	1	1
11	1	1
10		

B

d.) $\bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC + AB\bar{C}$

$\quad 0\ 0\ 0 \quad 1\ 0\ 1 \quad 0\ 1\ 1 \quad 1\ 1\ 0$

AB \ C	0	1
00	1	
01		1
11	1	
10		1

$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} + A\bar{B}C$

no minimization possible

Karnaugh Map and K-Map POS Minimization

Use a Karnaugh map to simplify each expression to a minimum POS form:

a. $(A+B+C)(\bar{A}+\bar{B}+\bar{C})(A+\bar{B}+C)$

0 0 0   1 1 1   0 1 0

AB \ C	0	1
0 0	0	
0 1	0	
1 1		0
1 0		

$(A+C)(\bar{A}+\bar{B}+\bar{C})$

b.) $(X+\bar{Y})(\bar{X}+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+\bar{Y}+Z)$

0 1   1 0   0 1 1   1 1 0

0 0 1   1 0 0   0 1 1   1 1 0

XY \ Z	0	1
0 0		0
0 1		0
1 1	0	
1 0	0	

$(X+\bar{Z})(\bar{X}+Z)$