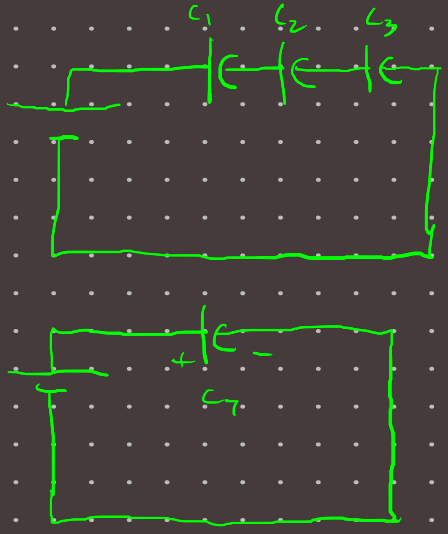
Chapter 12

Capacitors in series



$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$Q = CV$$

$$I = \frac{Q}{T} \qquad Q = I \cdot t$$

$$Q_1 = Q_2 = Q_3 = Q$$

$$-E + V_1 + V_2 + V_3 = 0$$

$$E = V_1 + V_2 + V_3$$

$$Q_1 = C_1 V_1 \qquad V_1 = \frac{Q_1}{C_1}$$

$$Q_2 = C_2 V_2 \qquad V_2 = \frac{Q_2}{C_2}$$

$$Q_3 = C_3 V_3 \qquad V_3 = \frac{Q_3}{C_3}$$

$$Q = E C_T \qquad E = \frac{Q}{C_T}$$

$$\frac{Q}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

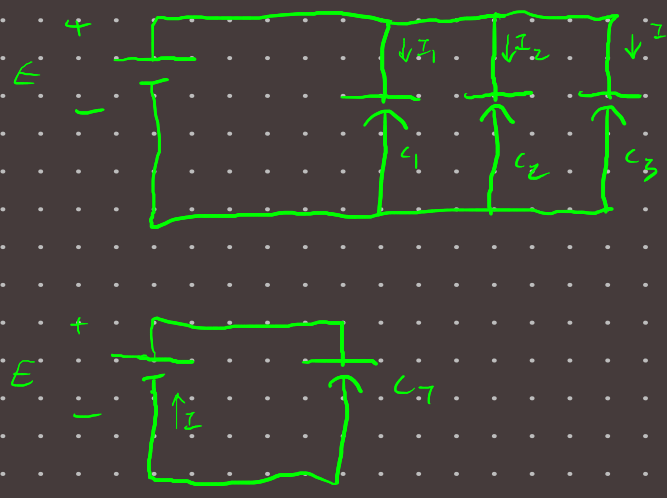
$$= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors act like $\frac{1}{R}$

So capacitors in series acts like resistors in parallel, where current is the same and voltage drops,

Whereas capacitors in parallel acts like resistors in series where voltage stays the same and current drops



KCL

$$I = I_1 + I_2 + I_3$$

$$I t = (I_1 + I_2 + I_3)\, t$$

$$Q = Q_1 + Q_2 + Q_3$$

$$Q = CV$$

$$Q = C_T \mathcal{E}$$

$$V_1 = V_2 = V_3 = E$$

$$C_T \mathcal{E} = C_1 \mathcal{E} + C_2 \mathcal{E} + C_3 \mathcal{E}$$

$$= \mathcal{E}(C_1 + C_2 + C_3)$$

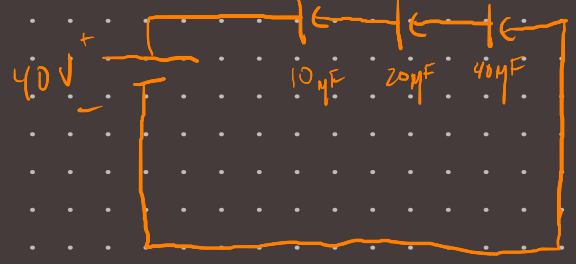
$$C_T = C_1 + C_2 + C_3$$

Energy that is stored in a capacitor

$$W = \frac{1}{2} CV^2 \text{ J} \qquad Q = CV$$

$$= \frac{1}{2C} C^2 V^2 = \frac{1}{2C} Q^2$$

Example: $V_x = \frac{C_T}{C_x} E$



$$\frac{1}{C_T} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40}$$

$$\frac{1}{C_T} = \frac{3}{70} \text{ μF}$$

$$C_T = 5.71 \text{ μF}$$

$$Q = (5.71 \text{ μF})(40 \text{ V}) = 228.4 \text{ μC}$$

$$Q_1 = Q_2 = Q_3 = 228.4 \text{ μC}$$

$Q = CV$

$$V = \frac{Q}{C}$$

$$V_1 = \frac{228.4 \text{ μC}}{10 \text{ μF}} = 22.28 \text{ V}$$

$$V_2 = \frac{228.4 \text{ μC}}{20 \text{ μF}} = 11.14 \text{ V}$$

$$V_3 = \frac{228.4 \text{ μC}}{40 \text{ μC}} = 5.71 \text{ V}$$

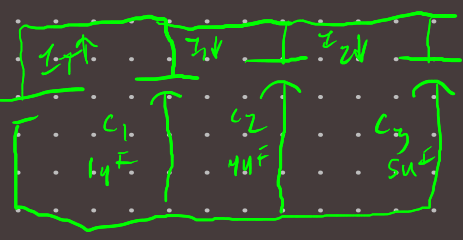
$$W = \frac{1}{2} CV^2 = \frac{1}{2} (5.71 \text{ mS})(40)^2$$

$$= 4.568 \text{ mS}$$

Example

$C_T = C_1 + C_2 + C_3$
$= 6\mu F + 4\mu F + 5\mu F$

$C_T = 10\mu F$



$Q = CV$

$Q_1 = (6\mu F)(50) = 300\mu C$

$Q_2 = (4\mu F)(50) = 200\mu C$

$Q_3 = (5\mu F)(50) = 250\mu C$

$Q_T = 500\mu C$

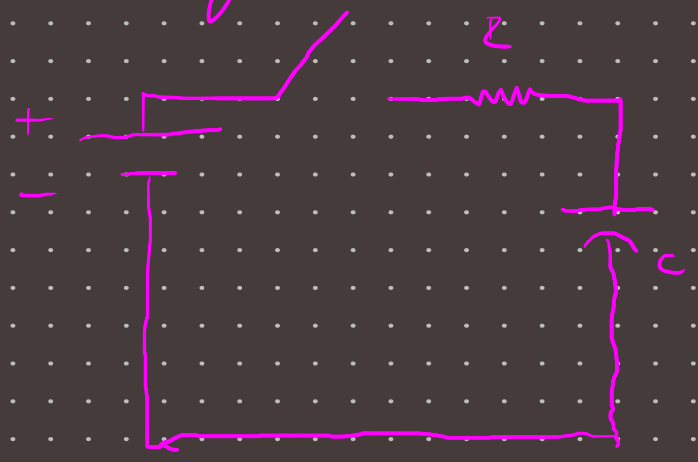
$W = \frac{1}{2} CV^2 = \frac{1}{2} (10\mu F)(50)^2$

C_T

Charging And Discharging

$t = 0^-$ right before we close the switch

$t = 0^+$ right after we close the switch



at $t = 0^-$ $V_C(\bar{0}) = 0$

$i_C = C \frac{dV}{dt}$

The voltage across the capacitor cannot change instantaneously

$V_C(\bar{0}) = V_C(\overset{+}{0})$

after closing the switch, $I = -E + IR + 0 = 0$

$I = \frac{E}{R}$

$I_i = \frac{E}{R}$

$I_{initial}$

At $t = 0^+$

$I_i = \frac{E}{R}$ $V_c(0^+) = 0$

After a long time $t \to \infty$

capacitor will get fully charged

$V_c(\infty) = E$

$-E + I_R + E = 0$ $I = 0$

$I_R = 0$ $V_R(\infty) = E$

$V_c(t) = V_f + (V_i - V_f)e^{-t/T}$

$= E + (0 - E)e^{-t/T}$

$V_{c(t)} = E(1 - e^{-t/T})$

$T = RC$

T_{AV} = Time Constant