Ren Nilsson - 10783 - Compulsory Assignment

Contents

Exercise 1

Optimization using the simplex method:

```
% The augmented matrix:
clc; clear;
A = [1  2  0  1  0  0  0  28;
     2  0  4  0  1  0  0  16;
     0  1  1  0  0  1  0  12;
    -2 -5 -3  0  0  0  1  0]
```

```
A =

     1     2     0     1     0     0     0    28
     2     0     4     0     1     0     0    16
     0     1     1     0     0     1     0    12
    -2    -5    -3     0     0     0     1     0
```

We want to bring x_2 in, since $A(4,2) < A(4,3) < A(4,1)$, and thus increases $A(4,8)$ the most. The pivot row should be $A(3,2)$, since

$$\frac{A(3,8)}{A(3,2)} = 12 < \frac{A(1,8)}{A(1,2)} = 14$$

```
A(1,:) = A(1,:)-2*A(3,:);

A(4,:) = A(4,:) + 5*A(3,:)
```

```
A =

     1     0    -2     1     0    -2     0     4
     2     0     4     0     1     0     0    16
     0     1     1     0     0     1     0    12
    -2     0     2     0     0     5     1    60
```

Next, we bring in x_1, since $A(4,1)$ is the only negative value in $A(4,:)$. $A(1,1)$ is the new pivot, since

$$\frac{A(1,8)}{A(1,1)} = 4 < \frac{A(2,8)}{A(2,1)} = 8$$

```
A(2,:) = A(2,:) -2*A(1,:);

A(4,:) = A(4,:) + 2*A(1,:)

A =

     1     0    -2     1     0    -2     0     4
     0     0     8    -2     1     4     0     8
     0     1     1     0     0     1     0    12
     0     0    -2     2     0     1     1    68
```

Last, we bring in x_3, with pivot $A(2,3)$, since $A(1,3)$ is negative and

$$\frac{A(2,8)}{A(2,1)} = 1 < \frac{A(3,8)}{A(3,1)} = 12$$

```
A(2,:) = A(2,:)/8;

A(1,:) = A(1,:) + 2*A(2,:);

A(3,:) = A(3,:) - A(2,:);

A(4,:) = A(4,:) + 2*A(2,:)

A =

  Columns 1 through 7

    1.0000         0         0    0.5000    0.2500   -1.0000         0
         0         0    1.0000   -0.2500    0.1250    0.5000         0
         0    1.0000         0    0.2500   -0.1250    0.5000         0
         0         0         0    1.5000    0.2500    2.0000    1.0000

  Column 8

    6.0000
    1.0000
   11.0000
   70.0000
```

Thus the maximum of the problem is 70, which is achieved when $x_1 = 6$, $x_2 = 1$ and $x_3 = 11$.

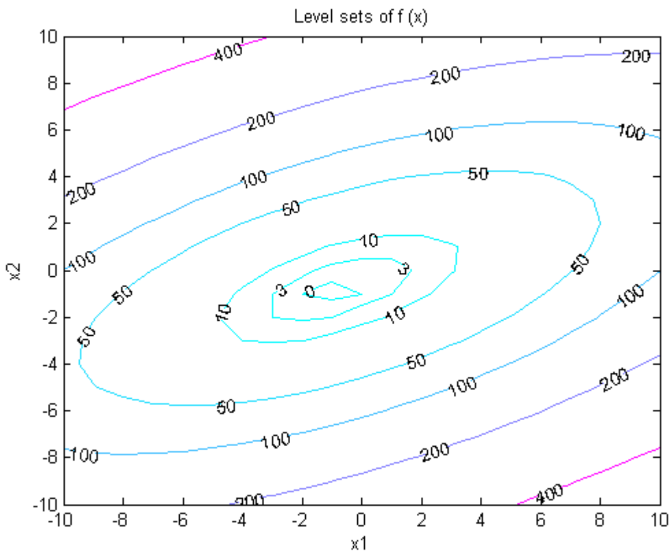
Exercise 2

Minimizing the function $f(x) = x_1^2 + 3 * x_2^2 - 2 * x_1 * x_2 + 3 * x_2$. 1. Write f on the form: $\frac{1}{2} * x^T * Q * x - x^T * b$:

```
Q = [2 -2;-2 6]
b = [0;-3]


Q =

     2    -2
    -2     6


b =

     0
    -3
```

2. Sketch the levels set for f and the gradient of f in the point $(1, 1)^T$ in a x_1, x_2-coordinate system.

```
[x1, x2] = meshgrid(-10:1:10,-10:1:10);
x = [x1;x2];
f1 = x1.^2+3*x2.^2-2*x1.*x2+3*x2;
[C,h1] = contour(x1,x2,f1,[0 3 10 50 100 200 400]);
set(h1,'ShowText','on','TextStep',get(h1,'LevelStep')*2)
colormap cool
title('Level sets of f (x)')
xlabel('x1')
ylabel('x2')
```



Gradient of f in the point $(1,1)^T$:

```
g1 = Q*[1;1]-b

g1 =

     0
     7
```

3. Find all point satisfying the FONC. Do these points satisfy the SONC?

The gradient of f is found to be:

$$f'(x) = Q * x - b$$

Finding the point satisfying the FONC equals solving the following:

$$Q * x - b = 0$$

This gives the following point:

```
x = inv(Q)*b
```

```
x =

   -0.7500
   -0.7500
```

The Hessian is found to be $F(x) = Q$. Thus, the SONC is to test whether Q > 0:

```
format short
eigs(Q)
```

```
ans =

    6.8284
    1.1716
```

Since all eigenvalues of Q is positive, Q > 0 then the SONC is satisfied in all points satisfiying the FONC.

4. Find the minimum of f over R^2

Since both the FONC and SOSC is satisfied by the previous piont, this is the global minimum:

```
x
```

```
x =

   -0.7500
   -0.7500
```

Exercise 3