(a) LIF Forward Propagation

Forward Flow

→ Charge
→ Decay
→ Reset
→ Fire

S_t

$\Theta(\cdot)$

U_t $\quad 1 - 1/\tau \quad$ U_{t+1}

C_t

(c) LIF Backward Propagation Though Time

Loss Function $\mathcal{L}$

Last Layer $\quad$ ...... $\quad$ ...... $\quad$ ......

Layer l

S_t $\quad$ S_{t+1} $\quad$ ...... $\quad$ S_T

$\mathcal{T}_{t+1}$ $\quad$ $\mathcal{T}_{t+\cdots}$

$\mathcal{P}_t$ $\quad$ $\sum$ $\quad$ $\mathcal{T}_T$

U_t $\quad$ U_{t+1} $\quad$ ...... $\quad$ U_T

$$\frac{\partial \mathcal{L}}{\partial U_t} = \mathcal{P}_t + \sum_{t'=t+1}^{T} \mathcal{T}_{t'} \approx \mathcal{P}_t + \mathcal{T}_{t+1}$$

(b) CLIF Forward Propagation

Forward Flow

→ Charge
→ Decay
→ Reset
→ Fire

S_t $\quad$ m_t

$\Theta(\cdot)$ $\quad$ $\times$

$\sigma(\cdot)$
$1/\tau$

U_t $\quad 1 - 1/\tau \quad$ U_{t+1}

C_t

(d) CLIF Backward Propagation Though Time

Loss Function $\mathcal{L}$

Last Layer $\quad$ ...... $\quad$ $\mathcal{T}_{t+1}$ $\quad$ ...... $\quad$ $\mathcal{T}_{t+\cdots}$ $\quad$ $\mathcal{T}_T$ ......

m $\quad$ S_t $\quad$ S_{t+1} $\quad$ ...... $\quad$ S_T

Layer l

$\mathcal{T}_{t+1}$ $\quad$ $\mathcal{T}_{t+\cdots}$

$\mathcal{P}_t$ $\quad$ $\sum$ $\sum$ $\quad$ $\mathcal{T}_T$

U_t $\quad$ U_{t+1} $\quad$ ...... $\quad$ U_T

$$\frac{\partial \mathcal{L}}{\partial U_t} = \mathcal{P}_t + \sum_{t'=t+1}^{T} \mathcal{T}_{t'} + \sum_{t'=t+1}^{T} \mathcal{T}_{t'} \approx \mathcal{P}_t + \mathcal{T}_{t+1} + \sum_{t'=t+1}^{T} \mathcal{T}_{t'}$$