

# **Lattice codes and sphere decoding**

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# Symbols and abbreviations

## Symbols

$\mathbb{C}$  Field of complex numbers

$\mathbf{X}$  Matrix

## Operators

$\|\cdot\|$  Euclidean norm

## Abbreviations

SNR Signal to noise ratio

# 1 Introduction

Wireless communication has been a crucial part of modern information technology for a couple of decades now. It is facing a lot of practical everyday problems which motivate the ongoing extensive research on the field. One of these problems is the noise and fading that occurs on wireless channels due to obstacles and radiation from the surroundings. To avoid data loss during transmission via wireless link one has to encode the data to be sent in such a robust way that it can still be recovered at the receiving end even in the presence of noise and fading of reasonable scale.

One way to tackle this problem, and the method this thesis focuses on, is the use of space-time lattice codes and sphere decoding. From a mathematical point of view the process of decoding can then be viewed as a problem of finding the closest lattice point to a given input vector, that is, the possibly noisy vector containing the data we receive from the wireless channel. This problem in its general form is known to be NP-hard but for communications applications, where the dimensionality and lattice shape are kept reasonable, there exist algorithms, like the sphere decoder, that offer polynomial expected complexity [1].

## 2 Lattices in communications technology

Before we go into lattice codes let us start with the definition of a lattice: Let  $\mathbf{a}_1, \dots, \mathbf{a}_n$

### 2.1 Space-time lattice codes

### 2.2 Closest vector problem

### 3 Sphere decoder

## 4 Simulations and results



## 5 Summary

## References

- [1] Mäki, M. *Space-time block codes and the complexity of sphere decoding*. Doria, Referenced 10.7.2017. Available: <https://www.doria.fi/bitstream/handle/10024/54404/gradu2008maki-miia.pdf>
- [2] Conway, J.H. and Sloane, N.J.A. *Sphere packings, lattices and groups*. Third edition, New York, Springer–Verlag, 1998.
- [3] Cassels, J.W.S. *An introduction to the Geometry of Numbers*, New York, Springer–Verlag, 1971.