Lattice codes and sphere decoding

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ABSTRACT OF THE BACHELOR'S SPECIAL ASSIGNMENT

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Symbols and abbreviations

Symbols

$\mathbb{C}$	Field of complex numbers
$\mathbf{x}$	Vector
$\mathbf{X}$	Matrix

Operators

$\|\cdot\|$	Euclidean norm

Abbreviations

SNR	Signal to noise ratio

1 Introduction

Wireless communication has been a crucial part of modern information technology for a couple of decades now. It is facing a lot of practical everyday problems which motivate the ongoing extensive research on the field. One of these problems is the noise and fading that occurs on wireless channels due to obstacles and radiation from the surroundings. To avoid data loss during transmission via wireless link one has to encode the data to be sent in such a robust way that it can still be recovered at the receiving end even in the presence of noise and fading of reasonable scale.

One way to tackle this problem, and the method this thesis focuses on, is the use of space–time lattice codes and sphere decoding. From a mathematical point of view the process of decoding can then be viewed as a problem of finding the closest lattice point to a given input vector, that is, the possibly noisy vector containing the data we receive from the wireless channel. This problem in its general form is known to be NP-hard but for communications applications, where the dimensionality and lattice shape are kept reasonable, there exist algorithms, like the sphere decoder, that offer polynomial expected complexity [1].

2 Lattices in communications technology

Before we go into communications applications of lattices such as lattice codes, let us start off with the definition of a lattice. Let n and m be positive integers such that $n \leq m$ and $\mathbf{b}_1, ..., \mathbf{b}_n \in \mathbb{R}^m$ be linearly independent vectors. A subset Λ of $\mathbb{R}^m$ is called a lattice of dimension n if it is defined as

$$\Lambda = \sum_{i=1}^{n} \mathbb{Z}\mathbf{b}_i = \{a_1\mathbf{b}_1 + ... + a_n\mathbf{b}_n | a_i \in \mathbb{Z}, 1 \leq i \leq n\} \tag{1}$$

where the set of vectors $\mathbf{b}_1, ..., \mathbf{b}_n \in \mathbb{R}^m$ is called the basis of the lattice Λ. All points of the lattice can be obtained as a linear combination of the basis vectors $\mathbf{b}_n$ and integer coefficients $\mathbf{a}_n$ as stated in (1). The number of basis vectors n is also called the rank of the lattice. The points of the lattice Λ form a group under addition which means that if $\mathbf{x} \in \Lambda$ then $-\mathbf{x} \in \Lambda$ and if $\mathbf{x}, \mathbf{y} \in \Lambda$ then $\mathbf{x} \pm \mathbf{y} \in \Lambda$. [3]

There is also a matrix representation for the same lattice

$$\Lambda = \{\mathbf{x} | \mathbf{x} = \mathbf{B}\mathbf{a}\} \tag{2}$$

where $\mathbf{B} = [\mathbf{b}_1, ..., \mathbf{b}_n]$ is an $m \times n$ matrix called the generator matrix of the lattice Λ and $\mathbf{a}$ is an n-dimensional integer column vector. Note that $\mathbf{B}$ is not uniquely determined by the lattice as in fact there exists infinitely many bases for the same lattice. If $\mathbf{B}$ is a basis for lattice Λ then $\mathbf{B}'$ is also a basis for the same lattice if the following holds

$$\mathbf{B}' = \mathbf{W}\mathbf{B}, \tag{3}$$

$$\det(\mathbf{W}) = \pm 1 \tag{4}$$

where $\mathbf{W}$ is an $m \times m$ matrix with integer entries.

2.1 Space–time lattice codes

2.2 Closest vector problem

3 Sphere decoder

4 Simulations and results

5 Summary

References

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