The solution to (**??**) is determined by the elementary quadratures

$$\sigma_0(t) = -\varepsilon c_1 c_0 c_3^2 \left[\sin c_1 t \int \cos c_1 t \mu dt - \cos c_1 t \int \sin c_1 t \mu dt\right]$$
$$+ A\cos c_1 t + B \sin c_1 t \tag{7.12}$$

where $\mu = \mu_1 \cos 3c_1 t + \mu_2 \sin 3c_1 t$.

8 The metric

All our equations being completely solved, we can explicitly construct the corresponding metric (**??**)

$$\mathrm{d}s^2 = (v_t + u_{xx})(\mathrm{d}t^2 + \mathrm{d}x^2) \mp \Lambda^{-1/2}(\mathrm{d}y^2 + \mathrm{d}z^2)$$
$$- 2b(\mathrm{d}t\,\mathrm{d}z - \mathrm{d}x\,\mathrm{d}y) + 2c(\mathrm{d}t\,\mathrm{d}y + \mathrm{d}x\,\mathrm{d}z). \tag{8.1}$$

The metric coefficients are defined as follows

$$u_{xx} = -\frac{1}{c_0\mu} e^{-c_1 x}\left(\pm\Lambda^{1/2} + \nu\right) \mp e^{-2c_1 x}\Lambda^{-1/2} \tag{8.2}$$

$$-b = v_z - u_{xy} = \frac{1}{c_0 c_1 c_3 \mu}\left[-\frac{\mu'}{\mu}\left(\nu \pm \Lambda^{1/2}\right) + \nu' \pm \frac{1}{2}\Lambda^{-1/2}\Lambda_t\right]\cos\theta$$
$$+ \left[\frac{1}{c_0 c_3 \mu}\left(\pm\Lambda^{1/2} + \nu\right) \pm c_3 e^{-c_1 x}\Lambda^{-1/2}\right]\sin\theta \tag{8.3}$$

$$c = v_y + u_{xz} = \frac{1}{c_0 c_1 c_3 \mu}\left[-\frac{\mu'}{\mu}\left(\nu \pm \Lambda^{1/2}\right) + \nu' \pm \frac{1}{2}\Lambda^{-1/2}\Lambda_t\right]\sin\theta$$
$$- \left[\frac{1}{c_0 c_3 \mu}\left(\pm\Lambda^{1/2} + \nu\right) \pm c_3 e^{-c_1 x}\Lambda^{-1/2}\right]\cos\theta \tag{8.4}$$

while for v_t we can use the r.h.s. of equation (**??**). Here Λ is defined in (**??**) whence it follows

$$\Lambda_t = 2c_0\mu'\left(c_3^2 e^{-c_1 x} - \sigma\right) - 2c_0\mu\sigma_t + 2\nu\nu'$$

where σ is defined in (**??**), $\theta = -c_1 t + \theta_0$ and the expression for σ_t is given right after the equation (**??**).

Since $\sigma(t)$ and $\sigma'(t)$ involve two different combinations of y and z and there is obviously no reduction in either x or t, there is no symmetry reduction of the metric (**??**) in the number of independent variables and hence