

or, equivalently

$$\begin{aligned} D_y[vu_{xy}] + D_z[vu_{xz}] - D_x[v\Delta[u]] &= \Phi(x, y, z, t) \\ \iff v_y u_{xy} + v_z u_{xz} - v_x \Delta[u] &= \Phi, \quad \text{where } \Delta[\Phi] = 0. \end{aligned} \tag{5.2}$$

On account of the relation (??) each of the relations (??) becomes

$$v_y u_{xz} - v_z u_{xy} = \Delta[u]u_{xx} - u_{xy}^2 - u_{xz}^2 + \chi(x, y, z, t) \tag{5.3}$$

where $\Phi_y = \chi_z$ and $\Phi_z = -\chi_y$ and hence $\Delta[\Phi] = \Delta[\chi] = 0$. Thus, we end up with the system of two equations (??) and (??) linear in derivatives of v . Solving this system algebraically for v_y and v_z and denoting $\delta = u_{xy}^2 + u_{xz}^2$, we obtain

$$\begin{aligned} v_y &= \frac{1}{\delta} \{ \Delta[u](v_x u_{xy} + u_{xx} u_{xz}) - \delta u_{xz} + \Phi u_{xy} + \chi u_{xz} \} \\ v_z &= \frac{1}{\delta} \{ \Delta[u](v_x u_{xz} - u_{xx} u_{xy}) + \delta u_{xy} + \Phi u_{xz} - \chi u_{xy} \}. \end{aligned} \tag{5.4}$$

In the following for simplicity we set $\Phi = 0$, $\chi = 0$ and refer to this case as *special first nonlocal symmetry*. In the following it is convenient to introduce the quantity

$$w = \frac{\delta}{\Delta[u]} - u_{xx}. \tag{5.5}$$

Equations (??) become

$$v_y = \frac{u_{xy}v_x - u_{xz}w}{w + u_{xx}}, \quad v_z = \frac{u_{xz}v_x + u_{xy}w}{w + u_{xx}} \tag{5.6}$$

with the immediate consequences

$$v_x = \frac{1}{\Delta[u]} (u_{xy}v_y + u_{xz}v_z), \quad w = \frac{1}{\Delta[u]} (u_{xy}v_z - u_{xz}v_y). \tag{5.7}$$

On account of the equations (??) and (??), the real two-component form (??) of *CMA* becomes

$$v_t = \frac{v_x^2 - u_{xx}w}{w + u_{xx}} + \frac{\varepsilon}{\Delta[u]}. \tag{5.8}$$

Integrability condition $(v_y)_z - (v_z)_y = 0$ of equations (??) yields

$$(u_{xz}w_y - u_{xy}w_z)v_x = u_{xx}(u_{xy}w_y + u_{xz}w_z) - \Delta[u](w + u_{xx})w_x. \tag{5.9}$$