

$$\sum_{\substack{\log p \\ \text{psv} \\ \text{mod } d}} \tilde{x}(a) = \frac{y}{g(d)} - \frac{1}{g(d)} \sum_{x \text{ mod } d} \tilde{x}(a) \sum_{\substack{y \mid \beta + \log p \\ y \geq 1/2, |y| \leq 1}} \frac{y^{\beta + \log p}}{1 + |y|}$$

$$+ O\left(y^{1/2} \log^2(Td) + \frac{y \log^2(Td)}{T}\right).$$

The double summation may be bounded by noting that each $|\tilde{x}(a)| = 1$, $|y^{\beta + \log p}| = y^\beta$ and $|\beta + \log p| \geq \sqrt{1/4 + \gamma^2} \geq (1 + |\gamma|)/3$. We let $T = x^3$ so that, using the hypothesis, $y \geq \theta x^{1-1/4+\delta} \geq \theta^2 x^{1-1/2+\delta/2} \geq \theta^2 x^{2\delta} \log^4 x$, and thus $y^{1/2} \log^2(Td) = O(y/\log^4 x)$. Also $T \geq x^{3/4} \geq x^{5/8}$ whereas $\log(Td) = O(\log y) = O(d)$, and thus $y \log^2(Td)/T = O(y/\log^4 x)$. Therefore

$$(2.4) \quad \left| \sum_{\substack{\log p \\ \text{mod } d}} \frac{\log p}{g(d)} \right| \leq \frac{3}{g(d)} \sum_{x \text{ mod } d} \sum_{\substack{y \mid \beta + \log p, y \geq 1/2, |y| \leq 1}} \frac{y^\beta}{1 + |y|} + O\left(\frac{y}{\log^4 x}\right).$$

Write \sum_a for a sum over all zeros $\beta + \log p$ of $\mathbb{I}(\mathbf{s}, \mathbf{x})$ and over all characters $x \text{ mod } d$, where $\sigma \leq \beta < \alpha$ and $|\gamma| \leq w^r$. (Each $\beta + \log p$ is counted with multiplicity equal to the number of these \mathbb{I} -functions for which it is a zero.) To estimate the double sum on the right side of (2.4), we use the upper bounds $y^\beta \leq y^{1-1/4}$ for $\beta \leq 1-1/4$; and $y^\beta \leq y$ for $\sigma \leq \beta \leq 1$, where $\sigma = 1-p/\log x$.

$$(a) \quad Rs=0.995, Ps=0.998, Fs=0.997$$

We need to form a notation for such partial arrays by selecting out certain indices from some other array. Let \mathbf{k} be the size of the original tree; for simplicity we keep it fixed throughout. Now we define $\mathbf{n}_\mathbf{k}$ for any $\mathbf{n} \in \mathbf{k}$, to be the set of indices corresponding to the subtree rooted at \mathbf{n} ; thus

$$\mathbf{n}_\mathbf{k} = \begin{cases} \mathbf{k} & \text{if } \mathbf{n} = \mathbf{k} \\ \{\mathbf{n}_1 \cup \{\mathbf{n}_2\} \cup \{\mathbf{n}_3 = \mathbf{k}\}\} & \text{otherwise.} \end{cases}$$

In the example, $\mathbf{k} = \{2, 4, 5, 8, 9, 10\}$, the indices of the left-hand subtree.

We call the set of partial arrays with subscripts in $\mathbf{n}_\mathbf{k}$ $\text{arrays}_{\mathbf{n}_\mathbf{k}}$; for example \mathbf{A}' above is in $\text{arrays}_{\mathbf{k}}$. We call the set of trees which they represent $\text{trees}_{\mathbf{n}_\mathbf{k}}$. To be precise, $\text{arrays}_{\mathbf{n}_\mathbf{k}}$ is the set of functions $\{\mathbf{n}_\mathbf{k} \rightarrow \text{atoms}\}$, and $\text{trees}_{\mathbf{n}_\mathbf{k}}$ is defined inductively by $\text{trees}_{\mathbf{n}_\mathbf{k}} = \{\text{trees}_{\mathbf{n}_\mathbf{k}}$ if $\mathbf{n} \geq \mathbf{k}$, and otherwise by $\text{trees}_{\mathbf{n}_\mathbf{k}} = \{\mathbf{n}_\mathbf{k}\}$ if $\mathbf{n} = \mathbf{k}$; \mathbf{A}' is in $\text{trees}_{\mathbf{k}}$, and $\mathbf{A}'_{\mathbf{n}_\mathbf{k}}$ is in $\text{trees}_{\mathbf{n}_\mathbf{k}}$.

An important operation corresponding to taking a subtree of a tree will be taking a subarray of an array. If \mathbf{A} is in $\text{arrays}_{\mathbf{n}}$ and \mathbf{m} is in \mathbf{n} , we write $\mathbf{A}_{\mathbf{m}}$ for the restriction of \mathbf{A} to indices in \mathbf{m} . So, for example, \mathbf{A}' in the above example could be written $\mathbf{A}'_{\mathbf{k}}$, being the partial subtree rooted at 2. In general, if \mathbf{A} is in $\text{arrays}_{\mathbf{n}}$, then $\mathbf{A}_{\mathbf{n}_\mathbf{k}}$ in $\text{arrays}_{\mathbf{n}_\mathbf{k}}$, and $\mathbf{A}_{\mathbf{n}_\mathbf{k}}$ in $\text{arrays}_{\mathbf{n}_\mathbf{k}}$ represent the left and right subtrees of the tree represented by \mathbf{A} . Clearly $\mathbf{m} \in \mathbf{n} \Leftrightarrow \mathbf{A}_{\mathbf{m}} = \mathbf{A}_{\mathbf{n}}$ and $\mathbf{A}_{\mathbf{n}_\mathbf{k}} = \mathbf{A}_{\mathbf{k}}$.

Now for each pair of domains $(\text{arrays}_{\mathbf{n}}, \text{trees}_{\mathbf{n}})$ we must define a representation function $\mathbf{r}_\mathbf{n}$ and a coding function $\mathbf{c}_\mathbf{n}$ inverse to it. Thus

$$\mathbf{r}_\mathbf{n}: \text{arrays}_{\mathbf{n}} \rightarrow \text{trees}_{\mathbf{n}} \quad (\text{representation, concrete to abstract})$$

$$\mathbf{c}_\mathbf{n}: \text{trees}_{\mathbf{n}} \rightarrow \text{arrays}_{\mathbf{n}} \quad (\text{coding, abstract to concrete})$$

They are defined recursively by

$$(c) \quad Rs=0.554, Ps=0.957, Fs=0.702$$

We put $\mathbf{i} \# \mathbf{a} = \mathbf{i}$ and use the notation

$$\mathbf{y}_\mathbf{n}(\mathbf{i}) = \mathbf{y}_{\mathbf{n}_\mathbf{k}}(\mathbf{i}) = \binom{\mathbf{n}}{\mathbf{i}} \mathbf{i}^{\mathbf{i}} (\mathbf{i} - \mathbf{i})^{-\mathbf{n}}, \quad \mathbf{r} = \mathbf{o}, \mathbf{e}_1, \dots, \mathbf{e}_n, \quad \mathbf{y}_{\mathbf{n}+1,\mathbf{n}}(\mathbf{i}) = \mathbf{o}.$$

Then the sum (16) becomes

$$\sum_{\mathbf{n}_\mathbf{k}} |\mathbf{y}_\mathbf{n}(\mathbf{i}) - \mathbf{y}_{\mathbf{n}+1,\mathbf{n}}(\mathbf{i})| = \frac{1}{1 - \mathbf{i}} \sum_{\mathbf{n}_\mathbf{k}} \frac{\mathbf{n} \# \mathbf{i}}{\mathbf{n} \# \mathbf{i}} \left| \frac{\mathbf{n} \# \mathbf{i}}{\mathbf{n} \# \mathbf{i}} - \mathbf{i} \right| |\mathbf{y}_\mathbf{n}(\mathbf{i})|.$$

We split this sum into two parts, let \mathbf{S}_1 be the sum for those \mathbf{n} , for which $|\mathbf{n} - \mathbf{i}| \leq \mathbf{n}^{-1}$, and let \mathbf{S}_2 be the remainder. For the evaluation of the sums we use the following known inequality, in which \mathbf{A} signifies an absolute constant:

$$\sum_{\substack{\mathbf{n} \# \mathbf{i} = \mathbf{A} \\ |\mathbf{n} - \mathbf{i}| \leq \mathbf{n}^{-1}}} \mathbf{y}_\mathbf{n}(\mathbf{i}) \leq \frac{\mathbf{A}}{\mathbf{n}^2}.$$

With the aid of this inequality we obtain

$$|\mathbf{S}_1| \leq \frac{\mathbf{A}}{\mathbf{n}^2} \max_{\mathbf{n} \# \mathbf{i} = \mathbf{A}} \left(\frac{\mathbf{n} \# \mathbf{i}}{\mathbf{n} \# \mathbf{i}} \left| \frac{\mathbf{n} \# \mathbf{i}}{\mathbf{n} \# \mathbf{i}} - \mathbf{i} \right| \right) \leq \frac{\mathbf{A}(\mathbf{n} + 1)}{\mathbf{n}^2}.$$

$$(e) \quad Rs=0.995, Ps=0.996, Fs=0.997$$

FIGURE 8. Examples of mathematical symbol and expression detection results for the proposed method. (a)–(f) show cropped regions from page images in different articles. The components in cyan, magenta and yellow denote the TP, FP and FN components, respectively. The sub-captions show the recall, precision and F-measure values for each page.

We created two types of subsets of the training dataset that consisted of a fixed number of document page images. The document page images in the first subset were collected in an article-wise manner from the full dataset. This means that all page images in the determined articles were included in the subset. The second subset was a collection of the same number of page images that were randomly selected from the full dataset. We expect that a diversity of font faces and

generated as a ring by $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 = \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3, \mathbf{e}_2 \mathbf{e}_3 = \mathbf{e}_3 \mathbf{e}_2 \mathbf{e}_1, \mathbf{e}_3 \mathbf{e}_1 = \mathbf{e}_1 \mathbf{e}_3 \mathbf{e}_2, \dots$, which are Schur polynomials in $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$. If \mathbf{M} is the sublattice of $\mathbf{V}_\mathbf{k}$ of elements of $\mathbf{V}_\mathbf{k}$ whose determinant is a multiple of d , then \mathbf{M} is a subring of $\mathbf{V}_\mathbf{k}$. As $\mathbf{V}_\mathbf{k}$ is a unique integral power of the determinant of \mathbf{R} and, in particular, as \mathbf{R} is unimodular then so is \mathbf{M} . \mathbf{M} is graded by the lattice \mathbf{R} with operators \mathbf{e}_i have degree i and letting \mathbf{e}_0 have degree 0.)

Section 3. Vertex Operators

For each $\mathbf{w} \in \mathbf{V}_\mathbf{k}$ will define a map \mathbf{u} from \mathbf{V} to the formal Laurent series $\mathbf{V}[[\mathbf{w}, \mathbf{w}^{-1}]]$. If \mathbf{u} is of the form $\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1 \mathbf{w} + \mathbf{u}_2 \mathbf{w}^2 + \dots$ then \mathbf{u} is a vertex operator. If \mathbf{u} is a product of $\mathbf{D}(\mathbf{w})$ then these operators have been constructed by Frenkel (3).

We can define $\mathbf{D}(\mathbf{w}, \mathbf{z})$ to be the formal expression

$$\sum_{\mathbf{i}} \mathbf{D}(\mathbf{w})^{\mathbf{i}} / \mathbf{i}! \mathbf{D}(\mathbf{w})^{\mathbf{i}} \mathbf{z}^{\mathbf{i}} = \mathbf{z}.$$

and define $\mathbf{D}(\mathbf{w}, \mathbf{z})$ to be $\mathbf{D}(\mathbf{w})^{\mathbf{z}} / (\mathbf{z} - 1)!$. If $\mathbf{u} = \mathbf{e}_i \mathbf{w}^i$ is an element of \mathbf{V} then we define $\mathbf{D}(\mathbf{w}, \mathbf{z})$ to be the formal expression

$$\sum_{\mathbf{i}} \mathbf{D}(\mathbf{w})^{\mathbf{i}} / \mathbf{i}! \mathbf{D}(\mathbf{w}, \mathbf{z})^{\mathbf{i}}.$$

This is not an operator from \mathbf{V} to $\mathbf{V}[[\mathbf{w}, \mathbf{w}^{-1}]]$ as it does not converge, but it can make sense as an operator from \mathbf{V} to $\mathbf{V}[[\mathbf{w}, \mathbf{w}^{-1}, \mathbf{z}, \mathbf{z}^{-1}]]$. This is the case if in each \mathbf{w} there are relations $\mathbf{D}(\mathbf{w}, \mathbf{z}) = \mathbf{D}(\mathbf{w}, \mathbf{z}^{-1})$ so that the “creation operators” $\mathbf{e}^{\mathbf{i}}$ and $\mathbf{e}^{\mathbf{i}}(\mathbf{z})$ occur to the left of all “annihilation operators” $\mathbf{e}^{\mathbf{j}}$ and $\mathbf{e}^{\mathbf{j}}(\mathbf{z})$ to the right of all $\mathbf{e}^{\mathbf{k}}$ and $\mathbf{e}^{\mathbf{k}}(\mathbf{z})$ which either do not all creation operators except for $\mathbf{e}^{\mathbf{0}}$ and $\mathbf{e}^{\mathbf{0}}(\mathbf{z})$. The normal ordering of $\mathbf{D}(\mathbf{w}, \mathbf{z})$ is denoted by $\mathbf{D}^{\mathbf{z}}(\mathbf{w}, \mathbf{z})$, and this is a universal vertex algebra with these generators and relations. It can be considered as a free vertex algebra on some set of generators \mathbf{u}_i for $i \in \mathbb{N}$ and integers n_i for $i \in \mathbb{N}$. However, if we consider \mathbf{u}_i for $i \in \mathbb{N}$ and n_i for $i \in \mathbb{N}$ and include the relations $\mathbf{u}_i(\mathbf{z}) = 0$ for $i > n_i$, then there is a universal vertex algebra with these generators and relations. It can be considered as a free vertex algebra on some set of generators \mathbf{u}_i for $i \in \mathbb{N}$ and integers n_i for $i \in \mathbb{N}$ by

Another example of a vertex algebra is given by taking anything with a derivation [i.e., maps $\mathbf{D}^{\mathbf{z}}$ with $\mathbf{D}^{\mathbf{z}} = 0$ for $\mathbf{z} \in \mathbf{0}$]. For $\mathbf{z} \in \mathbf{0}$,

$$\mathbf{D}^{\mathbf{z}} \mathbf{D}^{\mathbf{w}} = \left(\frac{\mathbf{z}}{\mathbf{w}} \right)^2 \mathbf{D}^{\mathbf{w}} \mathbf{D}^{\mathbf{z}},$$

$$\mathbf{D}^{\mathbf{z}} \mathbf{u}(\mathbf{w}) = \frac{\mathbf{z}}{\mathbf{w}} \mathbf{u}(\mathbf{w}) \mathbf{D}^{\mathbf{z}} \mathbf{w}.$$

and defining $\mathbf{u}_0(\mathbf{w})$ to be $\mathbf{D}^{\mathbf{w}} \mathbf{u}(\mathbf{w})$. This satisfies conditions

\mathbf{i} -nil and \mathbf{w} satisfies condition \mathbf{H} if and only if the ring is commutative. This is the case if $\mathbf{w} = \mathbf{0}$ and conversely any vertex algebra satisfying these comes from a unique ring with derivation. Hence vertex algebras are a generalization of commutative rings with derivations.

Frenkel (3).

We can define $\mathbf{D}(\mathbf{w}, \mathbf{z})$ to be the formal expression

$$\sum_{\mathbf{i}} \mathbf{D}(\mathbf{w})^{\mathbf{i}} / \mathbf{i}! \mathbf{D}(\mathbf{w}, \mathbf{z})^{\mathbf{i}} = \mathbf{z}.$$

and define $\mathbf{D}(\mathbf{w}, \mathbf{z})$ to be $\mathbf{D}(\mathbf{w})^{\mathbf{z}} / (\mathbf{z} - 1)!$. If $\mathbf{u} = \mathbf{e}_i \mathbf{w}^i$ is an element of \mathbf{V} then we define $\mathbf{D}(\mathbf{w}, \mathbf{z})$ to be the formal expression

$$\sum_{\mathbf{i}} \mathbf{D}(\mathbf{w})^{\mathbf{i}} / \mathbf{i}! \mathbf{D}(\mathbf{w}, \mathbf{z})^{\mathbf{i}}.$$

This is not an operator from \mathbf{V} to $\mathbf{V}[[\mathbf{w}, \mathbf{w}^{-1}]]$ as it does not converge, but it can make sense as an operator from \mathbf{V} to $\mathbf{V}[[\mathbf{w}, \mathbf{w}^{-1}, \mathbf{z}, \mathbf{z}^{-1}]]$. This is the case if in each \mathbf{w} there are relations $\mathbf{D}(\mathbf{w}, \mathbf{z}) = \mathbf{D}(\mathbf{w}, \mathbf{z}^{-1})$ so that the “creation operators” $\mathbf{e}^{\mathbf{i}}$ and $\mathbf{e}^{\mathbf{i}}(\mathbf{z})$ occur to the left of all “annihilation operators” $\mathbf{e}^{\mathbf{j}}$ and $\mathbf{e}^{\mathbf{j}}(\mathbf{z})$ to the right of all $\mathbf{e}^{\mathbf{k}}$ and $\mathbf{e}^{\mathbf{k}}(\mathbf{z})$ which either do not all creation operators except for $\mathbf{e}^{\mathbf{0}}$ and $\mathbf{e}^{\mathbf{0}}(\mathbf{z})$. The normal ordering of $\mathbf{D}(\mathbf{w}, \mathbf{z})$ is denoted by $\mathbf{D}^{\mathbf{z}}(\mathbf{w}, \mathbf{z})$, and this is a universal vertex algebra with these generators and relations. It can be considered as a free vertex algebra on some set of generators \mathbf{u}_i for $i \in \mathbb{N}$ and integers n_i for $i \in \mathbb{N}$ by

It is any vertex algebra that $\mathbf{D}^{\mathbf{z}}(\mathbf{w}, \mathbf{z})$ is a Lie algebra, where $\mathbf{D}^{\mathbf{z}}(\mathbf{w}, \mathbf{z})$ is the sum of all the spaces $\mathbf{D}^{\mathbf{z}}(\mathbf{V})$ for $\mathbf{z} \in \mathbf{z}$ and where the Lie algebra $\mathbf{D}^{\mathbf{z}}(\mathbf{V})$ is antisymmetric on \mathbf{V} . Any vertex algebra \mathbf{V} becomes a module for the Lie algebra $\mathbf{D}^{\mathbf{z}}(\mathbf{V})$ by letting $\mathbf{v} \in \mathbf{V}$ act on $\mathbf{w} \in \mathbf{V}$. If $\mathbf{v} \in \mathbf{D}^{\mathbf{z}}(\mathbf{V})$ then $\mathbf{v} \cdot \mathbf{w} = \mathbf{0}$. In particular \mathbf{V} is a $\mathbf{D}^{\mathbf{z}}(\mathbf{V})$ module and invariant under the action of $\mathbf{D}^{\mathbf{z}}(\mathbf{V})$. The operators $\mathbf{D}^{\mathbf{z}}$ and the products $\mathbf{u}_i(\mathbf{w})$ on \mathbf{V} are invariant under the action of $\mathbf{D}^{\mathbf{z}}(\mathbf{V})$. Note that the action of $\mathbf{D}^{\mathbf{z}}(\mathbf{V})$ on \mathbf{V} does not change the relations $\mathbf{D}(\mathbf{w}, \mathbf{z}) = \mathbf{D}(\mathbf{w}, \mathbf{z}^{-1})$ and the products $\mathbf{u}_i(\mathbf{w})$ invariant; see Section 3.3.

The free vertex algebra on some set of generators does not exist because there are no relations among the generators \mathbf{u}_i for $i \in \mathbb{N}$. However, if we consider $\mathbf{u}_i(\mathbf{z}) = 0$ for $i > n_i$, and include the relations $\mathbf{u}_i(\mathbf{z}) = 0$ for $i > n_i$, then there is a universal vertex algebra with these generators and relations. It can be considered as a free vertex algebra on some set of generators \mathbf{u}_i for $i \in \mathbb{N}$ and integers n_i for $i \in \mathbb{N}$ by

Let $\mathbf{D}^{\mathbf{z}}(\mathbf{w}, \mathbf{z})$ (respectively, $\mathbf{D}^{\mathbf{z}}(\mathbf{w}), \mathbf{D}^{\mathbf{z}}(\mathbf{w})$) be bounded (resp., bounded below, bounded above) for triangulated categories (of which we refer to [19]).

let $\mathbf{D}^{\mathbf{z}}(\mathbf{w})$ (respectively, $\mathbf{D}^{\mathbf{z}}(\mathbf{w}), \mathbf{D}^{\mathbf{z}}(\mathbf{w})$) be bounded (resp., bounded below, bounded above) for triangulated categories ($\mathbf{Z}', \mathbf{Z}, \mathbf{Z}''$, together with exact functors “recollement” conditions [1], 1.4.3.1–1.4.3.4) if and only if the following conditions are satisfied:

(t) adjoint \mathbf{i}^* and an exact right adjoint $\mathbf{i}^!$;

right adjoint \mathbf{l}_* and an exact left adjoint $\mathbf{l}_!$;

djoint properties, $\mathbf{i}^* \mathbf{l}_* = \mathbf{0}$ and $\mathbf{i}^! \mathbf{l}_* = \mathbf{0}$;

$$(d) \quad Rs=0.964, Ps=0.912, Fs=0.937$$

Proof. Let \mathbf{R} denote Haar measure on the unitary group of a finite dimensional \mathbf{G}^* -algebra. Then

$$\|\mathbf{E}_{\mathbf{u}_{n+1}, \dots, \mathbf{u}_1}(\mathbf{e}_1 - \mathbf{z})\|_1 = \left\| \prod_{i=1}^n (\mathbf{u}_{n+1-i} - \mathbf{z}) \mathbf{R} \mathbf{u}_i \right\|_1.$$

Applying the isomorphism $\mathbf{u}_{n+1} \dots \mathbf{u}_1$ of 4.1.11 we find that this expression equals

$$\left\| \prod_{i=1}^n (\mathbf{u}_{n+1-i} - \mathbf{z}) \mathbf{R} \mathbf{u}_i \right\|_1,$$

which equals $\|\mathbf{E}_{\mathbf{u}_{n+1}, \dots, \mathbf{u}_1}(\mathbf{e}_1 - \mathbf{z})\|_1$. Q.E.D.

Thus to show that $\mathbf{R}^* \mathbf{R} = \mathbf{G}$ it suffices to show that

$$\lim_{n \rightarrow \infty} \|\mathbf{E}_{\mathbf{u}_{n+1}, \dots, \mathbf{u}_1}(\mathbf{e}_1 - \mathbf{z})\|_1 = 0.$$

Since we know that the limit exists, it suffices to consider n odd; say $n = 2m-1$. From §5.1 let $\mathbf{B}_0, \mathbf{B}_1, \dots, \mathbf{B}_{m-1}$ be the minimal central projections in \mathbf{G}^* corresponding to $\mathbf{G}_0^{-1}, \mathbf{G}_1^{-1}, \dots, \mathbf{G}_{m-1}^{-1}$, similarly $\mathbf{B}_0, \mathbf{B}_1, \dots, \mathbf{B}_m$ for \mathbf{G}_0 and $\mathbf{B}_0, \mathbf{B}_1, \dots, \mathbf{B}_m$ for \mathbf{G}_1 . So $\mathbf{B}_0 = \mathbf{I} - \mathbf{e}_1 \mathbf{w} \mathbf{e}_2 \mathbf{w} \dots \mathbf{w} \mathbf{e}_{m-1}$, $\mathbf{B}_0 = \mathbf{I} - \mathbf{e}_1 \mathbf{w} \mathbf{e}_2 \mathbf{w} \dots \mathbf{w} \mathbf{e}_m$. We also know from §5.1 and (iii) of 3.3.1 that

$$\mathbf{e}_n, \mathbf{e}_1 = \mathbf{e}_{n+1}, \mathbf{B}_{n-1} \quad \text{for } n \geq 1. \quad (5.3.3)$$

Since all the embeddings on the Bratteli diagram are of multiplicity at most

$$(f) \quad Rs=1.000, Ps=0.971, Fs=0.985$$

mathematical notation styles is kept by the random selection. We adjusted the number of images in the subsets and repeated the experiment for U-Net training and evaluation.

Figure 10 shows the results of the experiments. The vertical and horizontal axes denote the symbol-based F -measure value F_s and number of pages in the subset, respectively. Whereas the F -measure value rapidly decreased when the number of pages in the article-based subset