Remark 5.1. *If we further assume the function λ to be Hölder continuous we enter the realm of Schauder's regularity theory and therefore*

$$\theta \in \mathcal{C}^{2,\alpha}(U),$$

for some $\alpha \in (0,1)$.

References

[1] B. Andreianov, M. Bendahmane and S. Ouaro, *Structural stability for variable exponent elliptic problems, I: the $p(x)$-Laplacian kind problems*, Nonlinear Anal. **73** (2010), 2–24.

[2] S.N. Antontsev and M. Chipot, *The thermistor problem: existence, smoothness uniqueness, blowup*, SIAM J. Math. Anal. **25** (1994), 1128–1156.

[3] D.J. Araújo, A. F. Maia and J.M. Urbano, *Sharp regularity for the inhomogeneous porous medium equation*, J. Anal. Math. **140** (2020), 395–407.

[4] D.J. Araújo, E. V. Teixeira and J.M. Urbano, *A proof of the $\mathcal{C}^{p'}-$regularity conjecture in the plane*, Adv. Math. **316** (2017), 541–553.

[5] D.J. Araújo, E. V. Teixeira and J.M. Urbano, *Towards the $\mathcal{C}^{p'}-$regularity conjecture in higher dimensions*, Int. Math. Res. Not. IMRN **2018** (2018), no. 20, 6481–6495.

[6] F. Demengel and G. Demengel, "Functional Spaces for the Theory of Elliptic Partial Differential Equations", Universitext, Springer, London, 2012.

[7] L. Diening, P. Harjulehto, P. Hästö and M. Růžička, "Lebesgue and Sobolev Spaces with Variable Exponents", Lecture Notes in Mathematics 2017, Springer, Heidelberg, 2011.

[8] X. Fan, *Global $\mathcal{C}^{1,\alpha}$ regularity for variable exponent elliptic equations in divergence form*, J. Differential Equations **235** (2007), 397–417.

[9] X. Fan and Q. Zhang, *Existence of solutions for $p(x)$-Laplacian Dirichlet problem*, Nonlinear Anal. **52** (2003), 1843–1852.

[10] H. Gao, B. Li and W. Sun, *Stability and convergence of fully discrete Galerkin FEMs for the nonlinear thermistor equations in a nonconvex polygon*, Numer. Math. **136** (2017), 383–409.

[11] S. D. Howison, J.F. Rodrigues and M. Shillor, *Stationary solutions to the thermistor problem*, J. Math. Anal. Appl. **174** (1993), 573–588.

[12] T. Kuusi and G. Mingione, *Gradient regularity for nonlinear parabolic equations*, Ann. Sc. Norm. Super. Pisa Cl. Sci. **12** (2013), 755–822.

[13] T. Kuusi and G. Mingione, *The Wolff gradient bound for degenerate parabolic equations*, J. Eur. Math. Soc. **16** (2014), 835–892.

[14] T. Kuusi and G. Mingione, *Riesz potentials and nonlinear parabolic equations*, Arch. Rational Mech. Anal. **212** (2014), 727–780.

[15] P. Lindqvist, *Stability for the solutions of* div $\left(|\nabla u|^{p-2}\nabla u\right) = f$ *with varying p*, J. Math. Anal. Appl. **127** (1987), 93–102.