with $\varepsilon = \pm 1$ respectively. Here u is a real-valued function of the two complex variables z^1, z^2 and their conjugates $\bar{z}^1, \bar{z}^2$, the subscripts denoting partial derivatives with respect to these variables, e.g. $u_{1\bar{1}} = \partial^2 u/\partial z^1 \partial \bar{z}^1$ and suchlike. A modern proof of this result one can find in the books by Mason and Woodhouse [**?**] and Dunajski [**?**].

To illustrate this property, we introduce the coframe of one-forms

$$\omega_1 = \frac{1}{\sqrt{u_{1\bar{1}}}}(u_{1\bar{1}}\mathrm{d}z_1 + u_{2\bar{1}}\mathrm{d}z_2), \quad \bar{\omega}_1 = \frac{1}{\sqrt{u_{1\bar{1}}}}(u_{1\bar{1}}\mathrm{d}\bar{z}_1 + u_{1\bar{2}}\mathrm{d}\bar{z}_2)$$
$$\omega_2 = \frac{1}{\sqrt{u_{1\bar{1}}}}\mathrm{d}z_2, \quad \bar{\omega}_2 = \frac{1}{\sqrt{u_{1\bar{1}}}}\mathrm{d}\bar{z}_2. \tag{1.3}$$

The metric (**??**) takes the canonical form

$$\mathrm{d}s^2 = \omega_1 \otimes \bar{\omega}_1 + \varepsilon\, \omega_2 \otimes \bar{\omega}_2 \tag{1.4}$$

where complex Monge-Ampère equation (**??**) has been used. Equation (**??**) makes obvious the claim about the signature of the metric.

We are mostly interested in ASD Ricci-flat metrics that describe gravitational instantons which asymptotically look like a flat space, so that their curvature is concentrated in a finite region of a Riemannian space-time (see [**?**] and references therein). The most important gravitational instanton is $K3$ which geometrically is Kummer surface [**?**], for which an explicit form of the metric is still unknown while many its properties and existence had been discovered and analyzed [**?**, **?**]. A characteristic feature of the $K3$ instanton is that it does not admit any Killing vectors, that is, no continuous symmetries which implies that the metric potential should be a noninvariant solution of CMA equation. As opposed to the case of invariant solutions, for noninvariant solutions of CMA there should be no symmetry reduction [**?**] in the number of independent variables. In this paper we achieve this goal by utilizing the invariance under an nonlocal symmetry.

The paper is organized as follows. In Section **??**, we convert CMA equation into real variables and two-component form. In Section **??**, we exhibit two alternative bi-Hamiltonian representations of CMA which we discovered earlier [**?**]. In Section **??**, we explicitly construct first nonlocal flows in the hierarchy of CMA system related to each of the two bi-Hamiltonian structures. In Section **??**, we formulate the invariance conditions with respect to both nonlocal symmetries appearing in each of the two alternative hierarchies. In this way, we keep in the invariance conditions the obvious discreet symmetry relating the two bi-Hamiltonian structures. Here for simplicity