

this metric does not admit any Killing vectors. Thus, we have obtained the Ricci-flat (anti-)self-dual metric without Killing vectors. It is generated by a non-invariant solution of the complex Monge-Ampère equation determined solely by its invariance with respect to the special first nonlocal symmetry of CMA without any additional assumptions. We see that such an invariance does not lead to a reduction in the number of independent variables in the solution, on the contrary to the invariance under Lie point symmetries. This explains our special interest to nonlocal symmetry flows in the hierarchies of bi-Hamiltonian systems of Monge-Ampère type which we constructed recently [?].

9 Conclusion

Our search for non-invariant solutions to the elliptic complex Monge-Ampère equation has been motivated by the fundamental problem of obtaining explicitly the metric of the gravitational instanton $K3$, since it will not admit any Killing vectors (continuous symmetries). Recently, we produced an example of such a metric, though not an instanton one, by combining our previous approaches to the problem and choosing at random a very particular solution to resulting equations [?].

Here we have demonstrated that a general requirement of invariance under nonlocal symmetries of CMA yields solutions which are not invariant with respect to any local symmetries and therefore no symmetry reduction results in the number of independent variables. We have explicitly constructed such a solution by a meticulous analysis of all integrability conditions of the invariance equations which made it possible a complete integration of these equations with no additional assumptions made. Thus, we have obtained the most general form of the solution of CMA which is invariant under the special first nonlocal symmetry in the hierarchy of CMA , not just a solution taken out by chance. We have also presented the corresponding ASD Ricci-flat metric without Killing vectors. It has rather a complicated form and further analysis is needed to study its properties.

This method seems to be a direct approach for obtaining noninvariant solutions of CMA from the invariance under other nonlocal flows in the hierarchy. We also point out that all our constructions and results have been obtained simultaneously for elliptic and hyperbolic CMA and so the corresponding metrics have either Euclidean or neutral (ultra-hyperbolic) signature, respectively.