EXISTENCE AND IMPROVED REGULARITY FOR A NONLINEAR SYSTEM WITH COLLAPSING ELLIPTICITY

EDGARD A. PIMENTEL AND JOSÉ MIGUEL URBANO

Abstract. We study a nonlinear system made up of an elliptic equation of blended singular/degenerate type and Poisson's equation with a lowly integrable source. We prove the existence of a weak solution in any space dimension and, chiefly, derive an improved $\mathcal{C}^{1,\text{Log-Lip}}$ regularity estimate using tangential analysis methods. The system illustrates a sophisticated version of the proverbial thermistor problem and our results are new even in simpler modelling scenarios.

Keywords: Elliptic singular/degenerate system, existence, improved regularity, thermistor problem.

AMS Subject Classifications MSC 2010: 35B65, 35J57, 35J92, 35Q79.

1. Introduction

There are many good reasons to investigate the regularity properties of solutions to nonlinear partial differential equations (pdes) and systems. Perhaps the most compelling is the enhancement of more efficient numerical schemes leading to concrete applications of what would otherwise be a purely theoretical endeavour. In recent years, there has been an intense activity around the development of a class of methods and techniques that culminate in the retrieval of improved regularity properties for solutions of a given pde imported from another pde which is *somehow close* to the original one. This

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