has been done, for example, in [?], where the sharp regularity for solutions of the inhomogeneous p-parabolic equation was derived from the regularity of p-caloric functions or in [?], where Sobolev regularity for viscosity solutions of fully nonlinear elliptic equations was obtained from the associated recession profile. Other instances of this approach to regularity can be found, for example, in [?, ?, ?, ?, ?, ?, ?, ?]. For pointwise gradient bounds in terms of potentials, see also [?, ?, ?].

In this paper, we extend this set of ideas, hitherto restricted to the analysis of single equations, to treat the nonlinear system of pdes

$$\begin{cases} -\operatorname{div}\left(|Du|^{\sigma(\theta(x))-2}\,Du\right) = f \\ \\ -\Delta\theta = \lambda(\theta(x))\,|Du|^{\sigma(\theta(x))}\,, \end{cases} \tag{1.1}$$

proposed by Zhikov in [?], describing the steady state distribution of the electrical potential u and the temperature θ in a *thermistor*, a portmanteau for a resistor whose electrical properties are thermally dependent. In (??), f is a given source and σ and λ are functions related to the electrical conductivity and resistance of the model, respectively. The thermistor problem, in its different versions, has been considered by many authors and there is an abundant literature around it, both in the physics/engineering and the mathematical communities. Far from being exhaustive, we mention here [?, ?, ?, ?, ?].

The mathematical analysis of the strongly coupled system (??) involves two major difficulties. On the one hand, the second equation has a right-hand side merely in L^1 and this low integrability is known to be a source of severe analytical hazards. On the other hand, the first equation is not uniformly elliptic as its modulus of ellipticity collapses at points where $|Du| = 0$, vanishing if the variable exponent $\sigma(\theta(x))$ is above two and blowing up if it is below that threshold. Since the exponent can vary in a range that crosses two, degeneracies and singularities are blended in our problem and one of the main achievements of the approach we use is to seamlessly treat the switching between regimes that otherwise correspond to two markedly different cases.

We first treat the existence of weak solutions under homogeneous Dirichlet boundary conditions. Contrary to the results of Zhikov in [?], which are valid only in dimensions up to three, we unlock the existence in any dimension. The key is in the use of a regularity result for the first equation in (??) that