4 Nonlocal flows

The first nonlocal flows of the hierarchy of CMA system are generated by J_1 and J^1 acting on the vector of variational derivatives of H_1

$$\begin{pmatrix} u_{\tau_2} \\ v_{\tau_2} \end{pmatrix} = J_1 \begin{pmatrix} \delta_u H_1 \\ \delta_v H_1 \end{pmatrix} \tag{4.1}$$

$$\begin{pmatrix} u_{\tau_{2'}} \\ v_{\tau_{2'}} \end{pmatrix} = J^1 \begin{pmatrix} \delta_u H_1 \\ \delta_v H_1 \end{pmatrix} \tag{4.2}$$

where τ_2, $\tau_{2'}$ are time variables of the flows (**??**), (**??**), respectively. Using the expressions (**??**), (**??**) and (**??**) for J_1, J^1 and H_1 we obtain explicit expressions for the flows (**??**) and (**??**)

$$\begin{aligned} u_{\tau_2} &= \Delta^{-1}\left\{D_z(au_{xx} - u_{xy}^2 - u_{xz}^2 - \varepsilon) - D_xD_y(av)\right\} + u_{xy}v \\ u_{\tau_{2'}} &= \Delta^{-1}\left\{D_y(au_{xx} - u_{xy}^2 - u_{xz}^2 - \varepsilon) + D_xD_z(av)\right\} - u_{xz}v. \end{aligned} \tag{4.3}$$

Second components of these flows are time derivatives of (**??**), $v_{\tau_2} = D_t[u_{\tau_2}]$, $v_{\tau_{2'}} = D_t[u_{\tau_{2'}}]$, so that the flows (**??**) and (**??**) commute with the flow (**??**) of CMA system and hence they are nonlocal symmetries of the CMA system.

5 Invariance conditions with respect to nonlocal symmetries

Solutions invariant with respect to nonlocal symmetries are determined by the conditions $u_{\tau_2} = 0$ and $u_{\tau_{2'}} = 0$ which due to (**??**) take the explicit form

$$\begin{aligned} D_z\left[\Delta[u]u_{xx} - u_{xy}^2 - u_{xz}^2\right] - D_xD_y\left[v\Delta[u]\right] + \Delta\left[vu_{xy}\right] = 0 \\ D_y\left[\Delta[u]u_{xx} - u_{xy}^2 - u_{xz}^2\right] + D_xD_z\left[v\Delta[u]\right] - \Delta\left[vu_{xz}\right] = 0. \end{aligned} \tag{5.1}$$

Here we impose both invariance conditions (**??**) on solutions of the CMA system in order to keep the discreet symmetry $z \mapsto y, y \mapsto -z$ between the two bi-Hamiltonian structures. Differentiating the first and second equations (**??**) with respect to y and z, respectively, and taking the difference of the results yields the integrability condition

$$\Delta\left\{D_y[vu_{xy}] + D_z[vu_{xz}] - D_x[v\Delta[u]]\right\} = 0$$