A second challenge comes from the eventual collapse of the ellipticity in (**??**). Indeed, along $\{Du = 0\}$, the first equation either degenerates or blows up, depending on $p(\cdot)$. In particular, we allow this variable exponent to oscillate around $p(\cdot) \equiv 2$; hence, the system may switch between the singular and the degenerate regimes. Our approach deals with this difficulty in a seamless fashion, which is in itself an unusual feature that deserves to be highlighted.

We now list the main assumptions on the data of the problem. Throughout the paper, we say a constant is *universal* if it only depends on the data.

A 1 (Uniform bound on σ)**.** *The function $\sigma : \mathbb{R} \to \mathbb{R}$ is bounded from below and there exists a constant σ^- such that*

$$1 \leq \frac{2d}{d+2} < \sigma^- \leq \sigma(x).$$

The lower bound on σ^- implies that solutions u to the first equation in (**??**) are locally bounded. But, by considering a possibly unbounded (from above) function σ, we completely detach the analysis from the constant-exponent setting. Indeed, were σ bounded and satisfying the next assumption, the variable exponent setting would not present extra challenges *vis-a-vis* the constant case.

A 2 (Lipschitz continuity of σ)**.** *The function σ is Lipschitz continuous and there exists $C_\sigma > 0$ such that*

$$\|\sigma\|_{\mathcal{C}^{0,1}(\mathbb{R})} \leq C_\sigma.$$

Notice that A**??** does not imply $p(\cdot)$ to be a Lipschitz continuous exponent. Even if θ is Hölder continuous, the composite function $p = \sigma \circ \theta$ is, *a priori*, merely Hölder continuous.

A 3 (Uniform bounds on λ)**.** *The function $\lambda : \mathbb{R} \to \mathbb{R}$ is bounded and there exists a constant λ^+, to be fixed later and depending only on the data, such that*

$$\|\lambda\|_{L^\infty(\mathbb{R})} \leq \lambda^+.$$

The upper bound on λ plays a critical role in the approximation methods put forward further in the paper. Perhaps more subtle is the fact that λ^+ is determined endogenously, in the context of the fixed-point argument in section **??**.