The metric (**??**) in real variables reads

$$
\begin{aligned}
& \mathrm{d}s^2 = (v_t + u_{xx})(\mathrm{d}t^2 + \mathrm{d}x^2) + \Delta(u)(\mathrm{d}y^2 + \mathrm{d}z^2) \\
& - 2b(\mathrm{d}t\,\mathrm{d}z - \mathrm{d}x\,\mathrm{d}y) + 2c(\mathrm{d}t\,\mathrm{d}y + \mathrm{d}x\,\mathrm{d}z).
\end{aligned} \tag{2.5}
$$

The coframe of one-forms becomes

$$
\begin{aligned}
& \Omega_1 = \frac{1}{2\sqrt{v_t + u_{xx}}}[(c + ib)(\mathrm{d}y + i\mathrm{d}z) + (v_t + u_{xx})(\mathrm{d}t + i\mathrm{d}x)] \\
& \Omega_2 = \bar{\Omega}_1, \quad \Omega_3 = \frac{\mathrm{d}y + i\mathrm{d}z}{2\sqrt{v_t + u_{xx}}}, \quad \Omega_4 = \bar{\Omega}_3
\end{aligned} \tag{2.6}
$$

with the metric

$$
\mathrm{d}s^2 = \Omega_1 \otimes \bar{\Omega}_1 + \varepsilon\,\Omega_2 \otimes \bar{\Omega}_2. \tag{2.7}
$$

3 Bi-Hamiltonian representations of CMA system

The *CMA* system (**??**) can be put in the Hamiltonian form

$$
\begin{pmatrix} u_t \\ v_t \end{pmatrix} = J_0 \begin{pmatrix} \delta_u H_1 \\ \delta_v H_1 \end{pmatrix} \tag{3.1}
$$

where δ_u and δ_v are Euler-Lagrange operators [**?**] with respect to u and v. Here J_0 is the Hamiltonian operator

$$
J_0 = \begin{pmatrix} 0 & \frac{1}{a} \\ -\frac{1}{a} & \frac{1}{a}(cD_y + D_y c - bD_z - D_z b)\frac{1}{a} \end{pmatrix} \tag{3.2}
$$

determining the structure of Poisson bracket and H_1 is the corresponding Hamiltonian density

$$
H_1 = \frac{1}{2}\left[v^2\Delta(u) - u_{xx}(u_y^2 + u_z^2)\right] - \varepsilon u. \tag{3.3}
$$

The first real recursion operator has the form

$$
\begin{aligned}
& R_1 = \begin{pmatrix} 0 & 0 \\ QD_z - cD_x & b \end{pmatrix} + \\
& \Delta^{-1}\begin{pmatrix} D_y\Big(-aD_x + bD_y + cD_z\Big) + D_z\Big(cD_y - bD_z\Big) & -D_z a \\ D_x\Big[D_y\Big(cD_y - bD_z\Big) + D_z\Big(aD_x - bD_y - cD_z\Big)\Big] & -D_x D_y a \end{pmatrix}
\end{aligned} \tag{3.4}
$$