where Δ^{-1} means operator multiplication, and the second recursion operator reads

$$R_2 = \begin{pmatrix} 0 & 0 \\ bD_x - QD_y & c \end{pmatrix} + \tag{3.5}$$
$$\Delta^{-1} \begin{pmatrix} D_y(bD_z - cD_y) + D_z(-aD_x + bD_y + cD_z) & D_y a \\ D_x\Big[D_y(-aD_x + bD_y + cD_z) + D_z(cD_y - bD_z)\Big] & -D_x D_z a \end{pmatrix}.$$

The two recursion operators R_1 and R_2 generate two alternative second Hamiltonian operators $J_1 = R_1 J_0$ and $J^1 = R_2 J_0$

$$J_1 = R_1 J_0 = \Delta^{-1} \begin{pmatrix} D_z & -D_x D_y \\ D_x D_y & D_x^2 D_z \end{pmatrix} + \tag{3.6}$$
$$\begin{pmatrix} 0 & \frac{b}{a} \\ -\frac{b}{a} & \frac{c}{a^2}\Big(bD_y - aD_x\Big) + \Big(D_y b - D_x a\Big)\frac{c}{a^2} + \frac{Q_-}{2a} D_z + D_z \frac{Q_-}{2a} \end{pmatrix}$$

where $Q_- = (c^2 - b^2 + \varepsilon)/a$, and

$$J^1 = R_2 J_0 = \Delta^{-1} \begin{pmatrix} D_y & D_x D_z \\ -D_x D_z & D_x^2 D_y \end{pmatrix} + \tag{3.7}$$
$$\begin{pmatrix} 0 & -\frac{c}{a} \\ \frac{c}{a} & \frac{b}{a^2}\Big(cD_z - aD_x\Big) + \Big(D_z c - D_x a\Big)\frac{b}{a^2} + \frac{Q^-}{2a} D_y + D_y \frac{Q^-}{2a} \end{pmatrix}$$

where $Q^- = (b^2 - c^2 + \varepsilon)/a$.

The flow (**??**) can be generated by the Hamiltonian operator J_1 from the Hamiltonian density

$$H_0 = zv\Delta(u) + u_x u_y \tag{3.8}$$

so that CMA in the two-component form (**??**) is a *bi-Hamiltonian system* [**?**]

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} = J_0 \begin{pmatrix} \delta_u H_1 \\ \delta_v H_1 \end{pmatrix} = J_1 \begin{pmatrix} \delta_u H_0 \\ \delta_v H_0 \end{pmatrix}. \tag{3.9}$$

The same flow (**??**) can also be generated by the Hamiltonian operator J^1 from the Hamiltonian density

$$H^0 = yv\Delta(u) - u_x u_z \tag{3.10}$$

which yields another bi-Hamiltonian representation of the CMA system (**??**)

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} = J_0 \begin{pmatrix} \delta_u H_1 \\ \delta_v H_1 \end{pmatrix} = J^1 \begin{pmatrix} \delta_u H^0 \\ \delta_v H^0 \end{pmatrix}. \tag{3.11}$$