where

$$b=\frac{f_\zeta}{f}A\omega'-\frac{a}{A}. \tag{6.18}$$

Integrability condition for the system (**??**) reads

$$(\theta_y)_z-(\theta_z)_y=b_z\cos\theta+b_y\sin\theta-\frac{f}{2A}b_x+b^2+\frac{f^2}{4A^2}\theta_x^2=0 \tag{6.19}$$

or in the explicit form

$$\left(\frac{f^2}{2A^3}u_{xxx}-\frac{f_\zeta}{2f}A\omega'+\frac{f^2}{2A^3}\frac{\omega''}{\omega'^3}-\frac{3f}{2A}\frac{\omega''}{\omega'^2}\right)^2+\frac{f^2}{A^2}\theta_x^2=0. \tag{6.20}$$

Reality condition for (**??**) implies that both quadratic terms vanish separately

$$u_{xxx}=\frac{f_\zeta}{f^3}A^4\omega'-\frac{\omega''}{\omega'^3}+\frac{3A^2}{f}\frac{\omega''}{\omega'^2} \tag{6.21}$$

and $\theta_x=0 \iff \theta=\theta(y,z,t)$. Using (**??**) in (**??**) for a and then in (**??**) for b we obtain

$$b=0,\quad a=\frac{f_\zeta}{f}A^2\omega' \tag{6.22}$$

and (**??**) with $\theta_x=0$ implies $\theta_y=\theta_z=0$, so that $\theta=\theta(t)$. Equations (**??**) become

$$A_y=a\sin\theta,\quad A_z=a\cos\theta,\quad A_x=\frac{f_\zeta}{f^2}A^3\omega'+A\frac{\omega''}{\omega'^2}. \tag{6.23}$$

From the definition (**??**) of A it follows

$$u_{xx}=\frac{A^2}{f}-\frac{1}{\omega'} \tag{6.24}$$

Since $f=f(\zeta,t)$ where $\zeta=\omega(u_x)+x$, we have

$$f_x=\frac{f_\zeta}{f}A^2\omega',\quad f_y=f_\zeta\omega' A\sin\theta\quad f_z=f_\zeta\omega' A\cos\theta \tag{6.25}$$

and hence $A_y/A=f_y/f$, $A_z/A=f_z/f$, so that $A=\alpha(x,t)f$. Then the last equation in (**??**) implies

$$\frac{A_x}{A}=\frac{f_x}{f}+\frac{\omega''}{\omega'^2}=\frac{f_x}{f}+\frac{\alpha_x(x,t)}{\alpha}\Longrightarrow\left(\frac{\omega''}{\omega'^2}\right)_y=0,\quad\left(\frac{\omega''}{\omega'^2}\right)_z=0 \tag{6.26}$$