

# Nonlocal symmetry of CMA generates ASD Ricci-flat metric with no Killing vectors

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## Abstract

The complex Monge-Ampère equation (*CMA*) in a two-component form is treated as a bi-Hamiltonian system. We present explicitly the first nonlocal symmetry flow in the hierarchy of this system. An invariant solution of *CMA* with respect to this nonlocal symmetry is constructed which, being a noninvariant solution in the usual sense, does not undergo symmetry reduction in the number of independent variables. We also construct the corresponding 4-dimensional anti-self-dual (ASD) gravitational metric with either Euclidean or neutral signature. It admits no Killing vectors which is one of characteristic features of the famous gravitational instanton *K3*.

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## 1 Introduction

In his pioneer paper [?], Plebański demonstrated that anti-self-dual (ASD) Ricci-flat metrics on four-dimensional complex manifolds are completely determined by a single scalar potential which satisfies his first or second heavenly equation. Such metrics are solutions to complex vacuum Einstein equations. Real four-dimensional hyper-Kähler ASD metrics

$$ds^2 = u_{1\bar{1}}dz^1d\bar{z}^1 + u_{1\bar{2}}dz^1d\bar{z}^2 + u_{2\bar{1}}dz^2d\bar{z}^1 + u_{2\bar{2}}dz^2d\bar{z}^2 \quad (1.1)$$

that solve the vacuum Einstein equations with either Euclidean or ultra-hyperbolic signature are governed by a scalar real-valued potential  $u = u(z^1, z^2, \bar{z}^1, \bar{z}^2)$  which satisfies elliptic or hyperbolic complex Monge-Ampère equation (*CMA*)

$$u_{1\bar{1}}u_{2\bar{2}} - u_{1\bar{2}}u_{2\bar{1}} = \varepsilon \quad (1.2)$$