

Classical Algebraic Geometry & Modern Computer Algebra: Innovative Software Design and its Applications

Martin Bies, Lars Kastner, Matthias Zach

Durham University

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Agenda

- 1** What is this session about?
- 2** OSCAR and MaRDI
- 3** Why is this topic close to our hearts?
- 4** Organizational Remarks & Schedule

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We are very excited to have all of you here, to listen to the fascinating talks and to discuss these topics we all care about deeply!

What is this session about?

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OSCAR and MaRDI

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Why is this topic close to our hearts?

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Organizational Remarks & Schedule

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- Capabilities exceed those of the individual systems.

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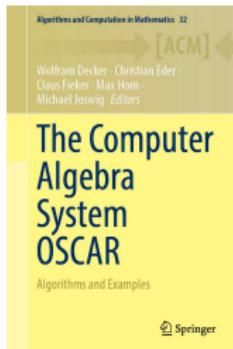
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- High-level integration of tools from different mathematical areas.
- Provides functions for groups, rings, fields, linear and commutative algebra, number theory, algebraic geometry, polyhedral geometry and much more.

The OSCAR Book: <https://link.springer.com/book/9783031621260>



- Comprehensive guide and reference.
- Chapters demonstrate applications of OSCAR in a broad range of mathematical domains:
 - Group Theory,
 - Number Theory,
 - Polyhedral Geometry,
 - Algebraic Geometry and Commutative Algebra,
 - Various specialized topics (GIT fans, Invariant Theory, Matroids, Tropical Geometry, Toric Geometry, ...)

Chapter preprints:

- Toric Geometry in OSCAR
M. Bies & L. Kastner, <https://arxiv.org/abs/2303.08110>.
- Polyhedral Geometry in OSCAR
T. Brysiewicz & M. Joswig, <https://arxiv.org/abs/2303.08110>.
- Elliptic fibrations on K3 surfaces
S. Brandhorst & M. Zach, <https://arxiv.org/abs/2311.11766>.

MaRDI

- Mathematical Research Data Initiative
- <https://www.mardi4nfdi.de/>



Why is this topic close to our hearts?

We – **Martin Bies**, **Lars Kastner**, and **Matthias Zach** – are all contributing to the OSCAR project (funded by DFG as part of the SFB-TRR 195) as well as MaRDI (funded by DFG project ID 460135501).

OSCAR builds on various established, but rather specialized subsystems:

GAP



 polymake



SINGULAR 

Images were generated using ChatGPT4o:
OpenAI. (2024). ChatGPT (July 2024 version) [Large language model].
Retrieved from <https://openai.com/chatgpt/>

- Flint
- Nemo /Antic
- Hecke
- Abstract Algebra

OSCAR Tutorials: <https://www.oscar-system.org/tutorials/>

OSCAR
SYMBOLIC TOOLS

The OSCAR project

Home
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Credits
Contributors
The OSCAR book
Oscar.jl on Github
MaRDI

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Tutorials

This page contains jupyter notebooks that demonstrate the functionality of the OSCAR project.

For each topic, you can decide to open a static version of the jupyter notebook, powered by [nbviewer](#). Alternatively, you can inspect the jupyter notebook directly on [github](#).

▶ How to interact with a "live" version

Click on one of the links below to filter notebooks (and re-click to disable filtering).

FTheory Tools	Group Theory	Number Theory	Polyhedral Geometry

Commutative Algebra	Toric Geometry



Polynomial Rings

Author(s): John Abbott, Martin Bies, Luca

Remke, Hans Schönemann

Last modified: July 12, 2024

[nbviewer](#)

[Github](#)



Polyhedral Geometry

Author(s): Martin Bies, Luca Remke

Last modified: July 12, 2024

[nbviewer](#)

[Github](#)

We are always open to collaboration

- At this point there have been many OSCAR success stories from various fields in and adjacent to mathematics.
- If you need something implemented in OSCAR, please get in touch
 - Contact one of the many OSCAR developers here at ICMS
 - Talk to us at the panel discussion
 - Get in touch at <https://www.oscar-system.org/community/>

Organizational Remarks & Schedule

■ Session picture – right here and now!



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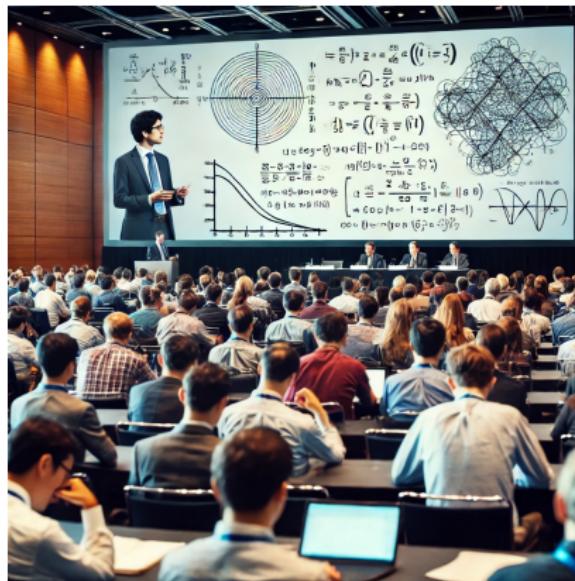
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- **Panel discussion:** “If I had the software to...” (15:00 to 15:30).
- Talk by Prof. Yang-Hui He concludes this session (16:00 to 17:00).

Thank You!



We appreciate your participation and look forward to the exciting talks and discussions ahead!