1) $X = \{2,-1,2,3,0,0,0,0\}$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$$

$$X_8[3] = \sum_{n=0}^{7} x[n] e^{\frac{-j\cdot 2\pi \cdot n \cdot 3}{8}} = \sum_{n=0}^{7} x[n] e^{-j3\pi n/4}$$

$$= 2 + (-1)e^{-j3\pi/4} + 2e^{-j3\pi/2} + 3e^{-j9\pi/4} + 0$$

$$= 2 + (-1)\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) + 2(0 - j) + 3\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$$

$$2 + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} - j2 + 3\frac{\sqrt{2}}{2} + j\frac{3\sqrt{2}}{2}$$

$$\boxed{(2 + 2\sqrt{2}) + j(2 - \sqrt{2})}$$

d) $W_N = e^{-j\frac{2\pi}{N}}$

a $W_N^{Mm} = \left(e^{-j\frac{2\pi}{N}}\right)^{Mm} = e^{-j\frac{2\pi}{N}\cdot Mm} = e^{-j\frac{2\pi}{\cancel{N}}\cdot\frac{m\cancel{N}}{L}} = e^{-j\frac{2\pi m}{L}} = \left(e^{-j\frac{2\pi}{L}}\right)^m$

$= W_L^m$

b

3) $h = \{4, 3, 2, 1\}$

$x = \{2, 3, 1, 1\}$

$$y_L[2] = h_0 x_2 + h_1 x_1 + h_2 x_0 = 4 \cdot 1 + 3 \cdot 3 + 2 \cdot 2 = 4 + 9 \cdot 4 = \boxed{17}$$

$$y_4[2] = y_L[2] + y_L[6] = h_0 x_2 + h_1 x_1 + h_2 x_0 + h_3 x_3 = 17 + 2 \cdot 1 = \boxed{19}$$

$$y_8[2] = \sum_m y_L[2 + m8] = y_L[2] + y_L[10] = h_0 x_2 + h_1 x_1 + h_2 x_0 + h_3 x_7 + h_4 x_6 + h_5 x_5 + h_6 x_4 + h_7 x_3$$

$$= 17 + 0 + 0 + 0 + 0 + 0 = \boxed{17}$$

5) $\pm 20 \pm 100n = \cancel{\quad} \boxed{20, 80, 120, 180 \text{ kHz}}$

a)

b) $\frac{f_s}{2} = \frac{100}{2} = \boxed{50\text{kHz}}$

c) $f = 20\text{kHz}$

$W_{analog} = 2\pi f = 2\pi \cdot 20\text{kHz} = 40000 = \boxed{4 \cdot 10^4 \text{ rad/sec}}$

$W_{digital} = \frac{2\pi f}{f_s} = \frac{2\pi \cdot 2\cancel{0}}{10\cancel{0}} = \frac{2\pi \cdot 2}{10} = \frac{4\pi}{10} = \boxed{\frac{2\pi}{5} \text{ rad}}$

$f_{digital\,sam} = \frac{f}{f_s} = \frac{20}{100} = \boxed{\frac{1}{5}}$

$f_{dig\,nyquist} = \frac{f}{f_s/2} = \frac{20}{100/2} = \frac{20}{50} = \boxed{\frac{2}{5}}$

d)

e) ~~$\frac{f}{f_s} \cdot 50 = \frac{20}{100} \cdot \frac{50}{100} = 20 \to \pm 20 \pm$~~ $\frac{f}{f_s} \cdot 50 = \frac{20}{100} \cdot 50 = 10$

$\pm 10 \pm n50 = \boxed{10, 40, 60, 90, 110, 140, 160, 190 \text{ kHz}}$

f) $\frac{f_s}{2} = \frac{100}{2} = \boxed{50\text{kHz}}$

7)

a
$$x(n) + 0.95x(n-1) + 0.9025x(n-2) = v(n) + 0.1v(n-1) - 0.72(n-2)$$
$$X[z]\left(1 + 0.95z^{-1} + 0.9025z^{-2}\right) = V[z]\left(1 + 0.1z^{-1} - 0.72z^{-2}\right)$$
$$H[z] = \frac{X(z)}{V(z)} = \frac{1 + 0.1z^{-1} - 0.72z^{-2}}{1 + 0.95z^{-1} + 0.9025z^{-2}} = \frac{z^2 + 0.1z - 0.72}{z^2 + 0.95z + 0.9025}$$

(Poles/zeros computed in MATLAB) $\cancel{(z-\frac{4}{5})(z+\frac{9}{10})}$

$$\boxed{\begin{array}{l}\text{poles: } -0.4750 \pm j0.8227 \\ \text{zeros: } \frac{4}{5},\ -\frac{9}{10}\end{array}}$$

b Innovations filter
- ARMA

c.
$$H(\omega) = \frac{1 + 0.1e^{-j\omega} - 0.72e^{-j2\omega}}{1 + 0.95e^{-j\omega} + 0.9025e^{-j2\omega}}$$
$$S_x(\omega) = \sigma_v^2 |H(\omega)|^2 = \boxed{\frac{\left|1 + 0.1e^{-j\omega} - 0.72e^{-j2\omega}\right|^2}{\left|1 - 0.9e^{-j\omega} + 0.81e^{-j2\omega}\right|^2} \cdot 4}$$