The Cooper Union Department of Electrical Engineering
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ECE310 Digital Signal Processing
Problem Set II: Transfer Function Analysis
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1. Given the following digital transfer function:

$$H(z) = \frac{(2z-1)(z+4)}{3(3z-1)(2z+5)^2}$$

   (a) Specify the ROC that corresponds to a stable system.

   (b) Write the general form of $h[n]$ for that ROC: do NOT find the exact signal (e.g., via partial fractions). My intention is for you to list the modes with unspecified constant coefficients.

   (c) Repeat the above for the case of a causal system.

   (d) Find an allpass factor $A(z)$ and a minimum-phase $H_{\min}(z)$ such that $H_{\min} = H \cdot A$. You can leave your answer in factored form. Also, superimpose graphs of the three phase responses (all unwrapped, in degree) in MATLAB.

2. Find minimum-phase $H(s)$ whose magnitude squared response is:

$$|H(\omega)|^2 = \frac{9+\omega^2}{(9+4\omega^2)(16+9\omega^2)}$$

3. The power spectral density of a discrete-time WSS random signal is given by:

$$S(\omega) = \frac{13+12\cos\omega}{(5+4\cos\omega)^2}$$

   (a) Express the PSD in the z-domain, $S(z)$. You do not have to multiply out the factors. In fact, your formula can have both positive and negative powers of z.

   (b) Determine whether $S(z)$ has poles and/or zeros (and their multiplicity) at 0 and/or ∞.

   (c) Use the method of spectral factorization to find the innovations filter $H(z)$ and the whitening filter $G(z)$, assuming the innovations signal is normalized to have unit variance. Again, you do not have to multiply out the factors. **Note:** You can use MATLAB or a calculator to help you avoid doing messy algebra by hand; e.g.,find roots of polynomials. I'm NOT asking you to write general-purpose spectral factorization code. As a hint, the (finite) poles and zeros are fairly simple rational numbers.