

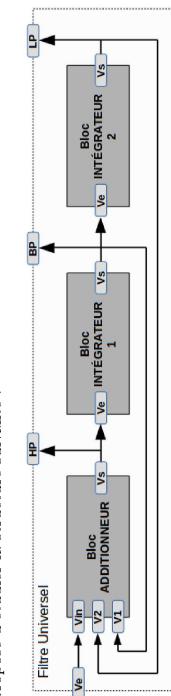
## BLOC 2 / FILTRAGE ACTIF

### Mission 2.1 - Filtrer des composantes fréquentielles - Ordre 1 Passif

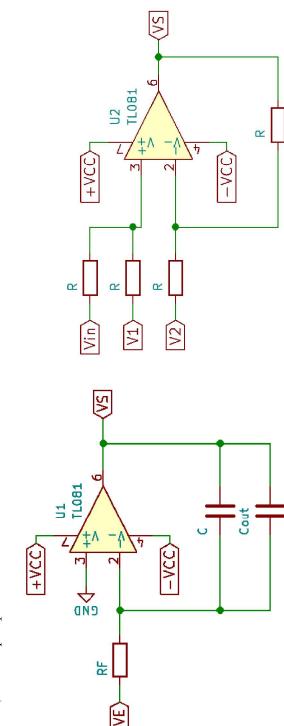
Proposer une structure de filtre du premier ordre qui laisse passer des signaux au dessus d'une fréquence  $f_c$ . Donner les principales caractéristiques et limitiations d'un tel filtre.

### Mission 2.2 - Filtrer des composantes fréquentielles - Ordre 2

On se propose d'étudier la structure suivante :



Pour cela, on se propose d'étudier les deux circuits suivants :



### Mission 2.4 - Réaliser un filtre à partir d'un gabarit

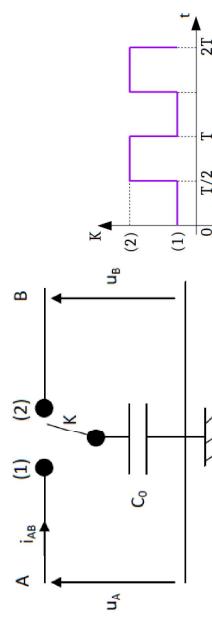
On s'intéresse ici aux filtres de Butterworth (voir annexe).

- On souhaite réaliser un filtre dont le gabarit est le suivant :  
 — gain supérieur à  $-1$  dB jusqu'à 10 kHz  
 — gain inférieur à  $-60$  dB à partir de 40 kHz

1. Tracer le gabarit du filtre.
2. Déterminer l'ordre du filtre minimal.
3. Déterminer la pulsation de coupure du filtre.
4. Déterminer la fonction de transfert du filtre

### Mission 2.3 - Filtrer des composantes fréquentielles autrement

On se propose d'étudier la structure suivante, dont l'interrupteur  $K$  est piloté par le signal de commande ci-dessous :



1. Calculer la charge stockée dans  $C_0$  entre les instants  $0$  et  $T/2$ , puis entre les instants  $T/2$  et  $T$ .
2. Quelle quantité de charges passe de A vers B entre les instants  $0$  et  $T$  ?
3. Calculer alors le courant moyen circulant du point A au point B pendant une période T.
4. Donner l'expression de la résistance équivalente  $R_{AB}$  vue entre les bornes A et B de cette cellule.

### Intégrateur

On réalise un intégrateur à partir du circuit de la figure 2.

1. Donner la fonction de transfert du circuit  $T(j\omega) = u_2/u_1$  en fonction de  $R_{AB}$  et de  $C$ .
2. Que devient alors la fonction de transfert  $T(j\omega) = u_2/u_1$  en fonction des éléments du système (C et  $C'$ ) ?
3. Quel est l'intérêt d'un tel circuit ?

### Etude du MAX296

On s'intéresse au composant MAX296 dont une partie de la documentation technique est donnée en annexe.

1. Quelles sont les fréquences maximales utilisables sur l'entrée INPUT ? Sur l'entrée CLOCK ? Quelles sont les applications visées ?
2. Quelle fréquence faut-il appliquer sur l'entrée CLOCK pour avoir une fréquence de coupure de 3 kHz ? Que vaut alors l'amplification théorique du signal à : (a) 300 Hz ; (b) 30 kHz ? (c) 5 kHz ?
3. Avec un filtre du second ordre (type Rauch) avec une pulsation de coupure à la même valeur, quelle aurait été l'amplification : (a) à 30 kHz ; (b) à 5 kHz ?

# MAX291/MAX292/ MAX295/MAX296

## 8th-Order, Lowpass, Switched-Capacitor Filters

### General Description

The MAX291/MAX292/MAX295/MAX296 are easy-to-use, 8th-order, lowpass, switched-capacitor filters that can be set up with corner frequencies from 0.1Hz to 50kHz (MAX291/MAX292) or 0.1Hz to 50kHz (MAX295/MAX296). The MAX291/MAX295 Butterworth filters provide maximally flat passband response, and the MAX292/MAX296 Bessel filters provide low overshoot and fast settling. All four filters have fixed responses, so the design task is limited to selecting the clock frequency that controls the filter's corner frequency.

An external capacitor is used to generate a clock using the internal oscillator, or an external clock signal can be used. An uncommitted operational amplifier (noninverting input grounded) is provided for building a continuous-time bypass filter for post-filtering or anti-aliasing.

Produced in an 8-pin DIP/SO and a 16-pin wide SO package, and requiring a minimum of external components, the MAX291 series delivers very aggressive performance from a tiny area.

### Applications

- ADC Anti-Aliasing Filter
- Noise Analysis
- DAC Post-Filtering
- 50Hz/60Hz Line-Noise Filtering

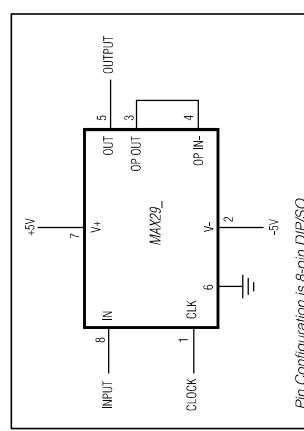
### Features

- ♦ 8th-Order Lowpass Filters:
- Butterworth (MAX291/MAX295)  
    Bessel (MAX292/MAX296)
- ♦ Clock-Tunable Corner-Frequency Range:
- 0.1Hz to 25kHz (MAX291/MAX292)  
    0.1Hz to 50kHz (MAX295/MAX296)
- ♦ No External Resistors or Capacitors Required
- ♦ Internal or External Clock
- ♦ Clock to Corner Frequency Ratio:
- 100:1 (MAX291/MAX292)  
    50:1 (MAX295/MAX296)
- ♦ Low Noise: -70dB THD + Noise (Typ)
- ♦ Operate with a Single +5V Supply or Dual ±5V Supplies
- ♦ Uncommitted Op Amp for Anti-Aliasing or Clock-Noise Filtering
- ♦ 8-Pin DIP and SO Packages

### Ordering Information

PART	TEMP. RANGE	PIN+PACKAGE
MAX291CPA	0°C to +70°C	8 Plastic DIP
MAX291CSA	0°C to +70°C	8 SO
MAX291CWE	0°C to +70°C	16 Wide SO
MAX291CD	0°C to +70°C	Dice*
MAX291EPA	-40°C to +85°C	8 Plastic DIP
MAX291ESA	-40°C to +85°C	8 SO
MAX291EWE	-40°C to +85°C	16 Wide SO
MAX291MJA	-55°C to +125°C	8 CERDIP**

### Typical Operating Circuit



For pricing, delivery, and ordering information, please contact Maxim Direct at 1-888-629-4642, or visit Maxim's website at [www.maximintegrated.com](http://www.maximintegrated.com).

Maxim Integrated

19-4526 Rev 5/10

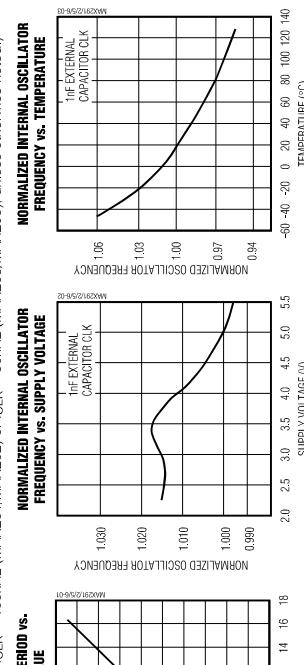
### ELECTRICAL CHARACTERISTICS (continued)

(V<sub>+</sub> = 5V, V<sub>-</sub> = -5V, filter output measured at OUT pin, 20kΩ load resistor to ground at OUT and OP OUT, f<sub>CLK</sub> = 100kHz (MAX291/MAX292) or f<sub>CLK</sub> = 50kHz (MAX295/MAX296), T<sub>A</sub> = T<sub>MIN</sub> to T<sub>MAX</sub>, unless otherwise noted.)

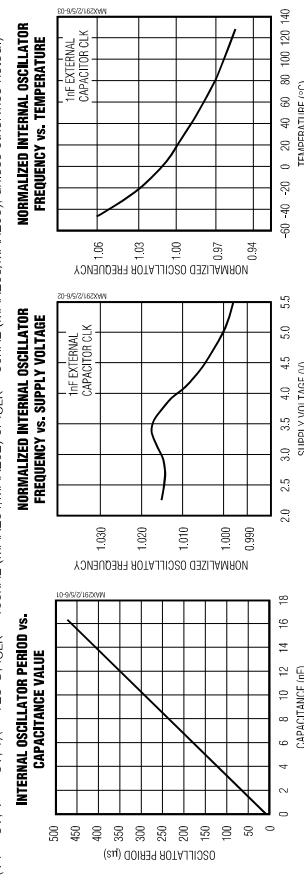
PARAMETER	CONDITIONS	MIN	TYP	MAX	UNITS
		±4		6	mVp-p
Output DC Swing	In = GND			±400	mV
Output Offset Voltage	DC Inversion Gain Error with Output Offset Removed	0.15	0	-0.15	dB
Total Harmonic Distortion plus Noise	T <sub>A</sub> = +25°C, f <sub>CLK</sub> = 100kHz			-70	dB
Clock Feedthrough	f <sub>CLK</sub> = 100kHz			6	mVp-p
<b>CLOCK</b>					
Internal Oscillator Frequency	C <sub>Osc</sub> = 100pF	29	35	43	kHz
Internal Oscillator Current Source/Sink	V <sub>CLK</sub> = 0V or 5V			+70	μA
Clock Input (High) (Note 1)				4.0	V
Low				1.0	V
<b>UNCOMMITTED OP AMP</b>					
Input Offset Voltage			±4	±50	mV
Output DC Swing			0.05	0.05	μA
Input Bias Current					
<b>POWER REQUIREMENTS</b>					
Supply Voltage				±2.375	mA
Dual Supply				±5.500	V
Single Supply	V <sub>-</sub> = 0V, GND = V ± 2			4.750	11,000
Supply Current	V <sub>+</sub> = 5V, V <sub>-</sub> = -5V, V <sub>CLK</sub> = 0V to 5V			15	22
	V <sub>+</sub> = 2.375V, V <sub>-</sub> = -2.375V, V <sub>CLK</sub> = -2V to 2V			7	12

Note 1. Guaranteed by design.

(V<sub>+</sub> = 5V, V<sub>-</sub> = -5V, T<sub>A</sub> = +25°C, f<sub>CLK</sub> = 100kHz (MAX291/MAX292) or f<sub>CLK</sub> = 50kHz (MAX295/MAX296), unless otherwise noted.)



### Typical Operating Characteristics

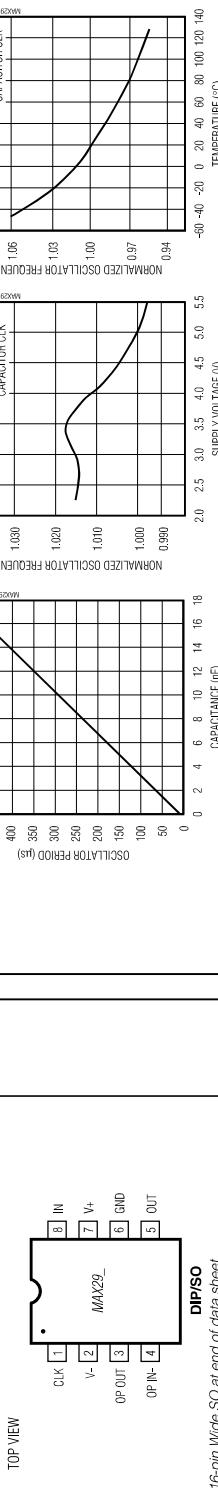


### Ordering Information continued at end of data sheet.

\* Contact factory for dice specifications.

\*\* Contact factory for availability and processing to MIL-STD-883.

### Pin Configurations



# MAX291/MAX292/MAX295/MAX296

## 8th-Order, Lowpass, Switched-Capacitor Filters

# MAX291/MAX292/MAX295/MAX296

## 8th-Order, Lowpass, Switched-Capacitor Filters

(V<sub>+</sub> = 5V, V<sub>-</sub> = -5V, TA = +25°C, f<sub>CLK</sub> = 100kHz (MAX291/MAX292) or f<sub>CLK</sub> = 50kHz (MAX295/MAX296), unless otherwise noted.)

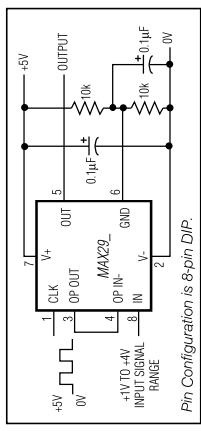
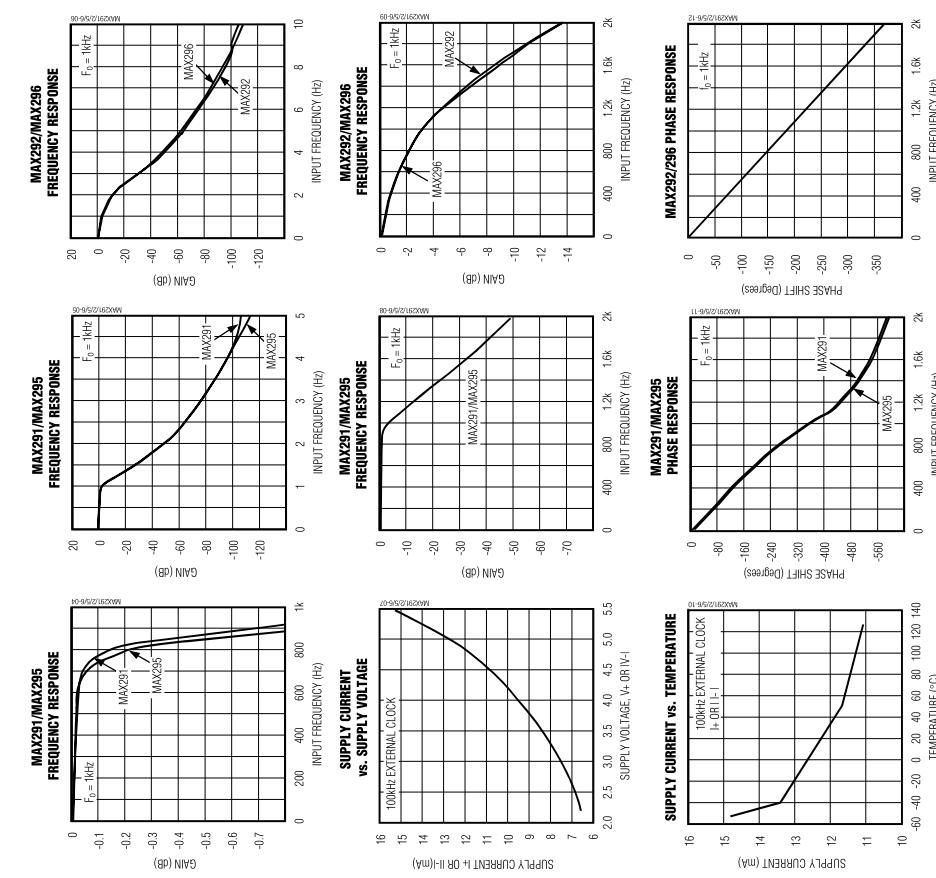


Figure 3. 8-pin DIP Pin Configuration

clock frequency over the clock range 100kHz to 1MHz. Varying the rate of an external clock will dynamically adjust the corner frequency of the filter.

Ideally, the MAX291/MAX292/MAX295/MAX296 should be clocked symmetrically (50% duty cycle). MAX291/MAX292/MAX295/MAX296 can be operated with clock asymmetry of up to 60/40% (or 40/60%) if the clock remains HIGH and LOW for at least 200ns. For example, if the part has a maximum clock rate of 2.5MHz, then the clock should be high for at least 200ns, and low for at least 200ns.

When using the internal oscillator, the capacitance (C<sub>Osc</sub>) from CLK to ground determines the oscillator frequency:

$$f_{Osc} (\text{kHz}) \approx \frac{10^5}{3C_{Osc} (\text{pF})}$$

The stray capacitance at CLK should be minimized because it will affect the internal oscillator frequency.

### Application Information

#### Power Supplies

The MAX291/MAX292/MAX295/MAX296 operate from either dual or single power supplies. The dual-supply voltage range is +2.375V to +5.50V. The +2.5V dual supply is equivalent to single-supply operation (Figure 3). Minor performance degradation could occur due to the external resistor divider network, where the GND pin is biased to mid-supply.

#### Input Signal Range

The ideal input signal range is determined by observing at what voltage level the total harmonic distortion plus noise (THD + Noise) ratio is maximized for a given corner frequency. The Typical Operating Characteristics show the MAX291/MAX292/MAX295/MAX296 THD + Noise response as the input signal's peak-to-peak amplitude is varied.

**Uncommitted Op Amp**  
The uncommitted op amp has its noninverting input tied to the GND pin, and can be used to build a 1st- or 2nd-

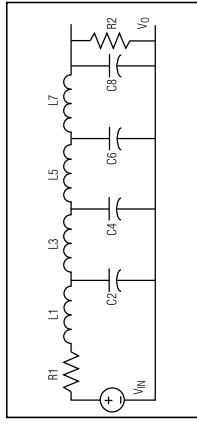


Figure 2. 8th-Order Ladder Filter Network

error on its respective poles, while the same mismatch in a ladder filter design will spread its error over all poles. The MAX291/MAX292/MAX295/MAX296 input impedance is effectively that of a switched-capacitor resistor (see equation below, and Table 1), and it is inversely proportional to frequency. The input impedance values determined below represent average input impedance, since the input current is not continuous. The input current flows in a series of pulses that charge the input capacitor every time the appropriate switch is closed. A good rule of thumb is that the driver's input source resistance should be less than 10% of the filter's input impedance. The input impedance of the filter can be estimated using the following formula:

$$Z = 1 / (f_{CLK} * C)$$

where: f<sub>CLK</sub> = Clock Frequency  
The input impedance for various clock frequencies is given below:

Table 1. Input Impedance for Various Clock Frequencies

PART	C (pF)	10kHz (MΩ)	100kHz (MΩ)	1000kHz (kΩ)
MAX291	2.24	44.6	4.46	446
MAX292	3.28	30.5	3.05	305
MAX295	4.47	22.4	2.24	224
MAX296	4.22	23.7	2.37	237

The MAX291/MAX292/MAX295/MAX296 maximum recommended clock frequency is 25MHz, producing a cutoff frequency of 25kHz for the MAX291/MAX292 and 50kHz for the MAX295/MAX296. The CLK pin can be driven by an external clock or by the internal oscillator with an external capacitor. For external clock applications, the clock circuitry has been designed to interface with +5V CMOS logic. Drive the CLK pin with a CMOS gate powered from OV and +5V when using either a single +5V supply or dual +5V supplies. The MAX291/MAX292/MAX295/MAX296 supply current increases slightly (<3%) with increasing

## UNIVERSAL ACTIVE FILTER

Check for Samples: [UAF42](#)

### FEATURES

- **VERSATILE:**
  - Low-Pass, High-Pass
  - Band-Pass, Band-Reject
- **SIMPLE DESIGN PROCEDURE**
- **ACCURATE FREQUENCY AND Q:**
  - Includes On-Chip 1000pF  $\pm 0.5\%$  Capacitors

### APPLICATIONS

- TEST EQUIPMENT
- COMMUNICATIONS EQUIPMENT
- MEDICAL INSTRUMENTATION
- DATA ACQUISITION SYSTEMS
- MONOLITHIC REPLACEMENT FOR UAF41

The UAF42 is a universal active filter that can be configured for a wide range of low-pass, high-pass, and band-pass filters. It uses a classic state-variable analog architecture with an inverting amplifier and two integrators. The integrators include on-chip 1000pF capacitors trimmed to  $0.5\%$ . This architecture solves one of the most difficult problems of active filter design—obtaining tight tolerance, low-loss capacitors.

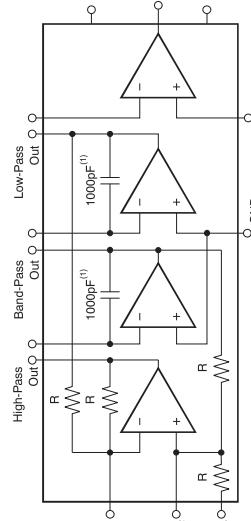
A DOS-compatible filter design program allows easy implementation of many filter types, such as Butterworth, Bessel, and Chebyshev. A fourth, uncommitted FET-input op amp (identical to the other three) can be used to form additional stages, or for special filters such as band-reject and inverse Chebyshev.

The classical topology of the UAF42 forms a time-continuous filter, free from the anomalies and switching noise associated with switched-capacitor filter types.

The UAF42 is available in 14-pin plastic DIP and

SOIC-16 surface-mount packages, specified for the

$-25^\circ\text{C}$  to  $+85^\circ\text{C}$  temperature range.



NOTE: (1)  $\pm 0.5\%$ .

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## ELECTRICAL CHARACTERISTICS

At  $T_A = +25^\circ\text{C}$ , and  $V_S = \pm 15\text{V}$ , unless otherwise noted.

PARAMETER	CONDITIONS	UAF42P, AU		UNIT
		MIN	MAX	
FILTER PERFORMANCE				
Frequency Range, $f_c$	$f = 1\text{kHz}$	0 to 100	1	kHz
Frequency Accuracy	vs Temperature	0.01	1	%
Maximum Q		400	—	% $^\circ\text{C}$
Maximum (Q · Frequency) Product		500	—	kHz
Q vs Temperature	$(f_0 \cdot Q) < 10^4$	0.01	—	% $^\circ\text{C}$
Q Repeatability	$(f_0 \cdot Q) < 10^5$	0.025	—	% $^\circ\text{C}$
Offset Voltage, Low-Pass Output	$(f_0 \cdot Q) < 10^6$	2	—	%
Resistor Accuracy		±5	—	mV
OFFSET VOLTAGE <sup>(1)</sup>		0.5	1	%
Input Offset Voltage	$V_S = \pm 15\text{V}$	±0.5	±5	mV
vs Temperature		±3	—	$\mu\text{V}/^\circ\text{C}$
vs Power Supply		96	96	dB
INPUT BIAS CURRENT <sup>(1)</sup>		10	50	pA
Input Bias Current	$V_{CM} = 0\text{V}$	5	—	pA
NOISE				
Input Voltage Noise		25	—	$\text{nV}/\sqrt{\text{Hz}}$
Noise Density, $f = 10\text{Hz}$		10	—	$\text{nV}/\sqrt{\text{Hz}}$
Input Offset Voltage	$f = 10\text{kHz}$	2	—	$\mu\text{V}_{pp}$
Voltage Noise: BW = $0.1\text{Hz}$ to $10\text{Hz}$		2	—	$\text{fA}/\sqrt{\text{Hz}}$
Input Bias Current Noise		2	—	$\text{fA}/\sqrt{\text{Hz}}$
Noise Density, $f = 10\text{kHz}$		2	—	$\text{fA}/\sqrt{\text{Hz}}$
INPUT VOLTAGE RANGE <sup>(1)</sup>		±11.5	—	V
Common-Mode Input Range		96	—	dB
Common-Mode Rejection		10 <sup>13</sup>   2	—	$\Omega \parallel \text{pF}$
INPUT IMPEDANCE <sup>(1)</sup>		10 <sup>13</sup>   6	—	$\Omega \parallel \text{pF}$
Differential		—	—	
Common-Mode		—	—	
OPEN-LOOP GAIN <sup>(1)</sup>				
Open-Loop Voltage Gain	$V_O = \pm 10\text{V}$ , $R_L = 2\text{k}\Omega$	90	126	dB
FREQUENCY RESPONSE				
Slew Rate		10	—	$\text{V}/\mu\text{s}$
Gain-Bandwidth Product	$G = +1$	4	—	MHz
Total Harmonic Distortion	$G = -1$ , $f = 1\text{kHz}$	0.1	—	%
OUTPUT <sup>(1)</sup>				
Voltage Output	$R_L = 2\text{k}\Omega$	±11	±11.5	V
Short Circuit Current		±25	—	mA

(1) Specifications apply to uncommitted op amp,  $A_3$ . The three op amps forming the filter are identical to  $A_3$  but are tested as a complete filter.

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## APPLICATION INFORMATION

The UAF42 is a monolithic implementation of the proven state-variable analog filter topology. This device is pin-compatible with the popular UAF41 analog filter, and it provides several improvements.

The slew rate of the UAF42 has been increased to  $10V/\mu s$ , versus  $1.6V/\mu s$  for the UAF41. Frequency  $\times$  Q product of the UAF42 has been improved, and the useful natural frequency extended by a factor of four to 100kHz. FET input op amps on the UAF42 provide very low input bias current. The monolithic construction of the UAF42 provides lower cost and improved reliability.

### DESIGN PROGRAM

Application report [SBFA002](#) (available for download at [www.ti.com](#)) and a computer-aided design program also available from Texas Instruments make it easy to design and implement many kinds of active filters. The DOS-compatible program guides you through the design process and automatically calculates component values.

Low-pass, high-pass, band-pass and band-reject (notch) filters can be designed. The program supports the three most commonly-used all-pole filter types: Butterworth, Chebyshev and Bessel. The less-familiar inverse Chebyshev is also supported, providing a smooth passband response with ripple in the stop band. With each data entry, the program automatically calculates and displays filter performance. This feature allows a spreadsheet-like *what-if* design approach. For example, a user can quickly determine, by trial and error, how many poles are required for a desired attenuation in the stopband. Gain/phase plots may be viewed for any response type.

The basic building element of the most commonly-used filter types is the second-order section. This section provides a complex-conjugate pair of poles. The natural frequency,  $\omega_n$ , and Q of the pole pair determine the characteristic response of the section. The low-pass transfer function is shown in **Equation 1**:

$$\frac{V_o(s)}{V_i(s)} = \frac{A_{LP}(\omega_n)^2}{s^2 + s(\omega_n/Q) + (\omega_n)^2} \quad (1)$$

The high-pass transfer function is given by **Equation 2**:

$$\frac{V_{HP}(s)}{V_i(s)} = \frac{A_{HP}s^2}{s^2 + s(\omega_n/Q) + (\omega_n)^2} \quad (2)$$

The band-pass transfer function is calculated using **Equation 3**:

$$\frac{V_{BP}(s)}{V_i(s)} = \frac{A_{BP}(\omega_n/Q)s}{s^2 + s(\omega_n/Q) + (\omega_n)^2} \quad (3)$$

A band-reject response is obtained by summing the low-pass and high-pass outputs, yielding the transfer function shown in **Equation 4**:

$$\frac{V_{BR}(s)}{V_i(s)} = \frac{A_{BR}(s^2 + \omega_n^2)}{s^2 + s(\omega_n/Q) + (\omega_n)^2} \quad (4)$$

The most common filter types are formed with one or more cascaded second-order sections. Each section is designed for  $\omega_n$  and Q according to the filter type (Butterworth, Bessel, Chebyshev, etc.) and cutoff frequency. While tabulated data can be found in virtually any filter design text, the design program eliminates this tedious procedure.

Second-order sections may be noninverting (**Figure 1**) or inverting (**Figure 2**). Design equations for these two basic configurations are shown for reference. The design program solves these equations, providing complete results, including component values.

## UAF42

## APPLICATION INFORMATION

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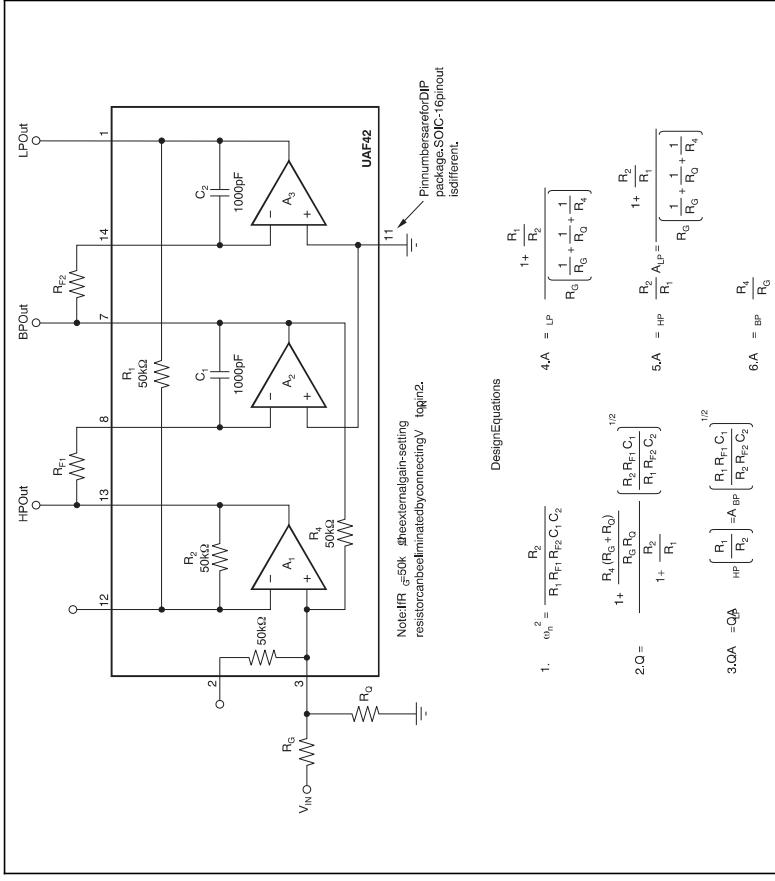
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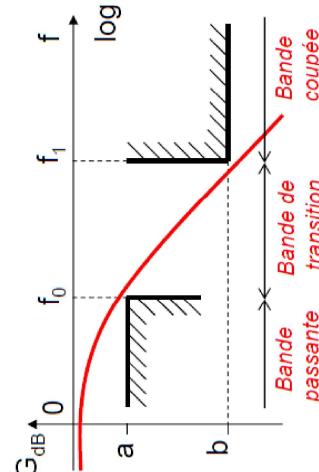


## Annexe : Filtres actif de Butterworth et de Chebychev

Document basé sur le cours de Sylvie Lebrun, *Filtrage analogique, 2015*.

### Gabarit d'un filtre

Le gabarit d'un filtre correspond aux contraintes fréquentielles et en gain que doit satisfaire le système à développer.



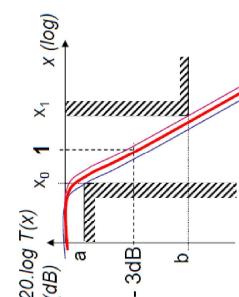
On souhaite souvent réaliser un système de filtrage qui possède les caractéristiques suivantes :

- transmission de fréquence inférieure à  $f_0$
- valeur minimale  $a$  de gain dans la bande de fréquence à transmettre
- valeur maximale  $b$  de gain dans la bande de fréquence à éliminer (à partir d'une fréquence  $f_1$ )

Le gabarit est caractérisé par 2 points ( $f_0, a$ ) et ( $f_1, b$ ).

A partir de ce gabarit, plusieurs types de filtres peuvent être utilisés : Butterworth, Chebychev, Bessel, Cauer

Pour la suite, on posera :  $X = \frac{\omega}{\omega_c}$  où  $\omega_c$  est la fréquence de coupure du système, définie à  $-3\text{dB}$  par rapport au gain dans la bande passante.



### Filtre de Butterworth

Ce type de filtre est utilisé pour sa réponse extrêmement plate dans la bande-passeante.

La réponse en fréquence d'un tel filtre est tel que son module vaut :

$$T(X) = \frac{1}{\sqrt{1 + X^{2n}}}$$

où  $n$  est l'ordre du filtre.

#### Détermination de $n$

En s'intéressant aux conditions aux limites :

$$\begin{cases} 20 \cdot \log_{10} T(x_0) > a \\ 20 \cdot \log_{10} T(x_1) < b \end{cases} \Leftrightarrow \begin{cases} x_0^{2n} < 10^{-a/10} - 1 \\ x_1^{2n} > 10^{-b/10} - 1 \end{cases} \quad (1)$$

En divisant (2) par (1) on obtient alors la valeur minimale de  $n$ . On choisira  $n$  la plus petite valeur entière qui satisfasse :

$$n \geq \frac{1}{2} \cdot \frac{\log_{10} \frac{10^{-a/10} - 1}{10^{-b/10} - 1}}{\log_{10} \frac{f_0}{f_1}}$$

#### Détermination de $f_c$

On calcule alors avec (1) et (2) les fréquences de coupure limites :

$$f_{c,0} = \frac{f_0}{(10^{-a/10} - 1)^{1/(2n)}} \quad f_{c,1} = \frac{f_1}{(10^{-b/10} - 1)^{1/(2n)}}$$

On choisit ensuite la fréquence de coupure comme étant la moyenne géométrique des deux fréquences précédentes :

$$f_c = \sqrt{f_{c,0} \cdot f_{c,1}}$$

#### Fonction de transfert

Il faut trouver une fraction rationnelle complexe  $T(p)$  (avec  $p = j \cdot x$ ) qui admette  $T(x)$  comme module. On factorise alors le polynôme :  $B_n(x) = 1 + x^{2n}$ .

On trouve alors que  $B_n(p)$  peut s'écrire sous la forme des polynômes obtenus par Butterworth :

n	Polynôme de Butterworth $B_n(p)$ pour $\omega_c = 1$
1	$(p+1)$
2	$p^2 + 1.4142p + 1$
3	$(p+1)(p^2 + p + 1)$
4	$(p^2 + 0.7054p + 1)(p^2 + 1.8478p + 1)$
5	$(p+1)(p^2 + 0.6180p + 1)(p^2 + 1.6180p + 1)$
6	$(p^2 + 0.5176p + 1)(p^2 + 1.4142p + 1)(p^2 + 1.9319p + 1)$
7	$(p+1)p^2 + 0.4450p + 1)(p^2 + 1.2470p + 1)(p^2 + 1.8019p + 1)$
8	$(p^2 + 0.3902p + 1)(p^2 + 1.1111p + 1)(p^2 + 1.6629p + 1)(p^2 + 1.9616p + 1)$

La fonction de transfert normalisée s'écrit alors ( $n$  impair et  $n$  pair) :

$$T(p) = \frac{1}{(1+p) \cdot (a^2 + b^2 + 2 \cdot bp + p^2) \cdot (\dots)} \quad T(p) = \frac{1}{(a^2 + b^2 + 2 \cdot bp + p^2) \cdot (\dots)}$$