Definition 1. *The sample mean of database X of size n is*

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Theorem 1. *Say database X has size n and is bounded above by M and bounded below by m. Using the distance where two databases differ by the edit of one record, $\bar{X}$ has sensitivity bounded above by*

$$\frac{M-m}{n}.$$

Proof. Say X and X' are neighboring databases which differ at data-point x_j. Then

$$\begin{aligned}
\Delta\bar{X} &= \max_{X,X'} \left|\bar{X} - \bar{X}'\right| \\
&= \max_{X,X'} \frac{1}{n}\left|\left(\sum_{\{i\in[n] | i\neq j\}} x_i\right) + x_j - \left(\sum_{\{i\in[n] | i\neq j\}} x'_i\right) + x'_j\right| \\
&= \max_{X,X'} \frac{1}{n}\left|x_j - x'_j\right| \\
&\leq \frac{M-m}{n}.
\end{aligned}$$

□

Theorem 2. *Say database X has size n and is bounded above by M and bounded below by m. Using the distance where two databases differ by the addition or removal of one record, $\bar{X}$ has sensitivity bounded above by*

$$\frac{M-m}{n-1}.$$

Proof. Say X and X' are neighboring databases where $X' = X \cup \{y\}$.

$$
\begin{aligned}
\Delta\bar{X} &= \max_{X,X'} \left|\bar{X} - \bar{X}'\right| \\
&= \max_{X,X'} \left| \frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n+1}\left(\sum_{i=1}^{n} x_i + y\right)\right| \\
&= \max_{X,X'} \left| \left(\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n+1}\sum_{i=1}^{n} x_i\right) - \frac{y}{n+1}\right| \\
&= \max_{X,X'} \left| \frac{1}{(n+1)n}\sum_{i=1}^{n} x_i - \frac{y}{n+1}\right| \\
&= \frac{1}{n+1}\max_{X,X'} \left| \frac{1}{n=1}\sum_{i=1}^{n} x_i - y\right| \\
&= \frac{1}{n+1}\max_{X,X'} \left|\bar{X} - y\right| \\
&\leq \frac{M-m}{n+1}.
\end{aligned}
$$

Say X and X' are neighboring databases where $y \in X$ and $X' = X \setminus \{y\}$.

$$
\begin{aligned}
\Delta\bar{X} &= \max_{X,X'} \left|\bar{X} - \bar{X}'\right| \\
&= \max_{X,X'} \left| \frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i - y\right)\right| \\
&= \max_{X,X'} \left| \left(\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n-1}\sum_{i=1}^{n} x_i\right) + \frac{y}{n-1}\right| \\
&= \max_{X,X'} \left| \frac{-1}{(n-1)n}\sum_{i=1}^{n} x_i + \frac{y}{n-1}\right| \\
&= \frac{1}{n-1}\max_{X,X'} \left| y - \frac{1}{n}\sum_{i=1}^{n} x_i\right| \\
&= \frac{1}{n-1}\max_{X,X'} \left|y - \bar{X}\right| \\
&\leq \frac{M-m}{n-1}.
\end{aligned}
$$

Say X and X' are neighboring databases that differ by the addition or removal of one record.

$$
\begin{aligned}
\Delta\bar{X} &\le \max\left(\frac{M-m}{n-1}, \frac{M-m}{n+1}\right) \\
&= \frac{M-m}{n-1}
\end{aligned}
$$

$\square$