Definition 1. Let z be the true value of the statistic and let X be the random variable the noisy release is drawn from, i.e. $X = z + c$ where $c \sim \mathrm{Lap}(\lambda)$. Let α be the statistical significance level, and let $Y = |X - z|$. Then, accuracy a is the a s.t.

$$\alpha = \Pr[Y > a].$$

Theorem 1. *The accuracy of an ϵ-differentially private statistic with sensitivity s, at statistical significance level α is*

$$a = \frac{s}{\epsilon} \ln(1/\alpha).$$

Proof. Recall that the probability density function f of the Laplace distribution, for $x \sim X$ with location parameter μ and scaling parameter λ is defined to be

$$f(x) = \frac{1}{2\lambda} e^{\frac{-|x-\mu|}{\lambda}}$$

Then, since the pdf g of Y is the same as the folded pdf of X, shifted over by μ and doubled,

$$g(y) = \frac{1}{\lambda} e^{-y/\lambda}.$$

Then,

$$\begin{aligned}
\alpha &= \Pr[Y > a] \\
&= 1 - \Pr[Y \leq a] \\
&= 1 - \int_{-\infty}^{a} g(y)dy \\
&= 1 - \int_{0}^{a} g(y)dy \\
&= 1 - (1 - e^{-a/\lambda})
\end{aligned}$$

Solving for a gives $a = \lambda \ln(1/\alpha)$. Then, since

$$\lambda = \frac{s}{\epsilon},$$

$$a = \frac{s}{\epsilon} \ln(1/\alpha)$$

□

Lemma 2. *An ϵ-differentially private histogram with sensitivity 2 has accuracy*

$$a = \frac{2}{\epsilon} \ln(1/\alpha)$$

Theorem 3. *An (ϵ, δ)-differentially private histogram using the stability mechanism has accuracy*

$$a = \frac{2}{\epsilon} \ln(\frac{2}{\alpha\delta})$$

Proof. Recall that the stability mechanism sets any counts smaller than

$$\frac{2}{\epsilon} \ln(2/\delta) + 1$$

to 0. Then, each count has accuracy bound by

$$\begin{aligned} a &= \frac{2}{\epsilon} \ln(1/\alpha) + \frac{2}{\epsilon} \ln(2/\delta) + 1 \\ &= \frac{2}{\epsilon} \ln(\frac{2}{\alpha\delta}) + 1 \end{aligned}$$

□