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# Wavelet analysis of quantitative easing in Japan

Diploma thesis

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Bc. et Bc. Jakub Škoda

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Diploma thesis

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# Abstract

ŠKODA, Jakub: Wavelet analysis of quantitative easing in Japan. [Diploma thesis] – University of Economics in Bratislava. Faculty of National Economy; Department of Banking and International Finance. – Supervisor: Ing. Barbora Stanová, PhD., Bratislava: NHF EUBA, 2022, 80 p.

The aim of the diploma thesis is to verify if there is a link between the rise of monetary base directly caused by Bank of Japan as part of quantitative easing and the intended rise of CPI. Bank of Japan has been now using quantitative easing in total for 14 years, from 2001 to 2006 and since 2013 to the present day. This makes Japan a good case for evaluating the effectiveness of this unconventional monetary policy. Various forms of VAR analysis are the main tool in the research of quantitative easing, however, wavelet analysis with its ability to identify changes that happen in a cycle over time looks at this phenomenon from a new perspective in order to discover new patterns and uncover hidden information. The diploma thesis consists of five chapters and an appendix. In Chapter 1 we introduce quantitative easing, its origin and the theory behind it as well as the economic-historical background that made Bank of Japan to use it. We also provide an overview of published literature on Japanese quantitative easing and on wavelet analysis. In Chapter 2 the aim of the thesis is specified in detail. Chapter 3 consists of mathematical introduction to wavelet analysis and practical instructions with examples on how to read its output. Chapter 4 thoroughly describes the data used in our wavelet analysis. This chapter is supplemented by Appendix where full Octave code used for the analysis can be found. Finally, in Chapter 5 we present the results which are consistent with the sentiment from the first chapter that the link between quantitative easing and CPI is far from clear and its effectiveness remains questionable. However, after start of quantitative easing in 2001 economic variables seem to be more stabilized and periodical. Furthermore, what consistently shows up in our results is a strong coherence between yen exchange rate and both CPI and GDP from 1985 to 2001, which matches the time frame from the Plaza Accord of 1985 to start of quantitative easing in 2001. This suggests that strong appreciation of yen after the Plaza Accord might have been the reason behind the financial assets bubble that later caused the Lost Decade, as it is often speculated, and quantitate easing might have somewhat helped in lessening the influence of yen exchange rate on the economy.

**Keywords:** quantitative easing, Bank of Japan, central banks, asset purchases, wavelet analysis, Plaza Accord

# Abstrakt

ŠKODA, Jakub: Wavelet analýza kvantitatívneho uvoľňovania v Japonsku. [Diplomová práca] – Ekonomická univerzita v Bratislave. Národohospodárska fakulta; Katedra bankovníctva a medzinárodných financií. – Vedúca diplomovej práce: Ing. Barbora Stanová, PhD., Bratislava: NHF EUBA, 2022, 80 s.

Cieľom diplomovej práce je overiť, či existuje súvislosť medzi rastom menovej bázy, ktorý priamo zapríčinila Bank of Japan v rámci kvantitatívneho uvoľňovania a zamýšľaným rastom CPI. Bank of Japan v súčasnosti využíva kvantitatívne uvoľňovanie celkovo už 14 rokov, od roku 2001 do roku 2006 a od roku 2013 dodnes. Z tohto dôvodu je Japonsko dobrým príkladom na hodnotenie účinnosti tejto nekonvenčnej menovej politiky. Rôzne formy VAR analýzy sú hlavným nástrojom vo výskume kvantitatívneho uvoľňovania, wavelet analýza so schopnosťou identifikovať zmeny, ku ktorým dochádza v cykle v priebehu času, sa však na tento fenomén pozerá z novej perspektívy s cieľom objaviť nové súvislosti a odhaliť skryté detaily. Práca je rozdelená na päť kapitol a prílohu. V kapitole 1 predstavujeme kvantitatívne uvoľňovanie, jeho pôvod a teoretický základ, ako aj ekonomicko-historické pozadie, ktoré prinútilo Bank of Japan používať ho. Poskytujeme tiež prehľad publikovanej literatúry o japonskom kvantitatívnom uvoľňovaní a wavelet analýze. V kapitole 2 je podrobne špecifikovaný cieľ práce. Kapitola 3 pozostáva z matematického úvodu do wavelet analýzy a praktických inštrukcií s príkladmi, ako čítať jej výstup. Kapitola 4 dôkladne popisuje dáta použité v našej wavelet analýze. Túto kapitolu dopĺňa príloha, v ktorej uvádzame úplný Octave kód použitý na analýzu. Na záver, v kapitole 5 uvádzame výsledky, ktoré sú v súlade s tézou z prvej kapitoly, že prepojenie medzi kvantitatívnym uvoľňovaním a CPI nie je úplne jasné a jeho účinnosť zostáva otázna. Po spustení kvantitatívneho uvoľňovania v roku 2001 sa však ekonomické premenné javia ako stabilnejšie a pravidelnejšie. Okrem toho sa v našich výsledkoch neustále prejavuje silná koherencia medzi výmenným kurzom jenu a CPI ako aj HDP v rokoch 1985 až 2001, čo zodpovedá obdobiu od dohody Plaza Accord z roku 1985 do začiatku kvantitatívneho uvoľňovania v roku 2001. To naznačuje, že silné zhodnocovanie jenu po Plaza Accord mohlo byť dôvodom bubliny finančných aktív, ktorá neskôr zapríčinila Stratenú dekádu, ako sa často predpokladá, a kvantitatívne uvoľnovanie mohlo do istej miery pomôcť pri zmierňovaní vplyvu výmenného kurzu jenu na ekonomiku.



**Kľúčové slová:** kvantitatívne uvoľňovanie, Bank of Japan, centrálne banky, nákupy aktív, wavelet analýza, dohoda Plaza Accord

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# 1 Quantitative easing – current state and topic overview

> “QE is controversial, the theory is muddy and the empirical evidence is open to interpretation, in part because there is little data to work with.”
> Stephen Williamson (2017), Federal Reserve Bank of St. Louis

Quantitative easing (QE) as it is understood today usually means large-scale **purchase of assets** by central banks in order to increase money supply. These unsterilized purchases, backed by creation of excess central bank reserves, should inject money into the economy. This increased quantity of money should increase asset prices, make credit more available as well as set more positive expectations in the market, which should result in increased nominal spending. (Benford et al. 2009; Bank of Canada 2009)

QE is categorized as non-standard monetary policy because it is used when other policies seem to be no longer efficient. The standard case for QE is expansionary monetary policy when interest rates are near zero. Usually, a central bank would lower its interest rate on interbank lending market, often overnight interest rate, which should decrease long-term market rates and interest rates set by financial institutions and via this stimulate consumption and investment in economy. However, with rates almost at zero, and with going deep into negative interest rates not considered an option, there is not much that the bank can do with interest rate alone and this is where it reaches for QE. (Bank of Canada 2009)

Purchase of assets and creation of excess central bank reserves made in the process of QE is prominently visible on the balance sheet, which rapidly expands. Assets side of the balance sheet increases as new assets were obtained by a central bank. Symmetrically to that central bank's liabilities side of balance sheet increases as new liabilities were created by a central bank to back the purchase of the assets. (Williamson 2017)

In communication with general public, banks often use the term quantitative easing, however, asset purchase is a term that is frequently used internally. In fact, the official names for implementation of quantitative easing programmes of the European Central

Bank and the Bank of England were both Asset Purchase Programme.

The origin of the term quantitative easing is far from simple. Richard A. Werner claims to have coined the term, when he used it in an article that first appeared on 2 September 1995 in the morning edition of Japanese language financial newspaper 日本経済新聞 (‘Nihon Keizai Shinbun’, ‘Japan Economics Newspaper’ nowadays known as Nikkei). The article named 景気回復量的金融緩和から (‘Keiki kaifuku ryōteki kinyū kanwa kara’, ‘How to Create a Recovery through Quantitative Monetary Easing’) was published as part of regular section 経済教室 (‘keizai kyoshitsu’, ‘Economic Classroom’), where economists can express their opinions and theories. Originally written in Japanese as 量的金融緩和 (‘ryōteki kinyū kanwa’, ‘quantitative monetary easing’), which was later literally translated to English as quantitative easing.

Werner meant by quantitative easing a direct targeting of the quantity of credit creation by the banking system. To complicate things further, one of the first quantitative easing that is now considered to be adopted on 19 March 2001 by the Bank of Japan (BOJ), was not originally called quantitative easing. This term was adapted by BOJ retroactively from 2002 onwards. (Werner 1995; Voutsinas and Werner 2011; BBC 2013) We find it also important to note a strange situation that the only sources that are concerned with the etymology of the term quantitative easing and the origin of its theory are those that are authored or co-authored by Werner and all other sources that we have found neither cite or mention Werner nor offer their own explanation for etymology and origin of QE. Usually, they just mention use of QE by the Bank of Japan. However, even the sources of the Bank of Japan as old as 1999 do not give further explanation on etymology and origin, even though one would expect that proper explanation will certainly appear there, as the theory of quantitative easing was a rather new concept at that time.

There are several macroeconomic theories that are attempting to explain how quantitative easing works. The one that seems to be generally accepted by central banks is **portfolio balance theory**. The large-scale purchase of assets done during QE by central banks should push up the prices of given assets, reducing yield on them. The newly created excess reserves used by central banks to pay for these assets are written on accounts of commercial banks as either central banks are purchasing assets directly from the commercial banks, or the commercial bank is writing this money to an account of its non-bank clients from whom these assets were bought by central banks. Regardless, the new money will be available to commercial banks that should encourage them to acquire assets or increase supply of credit to businesses and households. This should make loans cheaper via lowering interest rates on them, it should have increased the supply of deposits and further increase demand for financial assets, pushing their prices even further up and their yields down. This all shall encourage increased lending and investment and so help in economic expansion, which in turn should cause rise of the inflation. Alternatively, sort of back up explanation, is **signalling theory** that state QE might also work as a powerful gesture from a central bank, which demonstrates its commitment for the inflation goal. This alone should influence the expectations of the market participants and help to ensure the market, increase nominal spending and so cause the inflation. (Williamson 2017; Bank of Canada 2009; Benford et al. 2009)

## 1.1 Japanese economy after WW2 to asset price bubble

The circumstances that caused such novel extreme measure as QE to be used and still being used have origin in the Lost decade of the 1990s and the economic stagnation that continuous ever since. However, to get a clearer picture how Japan got there, it is best to briefly look at its economic history after the Second World War. Japanese postwar economy is usually divided into these periods: recovery period (1946–1950), rapid growth period (1950–1973, 9.3 percent annual growth), moderate growth period (including the bubble age) (1976–1991, 4.3 percent), and stagnation period (1992–, about 1 percent). (Iyoda 2010)

After the Second World War Japan experienced what is often called miraculous recovery, its economy focused primarily on production of raw materials including steel, coal and cotton as a part of 1946 policy called “Inclined Production Mode” (also translated as “Priority Production System”, keisha seisan hoshiki). Many reforms focused on democratization, demilitarization as well anti-trust measures, land and labour reform were done by occupying

Allied forces. In 1952 Japan regained political independence after signing San Francisco Peace Treaty and its economy was growing rapidly for next 20 years. This was also thanks to technology import, which allowed more efficient production. In 1960 also economic policy called “Income Doubling Plan” supervised by Prime Minster Ikeda started, which strongly focused on improving infrastructure, and the economy truly more than doubled in the size in that decade. This era was also accompanied by increased pollution and severe depopulation of rural areas. (Iyoda 2010)

Oil shocks during 1970s forced Japanese industries to be more energy efficient and also to refocus from raw materials to more value added goods such as semiconductors and electronics which helped Japanese economy to maintain a steady growth. This period also marked a big change in monetary policy, in 1973 yen’s fixed exchange rate was changed to floating. (Iyoda 2010)

Start of the decline was in late 1980s with Japanese asset price bubble, which is often said to have been caused by rapid appreciation of yen, from 237 to 159 yen to US dollar in terms of the monthly closing average during the 10-month period to July 1986. This rapid appreciation is usually attributed to the Plaza Accord of 1985, where G5 countries including Japan agreed to take actions to appreciate their currencies against US dollar on the global exchange market. Regardless of the cause, sudden appreciation of yen caused serious shock to Japanese export industries, created depression and BOJ had to reduce the official exchange rate to the lowest point yet of 2.5 percent to help the market. However, this caused major increase in loans for speculative trading in shares and real estate. (Iyoda 2010) Japanese banks that had already problems with over-loaning now “lent more, with less regard for quality of the borrower, than anyone else” and according to Paul Krugman “in doing so they helped inflate the bubble economy to grotesque proportions”. (Krugman 2009) Average share prices rose 4.8 times, average prices of real estate in 6 major cities rose 5.0 times for commercial and 2.9 times for residential buildings. (Iyoda 2010)

After decline of stock market and real estate prices in 1991 started an era of what is called the Lost Decade and is continuing ever since. Sometimes it is even called Lost 30 Years in Japanese media after 2020. From December 1989 to 1990 average share prices fell 40 percent, declined until 1992 and reached bottom in 2002. Capital gains were also rapidly decreasing and almost disappeared in 1998. The land prices in 6 major cities also declined until 2005. Amount of nonperforming loans in Japanese banks was also rapidly increasing, peaking at 52.4 trillion yen at the end of March 2002. During this time yen was still appreciating with rates 145 yen to 1 US dollar in 1990, 102 yen in 1994 and peaked at 79.75 yen to 1 US dollar in April 1995. This appreciation caused immense difficulty for Japanese export industries, many of them moving their factories abroad. (Iyoda 2010)

## 1.2 Quantitative easing in Japan

The Bank of Japan (BOJ) was the first central bank that called its monetary policy quantitative easing. QE came in two waves, the first from 19 March 2001 to 9 March 2006, the second from 4 April 2013 to the present day. Japan was dealing with problems caused by bad loans and financial assets bubbles of 1980s and 1990s, as well as the aftermath of later financial crisis and natural disasters.

The first QE was done by BOJ as the traditional monetary measures were unable to deal with the economic crisis, which broke out after burst of the asset price bubble. During the crisis 161 out of 1080 financial institutions failed between 1997 and 2001. It was also the first time since the Second World War that legal restructuring procedure and a resultant default in obligations in the interbank market had taken place as well as the first failure of a large bank in Japan. In 2000 and 2001 burst of the dot.com bubble had also occurred, which impacted Japan. Nonperforming loans were also a problem, creating 6.2% of total bank's credit in 1999 and peaking in 2002 with 8.4%, this problem was finally solved in 2005. The crisis caused further drop in economic growth, rise of unemployment to over 5% (which might seem low, but Japan usually has unemployment rate under 3%), decline in land prices, and most importantly effectively started the deflation after 1998. (Aramaki 2018)

BOJ changed main operating target for money market operations from the current uncollateralized overnight call rate to the outstanding balance of the current accounts. The outstanding balance, that was 4 trillion yen in February 2001, was originally planned to be increased to 5 trillion yen, this target was, however, progressively increased, with last change of target on 20 January 2004 to 30-35 trillion yen. This increase of outstanding balance was achieved through increased amount of outright purchase of long-term Japanese government bonds (JGB) from the current 400 billion yen per month to 1.2 trillion yen. One of effects of QE probably was that it brought uncollateralized overnight call rate to effectively zero percent. The QE was to continue until the consumer price index (excluding perishables, on a nationwide statistics) registers stably a zero percent or an increase year on year. On 9 March 2006 BOJ changed the operating target of money market operations from the outstanding balance of current accounts back to the uncollateralized overnight call rate, with target to hold the rate at effectively zero percent. BOJ also began to plan reduction in current account balance via short-term money market. (Bank of Japan 2001, 2006; Spiegel 2006) Interestingly, neither the original Japanese version nor the translation of the 2001 Announcement used the term quantitative easing, however, working paper analysing this new monetary policy a year later is called "One year under quantitative easing" and the term is used frequently in the text as well.

Main reason for the stop of QE in 2006 was the Long Recovery Period (2003-2006) with moderate economic growth that made it seem like the crisis was solved, even though deflation continued, and unemployment rate remained unchanged. After this came a series of economic shocks that have worsened the situation followed by Global Financial Crisis in 2008, Tōhoku earthquake and tsunami, and Fukushima nuclear disaster in 2011. The second wave of QE is also a direct part of Abenomics, a policy package named after then newly elected Prime Minster Shinzo Abe. Abenomics was adapted in 2013 with focus on 'bold monetary policy', 'flexible fiscal policy' and 'growth strategy'. Shinzo Abe also nominated the new governor of BOJ in February 2013. (Aramaki 2018; Wakatabe 2015)

On 4 April 2013 BOJ started a new policy called Quantitative and Qualitative Monetary Easing (QQE), which has again re-changed the main operating target for money market operations of BOJ from the uncollateralized overnight call rate to the monetary base. Monetary base, whose amount outstanding was 138 trillion yen at the end of 2012, should increase at an annual pace of about 60-70 trillion yen. Increase in monetary base was accompanied by increase in purchase of Japanese government bonds (JGB) to annual pace of 50 trillion yen, Exchange-traded funds (ETFs) to 1 trillion yen and Japan real estate investment trusts (J-REITs) to 30 billion yen, while at the end of 2012 BOJ held JGBs worth 89 trillion yen, ETFs 1.5 trillion and J-REITs 0.11 trillion yen. Furthermore, JGBs eligible for purchase were broadened to all maturities including 40-year bonds, while the average remaining maturity of JGB purchases was extended from slightly less than three years to about seven years. Announcement also included ambitious aim to achieve the price stability target of 2 percent in terms of the year-on-year rate of change in the consumer price index (CPI) with a time horizon of about two years. (Bank of Japan 2013) As this aim had not been achieved, on 29 January 2016 BOJ decided to introduce negative interest rate of minus 0.1 percent to current accounts that financial institutions held at BOJ. (Bank of Japan 2016) Finally, on 21 September 2016 BOJ adapted QQE with Yield Curve Control which is its active policy to the present day. This policy has two major components. One component is "inflation-overshooting commitment" in which BOJ continues with expanding the monetary base until the year-on-year rate of increase in the observed CPI (all items less fresh food) exceeds 2 percent and stays above the target in a stable manner. The other component is "yield curve control" in which BOJ tries to control short-term and long-term interest rates through market operations. (Bank of Japan 2020)

## 1.3 Previous research

Below we briefly present economic research done with wavelet analysis to proof viability of this method. Wavelet was already also used to Japan, however, it was more often discrete wavelet or macroeconomic policy and QE was not the main point of the study.

As representative of non-wavelet research on QE in Japan, we decided to focus on VAR analysis. It is a method closest to wavelet analysis and has already been repeatedly used to analyse QE in Japan from various angles.

Overall, the current results are rather vague and inconclusive, any evidence for or against QE is rather weak, no definitive results have been found. This is only natural as state economies are complex systems which we are unable to fully analyse, but it keeps us in the dark when it comes to correct use of the unconventional monetary policy.

As far as the wavelet analysis is concerned, we mainly focused on the work by L. Aguiar-Conraria and M. J. Soares, who have published variety of articles focusing on the use of continuous wavelet analysis on economic data. A good example that wavelet analysis is suitable for study of monetary policy is the first article published by the authors about decomposing the time–frequency effects of monetary policy. (Aguiar-Conraria, Azevedo, and Soares 2008) It has analysed monthly data on the interest rate, inflation rate, industrial production index, M1 and M2 monetary aggregates of the USA from 1921 to 2007. It for example showed that “the reduction in the US output and inflation volatility decreased in the 1960s at all frequencies (and not in the 1980s as is usually claimed), but that it was temporarily revived in the 1970s (especially at the business cycle frequency) probably because of the oil price shocks.” Similar observation was made for inflation rates as well, which lead the authors to conclusion that “great moderation” is also a nominal phenomenon. Moreover, especially its part 4 ‘Interest rates and macroeconomic activity’ can be considered as a good example of how to read wavelet graphs of real economic data. This article has also generated lots of citations and methods of wavelet analysis were applied to various fields from international stock market through energy commodities to Bitcoin.

Economic literature before this article focused more on the discrete wavelet, however, there were already some works published about continuous wavelet. Maybe the first use of continuous wavelet in the economic literature was to analyse business cycles of Slovenia (Jagri and Ovin 2004), although it was already using continuous wavelet, its form is somewhat different from the current standard. Another duo of researchers active in wavelet analysis for economic use are D. Mayes and P. Crowley. At first they were focused more on discrete wavelets, but their works often have brief mentions of continuous wavelet as well such as inclusion of wavelet multiple coherency and wavelet partial coherency (Mayes, Crowley, and Maraun 2006) or introduction to continuous wavelet (Crowley 2007). Around the same time as Aguiar-Conraria and Soares published an evaluation of growth cycle co-movement and synchronization in the euro area core, which analysed business cycles using continuous wavelet analysis. (Crowley and Mayes 2009).

Another important article from Aguiar-Conraria and Soares serves as a guide for application of wavelet analysis as well as review of the use of wavelet analysis in economics so far. It also meant to be an in-depth introduction of continuous wavelets for economics, explaining mathematics behind it, correct ways to read its output as well as place for introducing their own Matlab Toolbox for continuous wavelet analysis. It has two versions, 'The Continuous Wavelet Transform: Moving Beyond Uni- and Bivariate Analysis' (2014) and also pre-print 'The continuous wavelet transform: a primer' (2011) which is more frequently cited as it was available for longer time and is free of charge. Both versions also include three examples of using wavelet analysis on The Great Moderation in the USA; finding leader among stock markets by analysing Price Index for FTSE All-Share (UK), the S&P 500 (USA), and the DAX (Germany); and finding relationship between stock markets and oil prices.

There seems to be only one article, where QE in Japan was explored using discrete wavelet analysis. (Hutchison and Westermann 2006) It explored just a short period between 1998 and 2006, using various euro-yen Tokyo interbank offered rates. They found out that QE helped keep long-term interest rates low, complementing BOJ's zero interest rate policy, however, it "failed to reverse market expectations that deflation and low economic growth would persist well into the future" and it rather caused market to accept deflation as the new norm, making further monetary policy more difficult.

Cases of use of continuous wavelet analysis exist, however, they are much sparser and we were able to find them at the final stages of writing our thesis, so we were sadly not able to strongly reflect on them. (Tiwari et al. 2018) looked at the output, money and interest rate between 1972–2017 and found out that "output and money are highly coherent in low, middle and high frequencies, and coherence increases while controlling for interest rate, with money growth as the leading variable most of the time across frequencies." The second study (Ryczkowski 2019) focuses on the relation between nominal residential property price indices, M3, M2, bank credit and credit from all sectors of twelve developed countries, including Japan. They found out that the credit growth is correlated with house price inflation during house prices boom and such booms can be stopped by increasing lending restrictions. That is, they proved common economic intuition. Specifically for Japan, they arrived at the same conclusion for the 1990s and 2001-2006, however, for after 2007 “money growth and house price inflation were either anti-phase or the link was insignificant”. A more recent study (Meng and Huang 2021) analysed monthly time-series of interest rates, monetary aggregates M1 and M2, consumer price index for all items Japan, production of total industry in Japan from January 1960 to June 2020. Among their findings is that before 1999 M1 was simply reflecting inflation. After 1999 on scale of 3 years the inflation followed the trend of M1.

Rather than wavelet analysis, VAR is a much more common tool for analysing effects of quantitative easing, as we can see for example in the paper “Are there differences in the effectiveness of quantitative easing at the zero-lower-bound in Japan over time?” by Michaelis and Watzka (2017). The authors were using TVP-VAR to investigate whether the impact of a QE shock varied over time in Japan through a marginal likelihood estimation which compares a constant coefficient VAR with TVP-VAR. They confirmed that the TVP-VAR was a better fit for Japan and that a QE shock had changing effects over time. They have found through impulse response analysis that the effects on core CPI have become stronger over time. More specifically, the responses of prices in 2014, when Abenomics was under way, stay significant, during the Zero-Interest Rate Policy (ZIRP) and QE-1 the price level responses are smaller and less significant.
Theys did not find a significant impact on real GDP during the time of Abenomics strategy and the QE-1 period, whereas slightly significant effects, if any at all, are found for the ZIRP.
Overall, their findings supported variance decomposition analysis. Especially since Abenomics, the importance of QE shocks on the price level has increased, whilst it seems to have decreased for real GDP. These effects are likely to be driven to some extent by the current Abenomics programme.

For a more comprehensive review of VAR analysis of QE we recommend looking at the (Michaelis and Watzka 2017) mentioned before as well as at ‘From Japanese Effectiveness Quantitative Easing’ and ‘From Japanese Quantitative Easing Policy – Korea’.

# 2 Aim of the thesis

The aim of the diploma thesis is to verify by means of wavelet analysis if there is a link between the rise of monetary base directly caused by Bank of Japan as part of quantitative easing and the intended rise of CPI. Based on the aim of the thesis we have set the following research question: *Did the rise of monetary base during QE in Japan from 2001 to 2021 have an influence on CPI?*

Long stagnation of the Lost Decade forced Bank of Japan to undertake quantitative easing, a never before used method of monetary policy. The aim of the thesis is to verify if there is a link between the rise of monetary base directly caused by BOJ as part of QE and intended rise of CPI and consequently also rise of GDP. This question is of great importance to current monetary policy as once unconventional method is now intensively used worldwide by the European Central Bank, Federal Reserve, Bank of England and many others. To answer the question of the effectiveness of QE, it is best to look at Japan not only because we have more data than for other countries, but also as Japan has used QE since 2001 and so much longer than anyone else. Furthermore, Japan also undertook QE in times of relative global economic stability of the Great Moderation, when other countries did not have such problems, which means that this first QE was not influenced by QE of other countries or by the global economic crisis.

However, effectiveness of QE in Japan is something that was already widely discussed as it is one of the most important questions of current monetary policy. To have a chance to bring any new insight into the topic, we further specify that the aim of this diploma thesis is to analyse QE in Japan in a novel way using wavelet analysis. This approach being somewhat different from others can bring a new light on this topic, either by finding and confirming contemporary findings or providing evidence. It also tests for the method of wavelet analysis itself and it can provide a meaningful output from the macroeconomic data where two main variables, core CPI and GDP, stagnate around the same values as it happened in Japan.

# 3 Wavelet analysis

Wavelet analysis allows to transform time-series into frequency domain while preserving some information from the time domain[1] revealing how its periodic components change over time. This is unlike other methods such as Vector Autoregression (VAR) which works only with time domain or Fourier transform that extracts frequency domain from the time-series but loses information about time in the process. Moreover, unlike Fourier analysis, wavelet analysis does not require from the time-series to be stationary, which is often not the case with economic data.

In this thesis, we will specifically use continuous wavelet transform that "maps a function of time, the original time-series, into a function of two variables – time and frequency[2]". This provides redundant information which makes it possible to reconstruct original time-series from its transform. Continuous Wavelet Transform has the form shown in Equation (1), where $x(t)$ is the time-series, $t$ marks given point in time. $\psi$ is mother wavelet, which is scaled and translated in time domain by $\tau$ which also controls location, and in frequency domain by $s$ which controls its width. Mother wavelet must satisfy admissibility condition of Equation (2) which ensures that original time-series $x(t)$ is preserved by wavelet transform and so time-series $x(t)$ can be later recovered from the wavelet transform. $C_\psi$ is called the admissibility constant. When graphed, mother wavelet function usually looks as a small wave that slowly decays as it goes to positive and negative infinity as it can be seen in Figure 1. From this also comes its name – wavelet.

$$W_{X:\psi}(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^* \left( \frac{t-\tau}{s} \right) \mathrm{d}t \tag{1}$$

[1]Trade-off between accuracy in time and frequency domain must still be made as the Heisenberg's uncertainty principle would suggest.

[2]To be exact, wavelet transform provides time-scale and not time-frequency representation. Depending on mother wavelet used, scales can be either converted to frequencies or such conversion might be meaningless as wavelet is too irregular and without any dominant periodic component to establish relationship between its scale and the usual "Fourier" frequency.

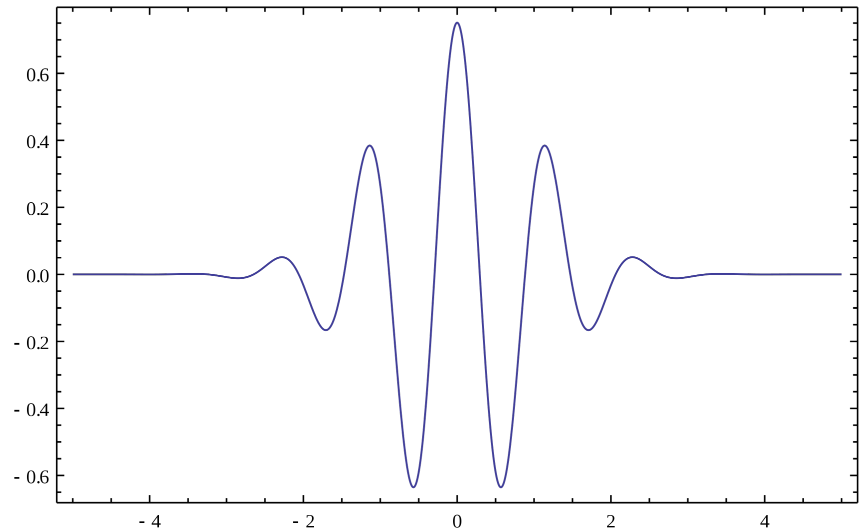


Figure 1: Morlet wavelet – an example of a typical wavelet; Source: JonMcLoone, Wikimedia Commons, CC BY-SA 3.0

$$C_\psi := \int_{-\infty}^{\infty} \frac{|\psi(\omega)|}{|\omega|} \, \mathrm{d}\omega$$
$$0 < C_\psi < \infty \tag{2}$$

This section is essentially retelling of Maria Joana Soares and Luís Aguiar-Conraria article (2014) with emphasis given on the reading from the wavelet graphs which are somewhat different of the output from other tools of the econometry.

## 3.1 Wavelet power spectrum

Wavelet power spectrum as defined by Equation (3) is often considered as one of the biggest advantages of wavelet analysis over Fourier analysis. To understand this advantage and to be able to use wavelet analysis in economics, it is at least required to be able to read a graph that wavelet power spectrum outputs.

$$(WPS)_x(\tau, s) = |W_x(\tau, s)|^2 \tag{3}$$

To explain the graph of wavelet power spectrum we will use a stylized example of single time-series with two periodic components which includes 50 years of monthly data. The first periodic component represents a stable 10-year cycle, second component is a 3-year cycle that between year 20 and 30 changes to a 5-year cycle. To generate dummy data that would simulate time-series with such properties we can use Equation (4), where $\varepsilon_t$ represents noise, random data with no periodicity, which is naturally present in measured data. The idea behind these examples is that if real-world time-series such as GDP, CPI and other measurable economic data had such cyclic characteristics, it would uncover them in the same way that it does for these dummy data, where cyclic characteristics are present by mathematical definition.

$$y_t = \cos\left(\frac{2\pi}{p_1}t\right) + \cos\left(\frac{2\pi}{p_2}t\right) + \varepsilon_t \tag{4}$$

$$t = \frac{1}{12}, \frac{2}{12}, \dots, \frac{50 \cdot 12}{12} \qquad p_1 = 10 \qquad p_2 = \begin{cases} 5 & \text{if } 20 \leq t \leq 30, \\ 3 & \text{otherwise.} \end{cases}$$

In the graph of wavelet power spectrum of Figure 2, the horizontal axis is time dimension, telling us when given periodic component happens, and in the vertical axes are depicted period cycles, telling us how often it repeats. White lines give estimate of cycle period as they mark "maxima of the undulations of the wavelet power spectrum". The colour represents power where warmer colours represent higher power which is more significant. It ranges from blue for low power to red for high power. Red colour around the white line means that the cycle is strong.

In Figure 2 we can see two cycles. At the bottom through the whole 10th period, a white line surrounded by red colour, which means it is a strong cycle that repeats every 10 years. Then there is the second white line on the 3rd period which breaks in the year 20, goes to the 5th period and that goes back to the 3rd period in year 30. This is a cycle that at first repeats every 3 years, then for 10 years it slows down to a 5-year cycle, and afterwards goes back again into a 3-year cycle.

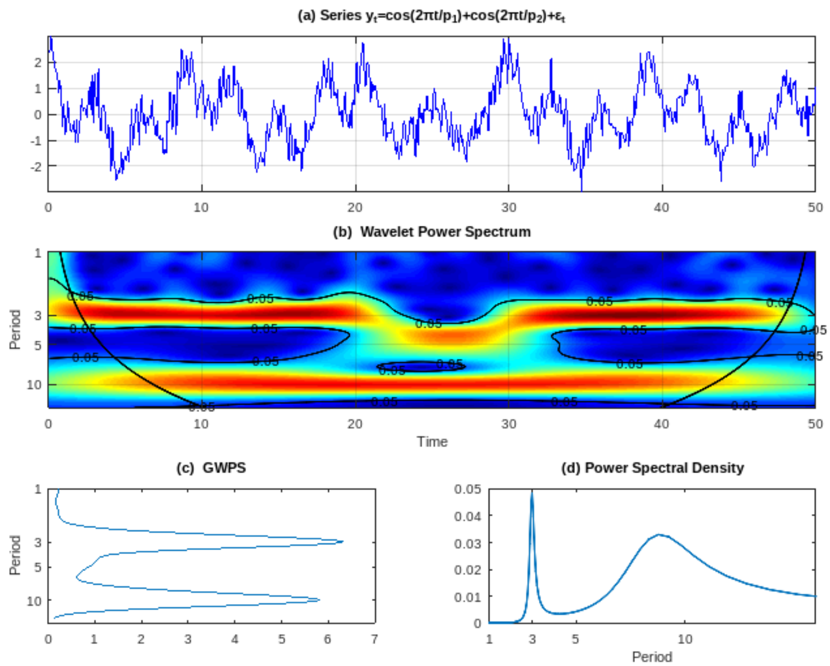


Figure 2: (a) Graph of the time-series; (b) Wavelet power spectrum; (c) Global wavelet power spectrum; (d) Power spectral density; generated using example code from ASToolbox2018

## 3.2 Coherency and phase-difference

Wavelet power spectrum is useful for analysing single time-series. There are also tools for analysing time-frequency dependencies between two time-series. When we multiply two continuous wavelet transforms of Equation (1) for two time-series x and y we get cross-wavelet transform Equation (5), which is the basic building block for analysing of the relationship between two time-series in the wavelet form. To simplify the notation, we write $W_x$ instead of $W_x(\tau, s)$, however, the meaning remains the same, wavelet transform of the time-series $x$ is still function of $\tau$ and $s$.

$$W_{xy} = W_x W_y^* \tag{5}$$

Cross-wavelet power spectrum is defined by Equation (6). If x = y, it simply results in wavelet power spectrum of Equation (3): $|W_{xx}| = |W_x W_x| = |W_x|^2 = (WPS)_x$.

$$(XWP)_{xy} = |W_{xy}| \tag{6}$$

Unlike the case with single time-series, we do not use Power Spectrum directly to generate a graph similar to that of Power Spectrum for single time-series. We use cross-wavelet power spectrum to compute wavelet coherency as given by Equation (7) the results of which are then graphed. $R_{xy}$ can attain values between 0 and 1, including 0 and 1. $S()$ is a smoothing operator in both time and scale which is necessary as otherwise the coherency would be identical to 1.

$$R_{xy} = \begin{cases} 0 & \text{if } \sqrt{S(|W_x|^2)S(|W_y|^2)} = 0 \\ \frac{|S(W_{xy})|}{\sqrt{S(|W_x|^2)S(|W_y|^2)}} & \text{otherwise} \end{cases} \tag{7}$$

Phase-difference defined by Equation (8) is more mathematically involved. $\Im()$ denotes imaginary part of the $S(W_{xy})$, $\Re()$ denotes the real part. As an example, if we have complex number $2+3i$ then $\Im(2+3i) = 3$ and $\Re(2+3i) = 2$. arctan is inverse function of tangent, it can have values between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, not including $\pm\frac{\pi}{2}$.

$$\phi_{xy} = \begin{cases} \arctan\left(\frac{\Im(S(W_{xy}))}{\Re(S(W_{xy}))}\right) & \Re(S(W_{xy})) > 0 \\ \arctan\left(\frac{\Im(S(W_{xy}))}{\Re(S(W_{xy}))}\right) + \pi & \Re(S(W_{xy})) < 0,\ \Im(S(W_{xy})) \geq 0 \\ \arctan\left(\frac{\Im(S(W_{xy}))}{\Re(S(W_{xy}))}\right) - \pi & \Re(S(W_{xy})) < 0,\ \Im(S(W_{xy})) < 0 \\ \frac{\pi}{2} & \Re(S(W_{xy})) = 0,\ \Im(S(W_{xy})) \leq 0 \\ -\frac{\pi}{2} & \Re(S(W_{xy})) = 0,\ \Im(S(W_{xy})) < 0 \end{cases} \tag{8}$$

Phase-difference can be understood as phase lead of $x$ over $y$, it can attain values between $-\pi$ and $\pi$ which tell us:

- series move in-phase at the specified time-frequency:
  - $\phi_{xy} = 0$, the time-series move together,
  - $\phi_{xy} \in (0, \frac{\pi}{2})$, the time-series x leads y,

- $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, y leads x,

- series moves out-of phase at the specified time-frequency:
  - $\phi_{xy} = \pm\pi$, antiphase relation,
  - $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, y leads x,
  - $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$, x leads y.

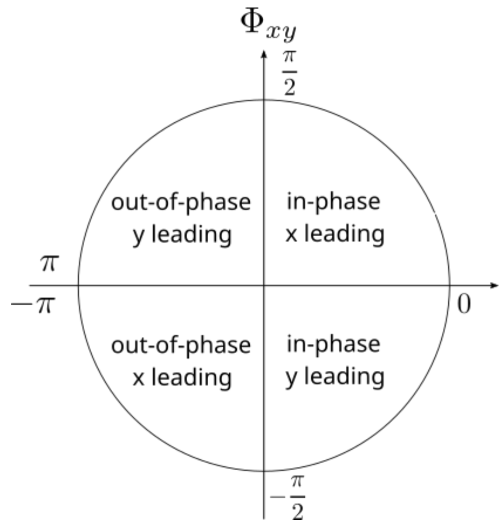


Figure 3: Phase-difference circle; according to Aguiar-Conraria, Soares, University of Minho, NIPE working paper; own work

Second stylized example consists of two time-series consisting of 50 years of monthly data. They share 3 and 6-year cycles, $x_t$ changes somewhat in year 25, while $y_t$ remains stable over time. Such dummy data can be generated with use of Equations (9) and (10).

$$x_t = \begin{cases} 4\sin\left(\frac{2\pi}{3}\left(t+\frac{5}{12}\right)\right) - 3\sin\left(\frac{2\pi}{6}\left(t-\frac{10}{12}\right)\right) + \varepsilon_{x,t}, & t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, \frac{25\cdot 12}{12} \\ 4\sin\left(\frac{2\pi}{3}\left(t-\frac{5}{12}\right)\right) - 3\sin\left(\frac{2\pi}{6}\left(t+\frac{10}{12}\right)\right) + \varepsilon_{x,t}, & t = 25+\frac{1}{12}, 25+\frac{2}{12}, \ldots, \frac{50\cdot 12}{12}, \end{cases} \tag{9}$$

$$y_t = \sin\left(\frac{2\pi}{3}t\right) + 3\sin\left(\frac{2\pi}{6}t\right) + \varepsilon_{y,t}, \quad t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, \frac{50\cdot 12}{12} \tag{10}$$

In the wavelet coherency graph of Figure 4 we can see red areas marking the region of strong coherency around frequencies 3 and 6. This means that time-series $x_t$ and $y_t$ are highly correlated around those frequencies.

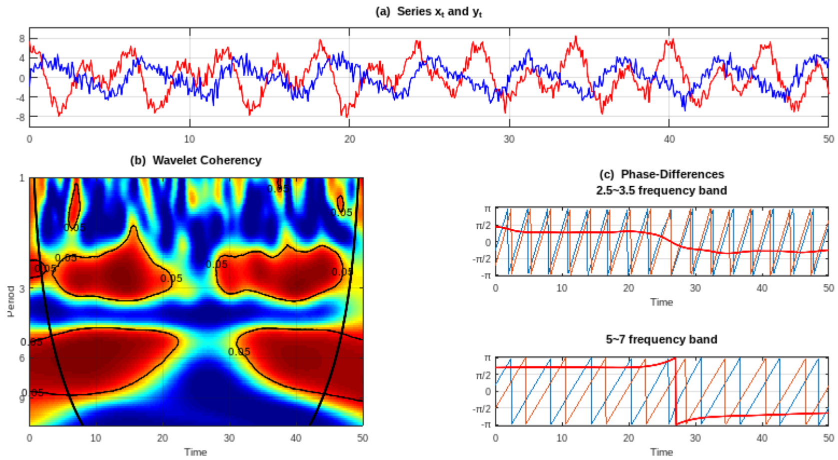

Figure 4: Wavelet coherency and phase-difference; generated using example code from ASToolbox2018

More information is revealed by phase differences of Figure 4. Red line of the phase-difference in the 2.5~3.5 frequency band is only between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ which means positive correlation between the 3-year cycles of the two time-series, these cycles are in phase. Phase-difference between 0 an $\frac{\pi}{2}$ in the first half means that 3-year $x_t$ cycle graphed in blue was leading over the green $y_t$ 3-year cycle, which is also visible in the graphed frequencies of these two cycles. In the second half the red phase-difference between $-\frac{\pi}{2}$ and 0 means that blue $x_t$ is lagging behind the green $y_t$, which is now leading in this half.

Red line of the phase-difference is significantly different for the 5~7 frequency band. Here the 6-year cycles are out of phase, that is they are negatively correlated. In the first half the red phase-difference is between $\frac{\pi}{2}$ and $\pi$ which means that green $y_t$ is leading. In the second half it is the other way around, with the red phase-difference between $-\pi$ and $-\frac{\pi}{2}$ which means that the blue $x_t$ is leading.

### 3.3 Multiple wavelet coherency and partial coherency

To analyse relationships between more than two time-series multiple wavelet coherency and partial wavelet coherency are used. Here the advantage lies not so much in computing wavelet coherency of three and more time-series, which while possible creates output that is harder to interpret. The main feature is the partial wavelet coherency which allows to analyse relationship of two (and more) time-series while eliminating influence that some other selected time-series might have on this relationship. If the (partial) coherency between two time-series increases after controlling for the third time-series, this third time-series was just obscuring their relationship to us. If opposite happens, part of interdependence between these two time-series was due to this third time-series.

The last example consists of three time-series of 50 years of monthly data. $x_t$ and $y_t$ share 3-year cycle, where $y_t$ leads $x_t$. $x_t$, $y_t$ and $z_t$ share 6-year cycle where $x_t$ leads $y_t$. Such dummy data can be generated with use of Equation (11).

$$
\begin{aligned}
x_t &= \sin\left(\frac{2\pi}{3}t\right) + 3\sin\left(\frac{2\pi}{6}t\right) + \varepsilon_{x,t} \\
y_t &= 4\sin\left(\frac{2\pi}{3}\left(t + \frac{5}{12}\right)\right) + 3\sin\left(\frac{2\pi}{6}\left(t - \frac{10}{12}\right)\right) + \varepsilon_{y,t}, \quad t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, \frac{50 \cdot 12}{12} \\
z_t &= 3\cos\left(\frac{2\pi}{6}t\right) + \varepsilon_{z,t}
\end{aligned}
\tag{11}
$$

In (a.1) of Figure 5 we can see simple wavelet coherency of two time-series as defined by Equation (7). In (b.1) we used partial wavelet coherency to eliminate effect of $z_t$ on the other two time-series. After controlling for $z_t$ in this way, only 3-year cycle relationships are captured and in much better way than they were on the simple coherency.

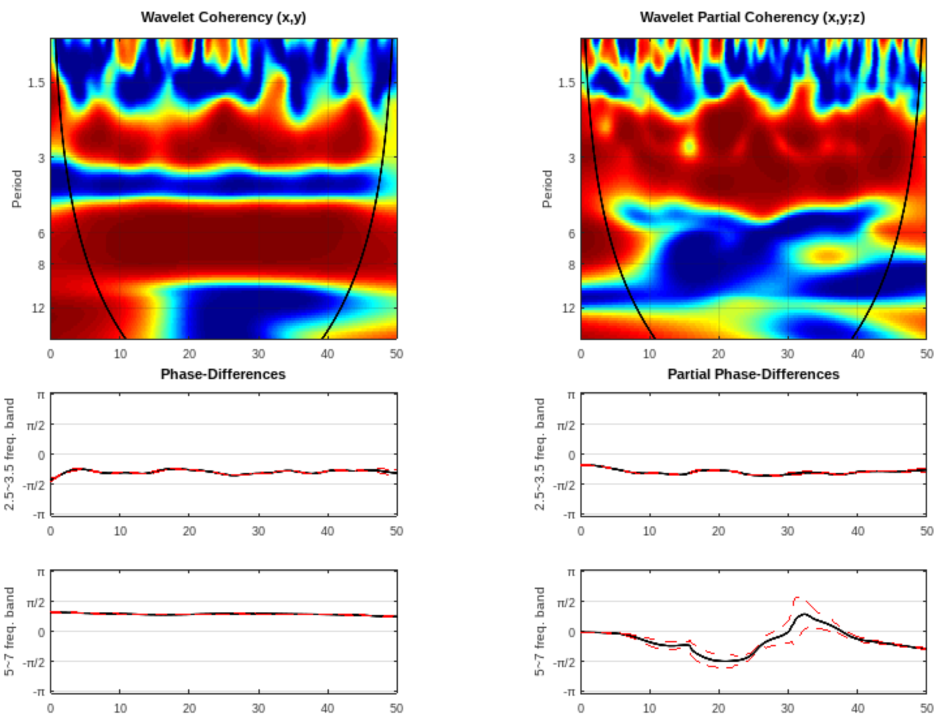


Figure 5: Multiple and partial wavelet coherency; generated using example code from ASToolbox2018

# 4 Methodology and data

Wavelet analysis will be used to analyse quarterly time-series of the Japanese core CPI, real GDP, monetary base and yen real effective exchange rate that covers a time horizon between the first quarter 1970 and the third quarter 2020. This dataset is similar to that used by Michaelis (2017).

Our data have been analysed with ASToolbox2018[3] by L. Aguiar-Conraria and M. J. Soares, a freely available MatLab toolbox designed specifically for wavelet analysis of economic data. Freely available GNU Octave, which is a language that aims to be fully compatible with any Matlab, was used to run the code. The full code is available in Appendix and we share it under MIT license.

We have also considered other software packages for wavelet analysis such as PyWavelets[4] for Python and collection of various libraries for R[5], especially WaveletComp[6], which seems designed for use in economic research. Main reason for choosing ASToolbox is that it has already been used in a number of publications to analyse economic data as it was specifically built for such task.

Below we present the tables that should serve as a quick overview of data we have used as well as a more detailed explanation accompanied by a graph of each time-series that we have used.

Table 1: First six entries in our dataset

| Date | fx | reserve | GDP | CPI |
|---|---|---|---|---|
| 1970-03-01 | 58.21 | 43975.00 | 32.21 | 30.90 |
| 1970-06-01 | 57.91 | 45686.00 | 32.56 | 31.37 |
| 1970-09-01 | 57.33 | 47236.00 | 35.36 | 31.67 |
| 1970-12-01 | 58.33 | 49323.33 | 40.21 | 32.50 |

[3]https://sites.google.com/site/aguiarconraria/joanasoares-wavelets
[4]https://github.com/PyWavelets/pywt
[5]See part *Frequency analysis* at https://cran.r-project.org/web/views/TimeSeries.html
[6]https://cran.r-project.org/web/packages/WaveletComp/index.html

| Date | fx | reserve | GDP | CPI |
|---|---|---|---|---|
| 1971-03-01 | 58.15 | 51134.67 | 33.84 | 32.83 |
| 1971-06-01 | 58.57 | 53118.00 | 33.87 | 33.60 |

Table 2: Last six entries in our dataset

| | Date | fx | reserve | GDP | CPI |
|---|---|---|---|---|---|
| 197 | 2019-03-01 | 75.76 | 5027542 | 109.01 | 101.20 |
| 198 | 2019-06-01 | 76.26 | 5062764 | 109.62 | 101.57 |
| 199 | 2019-09-01 | 78.73 | 5108374 | 109.66 | 101.63 |
| 200 | 2019-12-01 | 77.28 | 5159345 | 107.67 | 102.07 |
| 201 | 2020-03-01 | 76.70 | 5182689 | 106.99 | 101.87 |
| 202 | 2020-06-01 | 79.59 | 5270516 | 98.51 | 101.90 |

## 4.1 Monetary base

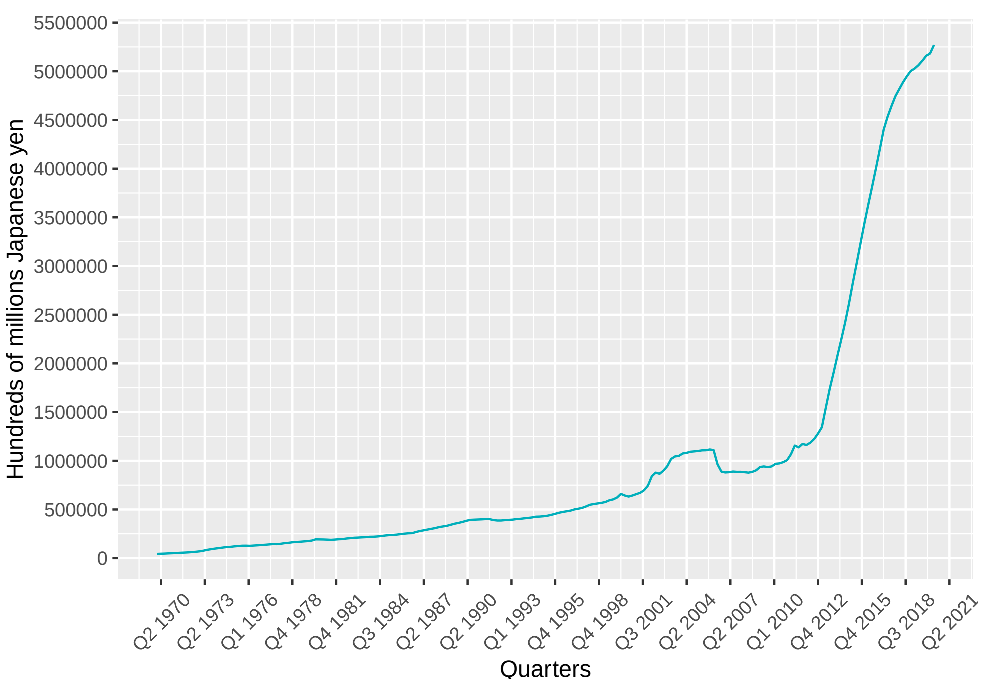


Figure 6: Monetary base of BOJ – average outstanding amounts, own work

- monetary base
- in hundreds of millions Japanese yen
- average outstanding amounts
- monthly series converted to a quarterly series (averaging over three respective months)
- source: BOJ Time-Series Data Search[7] `MD01'MABS1AA11X12` (Monetary Base/Seasonally Adjusted (X-12-ARIMA)/Average Amounts Outstanding)

BOJ is conducting QE by increasing outstanding balance which can be directly seen in this time-series. It is the only series that is not indexed, however, this is not a problem as we are only interested in periodicity of the data. Wavelet does not strictly need seasonally adjusted data, however, when it is available it is still better to use it, so seasonal cycles will not show up in the analysis.

## 4.2 Core CPI

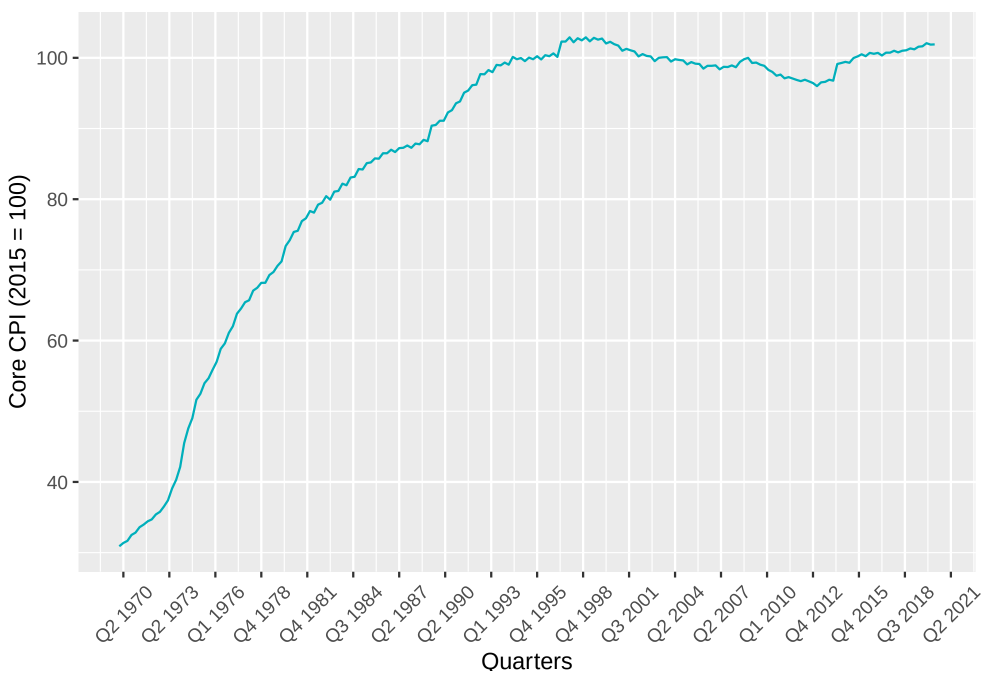


Figure 7: Core CPI of Japan – all items, less fresh food and energy, own work

- 2015-based
- monthly series converted to a quarterly series (averaging over three respective months)

[7] https://www.stat-search.boj.or.jp/index_en.html

- all items, less fresh food and energy
- area: all Japan
- source: https://www.e-stat.go.jp/en/dbview?sid=0003143513

The aim of QE is to increase the inflation to a desired level. We want to focus on those parts of the economy that can be influenced by monetary policy rather than those that can be strongly influenced by external factors such as wars, trade conflicts, natural disasters and the weather. Because of this we omitted both food and energy prices from the CPI. Furthermore, this is also a kind of CPI that for example ECB uses to measure inflation. Again we are only interested in the percentual rise and fall of the time-series, so the fact that here the data are indexed with base year 2015, while both GDP and exchange rate use year 2010 as the base is not a problem.

## 4.3 Real GDP

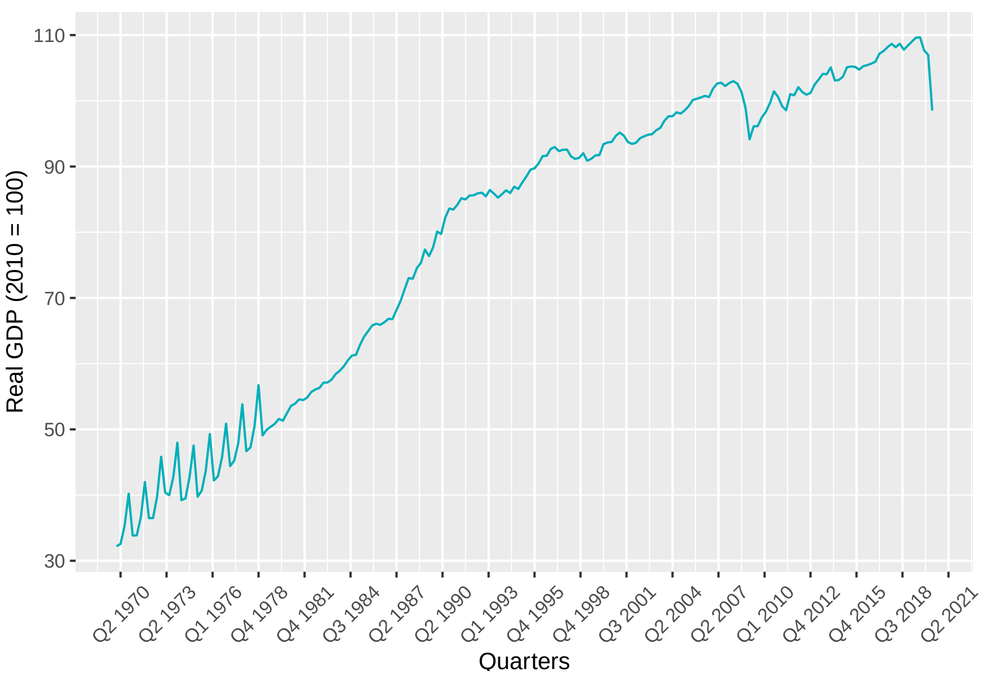


Figure 8: Real Gross Domestic Product of Japan, own work

- National Accounts, Constant Prices, Seasonally Adjusted
  - Gross Domestic Product, Volume, Seasonally Adjusted
- base year (2010 = 100)

- seasonally adjusted
- quarterly series
- source: IMF, https://data.imf.org/regular.aspx?key=61545852

If QE is successful and inflation rises to the desired level, economic balance should be achieved. This should also contribute to steady economic growth. We use real instead of nominal GDP as we want to decouple the influence of the inflation to the GDP. We were unable to find monthly time-series of real GDP in the given time frame; because of this, other series had to be converted to quarterly from monthly. However, even if we had the monthly data, they might have offered too much detail and would have made the final output hard to read, so we might have had to use quarterly data regardless.

## 4.4 Yen real effective exchange rate

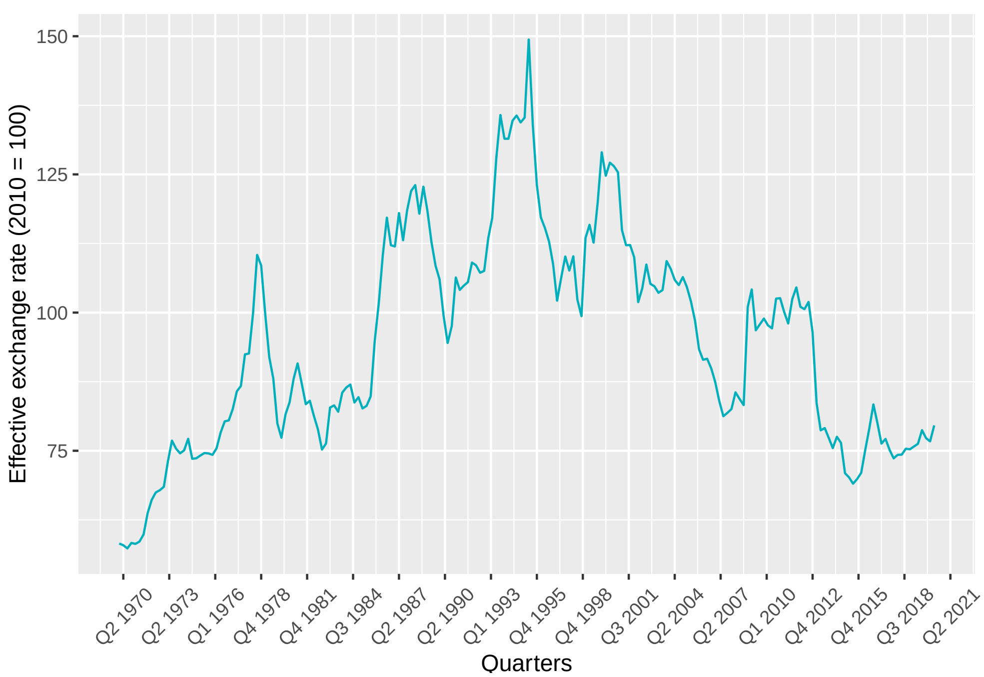


Figure 9: Yen real effective exchange rate, own work

- Japanese yen real effective exchange rate
- index (2010 = 100)
- trade weighted exchange rate
- monthly index converted to a quarterly series (averaging over three respective months)

- source: BOJ Time-Series Data Search[8] `FM09'FX180110002` (Real/Effective Exchange Rates)

Yen exchange rate together with the Plaza Accord seems to have a significant role in starting the Lost Decade, therefore it is important to look at its effect and connection to other variables as well. The advantage of trade weighted exchange rate is that it automatically selects currencies the most directly or indirectly involved in Japanese international trade. Instead of effective exchange rate for a currency basket it might be also entrusting to rate of Japanese yen to US dollar.

[8] https://www.stat-search.boj.or.jp/index_en.html

# 5 Results of wavelet analysis

When looking at the outputs of wavelet analysis of Japan, it will be important to pay attention to the time period of the first quantitative easing from March 2001 to March 2006. The second quantitative easing from April 2013 to the present day, which is also the time frame when Abenomics took place is important as well, however, it appears outside or just before the cone of influence depending on the period, so its effect will be hard to interpret at this point. Question about impact of the coronavirus pandemic cannot be answered from this analysis either, as years 2019 onwards are well outside of the cone of influence for any period and so any results we might see are not statistically significant.

## 5.1 Wavelet power spectrum

At first, we look at the individual time-series with use of the wavelet power spectrum, to see if they have any obvious periodic components.

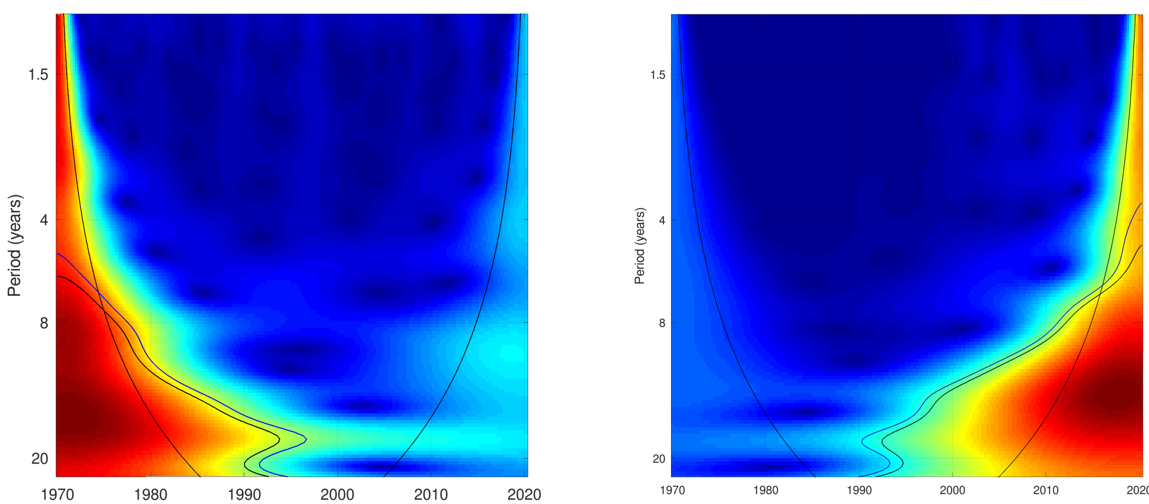


Figure 10: Wavelet power spectrum of core CPI (left) and monetary base (right), own work

Although, in both cases of CPI and monetary base most of the areas of higher power are outside of the cone of influence as we can see in Figure 10 and so cannot be interpreted, there is surprising similarity between power spectrums of the monetary base and CPI, which are almost the mirror images. On a second thought, this similarity is not so surprising as when we look at the plots of these data at Figures 6 and 7, they indeed were mirroring each other's development. CPI was rapidly rising and then became stable around 2001, which is also the time when the monetary base started to rise steadily because of the QE, while being stable for a long time before this date. There is no economic conclusion to be drawn from this, however, there is still a clear connection with these variables. Only because CPI became so stable and stopped rising for such a long time, did BOJ create such an unprecedented increase of monetary base.

GDP has only regions of low power in the cone of influence in Figure 11 which means that there is no periodicity in this time-series. No periodicity in GDP is something not usually expected, this might be the result of long economic stagnation of Japan. It might be interesting to look at periods higher than 20, however, this is not possible as we have just 50 years of data.

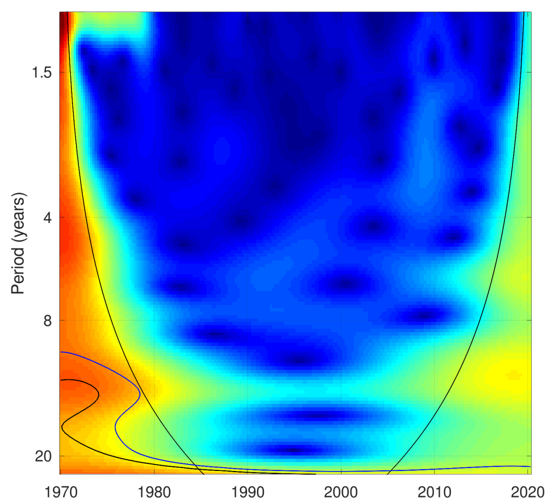


Figure 11: Wavelet power spectrum of real GDP, own work

The only time-series with clear periodicity is the yen effective exchange rate. The most prominent is the 8-year cycle in Figure 12 over the almost whole cone of influence. The most prominent era is 1980 to 2000, which is the time of economic growth in Japan. This is also the time when price of yen was rapidly rising due to the Plaza Accord of 1985.

In this era there were cycles with frequency ranging from 4-year to 8-year. Naturally,

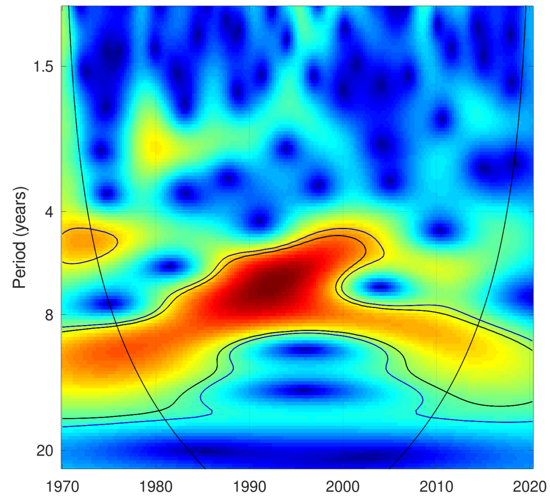


Figure 12: Wavelet power spectrum of yen real effective exchange rate, own work

exchange rates depend, even more than other economic variables, on the external factors rather than solely on what happens inside Japanese economy.

The main result from this part is that beside the yen effective exchange rate there is not much of periodicity in the individual time-series.

## 5.2 Coherency

Analysing dependencies between two time-series is likely to be more informative. As we are investigating effects of quantitate easing, we are mostly interested in how rise of the monetary base interacted with other variables, especially CPI, but we will also point out some other relations that appeared.

From left side of Figure 13 it seems as if after 1990 when CPI started to stagnate (see data plot of Figure 7), any connection between CPI and monetary base disappeared. There, however, seems to be 8 to 9-year cycle that was there before 1990 when CPI was rising steadily and reappeared after 2000 when the first QE took place. Although, it is not possible to say solely from this graph if reappearance of this cycle was due to QE. Similar can be said about the real GDP and monetary base graph on the right side of Figure 13.

In Figure 14 we can see the natural link between CPI and GDP, which interestingly gets

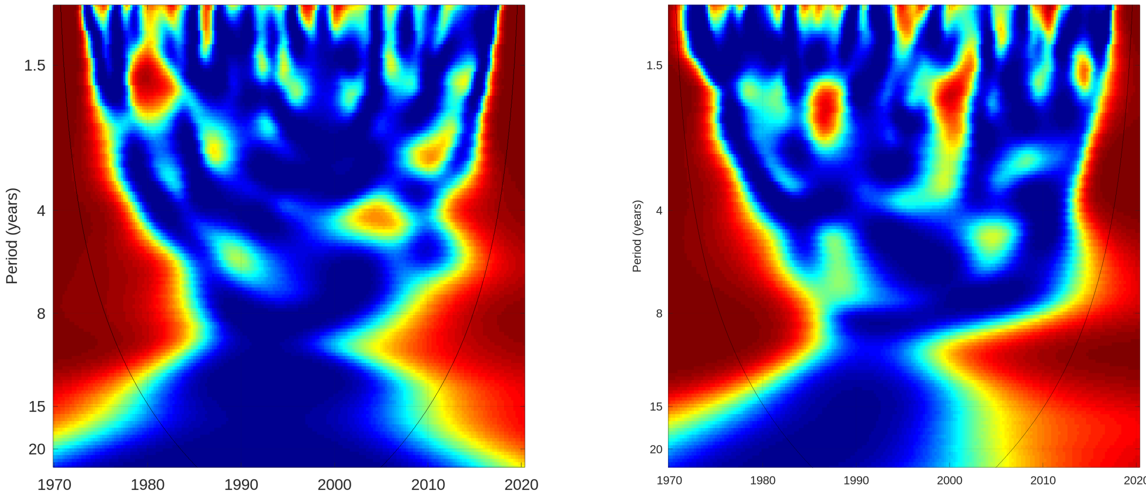


Figure 13: Wavelet coherency between core CPI and monetary base (left) and real GDP and monetary base (right), own work

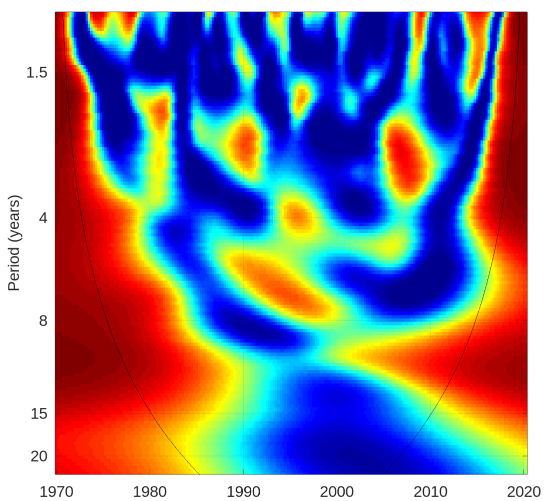


Figure 14: Wavelet coherency between real GDP and core CPI, own work

disconnected during the start of financial assets bubble in the late 1980s and does not connect again until around the start of QE in 2001.

There appears almost no clear link between foreign exchange rate and monetary base in Figure 15 until 2000s when the first QE started. This might be a positive sign that QE was effective as before 2000s monetary base was not rising quickly enough as you can see in Figure 6 to cause deprecation, with QE it was rising rapidly enough that it might have caused deprecation, which indeed happened in the 2000s (see the plot of exchange rate in Figure 9). However, QE would cause deprecation only if the money from it did not stay in the banking sector but got into the real economy. It also might be that other monetary measures that were launched together with QE as part of Abenomics caused this depreciation.

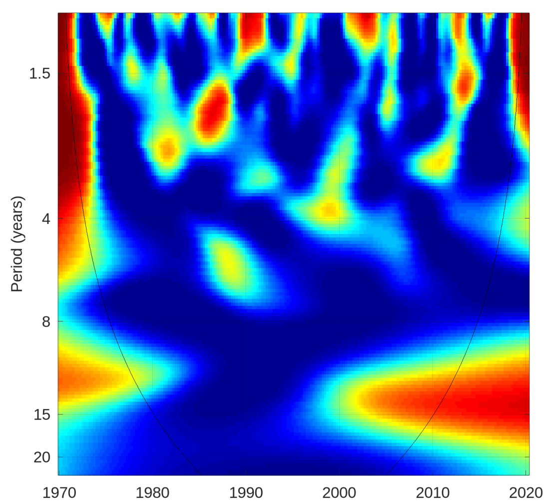


Figure 15: Wavelet coherency between yen real effective exchange rate and monetary base, own work

Both graphs in Figure 16 are strongly correlated from around 1983 to 2005 with 10 to 15-year cycle, this link might be there due to monetary policy and historic situation of Japan. The Plaza Accord of 1985 caused strong evaluation of yen and later start of the financial assets bubble, both GDP and CPI were rising together with the exchange rate, then after the burst of the bubble, GDP and CPI went down, later somewhat followed by the exchange rate.

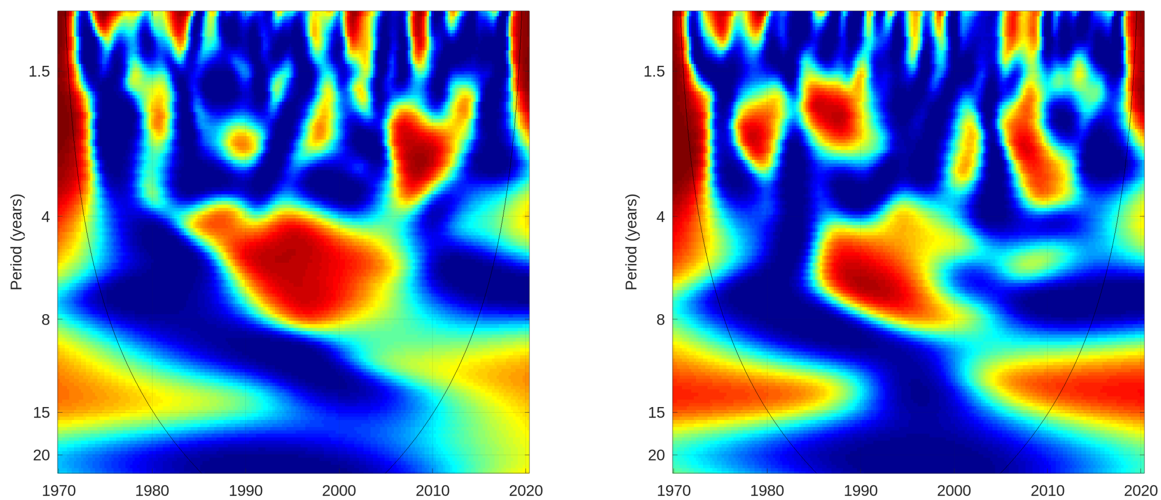


Figure 16: Wavelet coherency between yen real effective exchange rate and core CPI (left) and between exchange rate and real GDP (right), own work

This part of analysis cannot confirm that QE was effective, however, as the common element among graphs was the region of higher power in the 10th to 15th period that started around 2000 when the first QE started, there is at least suggestion that something important happened on this date. What it rather shows is that exchange rate had an important role between 1985 to about 2005 and was possibly influencing both GDP and CPI.

## 5.3 Multiple and partial coherency

There is not much new to see in Figure 17. It has features strongly similar to the reserve-CPI coherency of Figure 13 and this coherency has likely the strongest influence on its shape. We could find out more by looking at partial coherency where we control on the influence of these variables.

For both wavelet graphs in Figures 18 there seems to appear a 4 to 10-year cycle after the year 2000, while there was no such cycle before. This would mean that after 2000 monetary base started to influence both GDP and CPI. Start of QE in the year 2001 fits quite well as explanation of this phenomenon and it would suggest that QE really had some effect. After taking a closer look at the phase difference there indeed seems to be the case that at

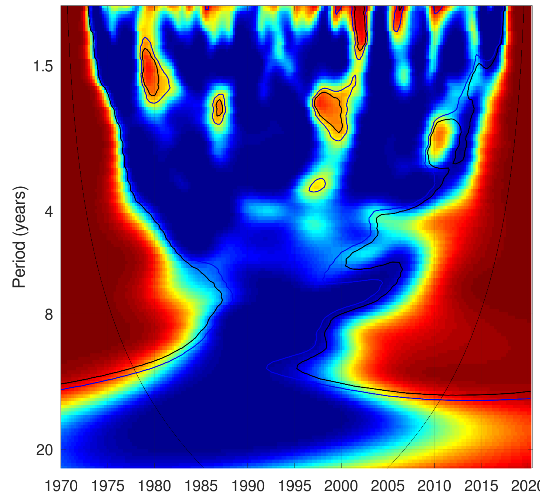


Figure 17: Multiple wavelet coherency between monetary base, core CPI and real GDP, own work

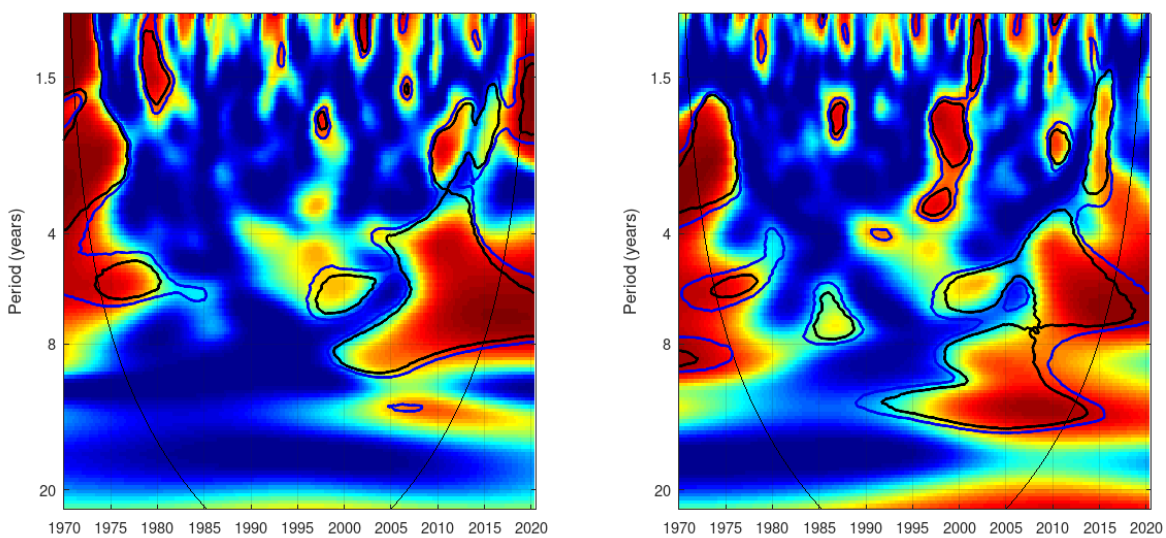


Figure 18: Partial wavelet coherency between monetary base and core CPI, controlling for real GDP (left) and monetary base and real GDP, controlling for core CPI (right), own work

least 4 to 8-year cycle development in rise of monetary reserve is followed by the rise of core CPI/GDP, which would indeed prove effectiveness of QE.

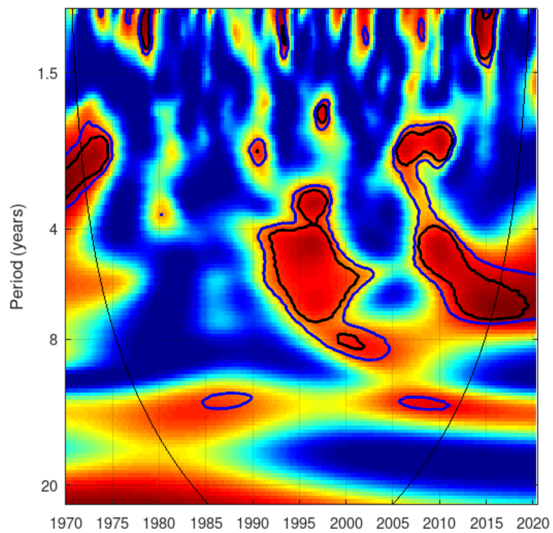


Figure 19: Partial wavelet coherency between core CPI and real GDP, controlling for monetary base (left) and controlling for effective exchange rate (right), own work

In Figure 19 there seems to be a long-term 10 to 20-year cycle between CPI and GDP, this suggests a strong link between these two economic variables and there is a hint that targeting inflation is good for BOJ to help with stable economic growth. Notable is that when we are controlling for yen exchange rate on the right side of the figure, it seems that from around 1985 when the Plaza Accord was signed to the start of QE in 2001 the relationship between GDP and CPI became visibly weaker.

The multiple wavelet coherency in Figure 20 summarizes many aspects that we have seen on other wavelet graphs. We can see strong coherency region between 1990 and 2000 in the 4th to 8th period that is most likely due to exchange rates and the Plaza Accord of 1985, as well as the strong coherency region on the left most likely due to CPI. However, there is also new region of high coherency on the right, which would suggest that economic variables of real CPI, real GDP and effective exchange rate indeed get more interconnected after 2000.

Partial coherencies in Figure 21 make things clearer. After controlling for effective exchange

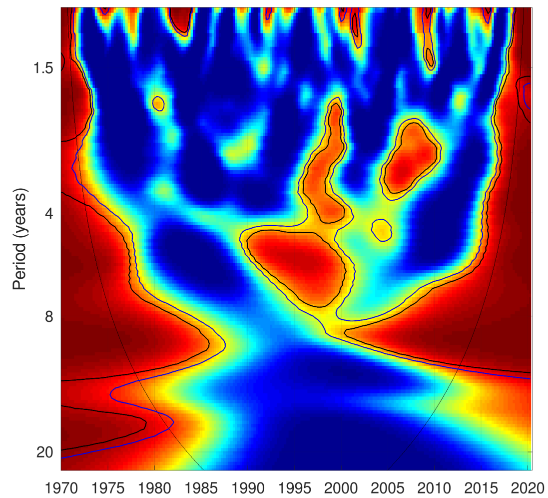


Figure 20: Multiple wavelet coherency between core CPI, real GDP and effective exchange rate, own work

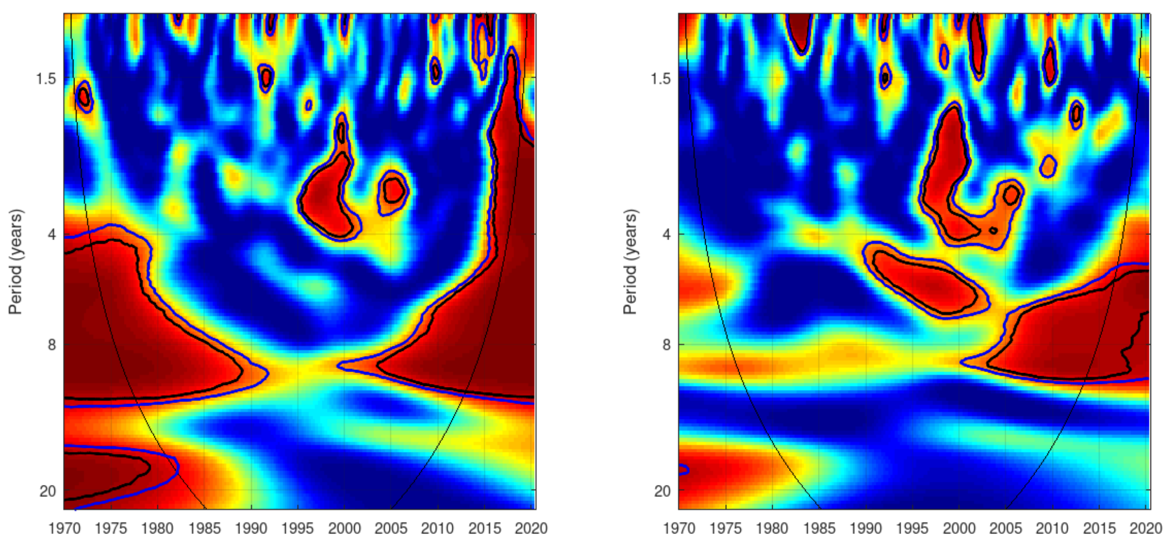


Figure 21: Partial wavelet coherency between core CPI and real GDP, controlling for effective exchange rate (left) and between core CPI and effective exchange rate, controlling for real GDP (right), own work

rate, there is strong coherency region on the whole timeline near the 9-year period, which gets somewhat weaker first in 1985, the year of the Plaza Accord, then significantly weaker in 1995 when Japanese yen was at peak of its strength, which is also 4 years after burst of asset price bubble. This wavelet might be telling us more about possible strong effect of exchange rates on Japanese economy rather than about QE, but it might be worth to look at it closer.

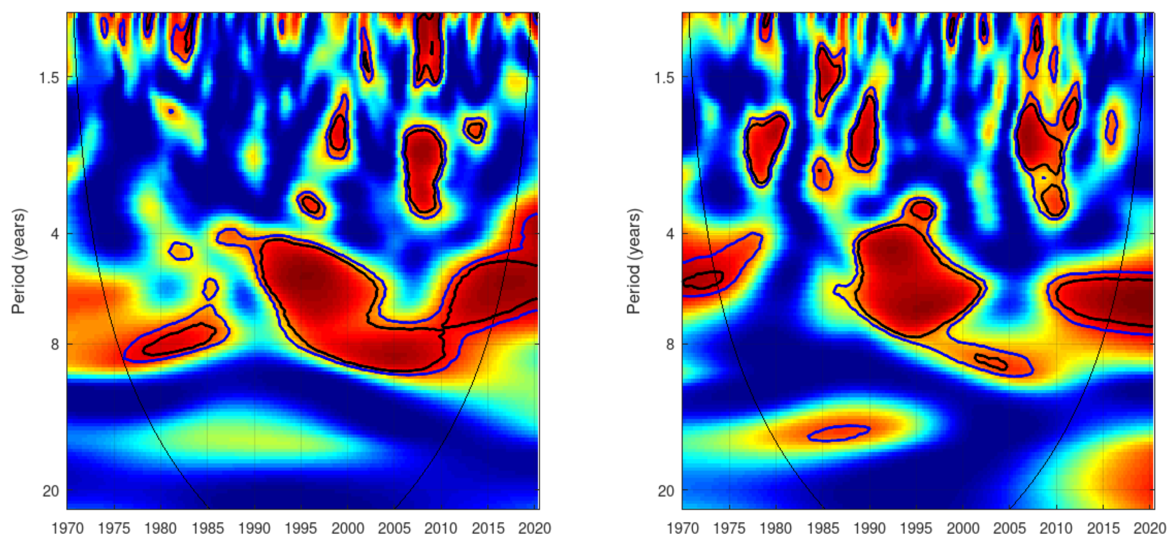


Figure 22: Partial wavelet coherency between effective exchange rate and core CPI (left) and real GDP (right), controlling for monetary base, own work

Figure 22 further makes it seem like the exchange rates had prolonged on effects on both GDP and CPI. We can also see from Figure 23 that there had not been a strong link between monetary reserve and yen exchange rate until the start of QE.

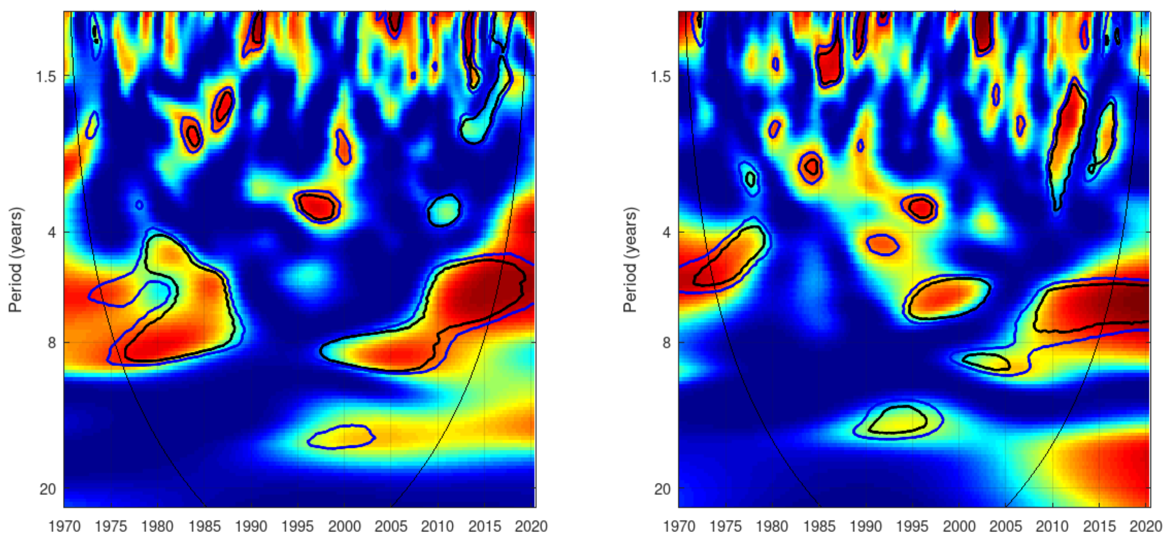


Figure 23: Partial wavelet coherency between effective exchange rate and monetary base, controlling for core CPI (left) and real GDP (right), own work

# 6 Conclusion

The aim of the diploma thesis was to verify by means of wavelet analysis if there was a link between the rise of monetary base directly caused by Bank of Japan as part of quantitative easing and the intended rise of CPI. While we cannot with certainty claim that QE is effective, our wavelet analysis shows that monetary base, which is directly increased by QE, started to be strongly linked with both GDP and CPI after start of QE in 2001, something that was not present there before and can be best seen in Figure 18. For us surprising finding was how prominently visible is exchange rate in the wavelet output and that its link to both CPI and GPD seems to be comparable to that of the monetary base as can be best seen in Figure 22.

Wavelet analysis almost always shows the start of strong coherency regions after year 2000 onwards, which overlap with start of QE in 2001. It also shows a possible strong influence of yen exchange rate on both inflation and GDP from the Plaza Accord of 1985 to 2001 when QE started. It might be that even if QE did not help with rise of inflation, it at least influenced economic variables and stabilized them through making them more periodic, and possibly, QE also helped with lessening the influence of the yen exchange rate. It would be certainly worthwhile to look at the topic of the Plaza Accord, influence of yen exchange rate on GDP and CPI and exchange rates relation with QE in the future.

However, it still remains true that no matter how much the monetary base was rising, the CPI remains largely unchanged. While we can say that QE did not cause macroeconomic change pronounced enough to cause desired effect of increased inflation, we do not have enough evidence to pronounce QE as an ineffective strategy either and it might be that both GDP and CPI would be much more unstable and possibly still influenced by the yen exchange rate if QE was not used.

Nature of wavelet analysis as well as many other economic analysis methods and data analysis methods in general is, they work the better the more data they have. We are able to gauge some preliminary information now, however, as we get reliable information inside the cone of influence only for years between 2000 and 2015, we will unfortunately have to wait a few decades for more data that will enable us a more thorough analysis. This is also the reason, why we treated impact of COVID-19 pandemic as off-topic as wavelet analysis of quarterly data is not suitable for analysing such recent events.

Our analysis only further proves that empirical analysing QE and other macroeconomic policies is not an easy task, ironically confirming the previous research that coming to a definite answer about the effectiveness of QE is at least for now not possible. This perfectly reflects the quote from Stephen Williamson we started Chapter 1 with: "QE is controversial, the theory is muddy and the empirical evidence is open to interpretation, in part because there is little data to work with." We might have stumbled upon possible clues that QE indeed effected Japanese economy, however, to be able to make any strong claims about our findings, we need longer time-series and also possible comparison with both other methods of analysis and QE in other countries.

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# Appendix: Octave code for wavelet analysis

Below we present Octave code that we have written with the use of ASToolbox2018, which resulted in Wavelet graphs that you can find in Chapter 5. We publish this code under MIT license. Please note that it relies on ASToolbox2018 which is itself a proprietary dependency. ASToolbox2018 is avaiable at Aguiar-Conraria's personal website[9].

To run ASToolbox2018 in Octave you have to omit `'Edgecolor',[.7 .7 .7]` option from the `contour()` function. If you want to use `xlsread()` function, you need to load io package by writing `pkg load io` in your script. In case it is not already installed, type `pkg install -forge io` in the command window. Beside that, ASToolbox2018 works just fine in Octave and produces output virtually identical to that of Matlab.

[9] https://sites.google.com/site/aguiarconraria/joanasoares-wavelets

Listing 1: Common Code for all scripts

```
% Copyright (c) 2022 Jakub Skoda
% Released under the MIT License.

% This code is run at start of every other script

clear; % clearing variables
clc; % clearing screen

% Path for loading the wavelet functions (has to be changed)
% use `if ispc else` to add file path for Windows, but Octave/Matlab
    should handle this themselves
% https://www.mathworks.com/help/matlab/ref/ispc.html  https://www.
    mathworks.com/matlabcentral/answers/117110-dealing-with-and-windows-
    vs-unix-for-path-definition
addpath('ASToolbox2018/Functions/WaveletTransforms');
addpath('ASToolbox2018/Functions/Auxiliary');

% Loading data
d = dlmread ('data.csv', "," ,1,1);
fx      = d(:,1);
reserve = d(:,2);
GDP     = d(:,3);
CPI     = d(:,4);

t =  transpose(1970:0.25:2020.25);

% sorting data in a nice matrix
% could have been done directly from dlmread()
% but this I find easier to read
X = [fx GDP CPI reserve];

Xsize = size(X,2);

% parameters for easy naming and saving of the output
names = cell(1,Xsize);
```

```
names{1} =      'fx';
names{2} =      'GDP';
names{3} =      'CPI';
names{4} =      'reserve';

% format for saving output
% use '-dsvg' for svg, '-dpng' for png
filform = '-dpng';

% ******** Common wavelet transforms parameters *********************
dt = 1/4;
dj = 1/30;
low_period = 1;
up_period = 23;

% -------------------- Choice of boundary conditions -----------
% To try other b.c.'s, make pad = 1 or pad = 2
pad = 0;     % Zero b.c.'s

% ---------------------- Choice of wavelet ---------------------
mother = 'Morlet'; % Morlet wavelet
beta = []; % 6.0; % omega parameter for the Morlet
gamma = [];  % Not important


% subfunction that checks if we are in octave
 function r = is_octave ()
   persistent x;
   if (isempty (x))
     x = exist ('OCTAVE_VERSION', 'builtin');
   end
   r = x;
 end
% https://web.archive.org/web/20190926055627/https://wiki.octave.org/Compatibility
```

Listing 2: Wavelet power spectrum

```
% Copyright (c) 2022 Jakub Skoda
% Released under the MIT License.

% Change the current folder to the folder of this m-file.
if(~isdeployed)
  cd(fileparts(which(mfilename)));
end

run('Common.m');

filpath = 'PowerSpectrumP/';
mkdir(filpath)

powername = 'PowerSpectrum-';

sig_type = 'AR0';

pictEnh = 2/5;  % Picture enhnacer
perc5 = 5/100;  % Percentil 5
perc10 = 10/100;% Percentil 10

%---------------- Computation ----------------------------------

% creating cells for saving the results
wave = cell(1,Xsize);
periods = cell(1,Xsize);
coi = cell(1,Xsize);
power = cell(1,Xsize);
pv_power = cell(1,Xsize);
maxPower = cell(1,Xsize);

for k = 1 : Xsize
      [wave{k},periods{k},coi{k},power{k},pv_power{k}] =...
        AWT(X(:,k),dt,dj,low_period,up_period,pad,mother,beta,gamma,
    sig_type);
```

```

      % Computation of Maxima (ridges)
      maxPower{k} = MatrixMax(power{k},3,.14);
end

% saves results of the computation into a loadable file
save(strcat(filpath,'Workspace-PowerSpectrum-',num2str(now)))
% to convert now to date use datestr(now,'yyyy-mmmm-dd_HH:MM')

%---------------- Plots ----------------------------------------

for k = 1 : Xsize
  %figure(k);
  figure(1);

    plotPOWER = subplot(1,1,1);

    ylim = log2([min(periods{k}),max(periods{k})]);
    yticks = [1.5 4 8 20];
    imagesc(t,log2(periods{k}),(power{k}).^pictEnh);
    ylabel('Period (years)');
    grid on;
    set(plotPOWER,'XLim',xlim,'XTick',xticks);
    set(plotPOWER,'YLim',ylim,'YTick',log2(yticks), ...
        'YTickLabel',yticks,'YDir','reverse');
    set(plotPOWER,'FontSize',20);
    %title(names{k},'Fontsize',10);
    colormap jet; % To obtain the colors of the paper;
    caxis('manual'); % To hold colors (when we superimpose the
                     % contours)
    hold on
    % Plot ridges
        contour(t,log2(periods{k}),maxPower{k},[1,1],'w-','LineWidth',1.5);
    % Pot COI
        plot(t,log2(coi{k}),'-k','LineWidth',1.25);
```

```
    % Plot levels of significance
        contour(t,log2(periods{k}),pv_power{k},[perc5,perc5],'k-','
   LineWidth',1.5);

     % 'Edgecolor',[.7 .7 .7], is not recognized by Octave
     if (is_octave)
     contour(t,log2(periods{k}),pv_power{k},[perc10 perc10],'LineWidth'
   ,1.5);
     else
     contour(t,log2(periods{k}),pv_power{k},[perc10 perc10],'Edgecolor'
   ,[.7 .7 .7],'LineWidth',1.5);
     end
    hold off
    print(strcat(filpath,powername,names{k}),filform) % saves output
end
```

Listing 3: Wavelet coherency phase-difference and gains

```
% Copyright (c) 2022 Jakub Skoda
% Released under the MIT License.

% Change the current folder to the folder of this m-file.
if(~isdeployed)
  cd(fileparts(which(mfilename)));
end
run('Common.m');

% parameters for easy saving of the output
filpath = 'CoherencyP/';
mkdir(filpath)

cohname = '-Coherency';
linname = '-PhaseDif_Gains';

% creating list of various combinations
Comb = nchoosek(1:Xsize,2)
CombSize = size(Comb,1)

% ******************  COHERENCY **********************************

% - Choice of size of windows for smoothing in coherency computation -
wt_size = 2; % Actual size used varies with scale s and
%              is given by:  wt_size*s/dt (with a minimum value of 5)
ws_size = 2; % Actual size used depends on dj and is
%              given by: ws_size/(2*dj) (with a minimum value of 5)


% levels of significance are not computed
% no need to specify these parameters

% creating cells for saving the results
WCO = cell(1,CombSize);
periods = cell(1,CombSize);
```

```
coi = cell(1,CombSize);
WGain = cell(1,CombSize);

for k = 1 : CombSize
% Index numbers of current varialbes
c1 = Comb(k,1)
c2 = Comb(k,2)

% Computation of Coherency
% [WCO,WCross,periods,coi,pv_WCO,WGain]
[WCO{k},~,periods{k},coi{k},~,WGain{k}] = ...
      AWCOG(X(:,c1),X(:,c2),dt,dj,low_period,up_period,pad,mother,beta,
   gamma,...
            wt_size,ws_size);
end

% saves results of the computation into a loadable file
save(strcat(filpath,'Workspace-Coherency-',num2str(now)))
% to convert now to date use datestr(now,'yyyy-mmmm-dd_HH:MM')

% ---------------- Plot of coherency ---------------------------

pict_enh = 5; % Picture enhancer
yticks_lab = [1.5 4 8 15 20];
yticks = log2(yticks_lab);

for k = 1 : CombSize
  c1 = Comb(k,1);
  c2 = Comb(k,2);

  logcoi = log2(coi{k});
  logperiods = log2(periods{k});

  %xlim = [t(1)-1 t(end)];
  %xticks = 0:40:200;
  ylim = [min(logperiods), max(logperiods)];
```

```

% figure(1+2*(k-1)); % use to plot each in a separte window
figure(1);
plotCOHER = subplot(1,1,1);
imagesc(t,log2(periods{k}), abs(WCO{k}).^pict_enh);
  colormap(jet)
  grid on

 % x-axis labels
 % most likely not needed, years get labeled just fine without it
 % set(plotCOHER,'XLim',xlim,'XTick',xticks);

 % y-axis labels
 % when removed it labels axis as 0, to 4.5
 % this is same value as ylim, but is not dependant on it
 % adding number to yticks_lab yeild unexpected result as well
 % for explanation see https://www.mathworks.com/help/matlab/graphics-object-properties.html
 % especially https://www.mathworks.com/help/matlab/ref/matlab.graphics.axis.axes-properties.html
   set(plotCOHER,'YLim',ylim,'YDir','reverse',...
      'YTick',yticks,'YTickLabel',yticks_lab);
 %     set(plotCOHER,'YDir','reverse');

  set(plotCOHER,'FontName','arial','FontSize',20);
  %title(strcat(names{c1},'-',names{c2}),'FontSize',9);
  ylabel('Period (years)','FontSize',20);

  % Plot the Cone Of Influence
  hold on
  plot(t,logcoi,'k');
  hold off

  % Saves output
  print(strcat(filpath,names{c1},'-',names{c2},cohname),filform)
end
```

```
%*******************  PHASE-DIFFERENCES & GAINS ********************

% ----------  Choice of  bands for phase-differences -----------
lpf1 = 7.5;
upf1 = 8.5;

lpf2 = 10;
upf2 = 12;

% creating cells for saving the results
phaseDif = cell(2,CombSize);
gain = cell(2,CombSize);

for k = 1 : CombSize
% Index numbers of current variables
c1 = Comb(k,1)
c2 = Comb(k,2)

% -------- Computation of (mean) phase-differences -------------

phaseDif{1,k}=MeanPHASE(WCO{k},periods{k},lpf1,upf1);
phaseDif{2,k}=MeanPHASE(WCO{k},periods{k},lpf2,upf2);

%  Computation of (mean) gains
gain{1,k} = MeanGAIN(WGain{k},periods{k},lpf1,upf1);
gain{2,k} = MeanGAIN(WGain{k},periods{k},lpf2,upf2);
end

% ---------------- Plots of phases and gains -------------------
ylim_phase = [-pi-0.1 pi+0.1];
yticks_phase = -pi:pi/2:pi;
yticks_phase_lab = {'-pi','-pi/2','0','pi/2','pi'};

for k = 1 : CombSize

```

```
c1 = Comb(k,1);
c2 = Comb(k,2);

logcoi = log2(coi{k});
logperiods = log2(periods{k});

%xlim = [t(1)-1 t(end)];
%xticks = 0:40:200;
ylim = [min(logperiods), max(logperiods)];

 % --------------------- Plots of phases -------------------------
figure(2*k);
%figure(2);
 plotPHASE1 = subplot(3,2,1); % 2.5~3.5 freq. band
  plot(t,phaseDif{1,k},'LineWidth',1,'Color','k');
    ylabel('7.5~8.5 freq. band');
    set(plotPHASE1,'XLim',xlim,'XTick',xticks)
    set(plotPHASE1,'YLim',ylim_phase,'YTick',yticks_phase,'YTickLabel'
    ,...
        yticks_phase_lab,'YGrid','on')
    set(plotPHASE1,'FontName','arial','FontSize',9)
    title(strcat('Phase-Difference of ',names{c1},'-',names{c2}),'
    FontSize',9)

plotPHASE2 = subplot(3,2,3); % 7.5~8.5 freq. band
    plot(t,phaseDif{2,k},'LineWidth',1,'Color','k');
    ylabel('10~12 freq. band');
    set(plotPHASE2,'XLim',xlim,'XTick',xticks)
    set(plotPHASE2,'YLim',ylim_phase,'YTick',yticks_phase,'YTickLabel'
    ,...
        yticks_phase_lab,'YGrid','on')
    set(plotPHASE2,'FontName','arial','FontSize',9)

% --------------------- Plots of  gains ------------------------
%ylimGain = [0 4];
%yticksGain = 0:4;
```

```
ylimGain1 = [min(gain{1,k}),max(gain{1,k})];
ylimGain2 = [min(gain{2,k}),max(gain{2,k})];

plotGAIN1 = subplot(3,2,2); % 2.5~3.5 freq. band
    plot(t,gain{1,k},'k-','LineWidth',1,'Color','k');
    ylabel('7.5~8.5 freq. band');
    set(plotGAIN1,'XLim',xlim,'XTick',xticks)
    set(plotGAIN1,'YLim',ylimGain1,'YGrid','on')
    % ,'YTick',yticksGain
    set(plotGAIN1,'FontName','arial','FontSize',9);
    title({' Y=\alpha+\beta X'},...
           'FontSize',7)
    legend('|\beta|','Location','Best')

plotGAIN2 = subplot(3,2,4);% 7.5~8.5 freq. band
    plot(t,gain{2,k},'k-','LineWidth',1,'Color','k');
    ylabel('10~12 freq. band');
   set(plotGAIN2,'XLim',xlim,'XTick',xticks)
   set(plotGAIN2,'YLim',ylimGain2,'YGrid','on')
   % 'YTick',yticksGain;
   set(plotGAIN2,'FontName','arial','FontSize',9);
   legend('|\beta|','Location','Best')

   %print(strcat(filpath,names{c1},'-',names{c2},linname),filform)
end
```

Listing 4: Multiple wavelet coherency

```
% Copyright (c) 2022 Jakub Skoda
% Released under the MIT License.

% Change the current folder to the folder of this m-file.
if(~isdeployed)
  cd(fileparts(which(mfilename)));
end
run('Common.m');

filpath = 'Multiple5000/';
mkdir(filpath)

% pkg load control

% Parameters for computing levels of significance
n_sur = 1; % Levels of sig. will be very inaccurate with this small
           % number of surrogates; use it just to test!
n_sur = 5000; % What must be used for credible significance levels!
%               WARNING: TAKES VERY LONG!


% Window sizes for smoothing (for coherency and gain)
windTime_size = 2;
windScale_size = 2;
p = 1;
q = 1; % Surrogates based on an ARMA(1,1) model

% PLOTS
fig = figure(1);
set(fig, 'PaperPositionMode', 'manual')
set(fig, 'PaperUnits', 'centimeters')
set(fig, 'PaperPosition', [0 0 10 10])

xlim = [t(1) t(end)+dt];
xticks = 1970:5:2020.25;
```

```

yticksLab = [1.5 4 8 20];
yticks = log2(yticksLab);

ylimPhase = [-pi-0.5 pi+0.5];
yticksPhase = -pi: pi/2 : pi;
yticksPhaseLab = {'-pi','-pi/2','0','pi/2','pi'};

perc5 = 5/100;  % Percentil 5
perc10 = 10/100;  % Percentil 10


% generates special combination-permutations of elements
% matters on the order of the first element
% the other two are just combinations where order does not matter
% 4 variables result in 12 different 'combinations'
%Comb = [];
%K = 1 : Xsize;
%for k = 1 : Xsize
% Comb = cat(1,Comb,cat(2, ones(size(Ck,1),1)*k, nchoosek(K(~ismember(K,
   k)),2)));
 % nchoose seek generates combination of two for all numbers from 1 to
   Xsize, beside number k
 % vetrical vector with all numbers of value k is added as the frist
   columns
 % this is added to matrix of previous 'combinations'.
%end

% Generates permutations for all various combinations of three elements
% 4 variables result in 24 different permutations
Comb = [];
K = 1 : Xsize;
for k = 1 : Xsize
 Comb = cat(1,Comb, perms(K(~ismember(K,k))));
end

```

```
%------------------------------------------------------------
% MAIN COMPUTATIONS
% Computation of multiple wavelet coherency

% and of complex partial wavelet coherency and complex partial gain
%coher_type = 'both';
%index_partial = 1;
%[WMCO,WPCO{1},periods,coi,pvM,pvP{1},PWGain{1}] = ...
%   MPAWCOG(X,dt,dj,low_period,up_period,pad,mother,beta,gamma,...
%            coher_type,index_partial,windTime_size,...
%            windScale_size,n_sur,p,q);


CombSize = size(Comb,1);

WMCO = cell(1,CombSize);
pvM = cell(1,CombSize);

periods = cell(1,CombSize);
coi = cell(1,CombSize);

%for l = 1:CombSize % to compute all possible combinations
for l = [1,19] % only these two are used in the thesis
  Y = X(:,Comb(l,:));
  Ysize = size(Y,2);

  c1 = Comb(l,1);
  c2 = Comb(l,2);
  c3 = Comb(l,3);

  coher_type = 'mult';
  index_partial = [];
  [WMCO{l},~,periods{l},coi{l},pvM{l}] = ...
     MPAWCOG(Y,dt,dj,low_period,up_period,pad,mother,beta,gamma,...
             coher_type,index_partial,windTime_size,...
             windScale_size,n_sur,p,q);
```

```
  fprintf("Computatation finished for l=%i.\n",l);
end

% saves results of the computation into a loadable file
save(strcat(filpath,'Workspace-n_sur_',num2str(n_sur),'-',num2str(now)))
% to convert now to date use datestr(now,'yyyy-mmmm-dd_HH:MM')

% -------------- Plot of multiple wavelet coherency ------------------

%for l = 1:CombSize % to plot all possible combinations
for l = [19, 1] % only these two are used in the thesis
  c1 = Comb(l,1);
  c2 = Comb(l,2);
  c3 = Comb(l,3);

  ylim = log2([min(periods{l}),max(periods{l})]);
  plotMC =  subplot(1,1,1);
  %fig = figure(l);
  fig = figure(1);
  % subplot(50,3,[1 40]);
    pictEnh = 20; % Picture enhancer
    imagesc(t,log2(periods{l}), abs(WMCO{l}).^pictEnh);
    ylabel('Period (years)');
    grid on
    set(plotMC,'XLim',xlim,'XTick',xticks);
    set(plotMC,'YLim',ylim, 'YTick',yticks,'YTickLabel',yticksLab, ...
        'YDir','reverse','FontSize',20);
    %title(strcat('Multiple Coherency:',names{c1},', ',names{c2},', ',names{c3}),'FontSize',8,'FontName','arial');
    colormap jet
    caxis('manual')
    hold on
    % Plot COI
    plot(t,log2(coi{l}),'k');
    % Plot levels of significance
        contour(t,log2(periods{l}),pvM{l},[perc5 perc5],'k-','LineWidth'
```

```
,1.5);

    % 'Edgecolor',[.5 .5 .5], is not recognized by Octave
    if (is_octave)
    contour(t,log2(periods{1}),pvM{1},[perc10 perc10],'LineWidth',1.5);
    else
    contour(t,log2(periods{1}),pvM{1},[perc10 perc10],'Edgecolor',[.5
   .5 .5],'LineWidth',1.5);
    end
   hold off

   print(strcat(filpath,names{c1},'_',names{c2},'_',names{c3}),'-dpng')
end

% All its periods are EQUAL
% isequal(periods{1,1},periods{1,2},periods{1,3},periods{1,4},periods
   {1,5},periods{1,6},periods{1,7},periods{1,8},periods{1,9},periods
   {1,10},periods{1,11},periods{1,12})
% ans = 1

% All its coi are EQUAL
% isequal(coi{1,1},coi{1,2},coi{1,3},coi{1,4},coi{1,5},coi{1,6},coi
   {1,7},coi{1,8},coi{1,9},coi{1,10},coi{1,11},coi{1,12})ans = 1
% ans = 1


% Permutation of same elements have different values
% in WMCO sometimes even up to 0.73
% max(max(abs(WMCO{1,6} - WMCO{1,9})))
% average error is up to 0.067
% (sum(sum(abs(WMCO{1,9} - WMCO{1,12})) )/(size(WMCO{1,6},1)*size(WMCO
   {1,6},2))


% This is however not dissimilar to permutations of elements with at
   least one different variable
```

```
%for k = 1:11
%(sum(sum(abs(WMCO{1,k} - WMCO{1,12}))) )/(size(WMCO{1,6},1)*size(WMCO
    {1,6},2))
%end
%ans = 0.1062
%ans = 0.1086
%ans = 0.1001
%ans = 0.092026
%ans = 0.070090
%ans = 0.063108
%ans = 0.093361
%ans = 0.085241
%ans = 0.067200
%ans = 0.061029
%ans = 0.074357

%for k = 1:11
%max(max(abs(WMCO{1,k} - WMCO{1,12})))
%end
%ans = 0.9078
%ans = 0.6739
%ans = 0.8050
%ans = 0.7108
%ans = 0.5899
%ans = 0.6010
%ans = 0.7787
%ans = 0.8612
%ans = 0.7317
%ans = 0.7176
%ans = 0.6839


% there is also difference in pvM
% pvM only attains values 0 or 1
% Difference between permutations is up to 0.32
% (sum(sum(abs(pvM{1,9} - pvM{1,12}))) )/(size(pvM{1,6},1)*size(pvM
```

```
   {1,6},2))

% Difference between different permutation of variables is comperable
%for k = 4:11
%(sum(sum(abs(pvM{1,k} - pvM{1,12}))) )/(size(pvM{1,6},1)*size(pvM
   {1,6},2))
%end
%ans = 0.3260
%ans = 0.3051
%ans = 0.2294
%ans = 0.3062
%ans = 0.2571
%ans = 0.3184
%ans = 0.3037
%ans = 0.2310
```

Listing 5: Partial wavelet coherency

```
% Copyright (c) 2022 Jakub Skoda
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% Change the current folder to the folder of this m-file.
if(~isdeployed)
  cd(fileparts(which(mfilename)));
end

run('Common.m');
filpath = 'Partial1000/';
mkdir(filpath)


% Computation parameters
coher_type = 'part';
windTime_size = 2;
windScale_size = 2;
n_sur = 1000; % must be used for credible significance levels
%n_sur = 1; %  innacurate results use for testing
p = 1;
q = 1;


% Plotting parameters
pictEnh = 5; %Picture enhancer

perc5 = 5/100;  % Percentil 5
perc10 = 10/100;  % Percentil 10

xlim = [t(1) t(end)+dt];
xticks = 1970:5:2020.25;

yticksLab = [1.5 4 8 20];
yticks = log2(yticksLab);

```

```
% Generates permutations for all various combinations of three elements
% 4 variables result in 24 different permutations
Comb = [];
K = 1 : Xsize;
for k = 1 : Xsize
 Comb = cat(1,Comb, perms(K(~ismember(K,k))));
end

CombSize = size(Comb,1);

WPCO = cell(2,CombSize);
periods = cell(2,CombSize);
coi = cell(2,CombSize);
pvP = cell(2,CombSize);
PWGain = cell(2,CombSize);

% ----- Computation of partial wavelet coherency --------

%for l = 1:CombSize % to compute all possible combinations
for l = [1, 3, 19, 12, 18] % compute only those used in the thesis
Y = X(:,Comb(l,:));
  for k = [2, 3]
  index_partial = k;

 [~,WPCO{k-1,l},periods{k-1,l},coi{k-1,l},~,pvP{k-1,l},PWGain{k-1,l}] = ...
 %[~,WPCO,~,~,~,pvP,PWGain] = ...
            MPAWCOG(Y,dt,dj,low_period,up_period,pad,mother,beta,gamma,...
            coher_type,index_partial,windTime_size,...
            windScale_size,n_sur,p,q);
            fprintf("Computatation finished for l=%i, k=%i.\n",l,k);
  end
end

save(strcat(filpath,'Workspace-n_sur_',num2str(n_sur),'-',num2str(now)))
```

```
% ----- Plot of partial wavelet coherency --------

fig_count =  1;

%for l = 1:CombSize % to plot all possible combinations
for l = [1, 3, 19, 12, 18] % plot only those used in the thesis
c1 = Comb(l,1);
c2 = Comb(l,2);
c3 = Comb(l,3);
   for k = [2, 3]
  Description = strcat(names{c1},'-',names{c2},'-',names{c3},'\_partial'
    ,num2str(k));
  filname = strcat(names{c1},'-',names{c2},'-',names{c3},'_partial',
    num2str(k));

  fig = figure(fig_count);
   set(fig, 'PaperPositionMode', 'manual')
   set(fig, 'PaperUnits', 'centimeters')
   set(fig, 'PaperPosition', [0 0 10 10])

   plotPCInf = subplot(1,1,1);

    ylim = log2([min(periods{k-1,l}),max(periods{k-1,l})]);

    imagesc(t,log2(periods{k-1,l}),abs(WPCO{k-1,l}).^pictEnh);
    grid on;
    set(plotPCInf,'XLim',xlim,'XTick',xticks);
    set(plotPCInf,'YLim',ylim, 'YTick',yticks,'YTickLabel',yticksLab,
   ...
        'YDir','reverse','FontSize',7)
    %title(Description,'FontSize',10,'FontName','arial');
    ylabel('Period (years)','FontSize',8);
    hold on
    colormap jet
```

```
    caxis('manual')
    % Plot COI
    plot(t,log2(coi{k-1,l}),'k');
    % Plot levels of significance
        contour(t,log2(periods{k-1,l}),pvP{k-1,l},[perc5 perc5],'k-','LineWidth',1.5);
     % 'Edgecolor',[.5 .5 .5], is not recognized by Octave
     if (is_octave)
     contour(t,log2(periods{k-1,l}),pvP{k-1,l},[perc10 perc10],'LineWidth',1.5);
     else
     contour(t,log2(periods{k-1,l}),pvP{k-1,l},[perc10 perc10],'Edgecolor',[.5 .5 .5],'LineWidth',1.5);
     end
    hold off

   %fig_count = fig_count + 1; % each plot in separate window
   print(strcat(filpath,filname),filform)
 end
end
```

Listing 6: Partial wavelet coherency phase-difference and gains

```
% Copyright (c) 2022 Jakub Skoda
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% Change the current folder to the folder of this m-file.
if(~isdeployed)
  cd(fileparts(which(mfilename)));
end

run('Common.m');
load('Partial/Workspace-n_sur_1-738401.7879');

filpath = 'Partial-PhaseDiff/';
mkdir(filpath)


% Computation of mean partial phase differences (with 95% CIs)
 %  and mean gains (in three different frequency bands)

% Short-cycles
    lpF0 = 1.5;
    upF0 = 4;
% Business-cycles
    lpF1 = 4;
    upF1 = 8;
% Long-cycles
    lpF2 = 8;
    upF2 = 20;
%
alfa = 0.05; % To compute 95% CI

phaseDif0 = cell(2,CombSize);
low_phaseDif0 = cell(2,CombSize);
up_phaseDif0 = cell(2,CombSize);

phaseDif1 = cell(2,CombSize);
```

```
low_phaseDif1 = cell(2,CombSize);
up_phaseDif1 = cell(2,CombSize);

phaseDif2 = cell(2,CombSize);
low_phaseDif2 = cell(2,CombSize);
up_phaseDif2 = cell(2,CombSize);

gain0 = cell(2,CombSize);
gain1 = cell(2,CombSize);
gain2 = cell(2,CombSize);

for l = 1 % :CombSize
  for k = [2, 3]
   % Computation of partial phase-differences
   % Message 'Cannot use Zar formula at point' is sometimes dysplayed
   % see line 176 of MeanPHASE.m form ASToolbox2018 for more information
  [phaseDif0{k-1,l},low_phaseDif0{k-1,l},up_phaseDif0{k-1,l}] = MeanPHASE(WPCO{k-1,l},periods{k-1,l},lpF0,upF0,alfa);
  [phaseDif1{k-1,l},low_phaseDif1{k-1,l},up_phaseDif1{k-1,l}] = MeanPHASE(WPCO{k-1,l},periods{k-1,l},lpF1,upF1,alfa);
  [phaseDif2{k-1,l},low_phaseDif2{k-1,l},up_phaseDif2{k-1,l}] = MeanPHASE(WPCO{k-1,l},periods{k-1,l},lpF2,upF2,alfa);

  % Computation of Gains
  gain0{k-1,l} = MeanGAIN(PWGain{k-1,l},periods{k-1,l},lpF0,upF0);
  gain1{k-1,l} = MeanGAIN(PWGain{k-1,l},periods{k-1,l},lpF1,upF1);
  gain2{k-1,l} = MeanGAIN(PWGain{k-1,l},periods{k-1,l},lpF2,upF2);
  end
end

save(strcat(filpath,'Workspace-n_sur_',num2str(n_sur),'-lp',num2str(lpF0),'_up',num2str(upF0),'-lp',num2str(lpF1),'_up',num2str(upF1),'-lp',num2str(lpF2),'_up',num2str(upF2),'-',num2str(now)))

% Phase-difference withOUT Partial coherency
% There seems to be still few bugs
```

```
ylimPhase = [-pi-0.5 pi+0.5];
yticksPhase = -pi: pi/2 : pi;
yticksPhaseLab = {'-pi','-pi/2','0','pi/2','pi'};

pause('on')
pause(2)

for l = 1 %:CombSize
c1 = Comb(l,1);
c2 = Comb(l,2);
c3 = Comb(l,3);
  for k = 3 % [2, 3]
  Description = strcat(names{c1},'-',names{c2},'-',names{c3},'\_partial'
    ,num2str(k));
  filname = strcat(names{c1},'-',names{c2},'-',names{c3},'_partial',
    num2str(k));
  fig = figure(1);
  set(fig, 'PaperPositionMode', 'manual')
  set(fig, 'PaperUnits', 'centimeters')
  set(fig, 'PaperPosition', [0 0 10 10])

  % Plots of partial phase-differences
  % 56 65
  plotDif0 = subplot(3,2,1);
  plot(t,phaseDif0{k-1,l},'LineWidth',1,'Color','k');
  % CIS
     hold on
     plot(t,low_phaseDif0{k-1,l},'k-.','LineWidth',0.75);
     plot(t,up_phaseDif0{k-1,l},'k-.','LineWidth',0.75);
     grid on;
     set(plotDif0,'XLim',xlim,'XTick',xticks)
     set(plotDif0,'YLim',ylimPhase,'YTick',yticksPhase,...
        'YTickLabel',yticksPhaseLab,'FontSize',7);
     title('Partial Phase-difference','FontSize',10,'FontName','arial');
     ylabel('1.5~4 years');
```

```
     hold off

    plotDif1 = subplot(3,2,3);;
    plot(t,phaseDif1{k},'LineWidth',1,'Color','k');
    hold on
    plot(t,low_phaseDif1{k},'k-.','LineWidth',0.75);
    plot(t,up_phaseDif1{k},'k-.','LineWidth',0.75);
    ylabel('4~8 years');
    grid on;
    set(plotDif1,'XLim',xlim,'XTick',xticks)
    set(plotDif1,'YLim',ylimPhase,'YTick',yticksPhase,...
        'YTickLabel',yticksPhaseLab,'FontSize',7)
    hold off

    plotDif2 =subplot(3,2,5);;
    plot(t,phaseDif2{k-1,l},'LineWidth',1,'Color','k');
    hold on
    plot(t,low_phaseDif2{k-1,l},'k-.','LineWidth',0.75);
    plot(t,up_phaseDif2{k-1,l},'k-.','LineWidth',0.75);
    ylabel('8~20 years');
    grid on;
    set(plotDif2,'XLim',xlim,'XTick',xticks)
    set(plotDif2,'YLim',ylimPhase,'YTick',yticksPhase,...
        'YTickLabel',yticksPhaseLab,'FontSize',7)
hold off


     % Plots of  Gains
     ylimGain0 = [min(gain0{k-1,l}),max(gain0{k-1,l})];
     ylimGain1 = [min(gain1{k-1,l}),max(gain1{k-1,l})];
     ylimGain2 = [min(gain2{k-1,l}),max(gain2{k-1,l})];

  plotGain0 = subplot(3,2,2);;
        plot(t,gain0{k-1,l},'LineWidth',1,'Color','k');
        ylabel('1.5~4 years');
        legend('| \beta |','Location','Best')
```

```
        % Octave warning: legend: 'best' not yet implemented for location specifier, using 'northeast' instead
        grid on;
        set(plotGain0,'XLim',xlim,'XTick',xticks)
        set(plotGain0,'YLim',ylimGain0,'FontSize',7);
        % 'YTick',0:.5:3,
        title('FFR = \alpha + \beta \pi + \gamma y','FontSize',10,'FontName','arial')

  plotGain1 = subplot(3,2,4);;
        plot(t,gain1{k-1,l},'LineWidth',1,'Color','k');
        ylabel('4~8 years');
        legend('| \beta |','Location','Best')
        grid on;
        set(plotGain1,'XLim',xlim,'XTick',xticks)
        set(plotGain1,'YLim',ylimGain1,'FontSize',7)
        % 'YTick', 0:.5:3

  plotGain2 = subplot(3,2,6);;
        plot(t,gain2{k-1,l},'LineWidth',1,'Color','k');
        ylabel('8~20 years');
        legend('| \beta |','Location','Best')
        grid on;
        set(plotGain2,'XLim',xlim,'XTick',xticks)
        set(plotGain2 ,'YLim',ylimGain2,'FontSize',7)
        % ,'YTick',0:.5:3

  axes( 'visible', 'off', 'title', Description);
  %print(strcat(filpath,filname),filform,'-r7086')
  % '-r7086; ,'-r0'
  end
end
```