Equations for BEC

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We are considering a system with a charged scalar field ϕ coupled to the electromagnetic field A_μ. The Lagrangian for this system includes contributions from both the scalar field and the electromagnetic field, and their interaction:

$$\mathcal{L} = |D_\mu \phi|^2 - \mu^2 |\phi|^2 + \lambda |\phi|^4 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \chi \phi^* \phi \tag{1}$$

Where:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{2}$$

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi \tag{3}$$

The equations of motion for this system are:
1. For the electromagnetic field A_μ:

$$\partial_\nu F^{\nu\mu} = j^\mu \tag{4}$$

With the current j^μ induced by the scalar field:

$$j^\mu = -2ie\left(\phi^* \partial^\mu \phi - (\partial^\mu \phi^*)\phi\right) - 2e^2 A^\mu |\phi|^2 \tag{5}$$

2. For the scalar field ϕ:

$$D_\mu D^\mu \phi + (\mu^2 - \chi)\phi - 2\lambda |\phi|^2 \phi = 0 \tag{6}$$