Traveling Salesperson

CS 312

Announcements

- Midterm curve is +2.
  - Subtract 0.1 from final grade bounds, so…
  - A now goes to 92.9 for the final grades
- Appeals are done
- Prayer
- Thought: what did you think of yesterday?

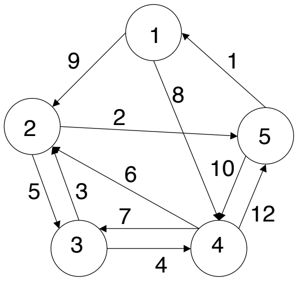
Schedule

- Today: TSP
- Friday: Knapsack B&B, minimax
- Monday: A* and variants
- Wednesday: Research in CS?
- Friday: Parallel algorithms?
- 1 week for proposals, 2 wks for code.

Objectives

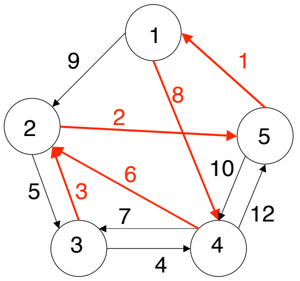
- Learn a more complex bounding function.

Bound on TSP Tour



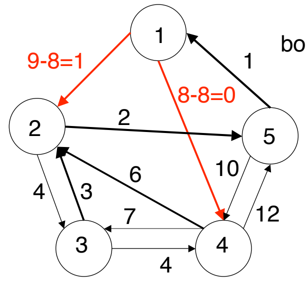
Every tour must leave every node and and arrive at every node.

Bound on TSP Tour



What's the cheapest way to leave each node?

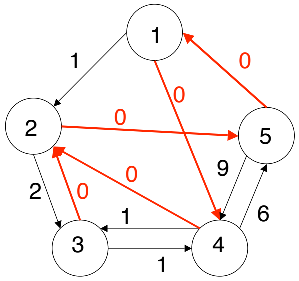
Bound on TSP Tour



bound=8+6+3+2+1

Subtract that cost from each edge leaving a node.

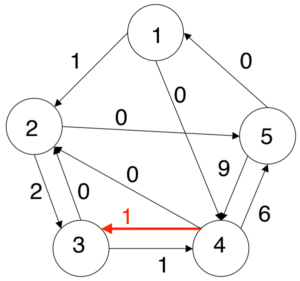
Bound on TSP Tour



bound=20

Save the sum of those costs. Subtract each cost from the cost of leaving each individual node.

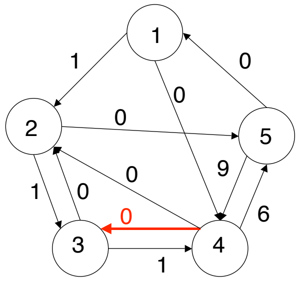
Bound on TSP Tour



bound=20

Does that tour arrive at every node? In this case, the tour never arrives at node 3.

Bound on TSP Tour



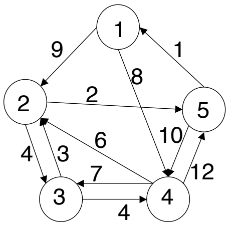
bound=21

We have to take an edge from node 1 to somewhere. Assume we take the cheapest.

The bound

- It will cost at least this much to visit all the nodes in the graph.
  - there's no cheaper way to get in and out of each node.
  - the edges are labeled with the extra cost of choosing another edge.

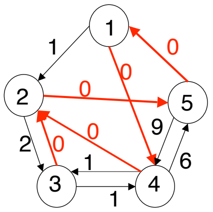
Bound on TSP Tour



$$\begin{pmatrix} 999 & 9 & 999 & 8 & 999 \\ 999 & 999 & 4 & 999 & 2 \\ 999 & 3 & 999 & 4 & 999 \\ 999 & 6 & 7 & 999 & 12 \\ 1 & 999 & 999 & 10 & 999 \end{pmatrix}$$

Algorithms do this using a matrix. Here's the representation.

Bound on TSP Tour



$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 2 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 1 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

Algorithms do this using a matrix. Here's the representation.

Using this bound for TSP

$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 21

1to2

$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 21+1

1to3

$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

1to4

$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

1to5

$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

Chose to go from 1 to 2

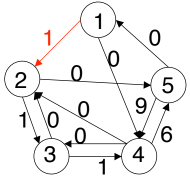
$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

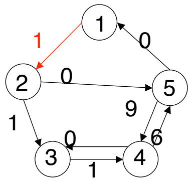
bound = 21

1to2

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 999 & 999 & 1 & 999 \\ 999 & 999 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 21+1





Add extra cost from 1 to 2, exclude edges from 1 or into 2.

Chose to go from 1 to 2

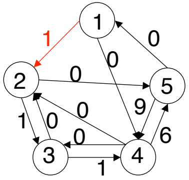
$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

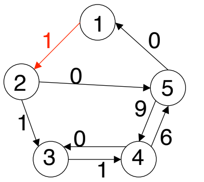
bound = 21

1to2

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 999 & 999 & 1 & 999 \\ 999 & 999 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 21+1+1





No edges into node 4 w/ 0 reduced cost.

Chose to go from 1 to 2

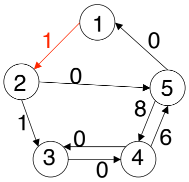
$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 21

1to2

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 999 & 999 & 0 & 999 \\ 999 & 999 & 0 & 999 & 6 \\ 0 & 999 & 999 & 8 & 999 \end{pmatrix}$$

bound = 21+1+1



Add cost of reducing edge into node 4.

Bounds for other choices.

1to3,1to5,1to2,1to4

$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 21

1to2

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 999 & 999 & 0 & 999 \\ 999 & 999 & 0 & 999 & 6 \\ 0 & 999 & 999 & 8 & 999 \end{pmatrix}$$

bound = 23

1to3

$$\begin{pmatrix} 999 & 999 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 999

1to4

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 999 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

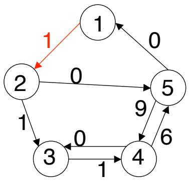
bound = 21

1to5

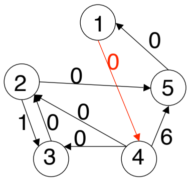
$$\begin{pmatrix} 999 & 1 & 999 & 0 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 1 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 9 & 999 \end{pmatrix}$$

bound = 999

Two possibilities.

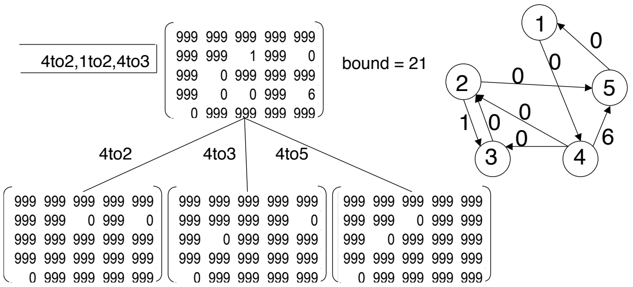


bound = 23



bound = 21

Leaving node 4



4to2,1to2,4to3

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 1 & 999 & 0 \\ 999 & 0 & 999 & 999 & 999 \\ 999 & 0 & 0 & 999 & 6 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

bound = 21

4to2

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 0 & 999 & 0 \\ 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

bound = 22

4to3

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 0 \\ 999 & 0 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

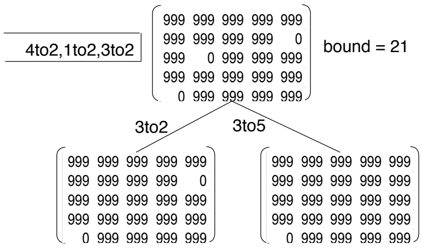
bound = 21

4to5

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 0 & 999 & 999 \\ 999 & 0 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

bound = 28

Leaving node 3



4to2,1to2,3to2

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 0 \\ 999 & 0 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

bound = 21

3to2

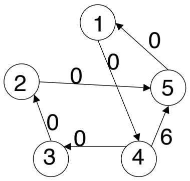
$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 0 \\ 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

bound = 21

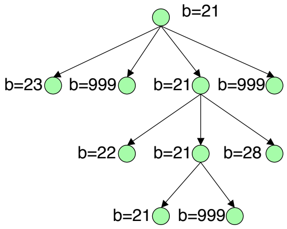
3to5

$$\begin{pmatrix} 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 999 & 999 & 999 & 999 & 999 \\ 0 & 999 & 999 & 999 & 999 \end{pmatrix}$$

bound = 999



Call tree for this problem.


b=21
b=23
b=999
b=21
b=999
b=22
b=21
b=28
b=21
b=999