Closest Antenna Constraint

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1 Introduction to the Problem

Assume that there are some customers and some antennas.
Let's define S as the set of antennas and C as the set of customers.

Parameters:

- *f(i)* is the activation cost of one antenna.
- *u(i)* is the capacity of an antenna.
- *c(i,j)* is the assignment cost for the antenna i to the customer *j*..
- *d(i,j)* is the distance from the antenna i to the customer *j*.

Variables:

- *x(i,j)* is a variable that assumes the value 1 if the antenna i is serving the customer j, 0 otherwise.
- *y(i)* is a binary indicator variable that is equal to 1 only and only if the antenna $i \in S$ is active.

Objective Function:

- min $\sum_{i \in S} f(i)y(i) + \sum_{i \in S} \sum_{j \in C} c(i,j)x(i,j)$

Constraints:

- $\sum_{i \in S} x(i,j) = 1 \ \forall j \in C$
- $\sum_{j \in C} x(i,j) \leqslant u(i)y(i) \ \forall i \in S$
- $x(i,j) \leq y(i) \ \forall i \in S, \forall j \in C$

The goal is to write a constraint that assigns to each customer the closest antenna.

2 Solution

We approach the problem as follows:

- We define a new parameter that we call MAX, that is equal to a number that is greater than the maximum value that $d(i,j)$ can be.
- We define the following constraint:
  $\forall i, k \in S, \forall j \in C : d(i,j)x(i,j) \leq d(k,j) + (1 - y(k)) * MAX$

This way we can select the closest antenna according to the other constraints and exploiting the objective function.
If $k = i$ than the equality is satisfied and the constraint holds.
If $k! = i$ than

- If k is active: $x(i,j)$ is equal to 1 iff $d(i,j) \leq d(k,j)$ (iff i is closer to j than k is).
- If k is not active: $x(i,j)$ can be either 1 or 0.