Supermarket and Clerks

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1 Introduction to the Problem

We are given a problem: we have to minimize the cost of clerks working in a supermarket using linear programming, find the dual problem and give an interpretation of it.
We are provided with:

- $d = \{M, T, W, Th, F, S, Su\}$, the set of days in the week.
- b_d, minimum number of clerks required in day d.
- c_d, cost of salary for the shift beginning in day d.

We also know that each clerk has to work 5 consecutive days and then has 2 days of rest.

2 Solution

I thought to model the problem as follows.
To start with I introduce a variable

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_7 \end{bmatrix} \tag{1}$$

that represents the number of clerks starting to work on the $i - th$ day. From now on I consider b and c as the column vectors with the meaning specified on the previous section.
It is possible to formulate the problem as:

- min $(\boldsymbol{x}\boldsymbol{c}^{\boldsymbol{T}})$
- $\boldsymbol{A}\boldsymbol{x} \geqslant \boldsymbol{b}$
- $\boldsymbol{x} \geqslant 0$

Since the number of clerks working on day i is equal to the sum of all the clerks that started to work in the 4 days before i and on i itself, the matrix A is defined as:

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{2}$$

To get the dual problem, I define the y (column vector) variable and following the table reported on the slides for the correspondence of primal-dual, the dual problem result as:

- max $(\boldsymbol{y}\boldsymbol{b}^{\boldsymbol{T}})$
- $\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{y} \leqslant \boldsymbol{c}$
- $\boldsymbol{y} \geqslant 0$

We can imagine y to represent the cost of work of clerks on day i. Clerks wants to maximize the total profit they can make, given the budget of the supermarket on the day i (and the number of working clerks on day i).