Multi-Armed Bandits: Exploration vs Exploitation

Fabricio Barth

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Outline

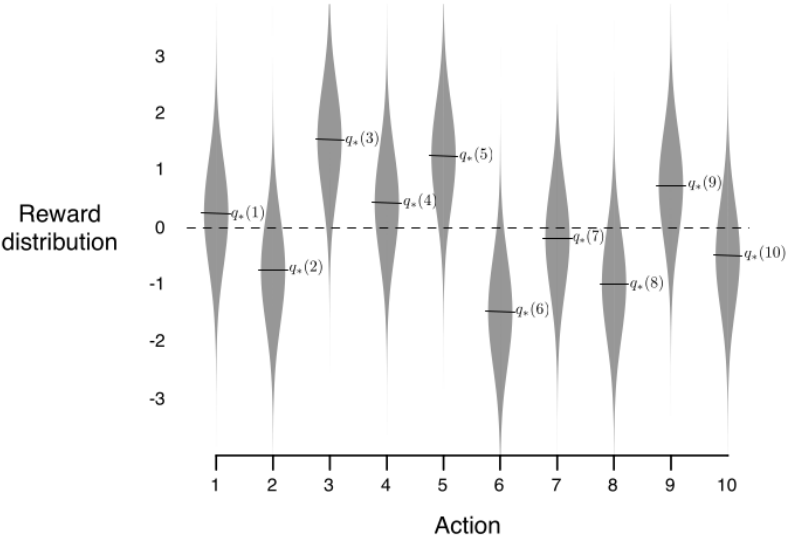
The Multi-Armed Bandit Problem

- A set of k actions (arms): $a \in \{1, \ldots, k\}$
- At time t, choose action A_t
- Observe a stochastic reward R_t
- Each action has an unknown reward distribution

Goal: maximize cumulative reward

$$\sum_{t=1}^{T} R_t$$

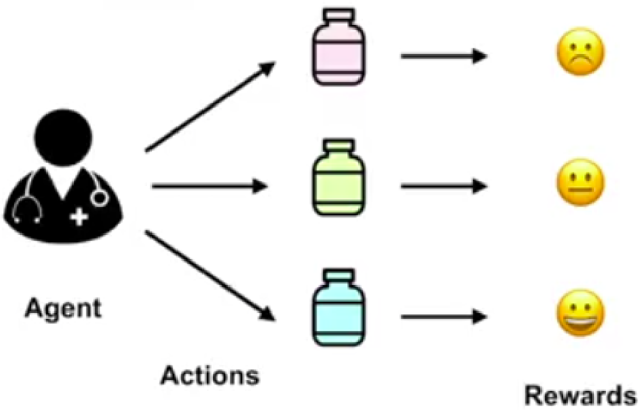
The Multi-Armed Bandit Problem: example



Code Activity

Go to the website, download the Jupyter notebook and execute the first activity.

Clinical Trial Example



It could be you in a restaurant.

Expected Reward of an Action

The true (unknown) value of action a is

$$q_*(a) = \mathbb{E}[R_t \mid A_t = a].$$

- $q_*(a)$ is the expected outcome if we always choose a
- The agent never observes $q_*(a)$ directly

Action-Value Estimates

Let $Q_t(a)$ denote the estimate of $q_*(a)$ at time t.
Using sample averages:

$$Q_t(a) = \frac{1}{N_t(a)} \sum_{i=1}^{N_t(a)} R_i^{(a)}$$

where $N_t(a)$ is the number of times action a has been selected.

Incremental Update Rule

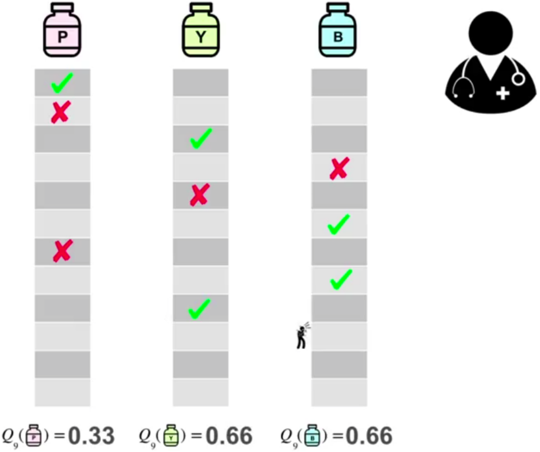
Instead of storing all rewards, we update incrementally:

$$Q_{t+1}(a) = Q_t(a) + \frac{1}{N_t(a)}(R_t - Q_t(a))$$

Estimating the value of Q(a) for the Clinical Trial Example

A reward of 1 if the treatment succeeds otherwise 0

$$Q_t(a) = \frac{\sum_{i=1}^{t-1} R_i}{t-1}$$



Code Activity

Go to the website and execute the second activity: implementing an incremental update rule.

Greedy Action Selection

If the true values were known:

$$A_t = \arg\max_a q_*(a)$$

In practice, we use estimates:

$$A_t = \arg\max_a Q_t(a)$$

- Exploits current knowledge
- Can get stuck with a suboptimal action

Code Activity

Execute the activity number 3: greedy action selection.

Explore vs Exploit

- **Exploit**: choose the best estimated action
- **Explore**: try actions with uncertain or lower estimates

Clinical trials must explore to discover better treatments, but also exploit to help current patients.

ε-Greedy Action Selection

With probability $1 - \varepsilon$: choose greedy action
With probability ε: explore uniformly

$$A = \begin{cases} \arg\max_a Q_t(a) & \text{with probability 1 - } \varepsilon \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$

Code Activity

Execute the activity number 4: ε-Greedy Action Selection

Clinical Trials: Why ε-greedy?

- Avoids prematurely committing to a suboptimal treatment
- Ensures all treatments are sampled
- Simple and effective baseline method

Common variants:

- Decaying ε_t
- Optimistic initialization

What Did We Learn in This Class?

- The **multi-armed bandit** is the simplest reinforcement learning problem: no states, only actions and rewards
- Real-world problems like **clinical trials** can be modeled as bandits
- Each action has an unknown **expected reward**:

$$q_*(a) = \mathbb{E}[R_t \mid A_t = a]$$

- We learn action values using data-driven estimates $Q_t(a)$ and incremental updates
- Purely greedy decisions can fail due to early randomness
- The core challenge is **exploration vs exploitation**
- ε-**greedy** balances this trade-off by exploring with probability ε

References

- Sutton, R. S., & Barto, A. G. (2018). *Reinforcement Learning: An Introduction*, 2nd ed., pp. 25–36.