Using a Cellular Automata Model to Simulate Motorway Traffic

1. Abstract

Looking at motorway traffic, often cars slow down, and then accelerate only to have to slow down again- resulting in clumps of cars packed closely and then times where there are very few cars around and everyone can drive at speed. Sometimes there are reasons for this such as lane closure, others seem to be no underlying cause. **We will investigate this by simulating one lane and three lane motorways and observing how the traffic flows.**

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2. Background

The approach used to tackle this problem is a cellular automaton a technique that involves the discretisation of space into cells. In this case it is the motorway road that is discretised into cells, and each cell can be in one of two states, occupied or unoccupied by a car. At each discrete timestep an update function is applied to each cell, and this is dictated by the state of the cell previously as well as local interactions with neighbouring cells in the system.

There are of course many different ways in which traffic flow could be modelled as well as a cellular automaton which include using objects as opposed to array elements, as well various machine learning techniques that look to mimic or exceed human driving levels. This model was chosen to its scalable ability. The implementation of a few simple rules can give rise to complex system, as each interaction on the microscopic level leads to patterns emerging on the macroscopic level. Depending on the initial conditions as well as the rules themselves the resulting patterns can be one of order, which we will see when there is ample space between the cars, or a chaotic system- for us that is traffic.

Cellular automaton models have been applied to a multitude of physical systems such as, lattices and those involving self-organisation. In particular growth problems, such as the spread of disease, which is depended on interactions between neighbouring bodies. Stephen Wolfram's paper used an example in biology where the pigmentation of a cone shell displays the pattern of a cellular automaton {2}. This class of cellular automaton is known as Rule 184 or a particle hopping method, as the system contains a one-dimensional array where the elements are update in accordance to a function and the neighbouring array elements {3}.

One of the potential practical implication for this system in particular is in the design on a driverless car system. While the technology required for autonomous driving is far more complex, and fundamentally different from a cellular automata simulation, what a cellular automaton allows is to model an entire system of traffic flow, and in particular look at the parameters that allow for the optimum flow of traffic. That is a road system whereby the least amount of fuel or energy is used, achieved by minimising starting and stopping due to traffic, or by looking at minimising the journey time for all drivers on the road- having the highest average velocity.

While the density of cars is a parameter that will be varied to observe the different flow patterns that emerge, in a real-world scenario there may be very little control over how many cars occupy a stretch of road, and thus looking at other factors, such as the speed limits, will be important. In a full-scale autonomous car system, the number of cars on individual roads is possible when observing the entire system as a whole.

While the manufactures of driverless cars are currently testing and producing vehicles that are designed to interact with the world around them by using technologies such as image recognition, if we look into the future, at a world where all vehicles on the road are driverless, then a centralised system controlling all of the vehicles would be an efficient way to ensure optimum flow. It would be able to determine the most efficient journeys for all the vehicles, and this system would be similar in nature to what the cellular automaton model looks to simulate. However, for simplicity sake we are looking specifically at when on a motorway, a future addition to this simulation could be interactions with other roads to create an entire system, and then look for the optimal conditions. This would be similar to how train systems are controlled, with the advantage of cars being able to move around broken down cars, and thus in this simulation will look at the effects accidents have on the remaining traffic.

Until we reach a stage where all cars on the road are autonomous, and even that is not a certainty, we must account for human irrationality. While we will not account for cars that are speeding beyond a limit, or reckless driving manoeuvres that lead to accidents, we will implement driver dawdling. This is where cars slow down when there is no requirement to, equivalent to not looking forward when at the wheel, or even checking a mobile device. The importance of this dawdling factor is that brings the model away from being completely deterministic due to how computers operate and gives a stochastic simulation more akin to properties nature. The resulting velocity fluctuations will account for unpredictable human behaviour and is similarly implemented in models in economics as well as in particle physics simulations to account for the probabilistic nature of quantum mechanics.

While this is not a true model of what motorway flow is like, as vehicles on a motorway are much freer flowing than we will allow them, and travel at much faster speeds than we will implement- this better model the occasions where motorways are densely populated and slow moving. It is also in systems of traffic that interesting flow patterns emerge and has previously been observed to be similar to non-Newtonian fluid flow {5}.

3. Hypothesis/ Expectations

It is fairly intuitive what it is that will lead to cars traveling slower; we would expect a higher density of cars on the road to result in slower velocities for the cars. This is simply as the distance between cars reduces, and this will cap the maximum velocity that they will be able to reach. One can make the argument that cars are able to travel at maximum velocity regardless of the distance between them and the next car if all cars are travelling at a high enough speed, however in a real world scenario this would be deemed as incredibly unsafe as the cars would not have the required stopping distance to avoid a collision should they be required to slow down and thus we have avoided this scenario in our simulation, and are expecting to observe that the velocities of our vehicles will be a decreasing function of the density of the traffic.

By changing the initial conditions to have fewer cars on the road we would expect to see some sort of lamina flow pattern, whereby the average speed of the vehicles converges to the max speed. This will be due to the cars having ample space between them to accelerate to their maximal velocity and will have no objects in front, within a distance that they will travel in the next timestep to decrease.

As the density increases, we will reach a point whereby the distance between vehicles is the same as the distance that each vehicle will travel in the next timestep. At this point a car dawdling will not only affect the speed of that car but also of any behind it, and thus lead to cars exhibiting an average velocity that is no longer tending to the maximum velocity of the vehicles. We can call this density of cars the critical density, $\rho_{critical}$.

What is less intuitive and predictable is the flow patterns that will emerge when the cars find themselves in a traffic build up. These will occur where the traffic densities are larger than the density, ρ, as stated we expect to observe the average speed to become a decreasing function of ρ.

We will denote the density of the traffic, ρ, as:

$$\rho = \frac{\text{number of cars on the road}}{\text{number of cells in the road}}, \quad [1]$$

and will have units of cars/cell.

We expect the flow when the density is below the critical density to display a chaotic and turbulent nature similar to disturbed fluid flow, and like a liquid we then expect the system to order itself. It is the

stochastic nature that will then disturb the system, and then cause a period of turbulence or traffic, and therefore we would expect shockwaves to arise, followed by periods of smooth flow, again followed by a traffic jam. These shockwaves caused by the dawdling of vehicles will be expected to increase both as density increases as well as the probability of dawdling is increased.
It will be interesting to see which factor appears to have more of an impact on the traffic, whether it be the density of cars on the road, or whether it be the probability of dawdling.

Another factor to consider is the ratio between the maximum speed of the vehicles and the period length, in our case the length of the road as we will impose periodic boundaries- that is considering the road to act as a loop where the cars that leave the road at one end will enter again at the other. We would expect that when the maximum velocity is high compared to the length of the road, the average velocity of the vehicles will now not be at, or close to, their maximal velocity as they will now require a greater distance between them to be able to travel at this speed. Thus, we should be able to determine that increasing the maximal velocity will have the same effect as increasing the density on the cars' ability to travel at this maximal velocity.
We will define the velocity of the cars, v, as:

$$v = \frac{\text{number of cells travelled}}{\text{number of timesteps}} \quad , \qquad [2]$$

and thus, will possess the units of *cells/timestep or, c/t for short.* We can clearly see how this is equivalent to the traditional definition of velocity:

$$v = \frac{\text{small change in distance}}{\text{small change in time}} = \frac{\Delta x}{\Delta t} \quad , \qquad [3]$$

and the average velocity of the cars, $\bar{v}$, is :

$$\bar{v} = \frac{\sum(\text{individual velocities})}{\text{number of cars}} \quad , \qquad [4]$$

and also possess the units of *c/t* .

In our simulation we are discretion time and space and will consider the change in time to be 1 timestep. Thus, we can see that the velocity of the car is equivalent to the distance it will travel in that next timestep, explaining how the average velocity of the vehicles will proportional to the average distance between cars, when the density is below $\rho_{critical.}$

Then when a system that contains multiple lanes, we will look to observe why cars will overtake others and the patterns that this will result in. We will expect those cars that will look to move the fastest to occupy the overtaking lanes. As cars now have the ability to move into other lanes if the car in front of them acts in a way that would force them to slow down in a single car system, we may expect to see that the average speed of the vehicles increases, but it will be interesting to see if this is the case when the overall road density is the same as in a single lane system, expecting to see the overtaking lane on the right will have a higher average vehicle speed.

When we introduce vehicles with different maximum speed, we may observe self-segregation of the faster vehicles into fast lanes and the slower vehicles into the slow lanes over the number of timesteps, as we will observe what effect having a mixture of cars will have to the average speed of the all vehicles, as well as the average speed of the vehicles of each individual type.

4. Method

4.1 Road Creation

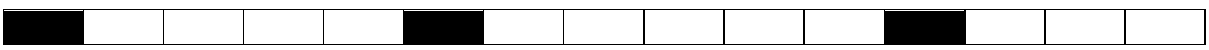

Figure 1. *Displaying a discretisation version of a road made up of 15 individual cells, with cars occupying the 0th, 5th and 11th elements.*

A NumPy array was used as the road with n number of spaces, then cars would populate the road, as can be seen in figure 1. An array was chosen over an alternative data structure such as a Python list, due to the speed and memory efficiencies that come with a NumPy array over a list or alternate structure. This reduction in memory size and increase in speed will become important when iterating over a large number of timesteps and then iterating the whole process a large number of times.

4	0	0	0	0	1	0	0	0	0	0	3	0	0	0

Figure 2. Displaying the array where an element of zero indicates no vehicles present, and an integer of 1 or greater represents the cell being occupied, mirroring what was represented in figure 1.

3					0						2			

Figure 3. *Displaying the velocities (c/t) of the elements populating the road in figure 2.*

As arrays must be filled by floats or integers, and to make the mathematics easiest, the road was created with every element set to zero. When populating the road, the velocities of the cars are represented in the element where that vehicle is on the road, but if the velocity were to be equal to the value of the element then a stationary would have the same value of that of a space in the road where there were no vehicles. To overcome this the elements in the array where a car is present are equal to the velocity of the car plus one, so that when a car is stationary its element value would be one, and thus differentializing it from an empty space in the road, as seen when comparing figure 2 to figure 3. We can call our element value, E:

$$E = v + 1. \qquad [5]$$

4.2 Car generation

There are two approaches to how the cars on the road can be generated and as to how the system deals with boundary of the system. In this situation that is cars that would travel beyond the final index of the array. The model was chosen to have a periodic boundary, also known as a closed boundary, whereby cars cycle in a loop, meaning that that the cars that leave at the end of the road, re-enter at the beginning.

For the open system the road would initially be empty, and with each iteration populated by cars travelling at random speeds between 1c/t and the maximum velocity assigned in the function, which we will call v_{max}. Initially v_{max} = 5c/t. The random speeds were determined using a random number generator between the two vales. This random generation of the positions and velocities of the vehicles gives rise to a stochastic simulation of the cars that travel on the roads. Thus, probability of two simulations being identical are extremely slim, however over time all systems are likely to show similar properties to those with identical initial conditions.

This populating of the road would continue until the desired density, ρ, assigned in the function, was achieved, this density will subsequently be altered to test different values. A new car will enter the road if the car density dropped below the assigned value, however this leads to a problem when trying to keep the road at a selected density. As there can be multiple cars that leave at the end of the road to maintain the density of the cars on the road, multiple cars will then have to be a generated at the beginning of the road, however there very well may be a constraint the space at the beginning of the road, and thus the situation can arise where fewer cars than are required to maintain the density are be generated. To guarantee having a constant density of vehicles on road one must adopt a closed system where by all the cars are generated initially, and the vehicles that leave the road at the end loop round and join at the start of the road, so the number of cars on the road always remains constant. This ability to maintain the density throughout a number of timesteps was the reason for adopting a closed system throughout the project.

In a closed system the cars are all generated initially to give a set density, ρ, by iterating through the number of cars required. Each was randomly placed in a space on the road, so long as that space was not already occupied, and again given a random velocity that was in the range of the v_{max}.

In order to create this continuous road, the road was duplicated, so that the cars at the end of road would then interact with copies of the cars at beginning, thus creating this continuity required for the closed system. The inbuilt roll function, np.roll, could not be implemented as different vehicles move at different velocities, something that the roll function cannot deal with as it moves every element by the same amount. Requiring the method of duplicating the road then bringing it back to size. In order to bring the road back to its original shape the first half of 'Road 2' (or the third quarter of this double road) precedes the second half of 'Road 1' (or the second quarter of this double road), where Road 1 is the original road, and Road 2 is the duplication.

As the cars in the 'Road 2 are a duplicate of the cars on the original road by placing these in front of second half of 'Road 1', positions of the vehicles are maintained. Any movement or interactions that would have taken place in the first half of the original road still takes place on the first half of the second road, with the exception that cars from the second half of the original road will now move onto and interact with the cars on duplicate road.

The outermost quarters cannot be used due to limits on elements potentially extending beyond the index of the array. This manipulation of road the takes place for each time step and gives the road its cyclic nature.

1st ½ of 'Road 1'	2nd ½ of 'Road 1'	1st ½ of 'Road 2'	2nd ½ of 'Road 2'

↓

1st ½ of 'Road 2'	2nd ½ of 'Road 1'

Figure 4. Displaying how the duplicated road is brought back to the roads original size.

4.3 Rules

Once the cars have been generated, they move and interact with their local neighbours, by iterating through each space and implementing a number of rules that dictate the speed of the vehicles and how they move. These rules were mostly implemented using if and else statements. These rules were used by Nagel and Schrekenberg in their paper modelling freeway traffic {4}.

Rule 1 *- If the velocity, v, of the car is lower than v_{max}, and the distance to the next car ahead is larger than v + 1, the speed is increased by one.*

Here we can see that the velocity of the car can be equivalent to the distance it will travel in the next timestep. Due to the discrete nature of the system, and that vehicles possess a velocity of an integer value, all cars accelerate at $1c/t^2$. This represents a car seeing an open space in front of it and the driver has the ability to accelerate.

This was implemented by summing the spaces that preceded the vehicle that correspond to the velocity of the car plus one. If this sum is equal to zero, indicating that there are no cars in this space, then the car's velocity is increased by one, subject to it not already traveling at the stated $v_{max,}$ an example can be seen in figure 5 figures display velocity, element would be v+1).

This first rule indicates how simplistic the model is with cars only traveling at discrete velocities and accelerating a discrete amount. As well as this all vehicles accelerate at the same rate regardless of whether they are moving from stationary or close to their top speed.

2					3	
3					3	

Figure 5. *A car's speed increasing under rule 1. The array below is the resulting array.*

Rule 2 *- If a driver at site i sees the next vehicle at site i+j, with j < v, she reduces speed to j −1.*

A count mechanism was used to determine how many spaces there are between the car and the next car in front, and the new velocity of the vehicle is the distance between minus one, this can be seen in figure 6 (figures display velocity, element value would be v+1).

This represents a car seeing a car in front of it that is a distance away less than the distance the car will travel in the next timestep. This indicates that if no change to its velocity took place the car could potentially collide into the car in front of it, if not in the following timesteps, and thus it slows down to a velocity that means it will not collide with the car. However, again this is a simplification and is not the most natural or realistic model, as a car can go from its maximum speed to stationary in one timestep as well as not take into account the velocity of the car that is ahead, simply that a car is there.

When not dramatically accelerating or decelerating these rules do a very good job of mimicking how people drive, as well as being very similar how adaptive cruise control operates in some vehicles- using radar to determine the distance between you and the next car and altering your velocity accordingly.

4			3	
2			3	

Figure 6. *A car's speed decreasing under rule 2. The array below is the resulting array.*

Rule 3 *- The velocity of each vehicle (if greater than zero) is decreased by one with probability p ('dawdling').*

Using a random number generator, and if this number is greater than a chosen value – creating a probability, then the velocity of the car is decreased by a value of one, giving us a p(dawdling) value. This represents the human like factor, giving rise to the stochastic nature of the simulation.

Rule 4- *Each vehicle is advanced by v sites.*

This is achieved by moving each element forward by its value, E, and then removing the element where it originally was, this results in the vehicle moving forward E-1 steps which is equivalent to moving forward by its velocity. While iterating over the entire road it was important to do so in reverse, that is starting at the end of the road through to the start. And this is as the cars must move to allow the ones behind to move, often into the spaces where those cars previously were. If this was not done cars will be written over as the ones behind them move forward, this can be seen in figure 7.

It was important that this rule was implemented before the previous rule (3) when using a closed, periodic system. This is because of how the periodic system was implemented, as a duplicate of the road, that is then brought back down. The road cannot be reduced back down to its original size until the cars have moved, and this rule (4) is implemented. If the dawdling were to take place the two versions the road become different, due to the probabilistic nature of rule 3, and bringing them back together to a single road incurs the problem that rogue cars are created and deleted, as cars from the different halves of the road move differently now having different velocities as a result of rule 3.

2			4				
		2					4

Figure 6. *Cars moving along the array according to their velocities under rule 4. The array below is the resulting array.*

4.4 Multiple Lanes

The model can then be extended to the shape of a motorway, containing three lanes. Now cars have the ability to move between lanes, either overtaking to have the opportunity to accelerate, or to move into a slower moving lane if a car is no longer looking to overtake different vehicles. Once again, we will introduce a probability, in the same was as was with the dawdling, to bring a human like element to these manoeuvres, giving us a p(lane change) value.

Overtaking Rules

For a car to overtake it must fulfil two conditions. The first being that it has an incentive: *the driver anticipates being able to drive more freely on the new lane,* and will overtaking if it is safe to do so: *not forcing other drivers to brake too hard*.

Looking at the first condition, the car must ensure that there is motivation to overtake,
if there is a car in front that is a distance in front, smaller than the distance the car would travel in the next time step and would cause the car to slow down, i.e. if it would otherwise undergo rule 2 if this were a single lane system.

If the car is travelling slower than its v_{max} speed it will look to move into the faster lane to accelerate towards its v_{max}. (Initially it was stated that a car must be traveling faster than the car in front of it to overtake, however this meant that cars with a higher v_{max}, as will be seen in the different vehicles section, were not incentivised to overtake those with a slower v_{max} , and look to travel faster. (This could be visualised as a sports car sitting behind a slow-moving lorry, and not overtaking as it was travelling slower than the lorry and by the rules of the cellular automata will remain so indefinitely). When a car overtakes the velocity of the vehicle is increased by one to simulate the car accelerating as it overtakes.

The car must also look at whether there are any cars in the other lane that are distance in front which is smaller than the distance the car would travel in the next time step, as this would cause the car to slow down, as seen in figure 8. Even if the car is forced to decelerate more by staying in its lane, it would have to remain for the purposes of adhering to our rules of overtaking.

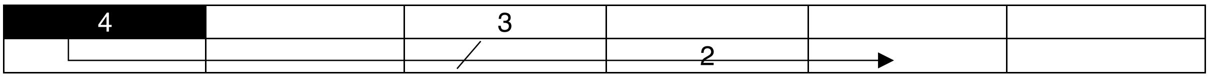

Figure 8. *the car at index 0 will not overtake and move forward, as it would be forced to decelerate upon moving to the overtaking lane. Values indicate velocities.*

The car can overtake if the space it will move into, and spaces in front that it will move through in one timestep (the velocity of the car) must be empty, or the cars in front must be moving faster or at the same speed of the than the vehicle moving into the lane, as seen in figure 9.

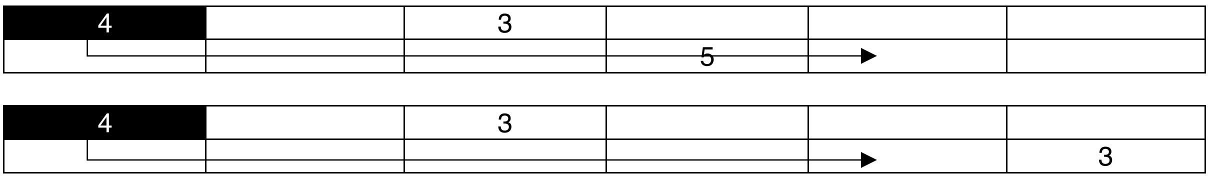

Figure 9. *Two acceptable conditions for overtaking to take place. The bottom array in each case represents the overtaking lane*

Looking at the second condition, any cars in the lane it is moving into should not have to slow down to accommodate the overtaking car. Thus, the car behind must be traveling at a speed less the distance between them. An example of an unacceptable overtaking manoeuvre can be seen as seen in figure 10.

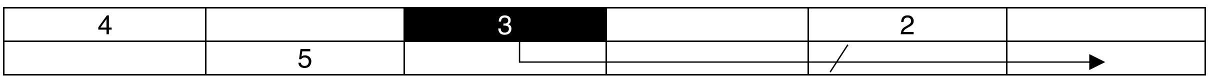

Figure 10. *An unacceptable overtaking manoeuvre, as it would force the vehicle in the right lane, travelling at high speed, to break.*

Rules for moving into the slower lane

For cars looking to move into a leftward lane must again adhere to the two conditions. They must have a reason to- while this is not carried out in practice on motorways, in an ideal world, cars should look to move back into the leftmost lane if they have no reason to be in the overtaking lanes- that is they will not be looking to overtake in the next timestep and that it is safe to do so. The p(lane change) here makes this more realistic to real world motorways.

They must also do so safely- like the first rule this is taken that either there are no cars in the space that the car will move into in the next timestep. If there are cars in the space that the car will move into in the next timestep, they must be traveling faster than the car, or else there is no reason for the car to move into that lane, as it will then look to overtake again immediately. An example of this can be seen in figure 11.

2					4	
		3				3

Figure 11. *An acceptable manoeuvre for moving into the slower lane as there is no incentive to stay in the overtaking lane.*

However, unlike the overtaking rule we will not take into consideration the car that will be behind the car which has moved into slower lane. This is because the car in the overtaking lane would have overtaken that car prior and thus moving back into the initial lane should be no problem, as the car behind should not have accelerated dramatically under the rules of the automata and will not be in the way of the car looking to move back into that lane. However of course in the real world, a driver would have to take this into consideration as other drivers do not always behave as one would expect and certainly do not adhere to the rules of logic and reason.

While this was a dual lane model what can be seen is that in the slower lane a build-up of cars still takes place, but not to the extent as in the single lane system, however the cars in the overtaking lane are either accelerating or moving at maximum velocity. Due to this the cars in the slower lane are struggling to find the space to be able to overtake and free themselves from the traffic, thus it made sense to simulate a motorway and extend this system to 3 lanes, allowing for move overtaking freedom, and what one would expect to see is less traffic build up when the initial conditions are similar.

In the extension of the model all the rules followed the same, only differing where cars in the middle lane could now either overtake or move into the left land depending on what velocity they were traveling at.

4.5 Different Vehicles

In order to again try and mimic what traffic on a motorway would be like, we will look to introduce different types of vehicles, that is vehicles with different maximum velocities. For label's sake we will assign the slowest vehicles to be lorries, the next fastest as cars and the fastest as motorbikes. By including this mixture of vehicles, we can see how they interact with each other- expecting the faster vehicles to look to overtake the slower ones, which would remain in leftmost lane.

To achieve this each type of vehicle was assigned an imaginary part, which would remain with the vehicle throughout. As the rules determining the motion and velocities of the vehicles only affect the real component (which references the velocity of the vehicle, plus one), the imaginary part would remain unchanged throughout and could simply couple with the maximum velocity assigned to that vehicle. This complex number will also affect how the array plots each vehicle so that it can be seen which vehicle which is, with the lorries appearing darker than the cars which are darker than the motorbikes.

4.6 Accidents

A common scenario when travelling on a motorway is a lane closure, whether this is due to road works, or an accident. By adding this into the simulation we will be able to observe how the traffic flow changes as a bottleneck is introduced, and how the velocity of the vehicles will change when reduced down to 2 lanes.

This was achieved by introducing a different object to the three types of vehicles that was not affected by the rules of the cellular automata. This lane closure can be introduced from the first timestep to replicate a lane closure due to road works, or part way throughout the iterations through time to simulate an accident that the vehicles in the road must then adapt to.

In order to have the cars behave closer to real vehicles, the overtaking rules were tweaked, to prevent vehicles from having to decelerate too much, which the breaks of a vehicle may not be able to withstand. This rule stated that if a vehicle is moving at greater than half of its v_{max} and would be forced to reduce its speed dramatically due to a stationary vehicle in front, it will have to move into the overtaking lane, and cars in the overtaking lane will have to reduce their speed sufficiently to allow this overtaking to take place without a collision. This is very common on the road where a vehicle will see that it will have to slow down in order to allow a car to move in front of it, either at a junction where a vehicle is moving onto the road, or in the case of an emergency as seen here. *If there is a car in the overtaking lane travelling at speed u and is a distance d behind the car looking to overtake, and u-d > v, this car must slow to u = v-d-1.* We must store the complex part of this car to add back on. We will refer to this as evasive overtaking.

Alternatively, if the car would ordinarily be unable to move into the other lane due to a car in front it should reduce its speed to make lane changing possible. *Looking at a car in the other lane traveling at velocity u, and is distance d in front of the car we are focused on, the car moving into the other lane must change its speed to at least v = d+u-1 if v > d+u.* In this case the cars won't need to increase speed as they change lane as they aren’t looking to overtake to be advantageous but to prevent an accident, thus we look at decreasing their speed to what is required to not crash.

As well as this the rule was altered so that a car would look to overtake if the velocity of the car is greater or equal to than the distance to the next car as opposed to just greater, and this was to incentivise stationary cars to look to overtake the stationary road closure in front of them.

One issue with the implementation of our cellular automata is that cars only look forward the distance that they will move in one timestep, and thus do not plan ahead as a human driver would, or a more sophisticated algorithm would. This can be seen in this scenario where the car must react to an unexpected object in front of it.

5. Results

5.1 Single Lane System

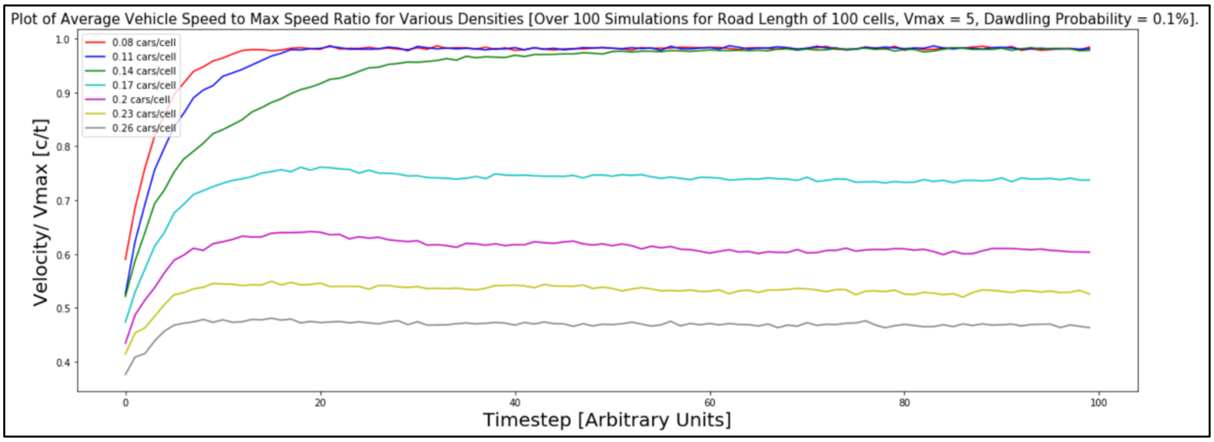

Figure 12. A graph showing how the different densities can affect the velocity to maximum velocity ratio over 100 timesteps, within a range of 0.08 to 0.26 cars/space. Constant parameters: V_{max} = 5c/t, dawdling probability = 10%, road length = 100 cells.

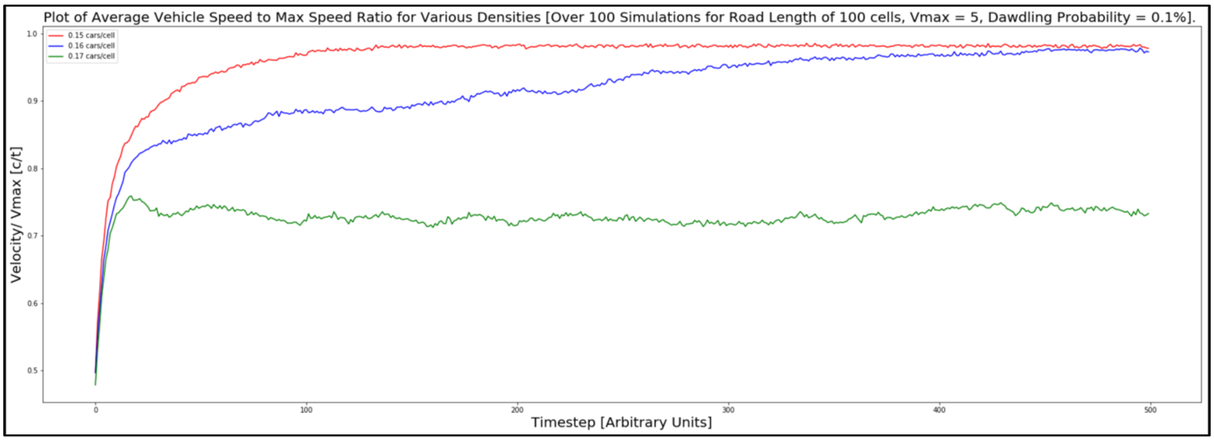

Figure 13. *A graph showing how the different densities can affect the velocity to maximum velocity ratio over 100 timesteps, within a range of 0.145. to 0.17 cars/space. Constant parameters: v_{max} = 5c/t, dawdling probability = 10%, road length = 100 cells.*

Ordinarily on a road the speed limit and the length of the road are fixed, and thus the variable that changes is the number of cars on the stretch of road. (The dawdling factor is another variable that will change, however we will take this to be a product of humans rather than the system and consider it later).

By altering different density possibilities, we are able to see how they affected the traffic flow. We used the metric of the ratio of the car's velocity to its assigned maximum velocity to determine the flow of the traffic. As the cars on the road at traveling at various speed to capture an understanding of the entire road we took the mean value for each car to give the average v/v_{max} at each timestep.

As the initial positions and velocities of the cars are random, and the dawdling of the cars is probabilistic, every iteration of the model will yield a different result. In order to obtain accurate results, the simulation was run 100 times and the average v/v_{max} was then calculated for the mean value at each timestep over the 100 simulations, and this was then plotted against the corresponding timestep.

Figure 12 confirms the predictions that were made, which stated that up until a critical density the velocity of the vehicles will tend to their vmax, however when the density of cars on the road is greater than the $\rho_{critical}$, the velocities of the cars will be inversely proportional to the density of cars on the road. Figure 13 shows the densities that are closer to the $\rho_{critical}$, and over a larger time step of 250 iterations in time to indicate that for a road with a system where the v_{max} = 5c/t, the $\rho_{critical}$ = 0.16 cars/cell. A larger number of iterations, 500, was taken in order to see where the values v/v_{max} tended to.

We are better able to understand the nature of the traffic flow by looking at the space time diagrams which show the positions of cars on the road at each timestep, with time increasing in the downward y direction. Figure 14 shows the traffic system when the traffic density is below the critical value, (ρ = 0.15 cars/cell), displaying that while a jam does arise due to the random initial speeds and positions of the cars on the road, over time this dispels and the cars all travel at their v_{max}, except for those which their speed has been reduced by 1 due to dawdling. The dawdling cars have very little effect on the rest of the road, not causing any further jam. We observe that the cars then move in a very ordered fashion, with the spaces between each car sufficient for it to remain traveling at its v_{max}, under rule 2 of the cellular automata.

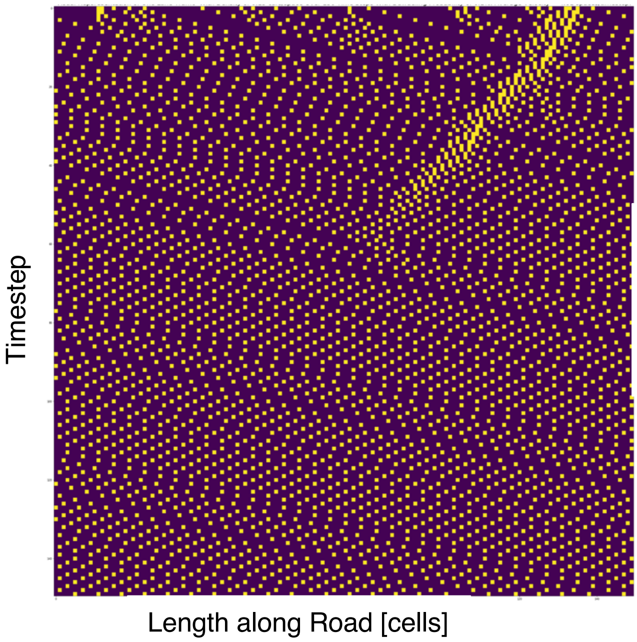

Figure 14. *A Space Time diagram, showing cars in yellow traveling along the road. Time increasing in the downward y direction. The density = 0.15 cars/ space. Constant parameters:* V_{max} *= 5c/t, dawdling probability = 15%, road length = 150 cells.*

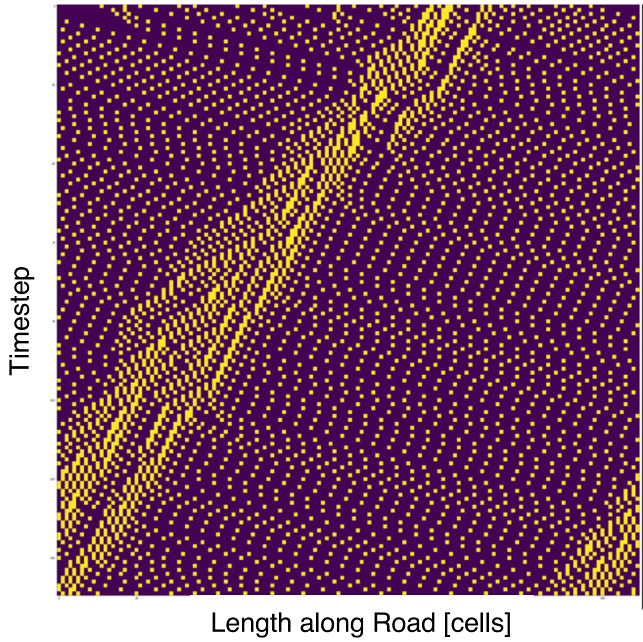

Figure 15. *Another spacetime diagram, with the density = 0.17 cars/ space. Constant parameters:* V_{max} *= 5c/t, dawdling probability = 15%, road length = 150 cells.*

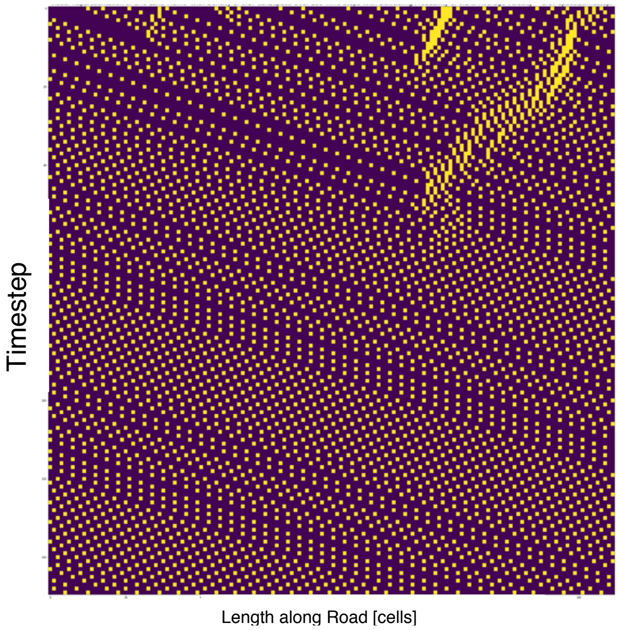

Figure 16. *A S pace Time diagram, with the The density = 0.17 cars/ space* V_{max} *= 4c/t. and Constant parameters: dawdling probability = 15%, road length = 150 cells.*

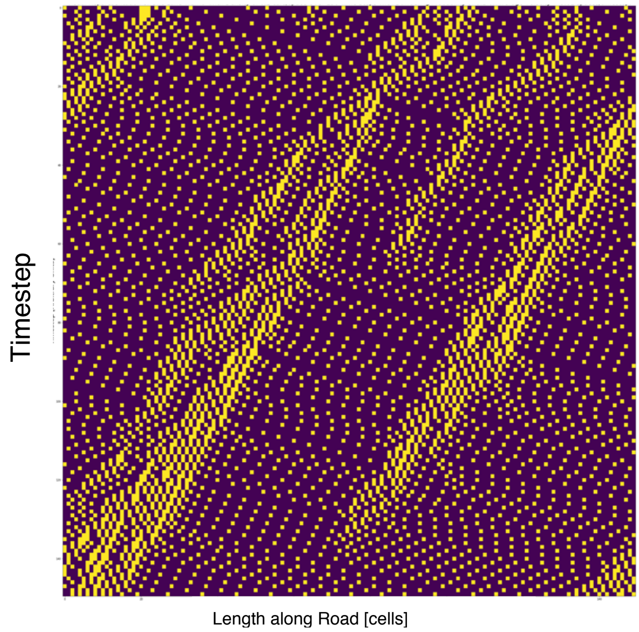

Figure 17. *A Space Time diagram, with the The density = 0.2 cars/space. Constant parameters:* V_{max} *= 5c/t, dawdling probability = 15,% road length = 150 cells.*

In figure 15, the density of the cars is greater than $\rho_{critical}$, (ρ = 0.17 cars/cell), and one can see that the initial traffic jam caused by the initial conditions of the cars is never resolved and remains over the 150 timesteps. It is interesting that this is the case rather than the traffic ordering itself but with each vehicle traveling at a slower velocity. From this it can be determined that (one must remember that this is under the specific rules of our cellular automata) that once a traffic jam forms on a portion of road it will not be irradiated until it has reached the end of the road. The reason for this is that all the cars have the incentive to travel at their top speed, as opposed to reduce traffic flow. Another factor that must be looked into is the periodic nature of the road, while this is unrealistic, as a far that has escaped from the traffic will not meet it again. Due to this in an open system the cars will then be able to move away at their v_{max}, and not encounter further traffic (caused by the same system), however what the periodic model does show is the effect on the entire system as opposed to an individual. To optimise traffic flow for the whole system the speed limit must be decreased, and the traffic could be irradiated. When thinking about bringing this back to a centralised driverless car system, if the density of the cars on the road is greater than the critical density for a given speed, the speed of all the cars in the system should be reduced to give the cars enough space between then to maintain this new velocity. This would make journeys more efficient as cars will no longer start and stop, as well the journeys being less frustrating for passengers, as no one likes to be stuck in traffic, as well as predicted journey times being easier to calculate and potentially more accurate. In figure 16 we can see that by lowering the v_{max} to 4 c/t for a system of density 16 cars/cell removes of this traffic jam.

When the density is much higher than the $\rho_{critical}$, in figure 17 the density is $\rho = 0.2$ cars/cell, and one can see that multiple traffic jams form, and while some does dispel, this only increases the size of the others. This produces the shockwave pattern that we predicted, with cars continually moving from areas of lamina flow to areas of traffic, and then being able to travel at close to or at their maximum speed. Here is another example where the entire system would be more efficient if the v_{max} was reduced as shown in figure 16.

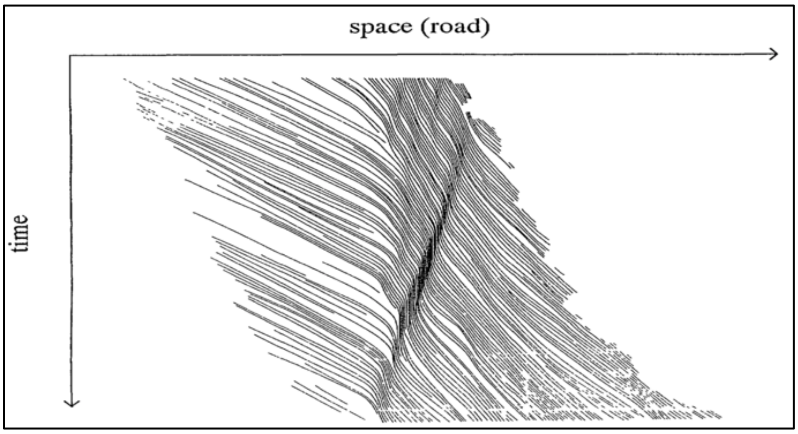

Figure 17. *Space Time diagram for cars from Aerial Photography (credited J. Treiterer). Each line represents the motion of one vehicle. (Taken from Kai Nagel and Michael Schreckenberg's 1992 paper) {4}*

Figure 17 shows the space time lines for actual traffic and we can observe the similarities between this figure and the output of the traffic flow from this simulation, in particular figure 15, depicting cars being held up in traffic before being able to accelerate away. This can use to validate that the model, despite its simplifications, does model traffic flow effectively and the measurements made can be applied to thinking about real world examples. In all figures the velocities of vehicles can be interpreted by diagonal lines they form moving towards the bottom right, as these represent the change in distance over the change in time, which is equivalent to the expression for velocity. Therefore, a shallower gradient represents a faster moving vehicle, and the closer to vertical a car's trajectory is the closer it is to stationary. This can also be seen move explicitly in figure 17.

As stated before, altering dawdling probability can be seen as arbitrary, as they do not represent a variable that would change depending on the system, but is simply a product of human behaviour. We can see what we expect to happen, that increasing the probability of dawdling taking place, decreases the average speed of the cars. This has a much greater effect when the density of the traffic is closer to the, as can been when comparing figures 18 and 19. This is because, as shown in figure 19, the cars have a greater space between them and the car in front, and thus have a higher probability of not being affected by the change in velocity of the vehicle in front.

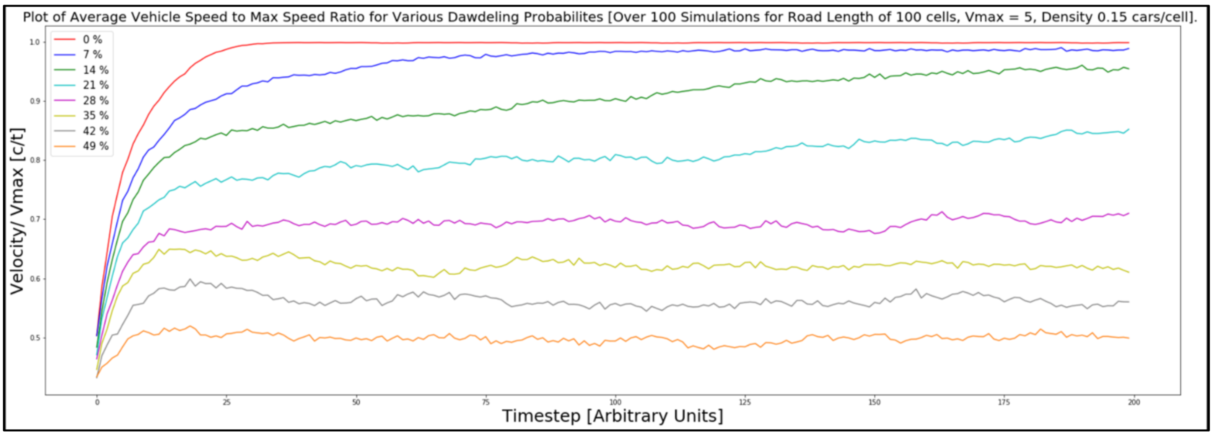

Figure 18. *A graph showing how the dawdling probabilities can affect the velocity to maximum velocity ratio over 100 timesteps, within a range of 0%. to 49%. Constant parameters: V_{max} = 5c/t, density = 0.15 cars/cell, road length = 100 cells.*

Another expected result is obtained when varying the v_{max} for a fixed length road, as can be seen in figure 19. The ratio of average velocity of the cars to their v_{max} decreases as the maximum velocity increases as the cars don't have space between them and the next vehicle to be able to travel as at their v_{max}. Naturally ratio drops as the v_{max} increases, as the ratio between the space between cars and the space required between cars decreases.

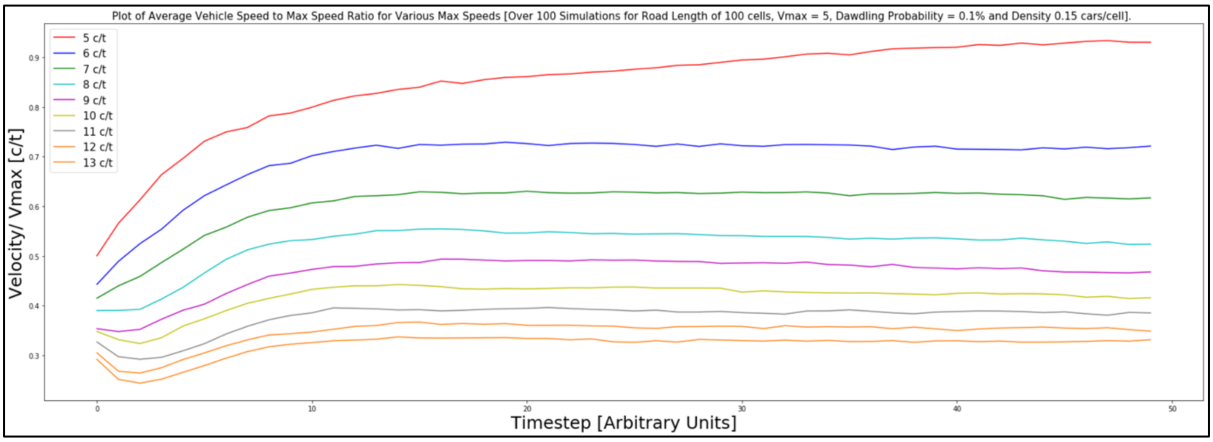

Figure 19. *A graph showing how different V_{max} values can affect the velocity to maximum velocity ratio over 100 timesteps, within a range of 5c/t to 13c/t. Constant parameters: V_{max} = 5c/t, density = 0.15 cars/cell, road length = 100 cells.*

When looking at the traffic jams that build up, it would be insightful to be able to describe any patterns they display. To quantify the positions of these fronts, areas of slow-moving vehicles were measured, and plotted in figure 20. An area of slow-moving vehicles is a group of 4 cells where within there are two adjacent stationary cars. We can observe clusters however it is very difficult to attempt to measure a wavelength, or any sort of pattern. And this may be due to the nature of the jams, they do not act as waves that oscillate, but are areas of turbulence and thus are very difficult to measure.

When attempting to measure the speed at which the traffic jam moved back through the road, it was seen that it moved in a non-uniform manner and thus it is very difficult to measure a velocity. This makes sense, as it can be seen clearly from the shape of the traffic jam, which seems to meander, changing shape and size non-uniformly. One would expect this is from the stochastic nature of the model, and again equating the traffic jam with turbulent motion makes most sense.

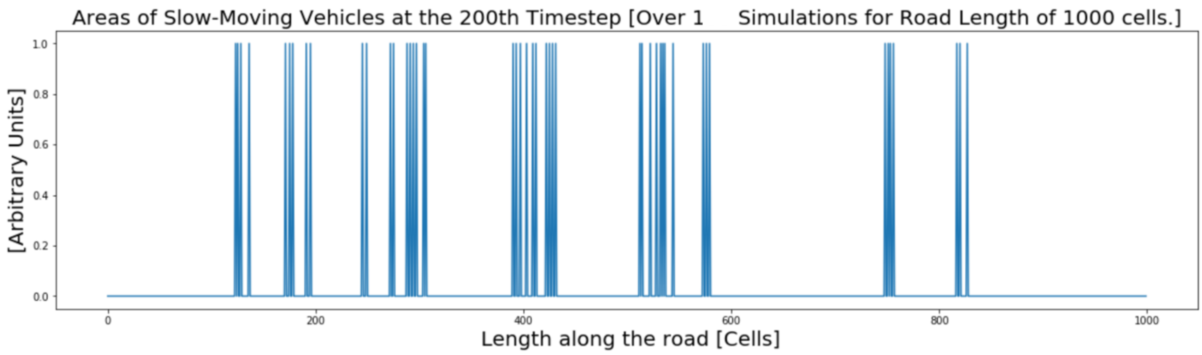

Figure 20. *A graph showing areas of slow-moving vehicles at the 200th timestep of a 1000 cell road.*

5.2 Multi Lane Systems

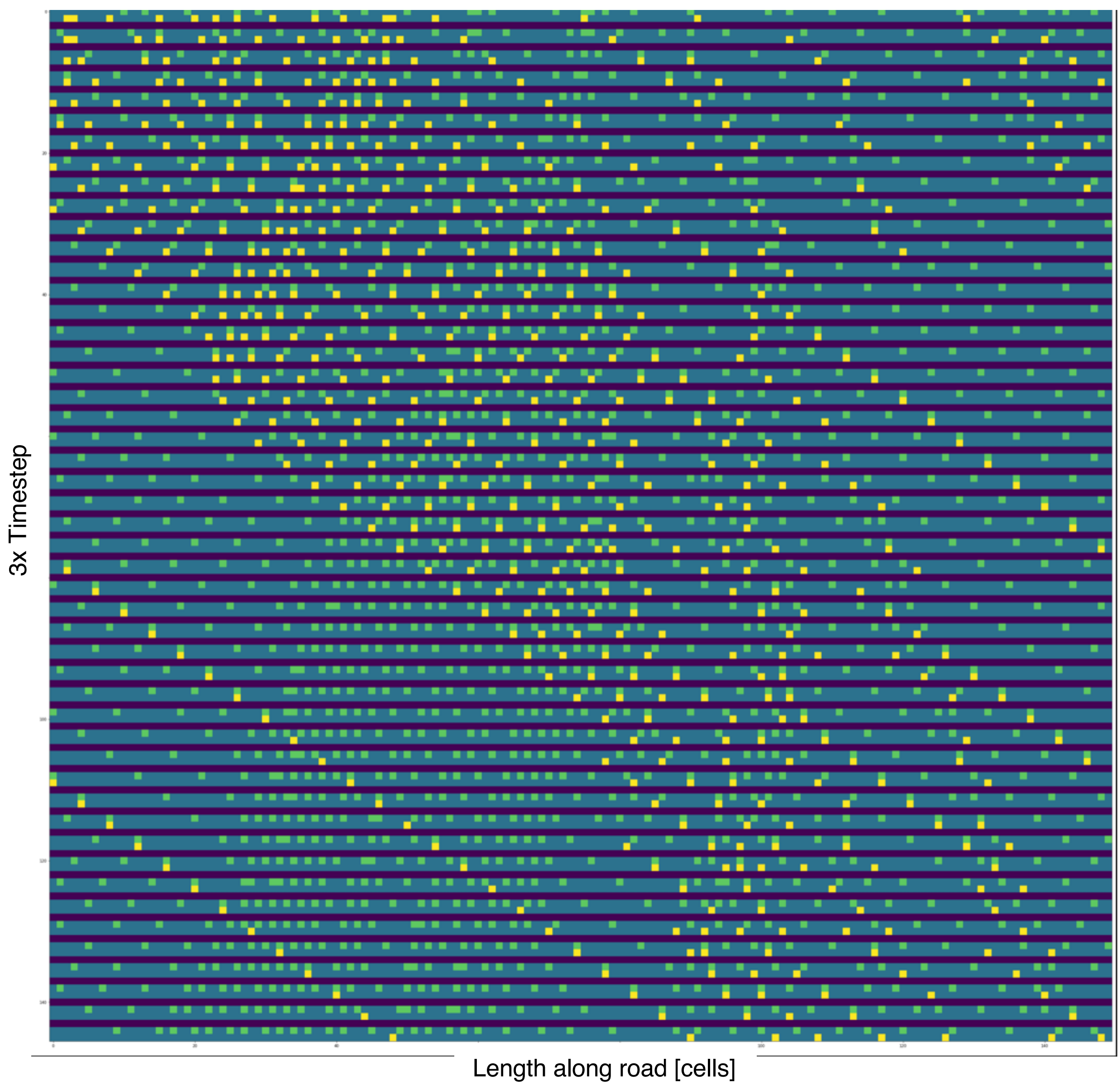

Figure 21. *A Space time diagram for a two-lane system. Cars in the left lane are shown as green, cars in the right lane are yellow, and the purple is a gap between each set of dual lanes. Density of the road was 0.15 cars/space with a v_{max} =5c/t, a dawdling probability of 10%, on a road with 150 cells over 50 timesteps.*

In introducing an overtaking lane, cars were able to overtake other vehicles if they were traveling faster than the car in front of them and it was safe to do so, here we would expect that the levels of traffic jam that formed would be smaller, a s any slower moving cars would be overtaken, allowing free flowing movement. In Figure 21 we see that in the two-lane system the cars in the overtaking lane, which are shown in yellow, are moving freely. Their velocities again can be interpreted by the gradients of their space-time lines, and other than vehicles that have just moved into the overtaking lanes, whose velocities cannot yet be interpreted, almost all vehicles are travelling at their maximum velocity, unimpeded by any traffic in their way. However while these vehicles are moving fast, it now makes it difficult for vehicles in the left lane to join the overtaking lane to leave the traffic and move more freely, as they need to be able to do so safely, and the fast moving traffic in the overtaking lane limits the space they have to be able to manoeuvre into right lane, thus we can observe in figure 21, that the cars in the left lane, coloured green, do not move as freely, however they are still moving at a moderate speed, indicated by their space-time lines than with single lane traffic, as anticipated.

In order to replicate motorway traffic, the model was extended to the familiar 3 lane system. We would expect this to allow for further lane movement to take place and a greater separation of vehicles. The density was set to 0.1 cars per space (to mimic having the same number of cars as the two lane system if the road was 100 cells), while all the overtaking models incorporated a lane changing probability of 80% to makes the system less deterministic, and in this case the different types of vehicles were introduced, with the lorries having the smallest v_{max}, shown in purple in figure 23, the next fastest- cars shown in blue and the fastest, motorbikes, show in yellow. As expected, the slowest vehicles the lorries are mainly found in the left-hand lane, while the fastest moving bikes are found in the overtaking lanes or are moving freely in the left-hand lane, as indicated by their space-time trajectories. When taken over 100 simulations as seen in figure 22 it can be confirmed that the lorries mostly occupy the assigned slower lanes, and over the number of timesteps, assign themselves to the leftmost lane. Some lorries can be seen found in the right-most lane and this is due to them being found in there and not being able to find a safe opening to be able to return to the slower lanes. This is due to the somewhat unrealistic overtaking rules where vehicles looking to move into a different lane could not interfere with the path of another vehicle, in essence this is how vehicles should manoeuvre, but in practice is not the case, as vehicles are often forced to react to another driver, or allow another to produce a manoeuvre, especially when there is a situation of traffic.

We can see that again the cars are most likely to occupy the left-most lane, however there is a larger probability of finding one in the middle or right-hand lane, but again over time confide to the left-hand most lane, as seen in figure 23. Figure 24 shows the number of motorbikes found in each lane, and interestingly for the most part the most likely location for a motorbike is the middle-lane. Of the 3 vehicles the most number of motorbikes were found in the right lane, however due to the lack of incentive to stay in that lane, these is still a small probability of finding a motorbike here, leaving large parts of this lane unused, while this is the safest configuration for if vehicles needed to react to something unexpected, it is the most inefficient if looking for vehicles to travel close to their respective v_{max}.

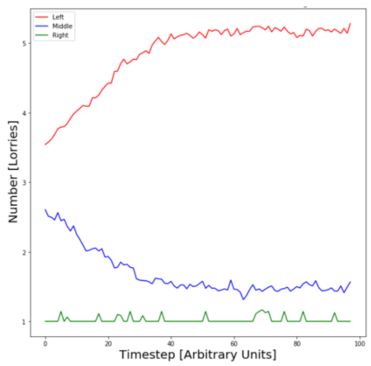

Figure 22. *Plot showing the number of lorries occupying each lane, averaged over 100 simulations.*

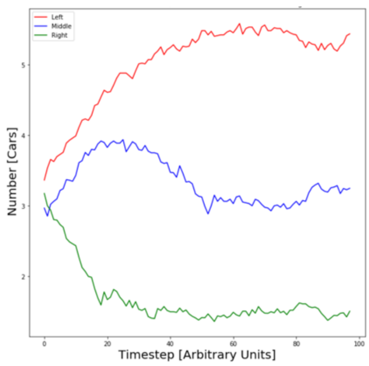

Figure 23. *Plot showing the number of cars occupying each lane, averaged over 100 simulations.*

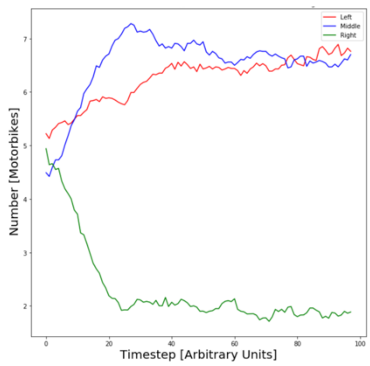

Figure 24. *The number of motorbikes occupying each lane, averaged over 100 simulations.*

What can also be seen is that the traffic build ups that are initially exhibited in figure 23, due to the random initial generation of vehicles somewhat dispel over time. This can be seen in figure 22, where the ratio for the average velocity of a vehicle to its v_{max} was recorded for each lane over 100 simulations. The vehicles in the right-most lane are free from any traffic and travel must closer to their v_{max}, they do not travel on average at their v_{max} however and this is because they are only incentivised to stay in this lane if they are in the process of overtaking- once they have overtaken a vehicle they will move back into the middle lane and continue to accelerate there, or in the left-hand lane if that is free.

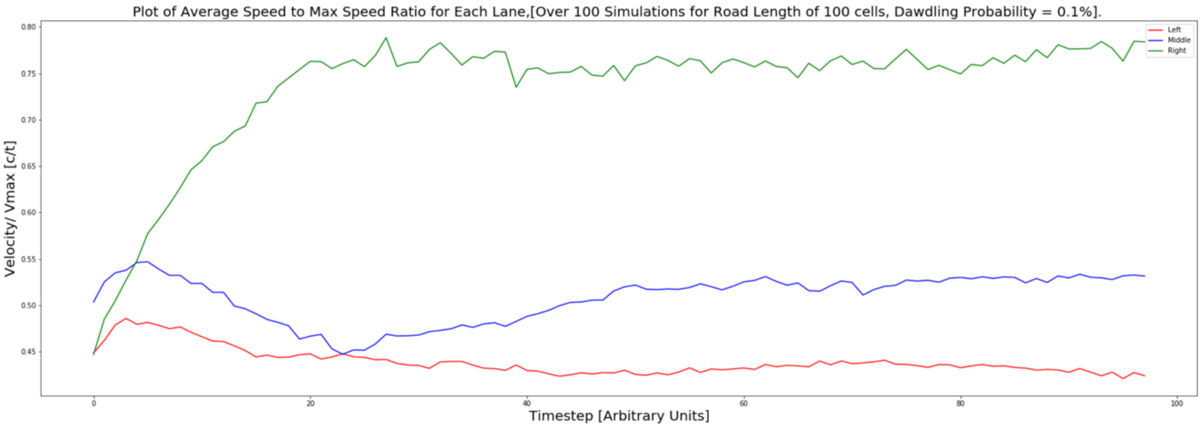

Figure 25. *Plot of the ratio of velocity to v_{max} for the vehicles in each of the lanes, averaged over 100 simulations, for a system of density 0.1 vehicles/cell.*

Looking at applying this to a centralised driverless car system, it would be inefficient to run a system where cars look to overtake other vehicles, as we can see that from figure 25, that in every lane vehicle are traveling at much less than their respective v_{max} than in a similar one laned system. Another reason for this inefficiency is that the fastest moving vehicles possess a v_{max} that is above the optimum v_{max} for the given length of road and density, which we saw in figure 20. However, in a real like situation this would not apply as the road would not be periodic in length, and the velocity of travel would be set depending on the density of vehicles on the road. The most efficient system for a driverless system can be inferred to be complete segregation of cars of different v_{max} bands, acting as individual lanes, each lane traveling at an optimised velocity for the cars in that lane with no lane changes. How practical this would be when including junctions cannot be determined, however for long stretches of motorway this system would be practical, and the most beneficial for the largest number of passengers.

To replicate the situation where an accident has occurred, resulting in a road block, an obstacle was introduced at the halfway point of the timestep iterations. Cars were then forced to react to this obstacle, and slow down accordingly. Due to the periodic nature of the road, the cars appear to travel in waves of high-density vehicles and low-density packets, and it can be seen when there is a large volume of cars moving towards the road closure in figure 27 that traffic does build up when a packet of high-density vehicles approaches the incident. Figure 27 displays vehicles as a different colour depending on their velocity at that point in time, and thus the slow down caused by the incident can be observed be noticing that the vehicles that are displayed as the darkest are those stuck in a traffic jam behind the accident. As well as this the number of fastest moving vehicles shown in yellow, drops to zero by the final timestep. This of course this could have been caused by other factors or by chance due to the stochastic nature of the simulation, however the likelihood of cars reducing in velocity when the density increases is greater as seen I figures 12 and 13, and thus is likely to be to contributing factor. Figure 28 confirms the decline in the velocity to v_{max} ratio for each lane, with the accident occurring at the 50th timestep, all three lanes see a reduction in velocity. The right-lane and the middle lane experiencing this do to increase in the density of the cars in each lane, and the left lane experiencing this reduction in average velocity due to a build-up on standstill cars behind the accident, looking to move into the middle lane when the opportunity arises.

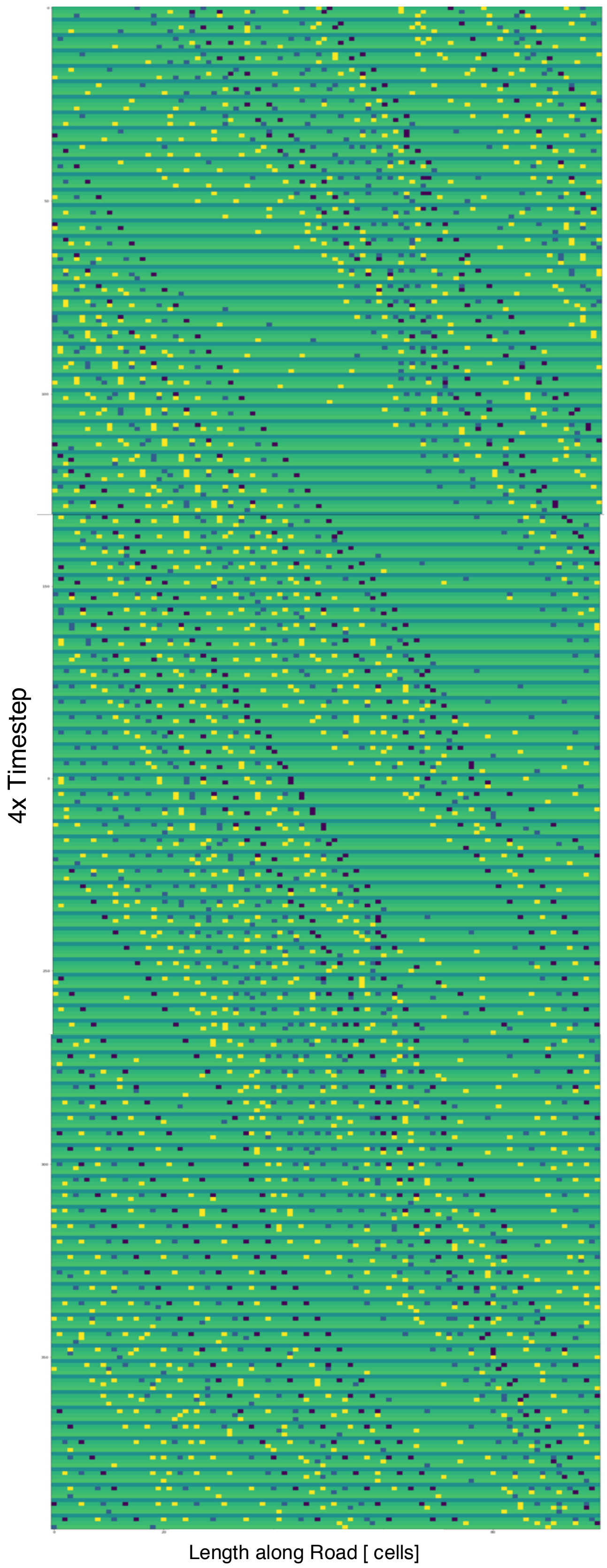

Figure 27. *Space-time diagram for a simulation of 3 lane traffic, over 100 timesteps, for a road of 100 cells, with an accident/ road closure was introduces at the 50th timestep- indicated in purple. Different colours indicate different vehicles- a lighter colour is equivalent to a faster moving vehicle.*

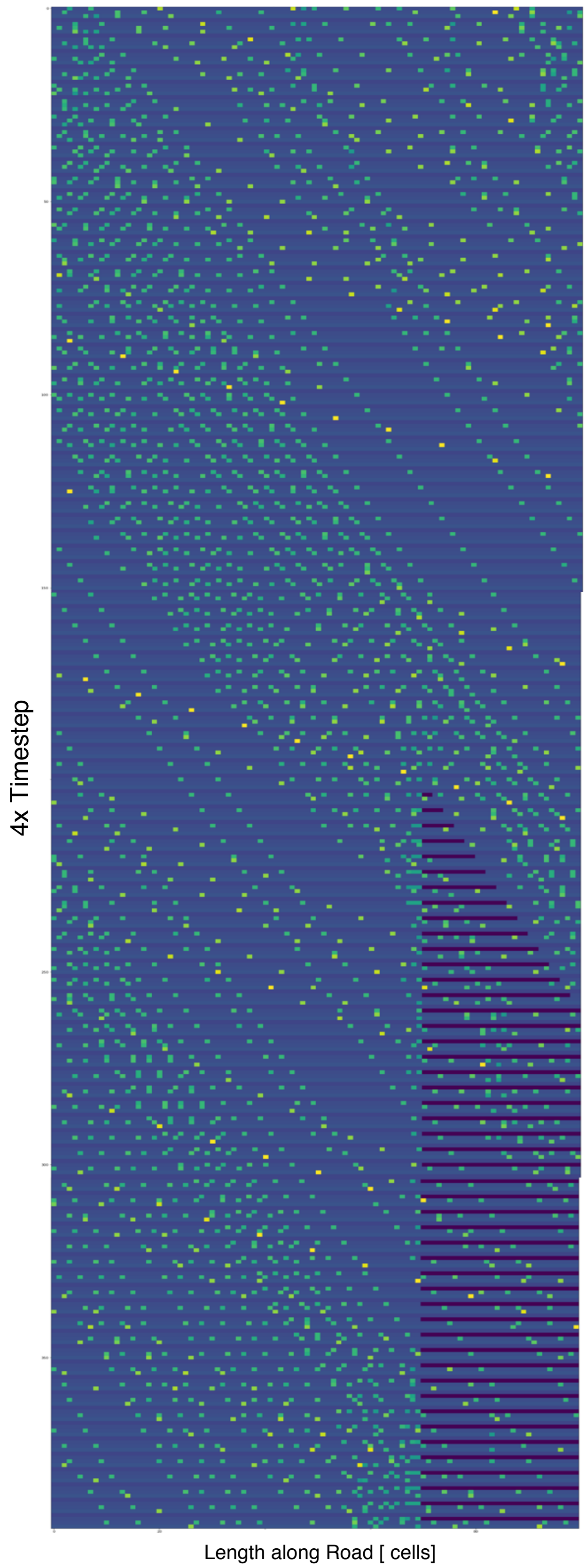

Figure 28. *Plot of the ratio of velocity to v_{max} for the vehicles in each of the lanes, averaged over 100 simulations, for a system of density 0.1 vehicles/cell, where an accident/ road closure was introduced at the 50th timestep.*

By implementing the slightly altered overtaking scheme whereby cars in the first lane will move into the second lane if they are traveling too quickly, we can observe again what happens when a road closure is introduced at the 50th timestep. As observed in the original model the velocities of the cars in the right lane decrease as expected, in figure 29, however we do observe that the vehicles in the middle lane see an increase in their velocity after the introduction of the blockage. The only explanation for this phenomenon is that vehicles in the left which then move into the middle lane are travelling faster than the average velocity of cars in the middle lane- for cars to move lanes regardless they must be travelling at least half of their v_{max}, and it can be seen that cars in the middle lane are traveling at less than this. Even though other cars or they may need to slow down to execute this overtaking manoeuvre, the overall effect of these cars moving lane with a significant velocity means that the average velocity increases, whereas cars in the original model will be looking to overtake, often from stationary. As well as this the velocity of vehicles in the left lane doesn't see a significant decrease. One can speculate that the reason behind this is that there is no large build-up of cars in the model at the bottleneck, or those that do form a queue were slow moving anyway and do not significantly change the average velocity of cars in that lane by decelerating to stationary.

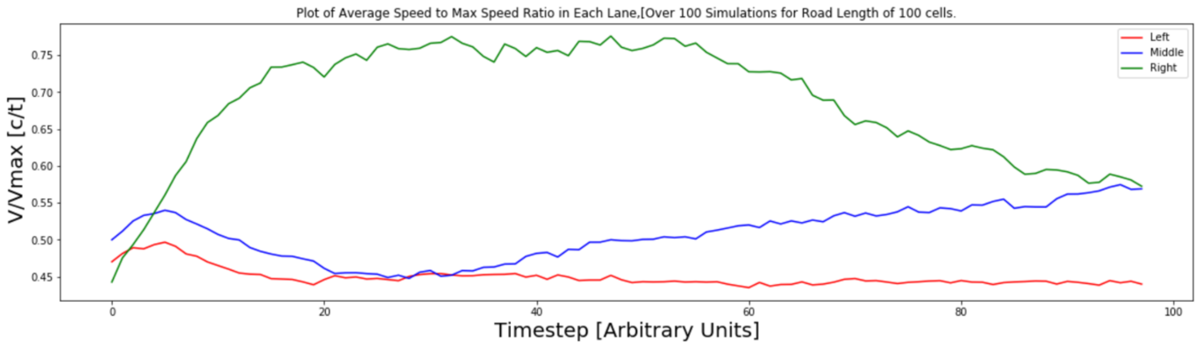

Figure 29. *Plot of the ratio of velocity to v_{max} for the vehicles in each of the lanes, averaged over 100 simulations, for a system of density 0.1 vehicles/cell, where an accident/ road closure was introduced at the 50th timestep, and the cars are overtaking using an evasive method.*

Conclusion

We have clearly been able to demonstrate that using a stochastic cellular automation model is effective at modelling traffic, particularly on a single lane system. A multi-lane system proved effective in demonstrating how vehicles ought to act, however it cannot account for the unpredictable nature of human behaviour, despite the inclusion of an arbitrary overtaking probability due to the strategies and dangerous nature of human driving. What the model does do very well is display how a driverless car system could be implemented and has looked at degerming in the optimum parameters regarding the velocity that a system of vehicles should travel at to run the most efficiently, depending on factors such as the density of the cars on the road, the number of lanes on the road, and the v_{max} of the vehicles that are on the road, determined by the nature of the vehicle, primarily its engine. Efficiency again determined by limiting the amount of starting and stopping, to reduce fuel or energy consumption, as well as reducing the average journey time for passengers. Safety would be built into the model, not allowing vehicles to crash in this closed system.

We have also successfully displayed that the traffic flow behaves, as expected, similarly to fluid flow, with the traffic jams acting like areas of turbulence in the system. The traffic jams propagate in a similar manner, and it is difficult to characterise their behaviours mathematically due to their unpredictable nature. This is very much true for the real-life traffic jams we were looking to model, where human unpredictability and irrationality play a large part in causing these inefficient driving patterns, and thus by human nature, will be difficult to eradicate while it is humans that are still those behind the wheel.

One difficult part of putting together this report has been the way in which I have displayed figures, primarily the space- time diagrams visualising traffic. As these are very large vertically, they were difficult to fit onto the pages, without having them take up multiple pages each. And in having them fit to the size they are at it becomes quite difficult to see individual vehicles are they are so small. There was also no way that reducing the number of timesteps, in order to make the plot dimensions more manageable, as the flow patterns aren't as easily displayed over shorter time periods. If I were to conduct a similar project in the future I would definitely require a rethink regarding how best to display such plots so that they are easy to understand for the observer.

To make the model again more realistic, one could include different sized vehicles to represent the different vehicles, as lorries and motorbikes do not take up the same amount of space on a road. This would allow for a more accurate description of road density and the available space between vehicles, as well as a more realistic description of the motion of the vehicles, as a lorry will overtake a car in a very different way to a motorcycle, and thus will require different conditions to those we stated, producing a different output. As well as this, different vehicles could possess different acceleration and deceleration properties that would depend on the power of the engine, as well as its size. The jumps in allowed velocities could be reduced to bring the whole system closer to moving with a continuous velocity and acceleration range as opposed to the discretised rules implanted here. The addition of these conditions would allow for a completer and more accurate model of vehicles.

To take the model system further, future implementations could include building up a road network, which could include, junctions, roundabouts, traffic lights, crossings, and a mixture of single and multilaned roads. As well as this network nodes or destinations could be added, and the automata be used to determine the optimum flow for the entire system, again likening this type of system to how trains are controlled on a track system. If this system was looking to model human driving still then the inclusion of traffic lights would be useful, however if the system were fully autonomous then traffic lights would not need to be an addition as the joining of cars at junctions would all be regulated internally reducing the time spent stationary (preferably to none) at junctions.

The model was built up by using a large number of nested conditional statements, and there is a question over whether this is the most effective method for the implantation of such a system, (the following improvement ideas are not within the scope of the course). We have already talked about the use of NumPy arrays over in-built lists for their speed, however it is unlikely that this method of large listed nests is the fastest approach when tackling this problem. When running the simulation 100 times to determine mean values to take into account the stochastic nature of the model, the time taken to run each test was significant, in the order of several minutes. When looking to scale this model, or implement new features, hard coding the conditional statements will not be the most efficient method in terms of programming time as well as the run time of the simulation itself, especially if it is needed to be run over a large number of iterations to extract data. While I am unsure of if these methods would work, one possibility is reworking the type of model from a cellular to automata to a form of supervised learning that took in data regarding the way humans drive and the tactics that humans use when on multilane systems or in more compilated road systems. This could be implemented in the form of a decision tree that would take in measurements regarding factors such as how fast the car is traveling, what its v_{max} is, the distance and speed of surrounding cars and the type of road it was on to compare with previous data and decisions to formulate what its next manoeuvre should be. A training data set obtained from traffic control stations in northern California was utilised by Fouladgar, Parchami, and Ghaderi's study using a Recurrent Neural Network to look at congestion due to road features such as bottlenecks {6} .

Another alternative to build a model that may run traffic in the most efficient method possible would be a form of reinforcing learning that included the v_{max} of the car as an input as well as giving the car a destination and assigning rewards along this route. The car would want to reach the destination as quickly as possible to gain the most rewards and thus would look for the most efficient journey. By ensuring that cars were not allowed to crash this could be trained and scaled far more efficient than hard coding any extensions to the system. However, as each individual vehicle would be competing with one another this model may not give the most efficient traffic flow but may mimic humans very well. A reinforcement model used by Alexandre Bayen looked at how air pollution could be reduced, much like how we looked at reducing energy wastage. Looking at a road loop it was similarly concluded that the most efficient flow involves vehicles moving in unison, "The automation essentially understands to not accelerate and catch up with the previous person – which would amplify the instability – but rather to behave as a flow pacifier, essentially smoothing down by restraining traffic so that it doesn't amplify the instability," {7}.

As stated, these alternatives were beyond the scope of the course, however I feel they provide a discussion for how vehicles, or any body of moving matter, such as grains or water could be modelled on a microscopic level and then scaled to show patterns at the macroscopic level just like this cellular automaton model looked to do and succeeded in doing so.

6. References

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{4} *A Cellular Automation Model For Freeway Traffic*- Kai Nagel, Michael Schrekenberg – 1992- Journal de Physique I

{5} *Fluid Dynamics Explains Some Traffic Jams* - Joel Shurkin, 2013- Inside Science

{6} *Scalable Deep Traffic Flow Neural Networks for Urban Traffic Congestion Prediction* - Mohammadhani Fouladgar, Mostafa Parchami, Ramez Elmasri and Amir Ghaderi - 2017

{7} *Machine Learning to Help Optimize Traffic and Reduce Pollution* – Alexandre Bayen et al - 2018