Kruskal-Szekeres coordinates for Schwarzschild spacetime

$$\begin{cases} U = \sqrt{\frac{r}{2GM} - 1} e^{\frac{r}{4GM}} \cosh\left(\frac{t}{4GM}\right) \\ V = \sqrt{\frac{r}{2GM} - 1} e^{\frac{r}{4GM}} \sinh\left(\frac{t}{4GM}\right) \end{cases} & \text{for } r > 2GM \\ U = \sqrt{1 - \frac{r}{2GM}} e^{\frac{r}{4GM}} \sinh\left(\frac{t}{4GM}\right) \\ V = \sqrt{1 - \frac{r}{2GM}} e^{\frac{r}{4GM}} \cosh\left(\frac{t}{4GM}\right) \end{cases} & \text{for } r < 2GM \\ V = \sqrt{1 - \frac{r}{2GM}} e^{\frac{r}{4GM}} \cosh\left(\frac{t}{4GM}\right) \end{cases} & \text{for } r < 2GM \\ \Rightarrow \begin{cases} U^2 - V^2 = \left(\frac{r}{2GM} - 1\right) e^{\frac{r}{2GM}} \\ V = \tanh\left(\frac{t}{M}\right) U & \text{for } r > 2GM \end{cases}$$

 $\Rightarrow \begin{cases} U^2 - V^2 = \left(\frac{r}{2GM} - 1\right) e^{\frac{r}{2GM}} \\ V = \tanh\left(\frac{t}{4GM}\right) U \\ V = \coth\left(\frac{t}{4GM}\right) U \end{cases}$ for r > 2GM

for r < 2GM





