ECE3051 – Analog and Digital Signal Processing, Fall Semester 2022-2023

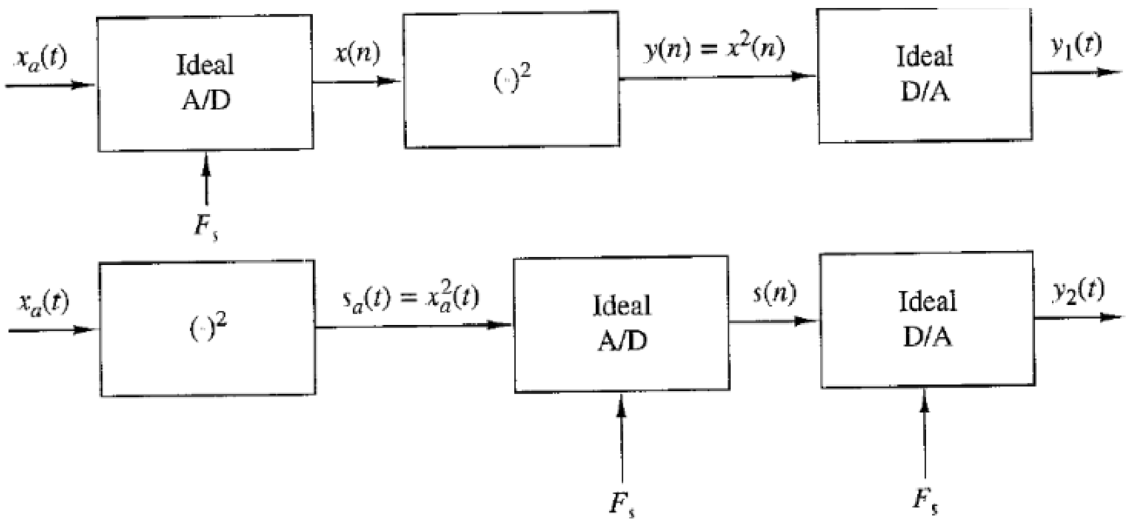
ELA DA – 2, Slot: L25-L26

By: Jonathan Rufus Samuel (20BCT0332) Date: 11.09.2022

ELA DA 2 – DOS: 11.09.2022

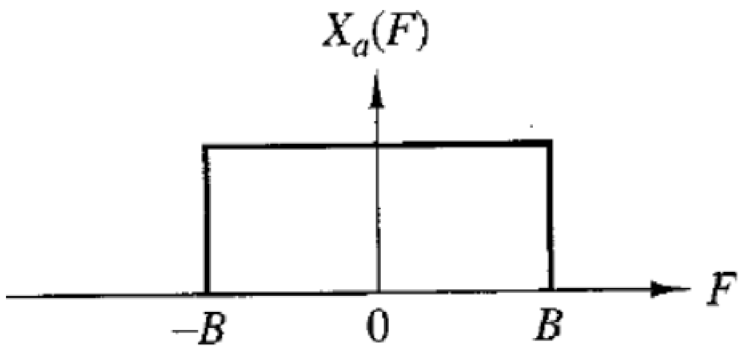
Task - 2: SAMPLING AND RECONSTRUCTION OF SIGNALS

Q1) Consider the two systems shown in the below fig:



a. Sketch the spectra of the various signals if $xa(t)$ has the Fourier transform shown in the below fig. and $Fs = 2B$. How are $y1(t)$ and $y2(t)$ related to $xa(t)$?

b. Determine $y1(t)$ and $y2(t)$ if $xa\ (t) = \cos 2\pi F0t$, $F0 = 20$Hz, and $Fs = 50Hz$ and $Fs = 30$Hz.



CODE:

```
%Task - 2: SAMPLING AND RECONSTRUCTION OF SIGNALS
%Name: Jonathan Rufus Samuel (20BCT0332)
```

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%Course: ECE3051 - ELA

%SubTask 1 - Consider the two systems shown in the below fig.

%a. Sketch the spectra of the various signals if xa(t) has the Fourier transform
shown
% in the below fig. and Fs = 2B. How are y1(t) and y2(t) related to xa(t)?
<SHOWN IN NOTES>

%b. Determine y1(t) and y2(t) if xa(t) = cos 2πF0t, F0 = 20Hz, and Fs = 50Hz and
Fs = 30Hz.
%Case a) Fs = 50Hz
n0 = -20:20;
syms x(n);
x(n) = cos(4*pi*n/5);
subplot(321),plot(n0,x(n0));
title('Sequence #1.1 - x(n)');
xlabel('x(n)');
ylabel('n0');

syms y(n);
y(n) = 1/2 + (1/2)*cos(8*pi*n/5);
subplot(322),plot(n0,y(n0));
title('Sequence #1.2 - y(n)');
xlabel('y(n)');
ylabel('n0');

syms y1(t);
y1(t) = 1/2 + cos(20*pi*t)*(1/2);
subplot(323),plot(n0,y1(n0));
title('Sequence #1.3 - y1(t) and y2(t)');
xlabel('y1(t)');
ylabel('t');

%Case b) Fs = 30Hz
n0 = -20:20;
syms x(n);
x(n) = cos(2*pi*n/3);
subplot(324),plot(n0,x(n0));
title('Sequence #2.1 - x(n)');
xlabel('x(n)');
ylabel('n0');

syms y(n);
y(n) = x(n)^2;
subplot(325),plot(n0,y(n0));
title('Sequence #2.2 - y(n)');
xlabel('y(n)');
ylabel('n0');

syms y1(t);
y1(t) = 1/2 + cos(20*pi*t)*(1/2);
subplot(326),plot(n0,y1(n0));
title('Sequence #2.3 - y1(t) and y2(t)');
xlabel('y1(t)');
ylabel('t');
```

OUTPUT:

>> %b. Determine $y1(t)$ and $y2(t)$ if $xa(t)$ = cos 2π$F0t$, $F0$ = 20Hz, and Fs = 50Hz and Fs = 30Hz.

```
%Case a) Fs = 50Hz
n0 = -20:20;
syms x(n);
x(n) = cos(4*pi*n/5);
subplot(321),plot(n0,x(n0));
title('Sequence #1.1 - x(n)');
xlabel('x(n)');
ylabel('n0');

syms y(n);
y(n) = 1/2 + (1/2)*cos(8*pi*n/5);
subplot(322),plot(n0,y(n0));
title('Sequence #1.2 - y(n)');
xlabel('y(n)');
ylabel('n0');

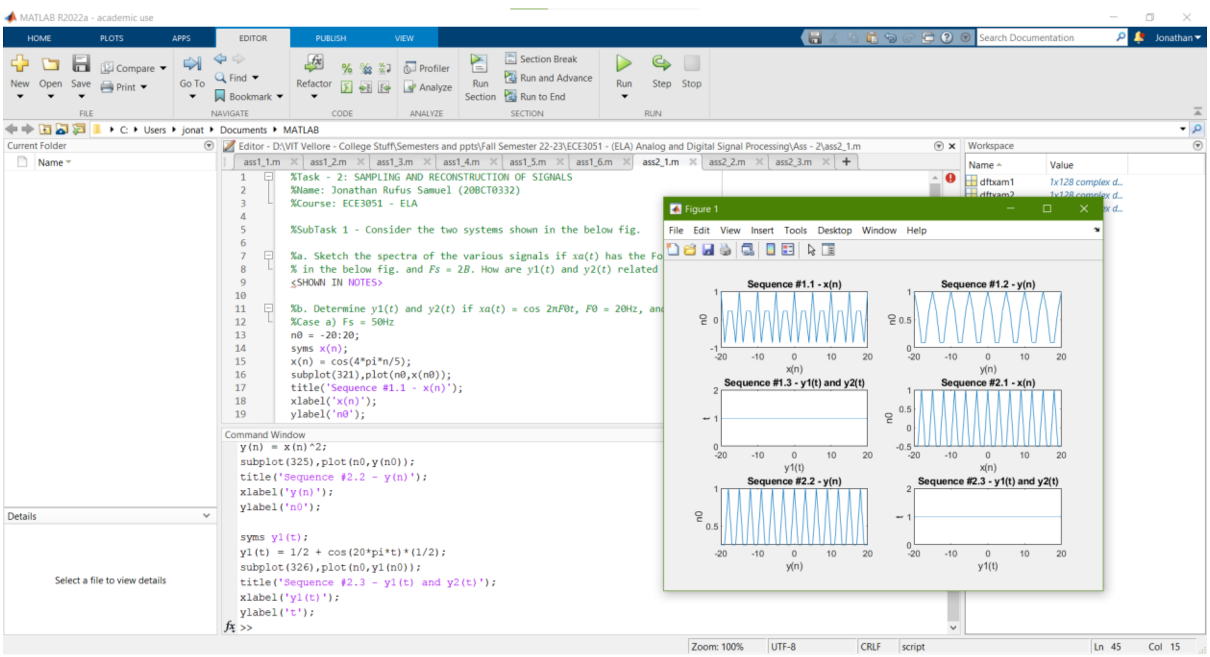
syms y1(t);
y1(t) = 1/2 + cos(20*pi*t)*(1/2);
subplot(323),plot(n0,y1(n0));
title('Sequence #1.3 - y1(t) and y2(t)');
xlabel('y1(t)');
ylabel('t');

%Case b) Fs = 30Hz
n0 = -20:20;
syms x(n);
x(n) = cos(2*pi*n/3);
subplot(324),plot(n0,x(n0));
title('Sequence #2.1 - x(n)');
```

```
xlabel('x(n)');
ylabel('n0');

syms y(n);
y(n) = x(n)^2;
subplot(325),plot(n0,y(n0));
title('Sequence #2.2 - y(n)');
xlabel('y(n)');
ylabel('n0');

syms y1(t);
y1(t) = 1/2 + cos(20*pi*t)*(1/2);
subplot(326),plot(n0,y1(n0));
title('Sequence #2.3 - y1(t) and y2(t)');
xlabel('y1(t)');
ylabel('t');
>>
```



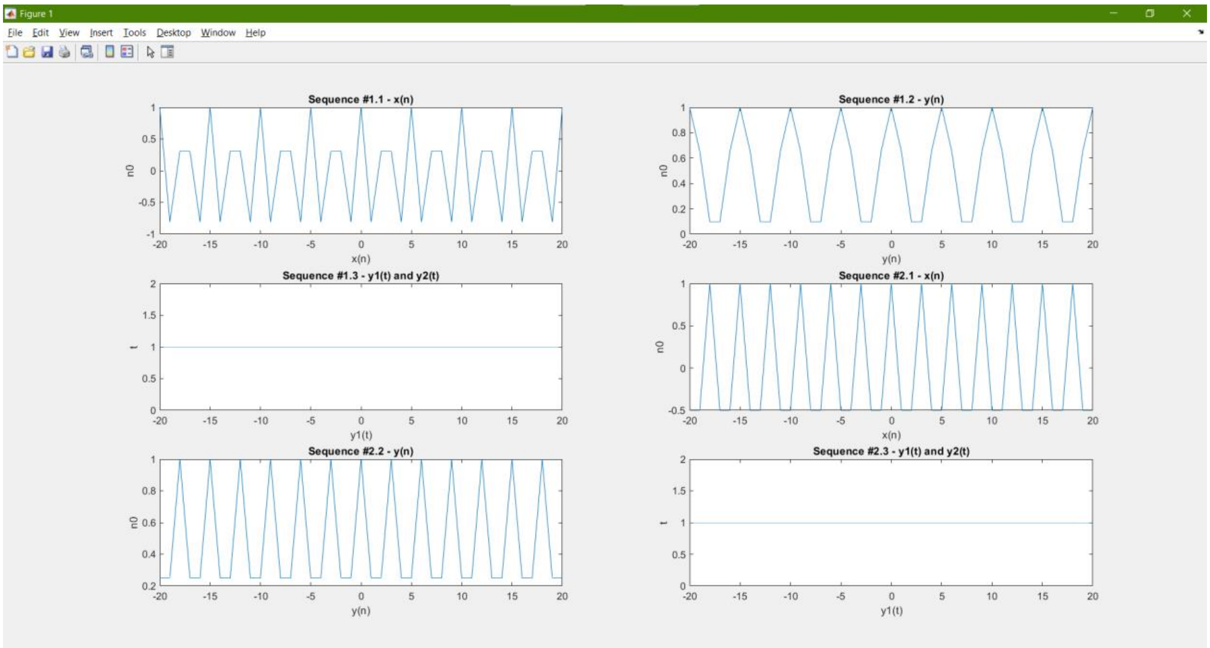
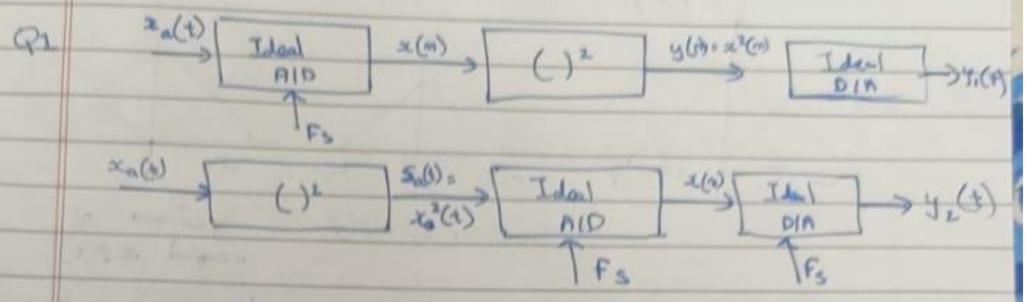


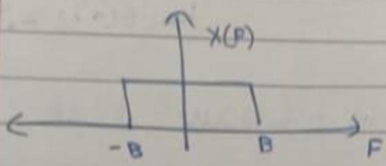
Figure 1
File Edit View Insert Tools Desktop Window Help
Sequence #1.1 - x(n)
n0
x(n)
Sequence #1.2 - y(n)
n0
y(n)
Sequence #1.3 - y1(t) and y2(t)
t
y1(t)
Sequence #2.1 - x(n)
n0
x(n)
Sequence #2.2 - y(n)
n0
y(n)
Sequence #2.3 - y1(t) and y2(t)
t
y1(t)

WORKING OUT:

Q1



a)



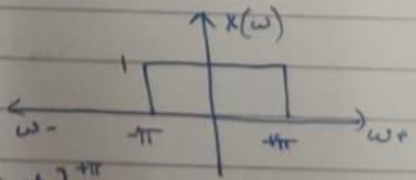
Consider $x(n)$ is the sequence of a signal & its fourier transform is $x(n) \rightarrow X(F)$ (or) $X(\omega)$

$\therefore \ x^2(n) \rightarrow X(\omega) * X(\omega)$

The o/p is generally the square of i/p signal $y_a(t)$

i.e let us assume $-B$ & B to be $-\pi$ & $+\pi$

$$\therefore \ x(n) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

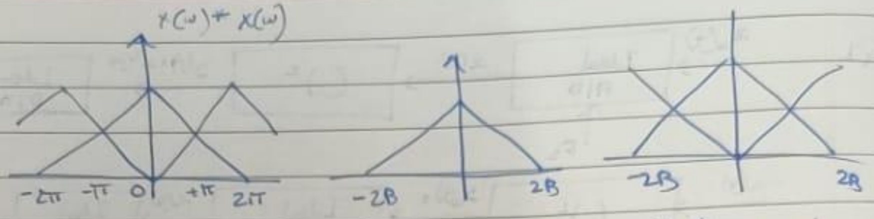


$$\Rightarrow \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{j\omega t}\, d\omega \ \Rightarrow \frac{1}{2\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-\pi}^{+\pi}$$

$$\Rightarrow \frac{1}{2\pi}\left[\frac{e^{j\pi t} - e^{-j\pi t}}{jt}\right] \Rightarrow x(t) = \frac{1}{\pi}\sin \pi t$$

$\therefore$ The o/p $y_1(t)$ is generally the square of the i/p signals $y_a(t)$. For the second system;

P.T.O



Spectrum of $x_a^2(t)$

Spectrum of sampled $x_a^2(t)$

i.e $S(n) = x_a^2(nT)$

$\therefore \; x_a^2(t) \longleftrightarrow X(\omega) * X(\omega)$; bandwidth is 2B

b) $x_a(t) = \cos 2\pi F_0 t$; $F_0 = 20Hz$, $F_s = 50Hz \mid 30Hz$

i) For $F_s = 50Hz$

$$x(n) = \cos \frac{40\pi n}{50} = \cos \frac{4\pi n}{5}$$

$$y(n) = x^2(n) = \cos^2 \frac{4\pi n}{5}$$

$$= \frac{1}{2} + \frac{1}{2} \cos \frac{8\pi n}{5}$$

$$\therefore \; y_1(t) = \frac{1}{2} + \frac{1}{2} \cos 20\pi t$$

$$S_a(t) = x_a^2(t)$$

$$= \cos^2 40\pi t$$

$$= \frac{1}{2} + \frac{1}{2} 80\pi t$$

$$S(n) = \frac{1}{2} + \frac{1}{2} \cos \frac{2\pi n}{5}$$

$$\therefore \; y_2(t) = \frac{1}{2} + \frac{1}{2} \cos 20\pi t$$

ii) For $F_s = 30Hz$

$$x(n) = \cos \frac{2\pi n}{3}$$

$$y(n) = x^2(n) = \cos^2 \frac{2\pi n}{3}$$

$$y_1(t) = \frac{1}{2} + \frac{1}{2} \cos 20\pi t$$

$$S_a(t) = x_a^2(t)$$

$$y_2(t) = \frac{1}{2} + \frac{1}{2} \cos 20\pi t$$

Q2) Frequency analysis of amplitude-modulated discrete-time signal-The discrete-time $x(n)$ = cos $2\pi f1n$ + cos $2\pi f2n$, $f1$ = 1/18, $f2$ = 5/128, modulates the amplitude of the carrier $xc(n)$ = cos $2\pi fcn$ with fc = 50/128. The resulting amplitude-modulated signal is $xam(n)$ = $x(n)xc$ (n) = $x(n)$cos $2\pi fc$?

Sketch the signals $x(n)$, $xc(n)$, and $xam(n)$, $0 \leq n \leq 255$. • Compute and sketch the 128-point DFT of the signal $xam(n)$, $0 \leq n \leq 127$. N=128 • Compute and sketch the 128-point DFT of the signal $xam(n)$, $0 \leq n \leq 99$. • Compute and sketch the 256-point DFT of the signal $xam(n)$, $0 \leq n \leq 179$. • Explain the results obtained in parts (b) through (d), by deriving the spectrum of the amplitude-modulated signal and comparing it with the experimental results.

CODE:

```
%SubTask 2 - Frequency analysis of amplitude-modulated discrete-time
%signal-The discrete-time 𝑥(𝑛) = cos 2𝜋𝑓1𝑛 + cos 2𝜋𝑓2𝑛, 𝑓1 = 1/18, 𝑓2 =
% 5/128, modulates the amplitude of the carrier 𝑥𝑐(𝑛) = cos 2𝜋𝑓𝑐𝑛 with
% 𝑓𝑐 = 50/128. The resulting amplitude-modulated signal is 𝑥am(𝑛) =
% 𝑥(𝑛)𝑥𝑐 (𝑛) = 𝑥(𝑛)cos 2𝜋𝑓𝑐𝑛

%a) Sketch the signals 𝑥(𝑛), 𝑥𝑐(𝑛), and 𝑥𝑎𝑚(𝑛), 0 ≤ 𝑛 ≤ 255.
n = 0:255;
f1 = 1/18; f2 = 5/128;
fc = 50/128;
xn = cos(2*pi*f1*n)+cos(2*pi*f2*n);%x[n]
xcn = cos(2*pi*fc*n);%xc[n]
xam = xn.*xcn;%xam[n]
subplot(3,1,1);
stem(n,xn);
title('Signal x(n)');
xlabel('x(n)');
ylabel('n');
subplot(3,1,2);
stem(n,xcn);
title('Signal xc(n)');
xlabel('xc(n)');
ylabel('n');
subplot(3,1,3);
stem(n,xam);
title('Signal xam(n)');
xlabel('xam(n)');
ylabel('n');

%b) Compute and sketch the 128-point DFT of the signal 𝑥𝑎𝑚(𝑛), 0 ≤ 𝑛 ≤ 127. N=128
dftxam1 = fft(xam(1:128),128);%calculating 128 point dft for 0<=n<=127
plot(abs(dftxam1));
title('128-point DFT of the signal 𝑥𝑎𝑚(𝑛), 0 ≤ 𝑛 ≤ 127');
xlabel('x(n)');
ylabel('n');

%c) Compute and sketch the 128-point DFT of the signal 𝑥𝑎𝑚(𝑛), 0 ≤ 𝑛 ≤ 99.
dftxam2 = fft(xam(1:100),128);%calculating 128 point dft for 0<=n<=99
plot(abs(dftxam2));
title('128-point DFT of the signal 𝑥𝑎𝑚(𝑛), 0 ≤ 𝑛 ≤ 99');
xlabel('x(n)');
ylabel('n');

%d) Compute and sketch the 256-point DFT of the signal 𝑥𝑎𝑚(𝑛), 0 ≤ 𝑛 ≤ 179.
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```
dftxam3 = fft(xam(1:180),256);%calculating 256 point dft for 0<=n<=179
plot(abs(dftxam3));
title('256-point DFT of the signal xam(n), 0 ≤ n ≤ 179');
xlabel('x(n)');
ylabel('n');
```

OUTPUT:

>> %SubTask 2 - Frequency analysis of amplitude-modulated discrete-time

%signal-The discrete-time $x(n)$ = cos $2\pi f1n$ + cos $2\pi f2n$, $f1$ = 1/18, $f2$ =

% 5/128, modulates the amplitude of the carrier $xc(n)$ = cos $2\pi fcn$ with

% fc = 50/128. The resulting amplitude-modulated signal is $x$am(n) =

% $x(n)xc\ (n)$ = $x(n)$cos $2\pi fcn$

%a) Sketch the signals $x(n)$, $xc(n)$, and $xam(n)$, $0 \leq n \leq 255$.

n = 0:255;

f1 = 1/18; f2 = 5/128;

fc = 50/128;

xn = cos(2*pi*f1*n)+cos(2*pi*f2*n);%x[n]

xcn = cos(2*pi*fc*n);%xc[n]

xam = xn.*xcn;%xam[n]

subplot(3,1,1);

stem(n,xn);

title('Signal x(n)');

xlabel('x(n)');

ylabel('n');

subplot(3,1,2);

stem(n,xcn);

title('Signal xc(n)');

xlabel('xc(n)');

ylabel('n');

subplot(3,1,3);

stem(n,xam);

title('Signal xam(n)');

xlabel('xam(n)');

ylabel('n');

%b) Compute and sketch the 128-point DFT of the signal $xam(n)$, $0 \le n \le 127$. N=128

dftxam1 = fft(xam(1:128),128);%calculating 128 point dft for 0<=n<=127

plot(abs(dftxam1));

title('128-point DFT of the signal $xam(n)$, $0 \le n \le 127$');

xlabel('x(n)');

ylabel('n');

%c) Compute and sketch the 128-point DFT of the signal $xam(n)$, $0 \le n \le 99$.

dftxam2 = fft(xam(1:100),128);%calculating 128 point dft for 0<=n<=99

plot(abs(dftxam2));

title('128-point DFT of the signal $xam(n)$, $0 \le n \le 99$');

xlabel('x(n)');

ylabel('n');

%d) Compute and sketch the 256-point DFT of the signal $xam(n)$, $0 \le n \le 179$.

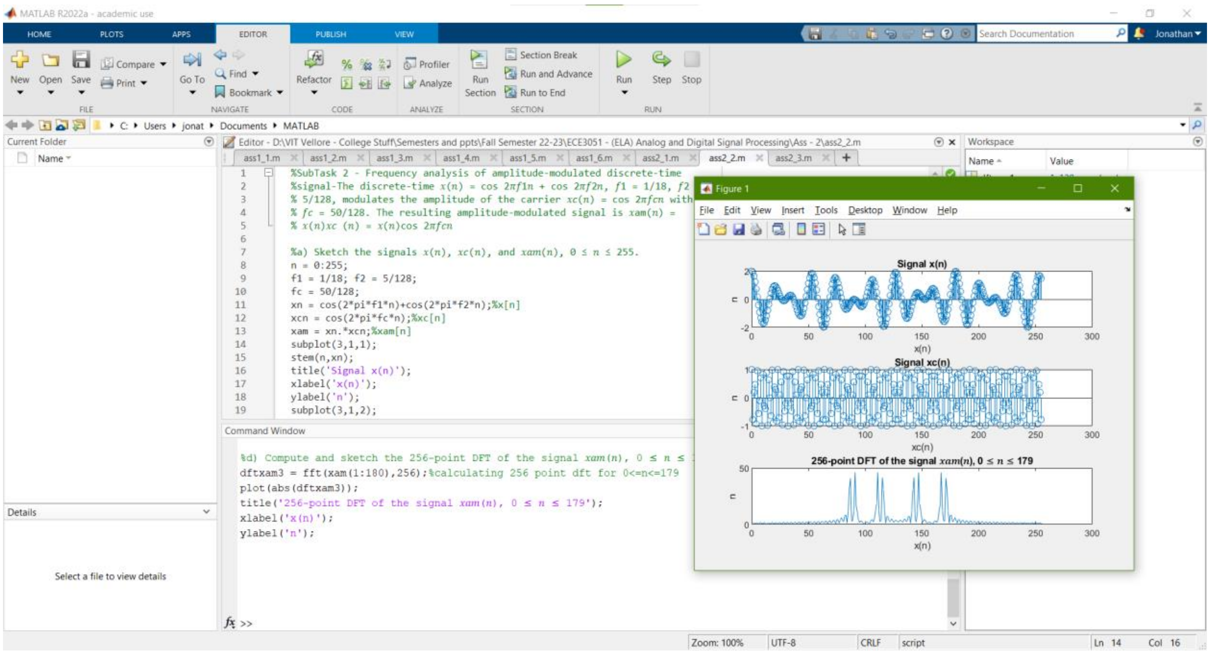
dftxam3 = fft(xam(1:180),256);%calculating 256 point dft for 0<=n<=179

plot(abs(dftxam3));

title('256-point DFT of the signal $xam(n)$, $0 \le n \le 179$');

xlabel('x(n)');

ylabel('n');


MATLAB R2022a - academic use
Editor - D:\VIT Vellore - College Stuff\Semesters and ppts\Fall Semester 22-23\ECE3051 - (ELA) Analog and Digital Signal Processing\Ass - 2\ass2_2.m
%SubTask 2 - Frequency analysis of amplitude-modulated discrete-time
%signal-The discrete-time x(n) = cos 2πf1n + cos 2πf2n, f1 = 1/18, f2
% 5/128, modulates the amplitude of the carrier xc(n) = cos 2πfcn with
% fc = 50/128. The resulting amplitude-modulated signal is xam(n) =
% x(n)xc (n) = x(n)cos 2πfcn
%a) Sketch the signals x(n), xc(n), and xam(n), 0 ≤ n ≤ 255.
n = 0:255;
f1 = 1/18; f2 = 5/128;
fc = 50/128;
xn = cos(2*pi*f1*n)+cos(2*pi*f2*n);%x[n]
xcn = cos(2*pi*fc*n);%xc[n]
xam = xn.*xcn;%xam[n]
subplot(3,1,1);
stem(n,xn);
title('Signal x(n)');
xlabel('x(n)');
ylabel('n');
subplot(3,1,2);
Command Window
%d) Compute and sketch the 256-point DFT of the signal xam(n), 0 ≤ n ≤
dftxam3 = fft(xam(1:180),256);%calculating 256 point dft for 0<=n<=179
plot(abs(dftxam3));
title('256-point DFT of the signal xam(n), 0 ≤ n ≤ 179');
xlabel('x(n)');
ylabel('n');
Figure 1
Signal x(n)
Signal xc(n)
256-point DFT of the signal xam(n), 0 ≤ n ≤ 179

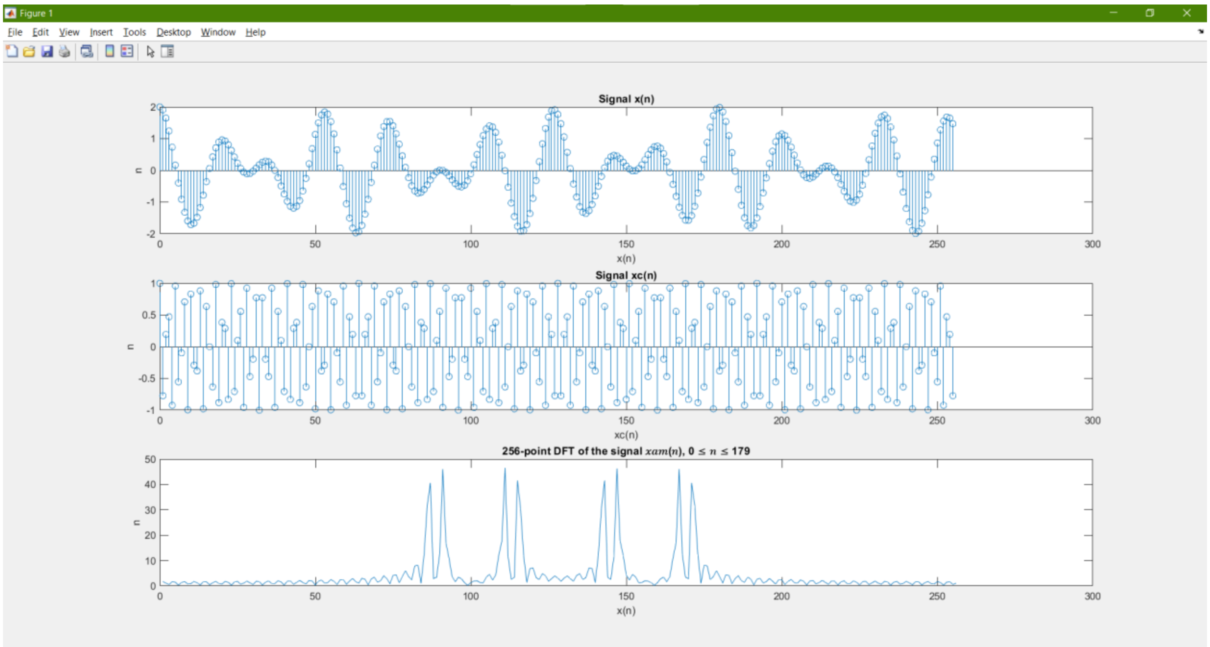
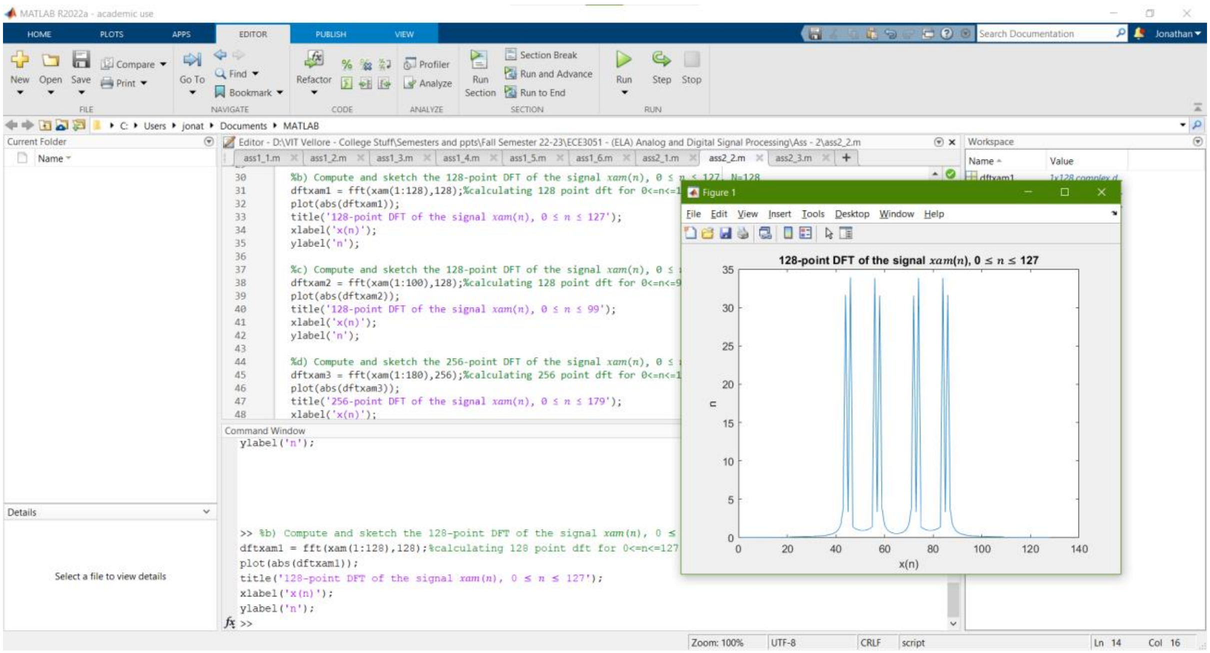
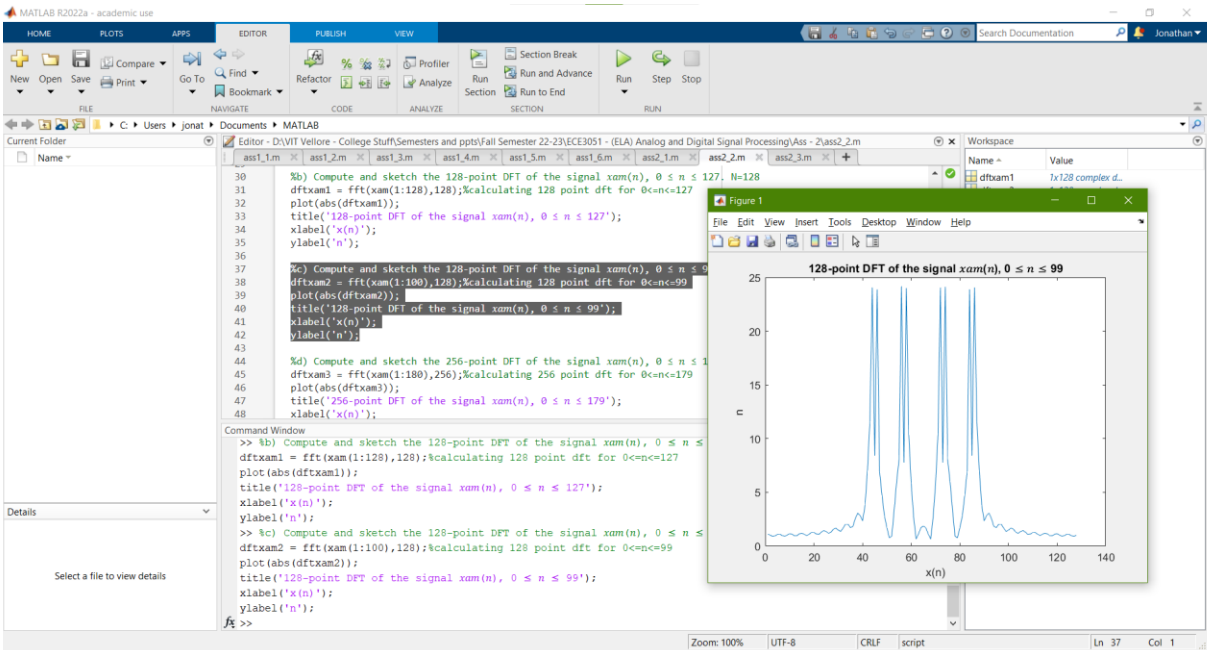
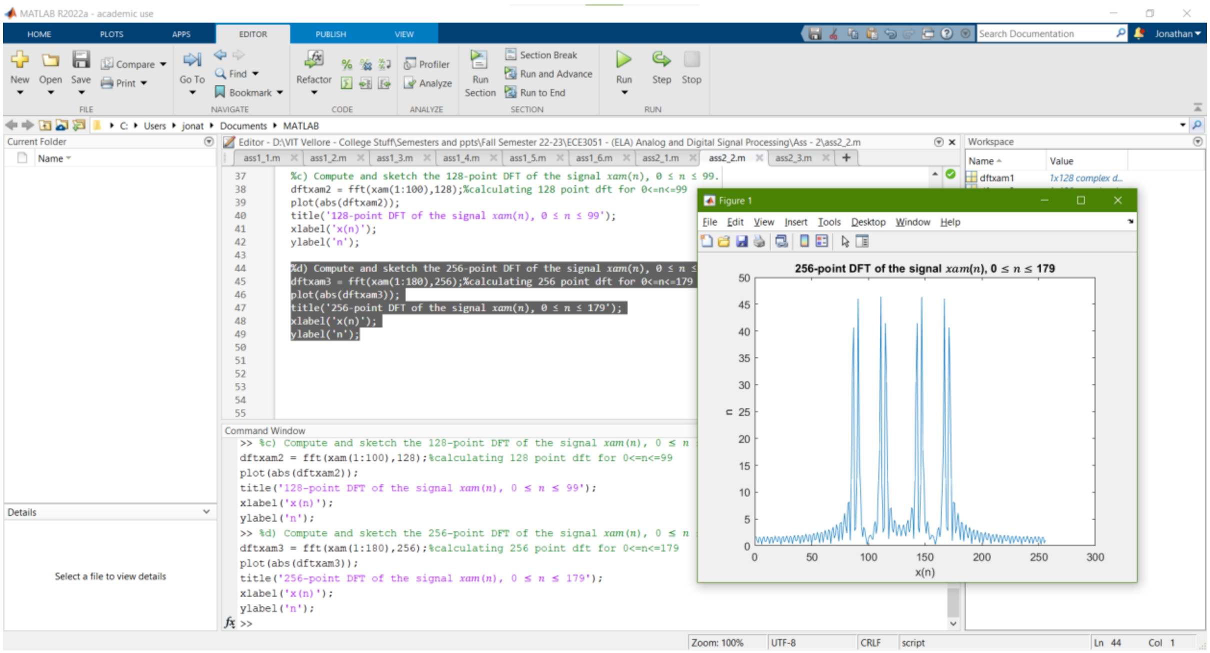


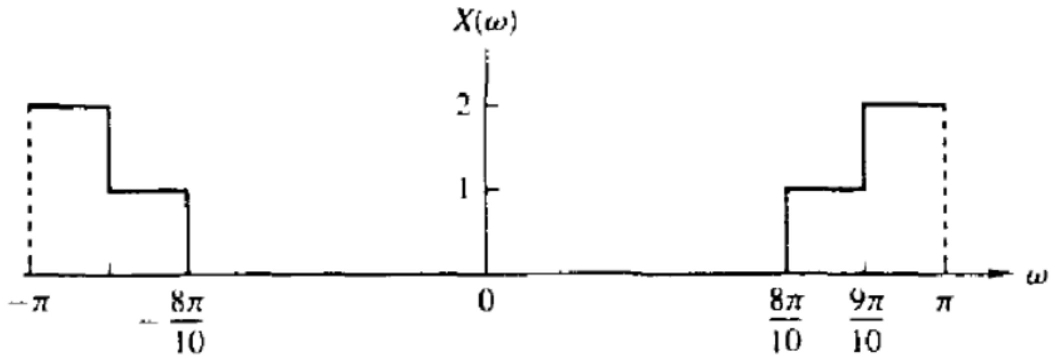
Figure 1
Signal x(n)
x(n)
n
Signal xc(n)
xc(n)
256-point DFT of the signal xam(n), 0 ≤ n ≤ 179
0
50
100
150
200
250
300


MATLAB R2022a - academic use
%b) Compute and sketch the 128-point DFT of the signal xam(n), 0 ≤ n ≤ 127, N=128
dftxam1 = fft(xam(1:128),128);%calculating 128 point dft for 0<=n<=127
plot(abs(dftxam1));
title('128-point DFT of the signal xam(n), 0 ≤ n ≤ 127');
xlabel('x(n)');
ylabel('n');
%c) Compute and sketch the 128-point DFT of the signal xam(n), 0 ≤
dftxam2 = fft(xam(1:100),128);%calculating 128 point dft for 0<=n<=9
plot(abs(dftxam2));
title('128-point DFT of the signal xam(n), 0 ≤ n ≤ 99');
xlabel('x(n)');
ylabel('n');
%d) Compute and sketch the 256-point DFT of the signal xam(n), 0 ≤
dftxam3 = fft(xam(1:180),256);%calculating 256 point dft for 0<=n<=1
plot(abs(dftxam3));
title('256-point DFT of the signal xam(n), 0 ≤ n ≤ 179');
xlabel('x(n)');
Figure 1
128-point DFT of the signal xam(n), 0 ≤ n ≤ 127
x(n)
n


MATLAB R2022a - academic use
%b) Compute and sketch the 128-point DFT of the signal xam(n), 0 ≤ n ≤ 127, N=128
dftxam1 = fft(xam(1:128),128);%calculating 128 point dft for 0<=n<=127
plot(abs(dftxam1));
title('128-point DFT of the signal xam(n), 0 ≤ n ≤ 127');
xlabel('x(n)');
ylabel('n');
%c) Compute and sketch the 128-point DFT of the signal xam(n), 0 ≤ n ≤ 9
dftxam2 = fft(xam(1:100),128);%calculating 128 point dft for 0<=n<=99
plot(abs(dftxam2));
title('128-point DFT of the signal xam(n), 0 ≤ n ≤ 99');
xlabel('x(n)');
ylabel('n');
%d) Compute and sketch the 256-point DFT of the signal xam(n), 0 ≤ n ≤ 1
dftxam3 = fft(xam(1:180),256);%calculating 256 point dft for 0<=n<=179
plot(abs(dftxam3));
title('256-point DFT of the signal xam(n), 0 ≤ n ≤ 179');
xlabel('x(n)');
Figure 1
128-point DFT of the signal xam(n), 0 ≤ n ≤ 99
x(n)
n



Q3) Determine the signal $x(n)$ if its Fourier transform is as given in the figure below:



CODE:

```
%SubTask 3 - Determine the signal x(n) if its Fourier transform is as
%given in the below figure (Figure shown in final document)

%(Problem Working shown in final Document)

%Inverse Fourier Transform for the given figure:
% x(n) = 1/pi[2sin(9pi/10) - sin(8pi/10)]; x(0) = 3/10

syms x(n);
x(n) = 1/pi * (2*sin(9*pi*n/10) - sin(8*pi*n/10));
t = -50:50; %t = -20:20;
subplot(211),plot(t,x(t));
title('Signal x(n) - Derived by Inverse Fourier Transform');
xlabel('time (t)');
ylabel('Magnitude (x(n))');
grid;

%Verification of x(0)
disp(x(0));
```

OUTPUT:

>> %SubTask 3 - Determine the signal $x(n)$ if its Fourier transform is as

%given in the below figure (Figure shown in final document)

%(Problem Working shown in final Document)

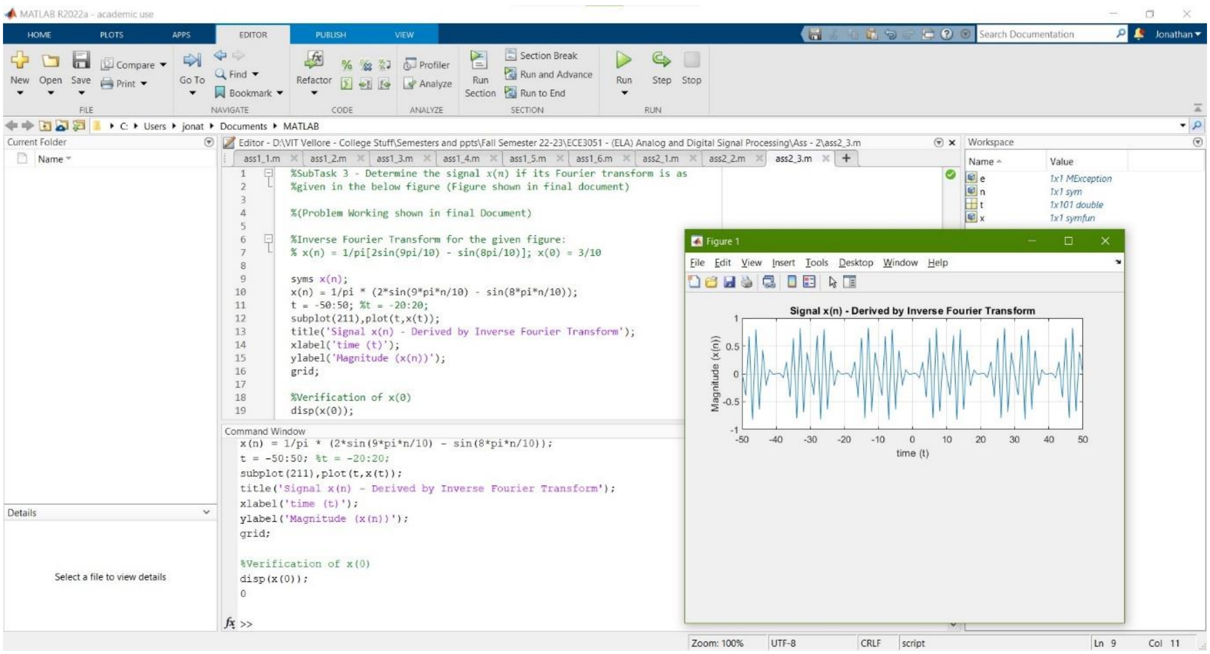
%Inverse Fourier Transform for the given figure:

% x(n) = 1/pi[2sin(9pi/10) - sin(8pi/10)]; x(0) = 3/10

syms x(n);

x(n) = 1/pi * (2*sin(9*pi*n/10) - sin(8*pi*n/10));

```
t = -50:50; %t = -20:20;

subplot(211),plot(t,x(t));

title('Signal x(n) - Derived by Inverse Fourier Transform');

xlabel('time (t)');

ylabel('Magnitude (x(n))');

grid;


%Verification of x(0)

disp(x(0));

0


>>
```



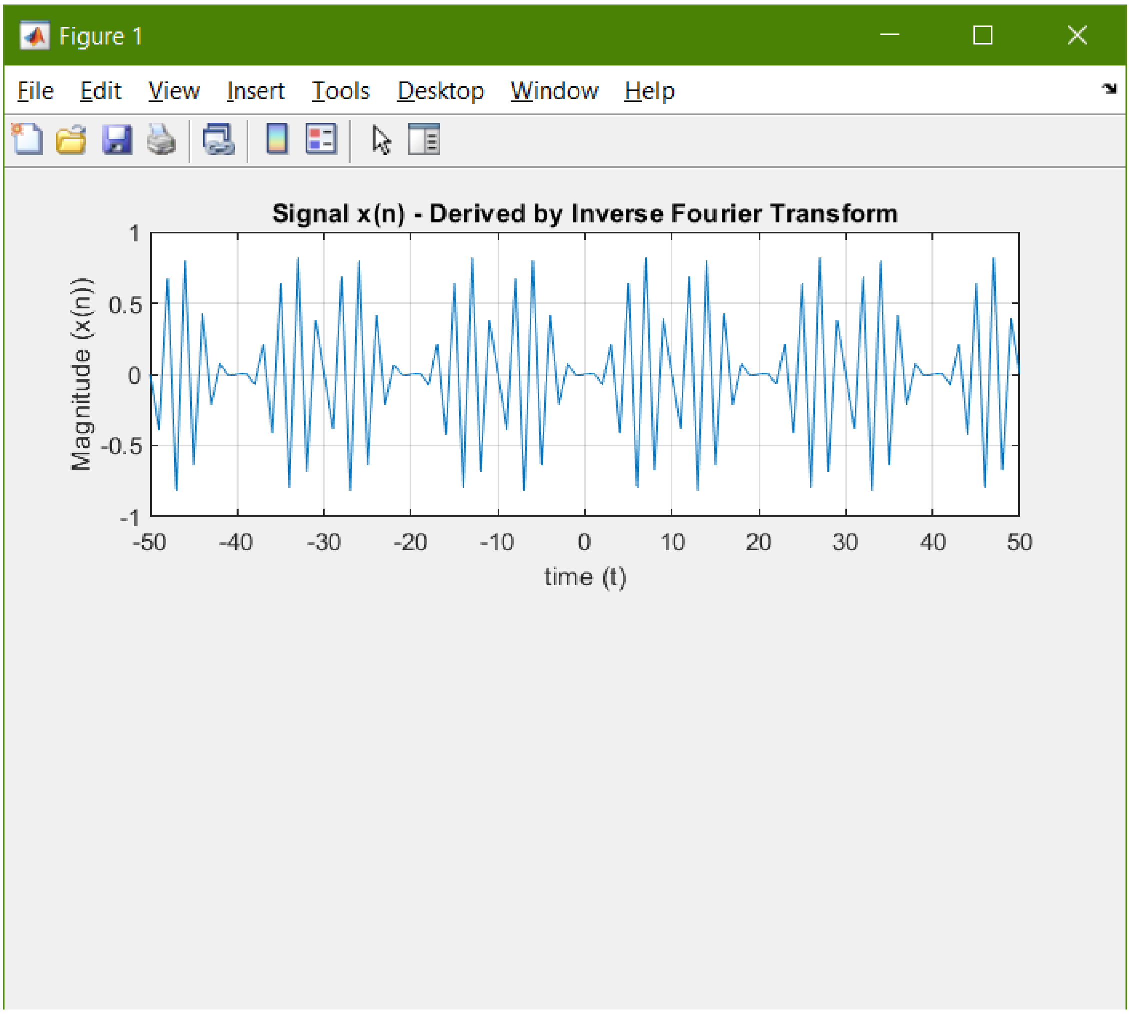
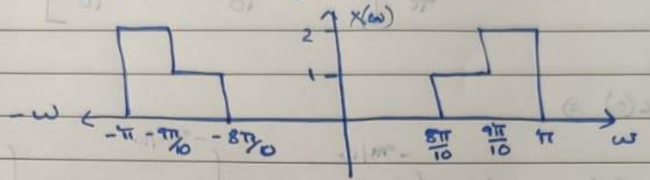


Figure 1
File Edit View Insert Tools Desktop Window Help
Signal x(n) - Derived by Inverse Fourier Transform
Magnitude (x(n))
time (t)
1
0.5
0
-0.5
-1
-50 -40 -30 -20 -10 0 10 20 30 40 50

WORKING OUT:

09.09.22 ECE 3051 Lab - DA2

Q3. Determine the signal $x(n)$ if it's fourier transform is given by the following figure:



$$\therefore x(n) = \frac{1}{2\pi}\int_{-\omega}^{+\omega} X(\omega)\, e^{j\omega n}\, d\omega$$

$$\therefore x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-9\pi/10} 2.e^{j\omega n}\, d\omega + \int_{-9\pi/10}^{-8\pi/10} e^{j\omega n}\, d\omega + 0\right.$$

$$\left. + \int_{8\pi/10}^{9\pi/10} e^{j\omega n} + \int_{9\pi/10}^{\pi} 2.e^{j\omega n}\, d\omega\right]$$

$$\Rightarrow \frac{1}{2\pi}\left[\frac{2}{jn}\left[e^{j\omega n}\right]_{-\pi}^{-9\pi/10} + \frac{1}{jn}\left[e^{j\omega n}\right]_{-9\pi/10}^{-8\pi/10}\right.$$

$$\left. + \frac{1}{jn}\left[e^{j\omega n}\right]_{8\pi/10}^{9\pi/10} + \frac{2}{jn}\left[e^{j\omega n}\right]_{9\pi/10}^{\pi}\right]$$

$$\Rightarrow \frac{1}{2\pi}\left[\frac{2}{jn}\left[e^{-9jn/10} - e^{-\pi jn}\right] + \frac{1}{jn}\left[e^{-8jn/10} - e^{-9\pi jn/10}\right]\right.$$

$$\left. + \frac{1}{jn}\left[e^{9\pi jn/10} - e^{8\pi jn/10}\right] + \frac{2}{jn}\cancel{\left[e^{-8\pi j n/10}\right]}\left[e^{j\pi n} - e^{j9\pi/10}\right]\right]$$

On grouping $\Rightarrow \frac{1}{2\pi}\left[2\sin\frac{9\pi n}{10} - \frac{2\sin 8\pi n}{10} + \frac{2\sin 9\pi n}{10} + \underbrace{2\sin n\pi}_{\simeq 0}\right]$

$$x(n) = \frac{1}{2\pi}\left[\cancel{4}^{2}\sin\frac{9\pi n}{10} - \cancel{2}\sin\frac{8\pi n}{10}\right]$$

$$\therefore \quad x(n) = \frac{1}{\pi}\left[2\sin\frac{9\pi n}{10} - \sin\frac{8\pi n}{10}\right]$$

for $(x(0) \Rightarrow$

$$x(0) = \frac{1}{2\pi}\int_{-\pi}^{-9\pi/10} 2\,d\omega + \int_{-9\pi/10}^{-8\pi/10} d\omega + 0 + \int_{+8\pi/10}^{9\pi/10} d\omega + \int_{9\pi/10}^{\pi} 2\,d\omega$$

$$\Rightarrow \frac{1}{2\pi}\left[2\left[\frac{-9\pi}{10} + \pi\right] + \left[\frac{-8\pi}{10} + \frac{9\pi}{10}\right] + \left[\frac{9\pi}{10} - \frac{8\pi}{10}\right] + 2\left[\pi - \frac{9\pi}{10}\right]\right]$$

$$\Rightarrow \frac{1}{2\pi}\left[\frac{2\pi}{10} + \frac{2\pi}{10} + \frac{2\pi}{10}\right] = \frac{1}{\cancel{2\pi}} \times \frac{\cancel{6\pi}^{3}}{10} = \frac{3}{10}$$