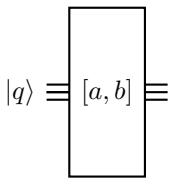
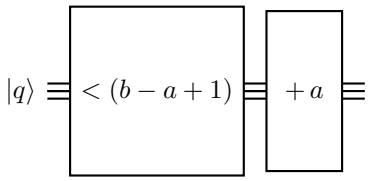
Documentation of Implementation B

Documentation for implementing the oracle 'Interval $[a, b]$' using the implementation B (previously described).

Oracle as Black Box



Oracle as its components



Unitary Matrix of Oracle

$$\begin{pmatrix} 0 & & \cdots & & 0 & 1 & & \\ \vdots & \ddots & & & & \ddots & \ddots & \\ 0 & & 0 & & 0 & & \ddots & 1 \\ -1 & \ddots & \vdots & \ddots & & & & 0 \\ & \ddots & 0 & & \ddots & & & \\ & & -1 & \ddots & & 0 & & \vdots \\ & 0 & & \ddots & 0 & & \ddots & \\ & & & & 1 & 0 & \cdots & 0 \end{pmatrix}$$

Reasoning behind the oracle algorithm

This oracle reuses the less-than oracle and the addition oracle. Firstly, it applies a less-than oracle (give a π-phase to a number of states) and then applies an addition oracle, shifting the marked states to the desired positions.Firstly it applies the oracle 'less-than $b - a + 1$', and then the oracle '$+a$', marking all the states in the interval $[a, b]$.

Classical algorithm which builds the oracle

Parameters needed for the classical algorithm which builds the oracle.

Parameters of the function:

- a: Lower boundary of the range of integers.
- b: Upper boundary of the range of integers.
- n: Number of qubits.

Parameters of the oracle:

- Which qubits of the general circuit is the oracle applied to.

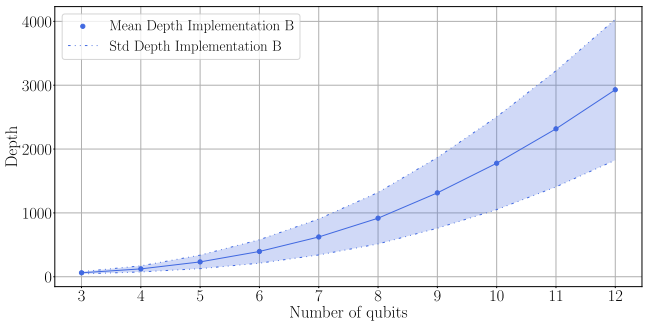
Conditions:

- Precondition: Input state must be the full superposed state with relative 0-phase.
- Postcondition: Full superposed state with a π-phase on states within range $[a, b]$.

Oracle Circuit

The details in this section are given with respect to a specific backend. In this case, FakeWashingtonV2 from Qiskit.

Depth:



Gate Set:

- The oracle requires a universal gate set (Clifford and T gates).
- The backend FackeWashingtonV2 has the following gate set: CX, RZ, S, X.

Assumptions over connections:

- The oracle assumes that each qubit is connected to the rest of the qubits.