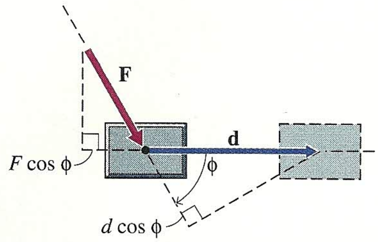


**Figure 11-1** Motion of a particle along a straight line.

### 11-2.1 Work of a Force

In mechanics, a force does work only when the particle to which the force is applied moves. For example, when a constant force **F** is applied to a particle that moves a distance **d** in a straight line, as shown in Fig. 11-1, the work done on the particle by the force **F** is defined by the scalar product

$$\begin{aligned} U = \mathbf{F} \cdot \mathbf{d} &= Fd \cos \phi \\ &= F_x d_x + F_y d_y + F_z d_z \end{aligned} \tag{11-1}$$

where $\phi$ is the angle between the vectors **F** and **d.** Equation 11-1 is usually interpreted as follows:

> The work done by a force **F** is the product of the magnitude of the force ($F$) and the magnitude of the rectangular component of the displacement in the direction of the force ($d \cos \phi$) (see Fig. 11-1).

However, $\cos \phi$ can also be associated with the force **F** instead of with the displacement **d.** Then Eq. 11-1 would be interpreted as follows:

> The work done by a force **F** is the product of the magnitude of the displacement ($d$) and the magnitude of the rectangular component of the force in the direction of the displacement ($F \cos \phi$) (see Fig. 11-1).

Since work is defined as the scalar product of two vectors, work is a scalar quantity with only magnitude and algebraic sign. When the sense of the displacement and the sense of the force component in the direction of the displacement are the same ($0 \leq \phi < 90°$), the work done by the force is positive. When the sense of the displacement and the sense of the force component in the direction of the displacement are opposite ($90° < \phi \leq 180°$), the work done by the force is negative. When the direction of the force is perpendicular to the direction of the displacement ($\phi = 90°$), the component of the force in the direction of the displacement is zero ($d = 0$) and the work done by the force is zero. Of course, the work done by the force is also zero if the displacement is zero ($d = 0$).

Work has the dimensions of force times length. In the SI system of units, this combination of dimensions is called a joule ($1 \text{ J} = 1 \text{ N} \cdot \text{m}$)[2]. In the U.S. Customary system of units, there is no special unit for work. It is expressed simply as ft · lb.

If the force is not constant or if the displacement is not in a straight line, Eq. 11-1 gives the work done by the force only during an infini-

[2]It may be noted that work and moment of a force have the same dimensions: They are both force times length. However, work and moment are two totally different concepts, and the special unit joule should be used only to describe work. The moment of a force must always be expressed as N · m.

tesimal part of the displacement, $d\mathbf{r}$ (see Fig. 11-2):

$$dU = \mathbf{F} \cdot d\mathbf{r} = F\, ds \cos\phi = F_t\, ds$$
$$= F_x\, dx + F_y\, dy + F_z\, dz \tag{11-2}$$

where $d\mathbf{r} = ds\, \mathbf{e}_t = dx\, \mathbf{i} + dy\, \mathbf{j} + dz\, \mathbf{k}$. The total work done by the force as the particle moves from position 1 to position 2 is obtained by integrating Eq. 11-2 along the path of the particle

$$U_{1\to2} = \int_1^2 dU = \int_{s_1}^{s_2} F_t\, ds$$
$$= \int_{x_1}^{x_2} F_x\, dx = \int_{y_1}^{y_2} F_y\, dy = \int_{z_1}^{z_2} F_z\, dz \tag{11-3}$$

For the special case in which the force $\mathbf{F}$ is constant (both in magnitude and direction), the force components can be taken outside the integral signs in Eq. 11-3. Then Eq. 11-13 gives

$$U_{1\to2} = F_x \int_{x_1}^{x_2} dx + F_y \int_{y_1}^{y_2} dy + F_z \int_{z_1}^{z_2} dz$$
$$= F_x(x_2 - x_1) + F_y(y_2 - y_1) + F_z(z_2 - z_1) \tag{11-4}$$

Note that the evaluation of the work done by a constant force depends on the coordinates at the end points of the particle's path but not on the actual path traveled by the particle. For the constant force $\mathbf{F}$ shown in Fig. 11-3, it doesn't matter if the particle moves along path $a$ from position 1 to position 2, along path $b$, or along some other path. The work done by the force is always the same. Forces for which the work done is independent of the path are called conservative forces.

The weight $W$ of a particle is a particular example of a constant force. When bodies move near the surface of the earth, the force of the earth's gravity is essentially constant ($F_x = 0$, $F_y = 0$, and $F_z = -W$). Therefore, the work done on a particle by its weight is $-W(z_2 - z_1)$. When $z_2 > z_1$ the particle moves upward (opposite the gravitational force), and the work done by gravity is negative. When $z_2 < z_1$ the particle moves downward (in the direction of the gravitational force), and the work done by gravity is positive.

Examples of forces that do work when a body moves from one position to another include the weight of the body, friction between the body and other surfaces, and externally applied loads. Examples of forces which do no work include forces at fixed points ($ds = 0$) and forces acting in a direction perpendicular to the displacement ($\cos\phi = 0$).

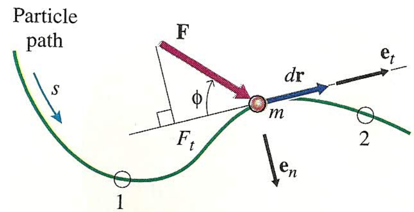



**Figure 11-2** Motion of a particle along a curved path.

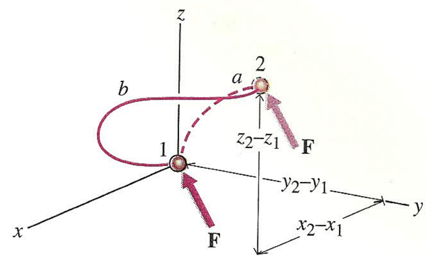
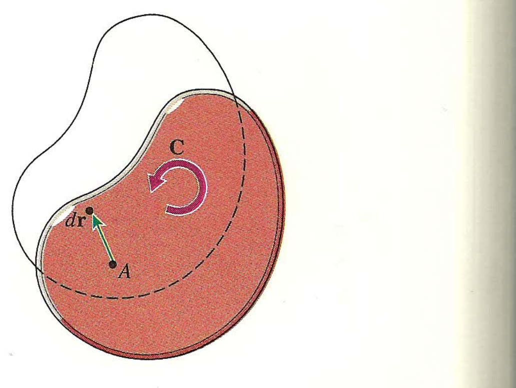



**Figure 11-3** Motion of a particle along a curved path.




(a)

## EXAMPLE PROBLEM 11-1

A 500-lb block $A$ is held in equilibrium on an inclined surface with a cable and weight system and a force $\mathbf{P}$ as shown in Fig. 11-5*a*. When the force $\mathbf{P}$ is removed, block $A$ slides down the incline at a constant velocity for a distance of 10 ft. The coefficient of friction between block $A$ and the inclined surface is 0.20. Determine

a. The work done by the cable on block $A$.
b. The work done by gravity on block $A$.
c. The work done by the surface of the incline on block $A$.
d. The total work done by all forces on block $A$.
e. The work done by the cable on block $B$.
f. The work done by gravity on block $B$.
g. The total work done by all forces on block $B$.

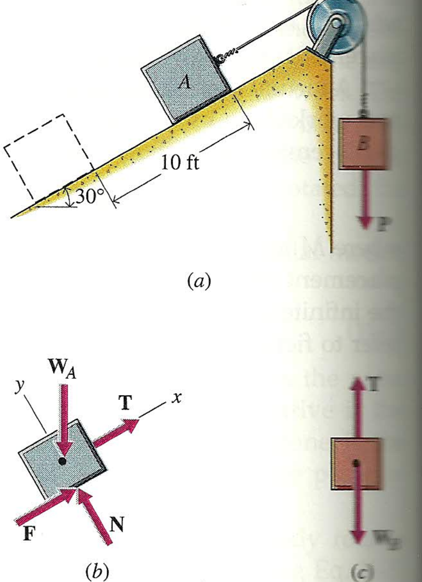



**Fig. 11-5**

### SOLUTION

Free-body diagrams for blocks $A$ and $B$ are shown in Fig. 11-5*b*. Four forces act on block $A$: the cable tension $\mathbf{T}$, the weight $\mathbf{W}_A$, and the normal and frictional forces $\mathbf{N}$ and $\mathbf{F}$ at the surface of contact between the block and the inclined surface. Two forces act on block $B$: the cable tension $\mathbf{T}$ and the weight $\mathbf{W}_B$. Since the blocks move at a constant velocity, they are in equilibrium and the equilibrium equations applied to block $A$ yield

$$N = W_A \cos 30° = 500 \cos 30° = 433 \text{ lb}$$
$$F = \mu N = 0.20(433) = 86.6 \text{ lb}$$
$$T = W_A \sin 30° - F = 500 \sin 30° - 86.6 = 163.4 \text{ lb}$$

The equilibrium equations applied to block $B$ yield

$$W_B = T = 163.4 \text{ lb}$$

Since all of the forces are constant in both magnitude and direction during the movements of the blocks, the work can be computed by using Eq. 11-4 (in two dimensions, so that $z_2 - z_1 = 0$)

The work done by a force $F$ during a linear displacement $ds$ in the direction of the force is $dU = F\, ds$.

$$U_{1\to 2} = F_x(x_2 - x_1) + F_y(y_2 - y_1)$$

Therefore, for block $A$, $x_2 - x_1 = -10$ ft, $y_2 - y_1 = 0$ ft, and

a. $U_T = 163.4(-10) = -1634 \text{ ft} \cdot \text{lb}$ Ans.
b. $U_W = (-500 \cos 60°)(-10) = 2500 \text{ ft} \cdot \text{lb}$ Ans.

Alternatively, the weight force acts vertically downward and block $A$ drops a vertical distance $h = 10 \sin 30°$ so that

$$U_W = W_A h = 500(10 \sin 30°) = 2500 \text{ ft} \cdot \text{lb}$$

c. $U_F = 86.6(-10) = -866 \text{ ft} \cdot \text{lb}$ Ans.

$$U_N = 0(-10) + 433(0) = 0 \text{ ft} \cdot \text{lb}$$ Ans.

d. $U_{\text{total}} = U_T + U_W + U_F + U_N = -1634 + 2500 - 866 + 0 = 0$ Ans.

The total work done by a system of forces is the algebraic sum of the work done by the individual forces.

and for block B, $x_2 - x_2 = 0$ ft, $y_2 - y_1 = 10$ ft, and

e. $U_T = 163.4(10) = 1634 \text{ ft} \cdot \text{lb}$ Ans.
f. $U_W = 163.4(-10) = -1634 \text{ ft} \cdot \text{lb}$ Ans.
g. $U_{\text{total}} = U_T + U_W = -1634 + 1634 = 0$ Ans.