9-3.1 Wedges

A wedge is just a block that has two flat faces that make a small angle with each other. Wedges are often used in pairs as shown in Fig. 9-18 to raise heavy loads. Depending on the angle between the two surfaces of the wedge, the weight being lifted (the output force) can be many times that of the force P (the input force) applied to the wedge. Also, a properly designed wedge will stay in place and support the load even after the force P is removed.

Wedge problems can often be solved using a semigraphical approach. Wedges are almost always constrained against rotation so that only force equilibrium need be considered. Also, the number of forces acting on a wedge is usually small (the friction and normal forces are usually combined into a single resultant force as in Fig. 9-19*a*), so force equilibrium can be expressed as a force polygon (Fig. 9-19*b*). The law of sines and the law of cosines can then be used to relate the forces and angles.

For the case of impending motion, the resultant of the normal and friction force is drawn at the angle of static friction (Fig. 9-20*a*) and the magnitude of the resultant or some other force determined. If motion is not impending, the resultant is drawn with whatever magnitude and at whatever angle ϕ_1 is required for equilibrium. This angle is then compared with the angle of static friction $\phi_1 \le \phi_{1s}$ (Fig. 9-20*b*) to determine whether or not equilibrium exists.

Like other machines, wedges are typically characterized by their **mechanical advantage** (M.A.), or the ratio of their output and input forces. In the case of wedges, the mechanical advantage is defined as

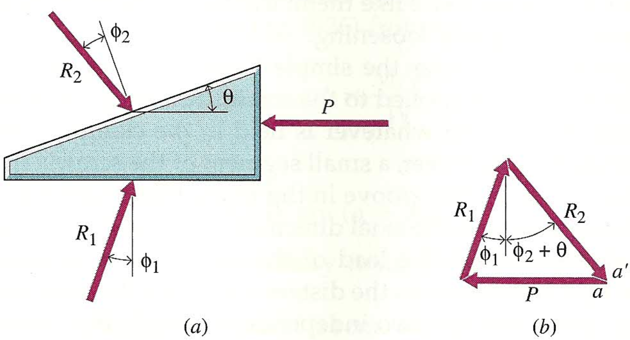


Figure 9-19 Free-body diagram and force polygon for a wedge.

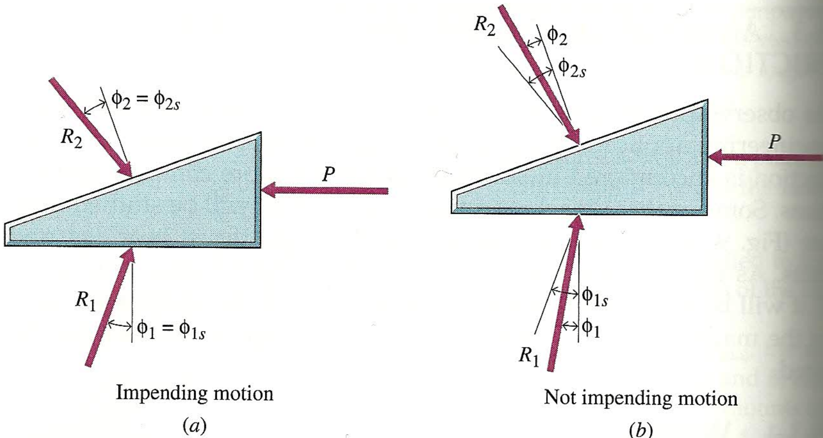


Figure 9-20 Free-body diagrams for a wedge when motion is impending and not impending.

the ratio

$$\text{M.A.} = \frac{\text{direct force}}{\text{wedge force}} \tag{9-6}$$

The numerator of Eq. 9-6 is the force that must be applied directly to some object to accomplish a desired task. In the case of the wedge in Fig. 9-18, this is just the weight of the object being raised. The denominator of Eq. 9-6 is the force that must be applied to the wedge to accomplish the same task. For the wedge of Fig. 9-18, this is *P*. Clearly, a well-designed wedge should have a mechanical advantage greater than one.

A wedge with a large mechanical advantage may not be the best overall design, however. A common design criterion for wedges is that the wedge remain in place after being forced under the load. A wedge that must be forcibly removed is called **self-locking.**

EXAMPLE PROBLEM 9-8

A wedge is to be used to slide the 3000 N safe of Fig. 9-36*a* across the floor. Determine the minimum force **P** necessary if the coefficient of friction is 0.35 at all surfaces and the weight of the wedges may be neglected.

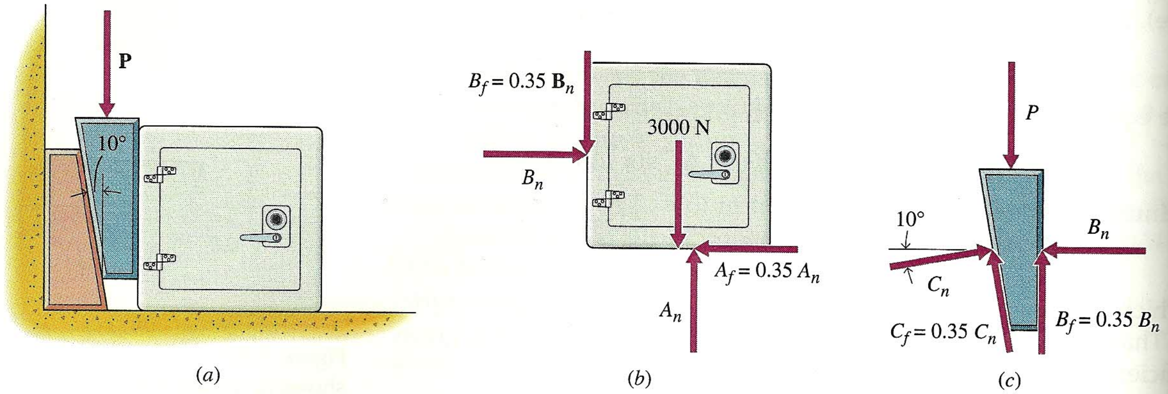


Fig. 9-36

SOLUTION

First draw the free-body diagram of the safe (Fig. 36*b*). Clearly, the safe will tend to move to the right and the friction force A_f must act to the left to oppose the motion. However, the direction of the friction force B_f is not as easy to ascertain. Even though the safe is not moving up or down, it appears to be moving up relative to the wedge. Hence, the friction force B_f must act downward on the safe to oppose this relative motion.

It may be easier to see the correct direction for this friction force on the equilibrium diagram of the wedge (Fig. 9-36*c*). The motion of the wedge is downward and the friction force B_f must act upward to oppose the motion. And if the safe exerts an upward frictional force on the wedge, then the wedge must exert an equal frictional force downward on the safe.

Since motion by slipping is impending, the friction force on each sliding surface of the wedge is the maximum available and it acts in a direction to oppose the motion.

The equilibrium equations for the safe are

$$+\rightarrow \Sigma F_x = B_n - 0.35A_n = 0$$
$$+\uparrow \Sigma F_y = A_n - 0.35B_n - 3000 = 0$$

which are solved to get

$$A_n = 3419\ N \qquad \text{and} \qquad B_n = 1197\ \text{N}$$

Equilibrium equations for the wedge are

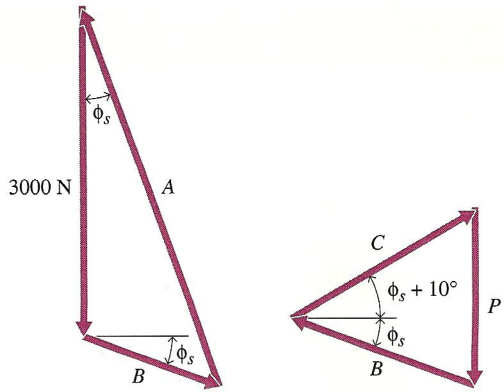
$$+\rightarrow \Sigma F_x = C_n \cos 10^\circ - 0.35C_n \sin 10^\circ - 1197 = 0$$
$$+\uparrow \Sigma F_y = C_n \sin 10^\circ + 0.35C_n \cos 10^\circ + (0.35)(1197) - P = 0$$

which give

$$C_n = 1295\ N \qquad \text{and} \qquad P = 1090\ N \qquad \text{Ans.}$$

SOLUTION 2. Using the force equilibrium triangle

The free-body diagrams are drawn as above (Figs. 9-36*b* and 9-36*c*). Then the force equilibrium triangles are drawn for the safe (Fig. 9-36*d*) and for the wedge



(*d*) (*e*)

Fig. 9-36*e*). In these diagrams, the normal and frictional forces have been combined into a single resultant force, which is drawn at the angle of static friction

$$\phi_s = \tan^{-1} 0.35 = 19.29°$$

since motion is impending.

Using the law of sines on the first force triangle (Fig. 9-36*d*)

$$\frac{B}{\sin (19.29°)} = \frac{3000}{\sin [90° - 2(19.29°)]}$$

gives immediately

$$B = 1268 \text{ N}$$

Then using the law of sines on the second force triangle (Fig. 9-36*e*)

$$\frac{P}{\sin [2(19.29°) + 10°]} = \frac{1268}{\sin (90° - 19.29° - 10°)}$$

gives

$$P = 1090 \text{ N} \qquad \text{Ans.}$$

Wedge problems can be solved using force triangles. Normal and frictional forces on each contact surface are replaced by resultants which act at the angle of static friction.