8-2 AXIAL FORCE AND TORQUE IN BARS AND SHAFTS

In Chapter 7, axially loaded two-force truss members and multiforce frame and machine elements were discussed in detail. Application of the equations of equilibrium to the various parts of the truss, frame, or machine allowed the determination of all forces acting at the smooth pin connections. In many other types of engineering applications, knowledge of the maximum axial force, maximum shear force, maximum twisting moment, or maximum bending moment transmitted by any cross section through the member is required to establish the ad-

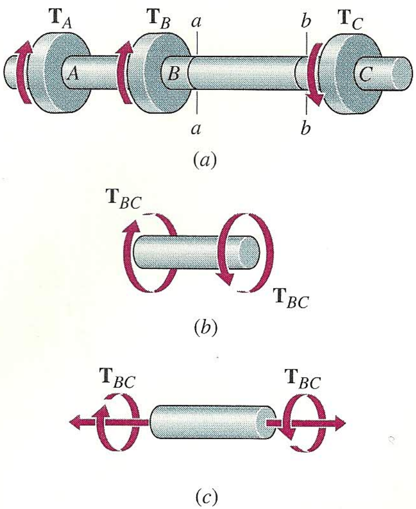

Figure 8-4 Torques transmitted by internal planes of a shaft.

equacy of the member for its intended use. In this section, the equations of equilibrium are used to establish the variation of internal axial force along the length of an axially loaded member and the variation of resisting torque transmitted by transverse cross sections of a shaft. Axial force and torque diagrams are introduced as a way to visualize the distributions for the full lengths of the members.

An axial force diagram is a graph in which abscissas represent distances along the member and ordinates represent the internal axial forces at the corresponding cross sections. In plotting an axial force diagram, tensile forces are positive and compressive forces are negative. Example Problem 8-1 illustrates the computations required to construct an axial force diagram for a simple tension member subjected to four axial loads.

In a similar manner, a torque diagram is a graph in which abscissas represent distances along the member and ordinates represent the internal resisting torques at the corresponding cross sections. The sign convention used for torques is illustrated in Fig. 8-4. In the shaft shown in Fig. 8-4*a*, torque is applied to the shaft at gear *C* and is removed at gears *A* and *B*. Torques transmitted by cross sections *aa* and *bb* in the interval between gears *B* and *C* are shown in a pictorial fashion in Fig. 8-4*b*. A vector representation of these torques is shown in Fig. 8-4*c*. Positive torques point outward from the cross section when represented as a vector, according to the right-hand rule. Example Problem 8-2 illustrates the computations required to construct a torque diagram for a shaft subjected to four torques at different positions along the length of the shaft.

CONCEPTUAL EXAMPLE 8-1: INTERNAL FORCES

Two teams A and B are engaged in a tug of war. At the instant shown in Fig. CE8-1a, which team is winning and what is the magnitude of the force being transmitted by the rope segment between team members A_3 and A_4?

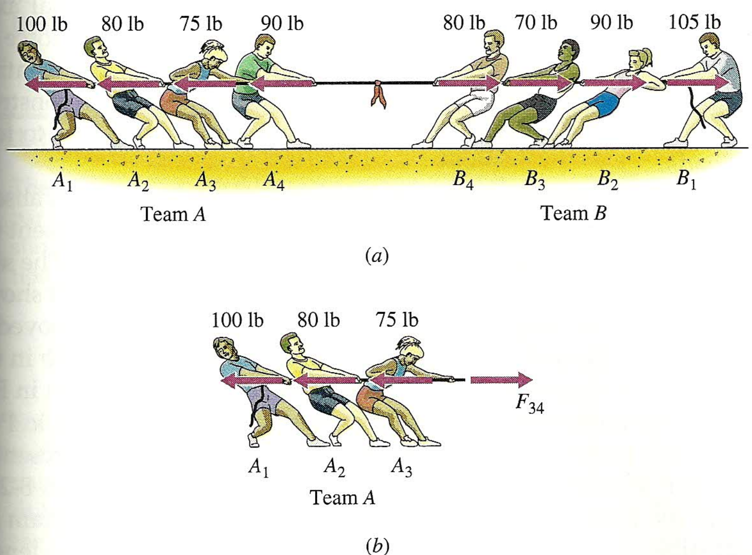

Fig. CE8-1

SOLUTION

Equilibrium of a system of collinear forces requires satisfaction of one force equation in the direction of the forces since all other equations are automatically satisfied. Thus, for the system shown in Fig. CE8-1a,

$$
\begin{aligned}
+\rightarrow \Sigma F &= A_1 + A_2 + A_3 + A_4 + B_1 + B_2 + B_3 + B_4 \\
&= -100 - 80 - 75 - 90 + 80 + 70 + 90 + 105 = 0 \qquad \text{Ans.}
\end{aligned}
$$

Therefore, the system is in equilibrium and neither team is winning.

Since the complete system consisting of teams A and B is in equilibrium, any part of the system isolated for consideration must also be in equilibrium under the action of the external loads applied to the part and the internal forces exposed during the isolating process. The force being transmitted by the rope segment between team members A_3 and A_4 can be determined by isolating team members A_1, A_2, and A_3 as a free-body diagram, as shown in Fig. CE8-1b. Force F_{34} on this diagram represents the action of the part of the system removed during the isolating process. Summing forces in the direction of the forces yields

$$
\begin{aligned}
+\rightarrow \Sigma F &= A_1 + A_2 + A_3 + F_{34} \\
&= -100 - 80 - 75 + F_{34} = 0 \\
F_{34} &= 255 \text{ lb} = 255 \text{ lb} \rightarrow \qquad \text{Ans.}
\end{aligned}
$$

In the event that the system is not in equilibrium, any unbalance of forces will produce an acceleration of the complete system in the direction of the unbalance in accordance with Newton's second law ($\mathbf{F} = m\mathbf{a}$). Such a situation represents a problem in dynamics.

EXAMPLE PROBLEM 8-1

A steel bar with a rectangular cross section is used to transmit four axial loads as shown in Fig. 8-5*a*.

a. Determine the axial forces transmitted by cross sections in intervals *AB*, *BC*, and *CD* of the bar.
b. Draw an axial force diagram for the bar.

> Equilibrium of a member subjected to a system of collinear forces requires only one force equation in the direction of the forces since all other equations are automatically satisfied.

SOLUTION

a. The forces transmitted by cross sections in intervals *AB*, *BC*, and *CD* of the bar shown in Fig. 8-5*a* are obtained by using the three free-body diagrams shown in Figs. 8-5*b*, 8-5*c*, and 8-5*d*. Applying the force equilibrium equation $\Sigma F = 0$ along the axis of the bar yields

$+\rightarrow \Sigma F = F_{AB} - 60 = 0$	$F_{AB} = +60$ kN	Ans.
$+\rightarrow \Sigma F = F_{BC} - 60 + 80 = 0$	$F_{BC} = -20$ kN	Ans.
$+\rightarrow \Sigma F = F_{CD} - 60 + 80 - 50 = 0$	$F_{CD} = +30$ kN	Ans.

For all the above calculations, a free-body diagram of the part of the bar to the left of the imaginary cut has been used. A free-body diagram of the part of the bar to the right of the cut would have yielded identical results. In fact, for the determination of F_{CD}, the free-body diagram to the right of the cut would have been more efficient, since only the unknown force F_{CD} and the 30-kN load would have appeared on the diagram.

b. The axial force diagram for the bar, constructed by using the results from part *a*, is shown in Fig. 8-5*e*. Note in the diagram that the abrupt changes in internal force are equal to the applied loads at pins *A*, *B*, *C*, and *D*. Thus, the axial force diagram could have been drawn directly below the sketch of the loaded bar of Fig. 8-5*a*, without the aid of the free-body diagrams shown in Fig. 8-5*b*, 8-5*c*, and 8-5*d*, by using the applied loads at pins *A*, *B*, *C*, and *D*.

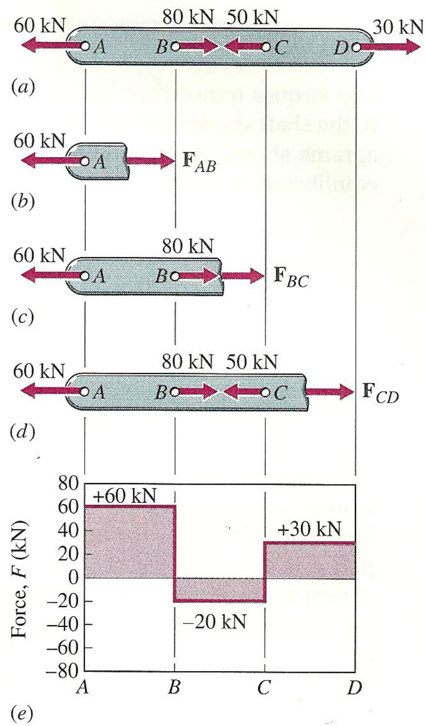

Fig. 8-5

EXAMPLE PROBLEM 8-2

A steel shaft is used to transmit torque from a motor to operating units in a factory. The torque is input at gear B (see Fig. 8-6*a*) and is removed at gears A, C, D, and E.

a. Determine the torques transmitted by cross sections in intervals AB, BC, CD, and DE of the shaft.

b. Draw a torque diagram for the shaft.

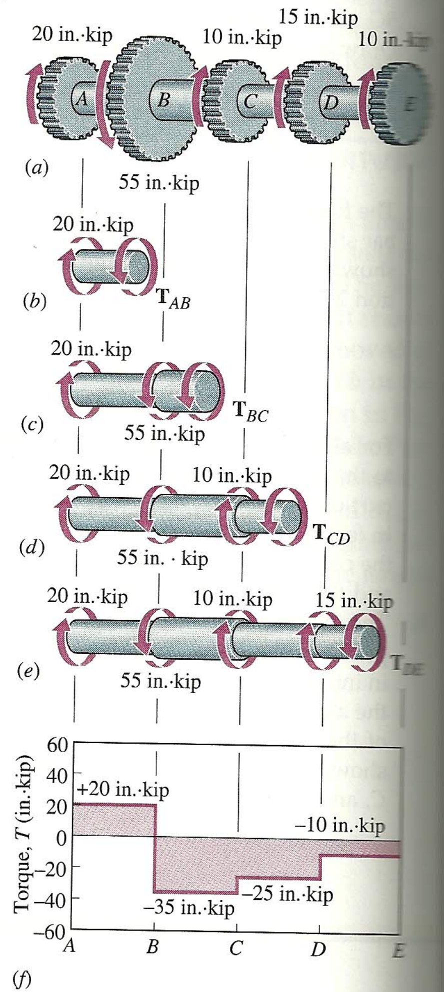

Fig. 8-6

SOLUTION

a. The torques transmitted by cross sections in intervals AB, BC, CD, and DE of the shaft shown in Fig. 8-6*a* are obtained by using the four free-body diagrams shown in Figs. 8-6*b*, 8-6*c*, 8-6*d*, and 8-6*e*. Applying the moment equilibrium equation $\Sigma M = 0$ about the axis of the shaft yields,

$$+\circlearrowleft \Sigma M = T_{AB} - 20 = 0 \qquad T_{AB} = +20 \text{ in.} \cdot \text{kip} \qquad \text{Ans.}$$
$$+\circlearrowleft \Sigma M = T_{BC} - 20 + 55 = 0 \qquad T_{BC} = -35 \text{ in.} \cdot \text{kip} \qquad \text{Ans.}$$
$$+\circlearrowleft \Sigma M = T_{CD} - 20 + 55 - 10 = 0 \qquad T_{CD} = -25 \text{ in.} \cdot \text{kip} \qquad \text{Ans.}$$
$$+\circlearrowleft \Sigma M = T_{DE} - 20 + 55 - 10 - 15 = 0 \qquad T_{DE} = -10 \text{ in.} \cdot \text{kip} \qquad \text{Ans.}$$

For all the above calculations, a free-body diagram of the part of the shaft to the left of the imaginary cut has been used. A free-body diagram of the part of the shaft to the right of the cut would have yielded identical results. In fact, for the determination of T_{CD} and T_{DE}, the free-body diagram to the right of the cut would have been more efficient, since fewer torques would have appeared on the diagram.

b. The torque diagram for the shaft, constructed by using the results from part *a*, is shown in Fig. 8-6*f*. Note in the diagram that the abrupt changes in torque are equal to the applied torques at gears A, B, C, D, and E. Thus, the torque diagram could have been drawn directly below the sketch of the shaft of Fig. 8-6*a*, without the aid of the free-body diagrams shown in Figs. 8-6*b*, 8-6*c*, 8-6*d*, and 8-6*e*, by using the applied torques at gears A, B, C, D, and E.

Equilibrium of a shaft subjected to a system of torques requires only one moment equation about the axis of the shaft since all other equations are automatically satisfied.