FORCES IN BEAMS AND CABLES

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*7.1. INTRODUCTION

In preceding chapters, two basic problems involving structures were considered: (1) determining the external forces acting on a structure (Chap. 4) and (2) determining the forces which hold together the various members forming a structure (Chap. 6). The problem of determining the internal forces which hold together the various parts of a given member will now be considered.

We will first analyze the internal forces in the members of a frame, such as the crane considered in Secs. 6.1 and 6.10, noting that whereas the internal forces in a straight two-force member can produce only *tension* or *compression* in that member, the internal forces in any other type of member usually produce *shear* and *bending* as well.

Most of this chapter will be devoted to the analysis of the internal forces in two important types of engineering structures, namely,

1. *Beams,* which are usually long, straight prismatic members designed to support loads applied at various points along the member.
2. *Cables,* which are flexible members capable of withstanding only tension and are designed to support either concentrated or distributed loads. Cables are used in many engineering applications, such as suspension bridges and transmission lines.

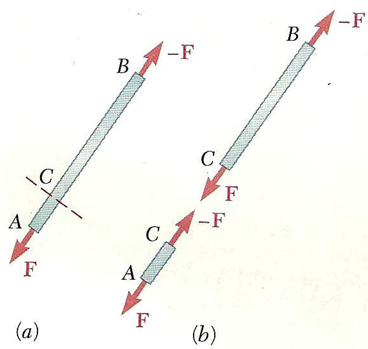


Fig. 7.1

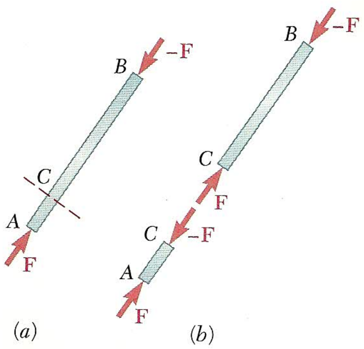


Fig. 7.2

*7.2. INTERNAL FORCES IN MEMBERS

Let us first consider a *straight two-force member AB* (Fig. 7.1*a*). From Sec. 4.6, we know that the forces $\mathbf{F}$ and $-\mathbf{F}$ acting at A and B, respectively, must be directed along AB in opposite sense and have the same magnitude F. Now, let us cut the member at C. To maintain the equilibrium of the free bodies AC and CB thus obtained, we must apply to AC a force $-\mathbf{F}$ equal and opposite to $\mathbf{F}$, and to CB a force $\mathbf{F}$ equal and opposite to $-\mathbf{F}$ (Fig. 7.1*b*). These new forces are directed along AB in opposite sense and have the same magnitude F. Since the two parts AC and CB were in equilibrium before the member was cut, *internal forces* equivalent to these new forces must have existed in the member itself. We conclude that in the case of a straight two-force member, the internal forces that the two portions of the member exert on each other are equivalent to *axial forces*. The common magnitude F of these forces does not depend upon the location of the section C and is referred to as the *force in member AB*. In the case considered, the member is in tension and will elongate under the action of the internal forces. In the case represented in Fig. 7.2, the member is in compression and will decrease in length under the action of the internal forces.

Next, let us consider a *multiforce member*. Take, for instance, member AD of the crane analyzed in Sec. 6.10. This crane is shown again in Fig. 7.3*a*, and the free-body diagram of member AD is drawn in Fig. 7.3*b*. We now cut member AD at J and draw a free-body diagram for each of the portions JD and AJ of the member (Fig. 7.3*c* and *d*). Considering the free body JD, we find that its equilibrium will

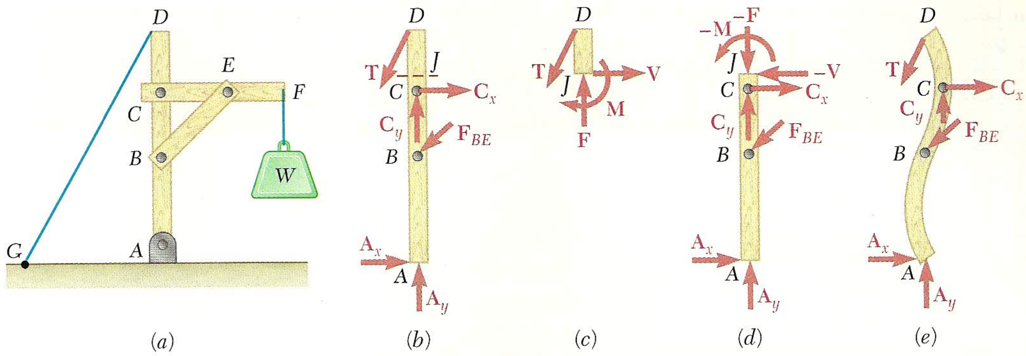


Fig. 7.3

be maintained if we apply at J a force $\mathbf{F}$ to balance the vertical component of $\mathbf{T}$, a force $\mathbf{V}$ to balance the horizontal component of $\mathbf{T}$, and a couple $\mathbf{M}$ to balance the moment of $\mathbf{T}$ about J. Again we conclude that internal forces must have existed at J before member AD was cut. The internal forces acting on the portion JD of member AD are equivalent to the force-couple system shown in Fig. 7.3*c*. According to Newton's third law, the internal forces acting on AJ must be equivalent to an equal and opposite force-couple system, as shown in Fig. 7.3*d*. It is clear that the action of the internal forces in member AD *is not limited to producing tension or compression* as in the case of straight two-force members; the internal forces *also produce shear and bending*. The force $\mathbf{F}$ is an *axial force;* the force $\mathbf{V}$ is called a *shearing force;* and the moment $\mathbf{M}$ of the couple is known as the *bending moment at* J. We note that when determining internal forces in a member, we should clearly indicate on which portion of the member the forces are supposed to act. The deformation which will occur in member AD is sketched in Fig. 7.3*e*. The actual analysis of such a deformation is part of the study of mechanics of materials.

It should be noted that in a *two-force member which is not straight,* the internal forces are also equivalent to a force-couple system. This is shown in Fig. 7.4, where the two-force member ABC has been cut at D.

Photo 7.1 The design of the shaft of a circular saw must account for the internal forces resulting from the forces applied to the teeth of the blade. At a given point in the shaft, these internal forces are equivalent to a force-couple system consisting of axial and shearing forces and a couple representing the bending and torsional moments.

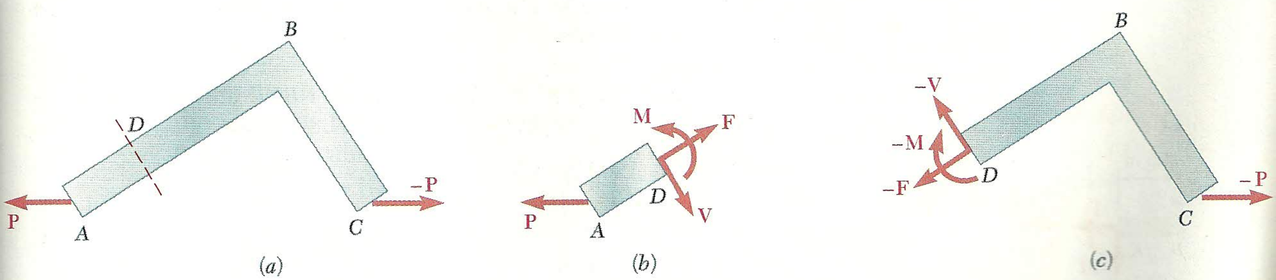
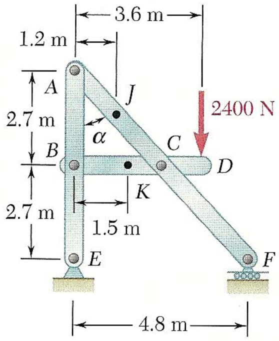
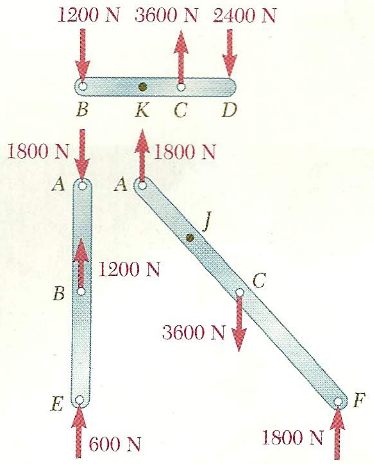


Fig. 7.4



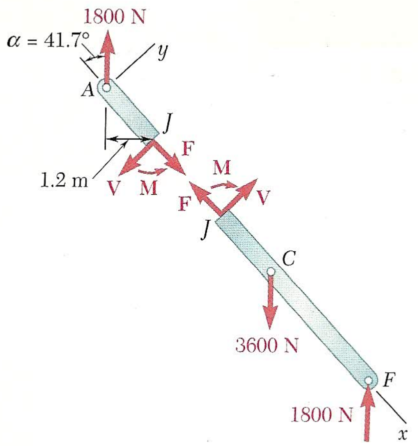
SAMPLE PROBLEM 7.1

In the frame shown, determine the internal forces (*a*) in member *ACF* at point *J*, (*b*) in member *BCD* at point *K*. This frame has been previously considered in Sample Prob. 6.5.



SOLUTION

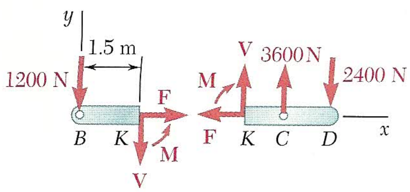
Reactions and Forces at Connections. The reactions and the forces acting on each member of the frame are determined; this has been previously done in Sample Prob. 6.5, and the results are repeated here.



***a.* Internal Forces at *J*.** Member *ACF* is cut at point *J*, and the two parts shown are obtained. The internal forces at *J* are represented by an equivalent force-couple system and can be determined by considering the equilibrium of either part. Considering the *free body AJ*, we write

$$+\circlearrowleft \Sigma M_J = 0: \qquad -(1800 \text{ N})(1.2 \text{ m}) + M = 0$$
$$M = +2160 \text{ N} \cdot \text{m} \qquad \mathbf{M} = 2160 \text{ N} \cdot \text{m} \circlearrowleft \blacktriangleleft$$
$$+\searrow \Sigma F_x = 0: \qquad F - (1800 \text{ N}) \cos 41.7° = 0$$
$$F = +1344 \text{ N} \qquad \mathbf{F} = 1344 \text{ N} \searrow \blacktriangleleft$$
$$+\nearrow \Sigma F_y = 0: \qquad -V + (1800 \text{ N}) \sin 41.7° = 0$$
$$V = +1197 \text{ N} \qquad \mathbf{V} = 1197 \text{ N} \swarrow \blacktriangleleft$$

The internal forces at *J* are therefore equivalent to a couple **M**, an axial force **F**, and a shearing force **V**. The internal force-couple system acting on part *JCF* is equal and opposite.



***b.* Internal Forces at *K*.** We cut member *BCD* at *K* and obtain the two parts shown. Considering the *free body BK*, we write

$$+\circlearrowleft \Sigma M_K = 0: \qquad (1200 \text{ N})(1.5 \text{ m}) + M = 0$$
$$M = -1800 \text{ N} \cdot \text{m} \qquad \mathbf{M} = 1800 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$
$$\xrightarrow{+} \Sigma F_x = 0: \qquad F = 0 \qquad \mathbf{F} = 0 \blacktriangleleft$$
$$+\uparrow \Sigma F_y = 0: \qquad -1200 \text{ N} - V = 0$$
$$V = -1200 \text{ N} \qquad \mathbf{V} = 1200 \text{ N} \uparrow \blacktriangleleft$$