*10.5. REAL MACHINES. MECHANICAL EFFICIENCY

In analyzing the toggle vise in the preceding section, we assumed that no friction forces were involved. Thus, the virtual work consisted only of the work of the applied force **P** and of the reaction **Q**. But the work of the reaction **Q** is equal in magnitude and opposite in sign to the work of the force exerted by the vise on the block. Equation (10.5), therefore, expresses that the *output work* $2Ql \cos \theta \, \delta\theta$ is equal to the *input work* $Pl \sin \theta \, \delta\theta$. A machine in which input and output work are equal is said to be an "ideal" machine. In a "real" machine, friction forces will always do some work, and the output work will be smaller than the input work.

Consider, for example, the toggle vise of Fig. 10.7a, and assume now that a friction force **F** develops between the sliding block B and the horizontal plane (Fig. 10.9). Using the conventional methods of statics and summing moments about A, we find $N = P/2$. Denoting by μ the coefficient of friction between block B and the horizontal
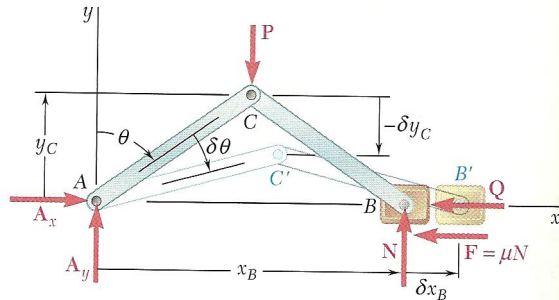

Fig. 10.9

plane, we have $F = \mu N = \mu P/2$. Recalling formulas (10.4), we find that the total virtual work of the forces $\mathbf{Q}$, $\mathbf{P}$, and $\mathbf{F}$ during the virtual displacement shown in Fig. 10.9 is

$$\begin{aligned} \delta U &= -Q\, \delta x_B - P\, \delta y_C - F\, \delta x_B \\ &= -2Ql \cos \theta\, \delta\theta + Pl \sin \theta\, \delta\theta - \mu Pl \cos \theta\, \delta\theta \end{aligned}$$

Setting $\delta U = 0$, we obtain

$$2Ql \cos \theta\, \delta\theta = Pl \sin \theta\, \delta\theta - \mu Pl \cos \theta\, \delta\theta \qquad (10.7)$$

which expresses that the output work is equal to the input work minus the work of the friction force. Solving for Q, we have

$$Q = \tfrac{1}{2}P(\tan \theta - \mu) \qquad (10.8)$$

We note that $Q = 0$ when $\tan \theta = \mu$, that is, when θ is equal to the angle of friction ϕ, and that $Q < 0$ when $\theta < \phi$. The toggle vise may thus be used only for values of θ larger than the angle of friction.

The *mechanical efficiency* of a machine is defined as the ratio

$$\eta = \frac{\text{output work}}{\text{input work}} \qquad (10.9)$$

Clearly, the mechanical efficiency of an ideal machine is $\eta = 1$, since input and output work are then equal, while the mechanical efficiency of a real machine will always be less than 1.

In the case of the toggle vise we have just analyzed, we write

$$\eta = \frac{\text{output work}}{\text{input work}} = \frac{2Ql \cos \theta\, \delta\theta}{Pl \sin \theta\, \delta\theta}$$

Substituting from Eq. (10.8) for Q, we obtain

$$\eta = \frac{P(\tan \theta - \mu)l \cos \theta\, \delta\theta}{Pl \sin \theta\, \delta\theta} = 1 - \mu \cot \theta \qquad (10.10)$$

We check that in the absence of friction forces, we would have $\mu = 0$ and $\eta = 1$. In the general case, when μ is different from zero, the efficiency η becomes zero for $\mu \cot \theta = 1$, that is, for $\tan \theta = \mu$, or $\theta = \tan^{-1} \mu = \phi$. We note again that the toggle vise can be used, only for values of θ larger than the angle of friction ϕ.