9.4. POLAR MOMENT OF INERTIA

An integral of great importance in problems concerning the torsion of cylindrical shafts and in problems dealing with the rotation of slabs is

$$J_O = \int r^2 \, dA \tag{9.3}$$

where r is the distance from O to the element of area dA (Fig. 9.6). This integral is the *polar moment of inertia* of the area A with respect *to the "pole"* O.

The polar moment of inertia of a given area can be computed from the rectangular moments of inertia I_x and I_y of the area if these quantities are already known. Indeed, noting that $r^2 = x^2 + y^2$, we write

$$J_O = \int r^2 \, dA = \int (x^2 + y^2) \, dA = \int y^2 \, dA + \int x^2 \, dA$$

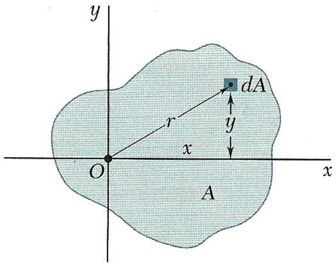
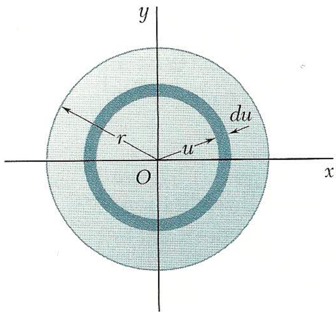


Fig. 9.6

SAMPLE PROBLEM 9.2

(a) Determine the centroidal polar moment of inertia of a circular area by direct integration. (b) Using the result of part a, determine the moment of inertia of a circular area with respect to a diameter.



SOLUTION

a. Polar Moment of Inertia. An annular differential element of area is chosen to be dA. Since all portions of the differential area are at the same distance from the origin, we write

$$dJ_O = u^2\, dA \qquad dA = 2\pi u\, du$$

$$J_O = \int dJ_O = \int_0^r u^2(2\pi u\, du) = 2\pi \int_0^r u^3\, du$$

$$J_O = \frac{\pi}{2} r^4 \blacktriangleleft$$

b. Moment of Inertia with Respect to a Diameter. Because of the symmetry of the circular area, we have $I_x = I_y$. We then write

$$J_O = I_x + I_y = 2I_x \qquad \frac{\pi}{2} r^4 = 2I_x \qquad I_{\text{diameter}} = I_x = \frac{\pi}{4} r^4 \blacktriangleleft$$