11-4.1 Elastic Potential Energy

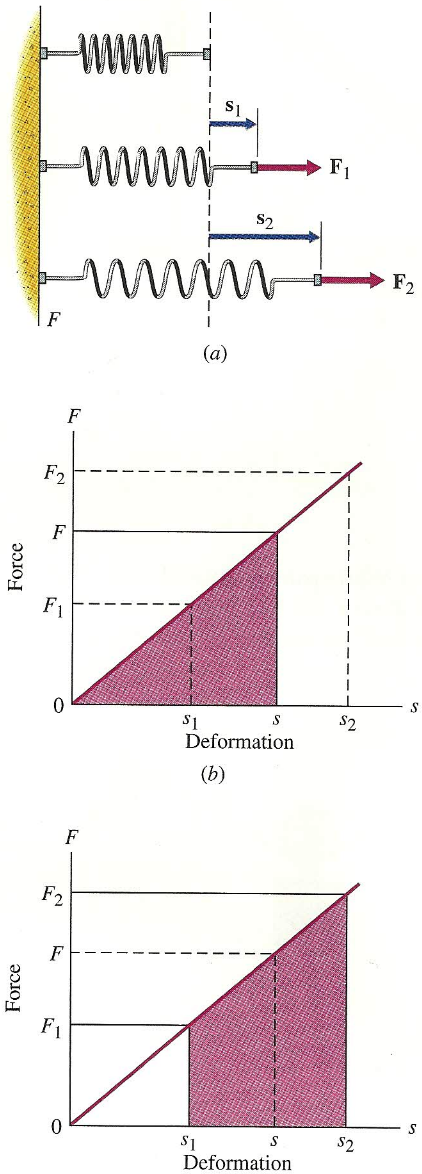

Figure 11-11 Deformation of a spring under the action of a force **F**.

A deformable body that changes shape under load but resumes its original shape when the loads are removed is known as an elastic body. The spring shown in Fig. 11-11*a* is an example of an elastic body that is widely used in engineering applications. When a tensile force is applied to the spring, the length of the spring increases. Similarly, when a compressive force is applied to the spring, the length decreases. The magnitude F of the force that must be applied to the spring to stretch it by an amount s is given by the expression

$$F = ks \tag{11-18}$$

where s is the deformation of the spring from its unloaded position and k is a constant known as the modulus of the spring. Any spring whose behavior is governed by Eq. 11-18 is an ideal linear elastic spring.

The work done in stretching an ideal spring from an initial unstretched position to a stretched position s can be determined from Eq. 11-1. Since the force F and displacement s are in the same direction, the angle α is zero and Eq. 11-1 becomes

$$U = \int_0^s ks\, ds = \frac{1}{2} ks^2 \tag{11-19}$$

For this case, the work done by the force F as it stretches the spring can be represented as the shaded triangular area under the load versus deformation curve shown in Fig. 11-11*b*. This area also represents the elastic potential energy V_e stored in the spring as a result of the change in shape of the spring.

In a similar manner, the work done in stretching an ideal spring from an initial position s_1 to a further extended position s_2 can be determined from Eq. 11-1. Thus,

$$U = V_e = \int_{s_1}^{s_2} ks\, ds = \frac{1}{2} k(s_2^2 - s_1^2) \tag{11-20}$$

In this case, the work done by the force F as it stretches the spring can be represented as the shaded trapezoidal area under the load versus deformation curve shown in Fig. 11-11*c*. It is important to note here that Eq. 11-19 is valid only if the deflection of the spring is measured from its undeformed position.

As a spring is being deformed (either stretched or compressed), the force on the spring and the displacement are in the same direction; therefore, the work done on the spring is positive, which increases its potential energy. If a spring is initially deformed and gradually released, the force and displacement are in opposite directions, the work is negative, and the potential energy decreases. The force of a spring on a body is opposite to the force of the body on the spring. Since both the body and the end of the spring have the same displacement, the work done by the spring on the body results in an equal decrease in the potential energy of the spring.

A torsional spring, which is used to resist rotation rather than linear displacement, can also store and release potential energy. The magnitude of the torque (twisting moment) T that must be applied to a torsional spring to produce a rotation θ is given by the expression

$$T = k\theta \qquad \text{(11-21)}$$

where T is the torque, θ is the angular deformation of the spring from its unloaded position, and k is a constant known as the modulus of the spring. The work U done by the torque T, which is equal to the potential energy V_e stored in the spring, is given by the expression

$$U = V_e = \int_0^\theta T\,d\theta = \int_0^\theta k\theta\,d\theta = \frac{1}{2}k\theta^2 \qquad \text{(11-22)}$$

which is analogous to the expression for the linear spring.

During a virtual displacement δs of a spring, the virtual work δU done on the spring and the change in virtual elastic potential energy δV_e of the spring are given by the expression

$$\delta U = \delta V_e = F\,\delta s = ks\,\delta s \qquad \text{(11-23)}$$