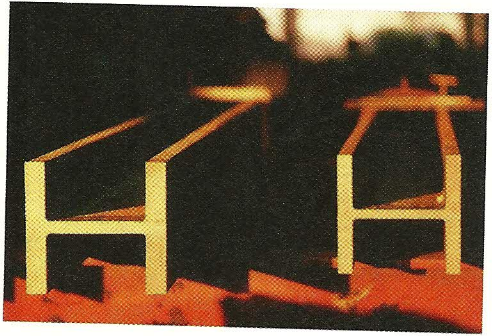


Photo 9.1 Figure 9.13 tabulates data for a small sample of the rolled-steel shapes that are readily available. Shown above are two examples of wide-flange shapes that are commonly used in the construction of buildings.

9.7. MOMENTS OF INERTIA OF COMPOSITE AREAS

Consider a composite area A made of several component areas A_1, A_2, A_3, . . . Since the integral representing the moment of inertia of A can be subdivided into integrals evaluated over A_1, A_2, A_3, . . . , the moment of inertia of A with respect to a given axis is obtained by adding the moments of inertia of the areas A_1, A_2, A_3, . . . with respect to the same axis. The moment of inertia of an area consisting of several of the common shapes shown in Fig. 9.12 can thus be obtained by using the formulas given in that figure. Before adding the moments of inertia of the component areas, however, the parallel-axis theorem may have to be used to transfer each moment of inertia to the desired axis. This is shown in Sample Probs. 9.4 and 9.5.

The properties of the cross sections of various structural shapes are given in Fig. 9.13. As noted in Sec. 9.2, the moment of inertia of a beam section about its neutral axis is closely related to the computation of the bending moment in that section of the beam. The determination of moments of inertia is thus a prerequisite to the analysis and design of structural members.

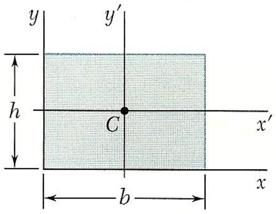
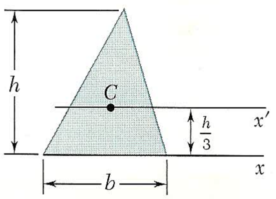
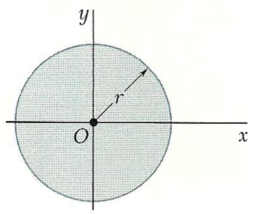
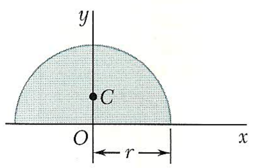
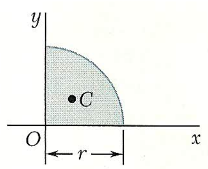
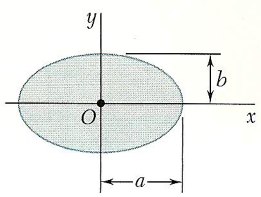
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Fig. 9.12 Moments of inertia of common geometric shapes.

It should be noted that the radius of gyration of a composite area is *not* equal to the sum of the radii of gyration of the component areas. In order to determine the radius of gyration of a composite area, it is first necessary to compute the moment of inertia of the area.

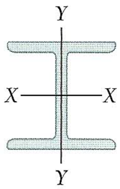
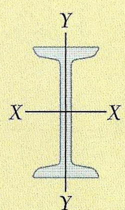
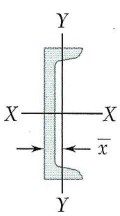
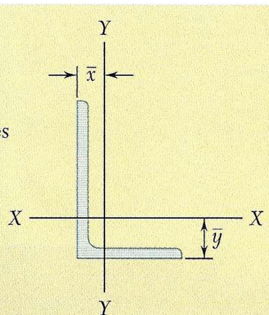
		Designation	Area in^2	Depth in.	Width in.	Axis X-X $\bar{I}_x$, in^4	$\bar{k}_x$, in.	$\bar{y}$, in.	Axis Y-Y $\bar{I}_y$, in^4	$\bar{k}_y$, in.	$\bar{x}$, in.
W Shapes (Wide-Flange Shapes)		W18 × 50†	14.7	17.99	7.495	800	7.38		40.1	1.65	
		W16 × 40	11.8	16.01	6.995	518	6.63		28.9	1.57	
		W14 × 30	8.85	13.84	6.730	291	5.73		19.6	1.49	
		W8 × 24	7.08	7.93	6.495	82.8	3.42		18.3	1.61	
S Shapes (American Standard Shapes)		S18 × 70†	20.6	18.00	6.251	926	6.71		24.1	1.08	
		S12 × 50	14.7	12.00	5.477	305	4.55		15.7	1.03	
		S10 × 35	10.3	10.00	4.944	147	3.78		8.36	0.901	
		S6 × 17.25	5.07	6.00	3.565	26.3	2.28		2.31	0.675	
C Shapes (American Standard Channels)		C12 × 25†	7.35	12.00	3.047	144	4.43		4.47	0.780	0.674
		C10 × 20	5.88	10.00	2.739	78.9	3.66		2.81	0.692	0.606
		C8 × 13.75	4.04	8.00	2.343	36.1	2.99		1.53	0.615	0.553
		C6 × 10.5	3.09	6.00	2.034	15.2	2.22		0.866	0.529	0.499
Angles		L6 × 6 × $\frac{3}{4}$‡	8.44			28.2	1.83	1.78	28.2	1.83	1.78
		L4 × 4 × $\frac{1}{2}$	3.75			5.56	1.22	1.18	5.56	1.22	1.18
		L3 × 3 × $\frac{1}{4}$	1.44			1.24	0.930	0.842	1.24	0.930	0.842
		L6 × 4 × $\frac{1}{2}$	4.75			17.4	1.91	1.99	6.27	1.15	0.987
		L5 × 3 × $\frac{1}{2}$	3.75			9.45	1.59	1.75	2.58	0.829	0.750
		L3 × 2 × $\frac{1}{4}$	1.19			1.09	0.957	0.993	0.392	0.574	0.493

Fig. 9.13A Properties of Rolled-Steel Shapes (U.S. Customary Units).*

*Courtesy of the American Institute of Steel Construction, Chicago, Illinois

†Nominal depth in inches and weight in pounds per foot

‡Depth, width, and thickness in inches

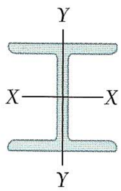
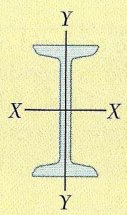
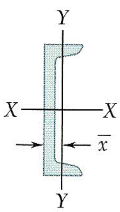
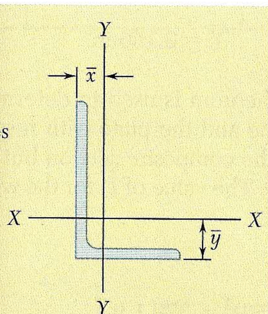
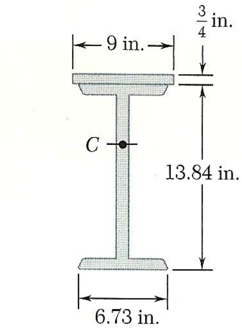
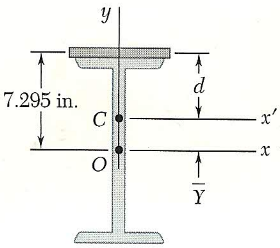
	Designation	Area mm^2	Depth mm	Width mm	Axis X-X $\bar{I}_x$ 10^6 mm^4	$\bar{k}_x$ mm	$\bar{y}$ mm	Axis Y-Y $\bar{I}_y$ 10^6 mm^4	$\bar{k}_y$ mm	$\bar{x}$ mm
W Shapes (Wide-Flange Shapes) 	W460 × 74†	9450	457	190	333	188		16.6	41.9	
	W410 × 60	7580	407	178	216	169		12.1	40.0	
	W360 × 44	5730	352	171	122	146		8.18	37.8	
	W200 × 35.9	4580	201	165	34.4	86.7		7.64	40.8	
S Shapes (American Standard Shapes) 	S460 × 104†	13300	457	159	385	170		10.4	27.5	
	S310 × 74	9480	305	139	126	115		6.69	26.1	
	S250 × 52	6670	254	126	61.2	95.8		3.59	22.9	
	S150 × 25.7	3270	152	91	10.8	57.5		1.00	17.2	
C Shapes (American Standard Channels) 	C310 × 37†	5690	305	77	59.7	112		1.83	19.7	17.0
	C250 × 30	3780	254	69	32.6	92.9		1.14	17.4	15.3
	C200 × 27.9	3560	203	64	18.2	71.5		0.817	15.1	14.3
	C150 × 15.6	1980	152	51	6.21	56.0		0.347	13.2	12.5
Angles 	L152 × 152 × 19.0‡	5420			11.6	46.3	44.9	11.6	46.3	44.9
	L102 × 102 × 12.7	2430			2.34	31.0	30.2	2.34	31.0	30.2
	L76 × 76 × 6.4	932			0.517	23.6	21.4	0.517	23.6	21.4
	L152 × 102 × 12.7	3060			7.20	48.5	50.3	2.64	29.4	25.3
	L127 × 76 × 12.7	2420			3.93	40.3	44.4	1.06	20.9	19.0
	L76 × 51 × 6.4	772			0.453	24.2	25.1	0.166	14.7	12.6

Fig. 9.13B Properties of Rolled-Steel Shapes (SI Units).
†Nominal depth in millimeters and mass in kilograms per meter
‡Depth, width, and thickness in millimeters



SAMPLE PROBLEM 9.4

The strength of a W14 × 30 rolled-steel beam is increased by attaching a 9 × $\frac{3}{4}$-in. plate to its upper flange as shown. Determine the moment of inertia and the radius of gyration of the composite section with respect to an axis which is parallel to the plate and passes through the centroid C of the section.



SOLUTION

The origin O of the coordinates is placed at the centroid of the wide-flange shape, and the distance $\overline{Y}$ to the centroid of the composite section is computed using the methods of Chap. 5. The area of the wide-flange shape is found by referring to Fig. 9.13A. The area and the y coordinate of the centroid of the plate are

$$A = (9 \text{ in.})(0.75 \text{ in.}) = 6.75 \text{ in}^2$$
$$\overline{y} = \tfrac{1}{2}(13.84 \text{ in.}) + \tfrac{1}{2}(0.75 \text{ in.}) = 7.295 \text{ in.}$$

Section	Area, in^2	$\overline{y}$, in.	$\overline{y}A$, in^3
Plate	6.75	7.295	49.24
Wide-flange shape	8.85	0	0
	$\Sigma A = 15.60$		$\Sigma\overline{y}A = 49.24$

$$\overline{Y}\Sigma A = \Sigma\overline{y}A \qquad \overline{Y}(15.60) = 49.24 \qquad \overline{Y} = 3.156 \text{ in.}$$

Moment of Inertia. The parallel-axis theorem is used to determine the moments of inertia of the wide-flange shape and the plate with respect to the x' axis. This axis is a centroidal axis for the composite section but *not* for either of the elements considered separately. The value of $\overline{I}_x$ for the wide-flange shape is obtained from Fig. 9.13A.

For the wide-flange shape,

$$I_{x'} = \overline{I}_x + A\overline{Y}^2 = 291 + (8.85)(3.156)^2 = 379.1 \text{ in}^4$$

For the plate,

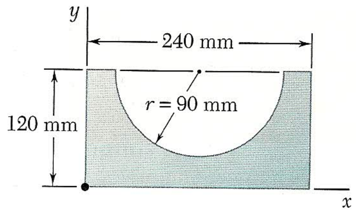
$$I_{x'} = \overline{I}_x + Ad^2 = (\tfrac{1}{12})(9)(\tfrac{3}{4})^3 + (6.75)(7.295 - 3.156)^2 = 116.0 \text{ in}^4$$

For the composite area,

$$I_{x'} = 379.1 + 116.0 = 495.1 \text{ in}^4 \qquad I_{x'} = 495 \text{ in}^4 \blacktriangleleft$$

Radius of Gyration. We have

$$k_{x'}^2 = \frac{I_{x'}}{A} = \frac{495.1 \text{ in}^4}{15.60 \text{ in}^2} \qquad k_{x'} = 5.63 \text{ in.} \blacktriangleleft$$

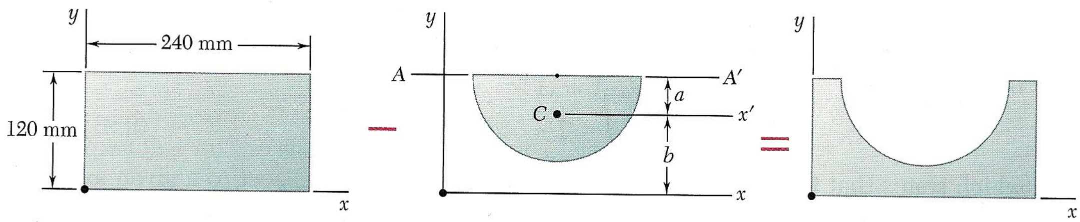


SAMPLE PROBLEM 9.5

Determine the moment of inertia of the shaded area with respect to the x axis.

SOLUTION

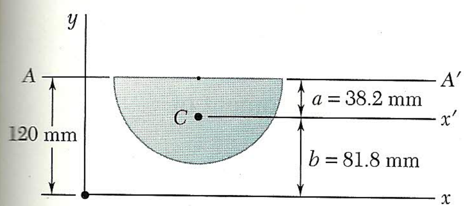
The given area can be obtained by subtracting a half circle from a rectangle. The moments of inertia of the rectangle and the half circle will be computed separately.



Moment of Inertia of Rectangle. Referring to Fig. 9.12, we obtain

$$I_x = \tfrac{1}{3}bh^3 = \tfrac{1}{3}(240 \text{ mm})(120 \text{ mm})^3 = 138.2 \times 10^6 \text{ mm}^4$$

Moment of Inertia of Half Circle. Referring to Fig. 5.8, we determine the location of the centroid C of the half circle with respect to diameter AA'.



$$a = \frac{4r}{3\pi} = \frac{(4)(90 \text{ mm})}{3\pi} = 38.2 \text{ mm}$$

The distance b from the centroid C to the x axis is

$$b = 120 \text{ mm} - a = 120 \text{ mm} - 38.2 \text{ mm} = 81.8 \text{ mm}$$

Referring now to Fig. 9.12, we compute the moment of inertia of the half circle with respect to diameter AA'; we also compute the area of the half circle.

$$I_{AA'} = \tfrac{1}{8}\pi r^4 = \tfrac{1}{8}\pi(90 \text{ mm})^4 = 25.76 \times 10^6 \text{ mm}^4$$
$$A = \tfrac{1}{2}\pi r^2 = \tfrac{1}{2}\pi(90 \text{ mm})^2 = 12.72 \times 10^3 \text{ mm}^2$$

Using the parallel-axis theorem, we obtain the value of $\bar{I}_{x'}$:

$$I_{AA'} = \bar{I}_{x'} + Aa^2$$
$$25.76 \times 10^6 \text{ mm}^4 = \bar{I}_{x'} + (12.72 \times 10^3 \text{ mm}^2)(38.2 \text{ mm})^2$$
$$\bar{I}_{x'} = 7.20 \times 10^6 \text{ mm}^4$$

Again using the parallel-axis theorem, we obtain the value of I_x:

$$I_x = \bar{I}_{x'} + Ab^2 = 7.20 \times 10^6 \text{ mm}^4 + (12.72 \times 10^3 \text{ mm}^2)(81.8 \text{ mm})^2$$
$$= 92.3 \times 10^6 \text{ mm}^4$$

Moment of Inertia of Given Area. Subtracting the moment of inertia of the half circle from that of the rectangle, we obtain

$$I_x = 138.2 \times 10^6 \text{ mm}^4 - 92.3 \times 10^6 \text{ mm}^4$$

$$I_x = 45.9 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

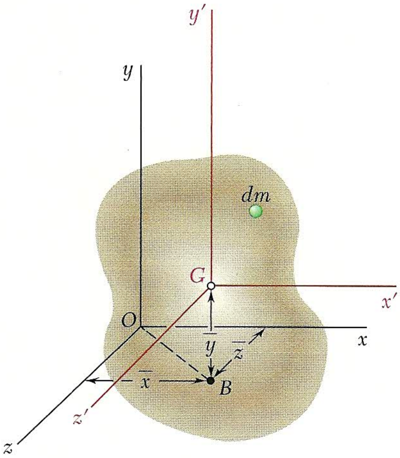


Fig. 9.22

9.12. PARALLEL-AXIS THEOREM

Consider a body of mass m. Let $Oxyz$ be a system of rectangular coordinates whose origin is at the arbitrary point O, and $Gx'y'z'$ a system of parallel *centroidal axes*, that is, a system whose origin is at the center of gravity G of the body† and whose axes x', y', and z' are parallel to the x, y, and z axes, respectively (Fig. 9.22). Denoting by $\bar{x}$, $\bar{y}$, and $\bar{z}$ the coordinates of G with respect to $Oxyz$, we write the following relations between the coordinates x, y, and z of the element dm with respect to $Oxyz$ and its coordinates x', y', and z' with respect to the centroidal axes $Gx'y'z'$:

$$x = x' + \bar{x} \qquad y = y' + \bar{y} \qquad z = z' + \bar{z} \tag{9.31}$$

Referring to Eqs. (9.30), we can express the moment of inertia of the body with respect to the x axis as follows:

$$\begin{aligned} I_x &= \int (y^2 + z^2)\, dm = \int [(y' + \bar{y})^2 + (z' + \bar{z})^2]\, dm \\ &= \int (y'^2 + z'^2)\, dm + 2\bar{y} \int y'\, dm + 2\bar{z} \int z'\, dm + (\bar{y}^2 + \bar{z}^2) \int dm \end{aligned}$$

The first integral in this expression represents the moment of inertia $\bar{I}_{x'}$ of the body with respect to the centroidal axis x'; the second and third integrals represent the first moment of the body with respect to the $z'x'$ and $x'y'$ planes, respectively, and, since both planes contain G, the two integrals are *zero;* the last integral is equal to the total mass m of the body. We write, therefore,

$$I_x = \bar{I}_{x'} + m(\bar{y}^2 + \bar{z}^2) \tag{9.32}$$

and, similarly,

$$I_y = \bar{I}_{y'} + m(\bar{z}^2 + \bar{x}^2) \qquad I_z = \bar{I}_{z'} + m(\bar{x}^2 + \bar{y}^2) \tag{9.32$'$}$$

We easily verify from Fig. 9.22 that the sum $\bar{z}^2 + \bar{x}^2$ represents the square of the distance OB, between the y and y' axes. Similarly, $\bar{y}^2 + \bar{z}^2$ and $\bar{x}^2 + \bar{y}^2$ represent the squares of the distance between the x and x' axes and the z and z' axes, respectively. Denoting by d the distance between an arbitrary axis AA' and the parallel centroidal axis BB' (Fig. 9.23), we can, therefore, write the following general relation between the moment of inertia I of the body with respect to AA' and its moment of inertia $\bar{I}$ with respect to BB':

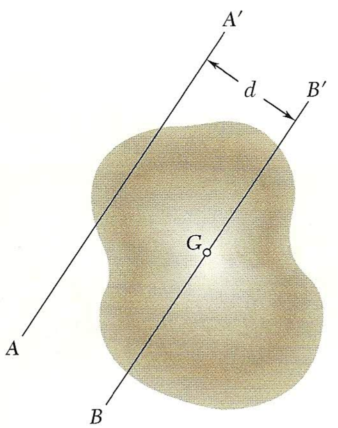


Fig. 9.23

$$I = \bar{I} + md^2 \tag{9.33}$$

Expressing the moments of inertia in terms of the corresponding radii of gyration, we can also write

$$k^2 = \bar{k}^2 + d^2 \tag{9.34}$$

where k and $\bar{k}$ represent the radii of gyration of the body about AA' and BB', respectively.

†Note that the term *centroidal* is used here to define an axis passing through the center of gravity G of the body, whether or not G coincides with the centroid of the volume of the body.

9.15. MOMENTS OF INERTIA OF COMPOSITE BODIES

The moments of inertia of a few common shapes are shown in Fig. 9.28. For a body consisting of several of these simple shapes, the moment of inertia of the body with respect to a given axis can be obtained by first computing the moments of inertia of its component parts about the desired axis and then adding them together. As was the case for areas, the radius of gyration of a composite body *cannot* be obtained by adding the radii of gyration of its component parts.

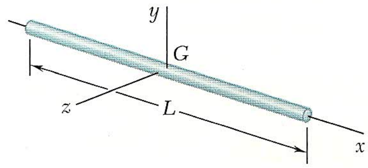
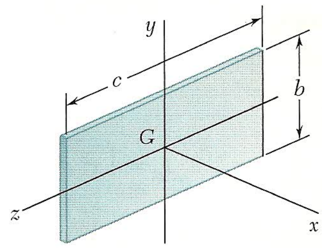
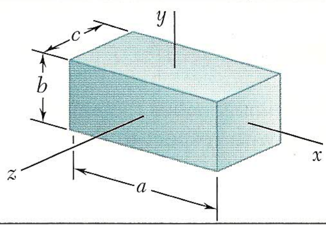
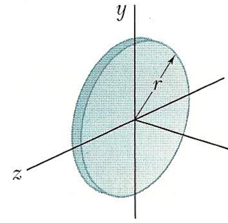
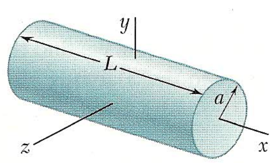
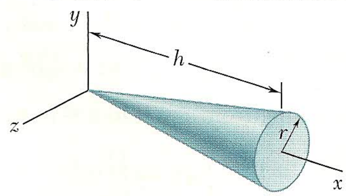
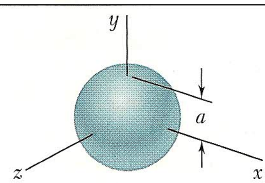
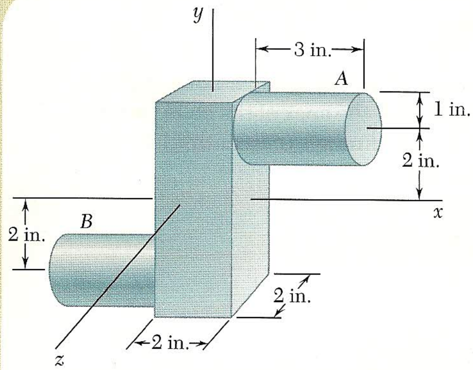
Slender rod		$I_y = I_z = \frac{1}{12}mL^2$
Thin rectangular plate		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}mc^2$ $I_z = \frac{1}{12}mb^2$
Rectangular prism		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}m(c^2 + a^2)$ $I_z = \frac{1}{12}m(a^2 + b^2)$
Thin disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$
Circular cylinder		$I_x = \frac{1}{2}ma^2$ $I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$
Circular cone		$I_x = \frac{3}{10}ma^2$ $I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$
Sphere		$I_x = I_y = I_z = \frac{2}{5}ma^2$

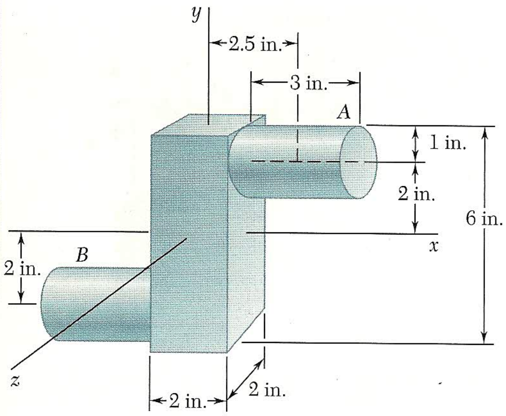
Fig. 9.28 Mass moments of inertia of common geometric shapes.

SAMPLE PROBLEM 9.12



A steel forging consists of a 6 × 2 × 2-in. rectangular prism and two cylinders of diameter 2 in. and length 3 in. as shown. Determine the mass moments of inertia of the forging with respect to the coordinate axes knowing that the specific weight of steel is 490 lb/ft^3.

SOLUTION



Computation of Masses

Prism

$$V = (2 \text{ in.})(2 \text{ in.})(6 \text{ in.}) = 24 \text{ in}^3$$

$$W = (24 \text{ in}^3)(490 \text{ lb/ft}^3)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 = 6.81 \text{ lb}$$

$$m = \frac{6.81 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.211 \text{ lb} \cdot \text{s}^2\text{/ft}$$

Each Cylinder

$$V = \pi(1 \text{ in.})^2(3 \text{ in.}) = 9.42 \text{ in}^3$$

$$W = (9.42 \text{ in}^3)(490 \text{ lb/ft}^3)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 = 2.67 \text{ lb}$$

$$m = \frac{2.67 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.0829 \text{ lb} \cdot \text{s}^2\text{/ft}$$

Mass Moments of Inertia. The mass moments of inertia of each component are computed from Fig. 9.28, using the parallel-axis theorem when necessary. Note that all lengths are expressed in feet.

Prism

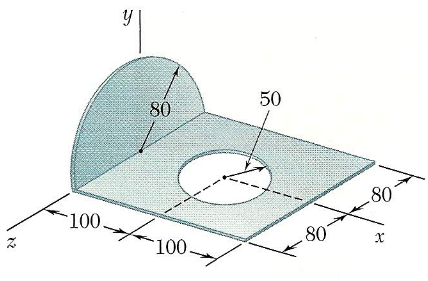
$$I_x = I_z = \tfrac{1}{12}(0.211 \text{ lb} \cdot \text{s}^2\text{/ft})[(\tfrac{6}{12} \text{ ft})^2 + (\tfrac{2}{12} \text{ ft})^2] = 4.88 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$
$$I_y = \tfrac{1}{12}(0.211 \text{ lb} \cdot \text{s}^2\text{/ft})[(\tfrac{2}{12} \text{ ft})^2 + (\tfrac{2}{12} \text{ ft})^2] = 0.977 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Each Cylinder

$$I_x = \tfrac{1}{2}ma^2 + m\bar{y}^2 = \tfrac{1}{2}(0.0829 \text{ lb} \cdot \text{s}^2\text{/ft})(\tfrac{1}{12} \text{ ft})^2 + (0.0829 \text{ lb} \cdot \text{s}^2\text{/ft})(\tfrac{2}{12} \text{ ft})^2$$
$$= 2.59 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$
$$I_y = \tfrac{1}{12}m(3a^2 + L^2) + m\bar{x}^2 = \tfrac{1}{12}(0.0829 \text{ lb} \cdot \text{s}^2\text{/ft})[3(\tfrac{1}{12} \text{ ft})^2 + (\tfrac{3}{12} \text{ ft})^2]$$
$$+ (0.0829 \text{ lb} \cdot \text{s}^2\text{/ft})(\tfrac{2.5}{12} \text{ ft})^2 = 4.17 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$
$$I_z = \tfrac{1}{12}m(3a^2 + L^2) + m(\bar{x}^2 + \bar{y}^2) = \tfrac{1}{12}(0.0829 \text{ lb} \cdot \text{s}^2\text{/ft})[3(\tfrac{1}{12} \text{ ft})^2 + (\tfrac{3}{12} \text{ ft})^2]$$
$$+ (0.0829 \text{ lb} \cdot \text{s}^2\text{/ft})[(\tfrac{2.5}{12} \text{ ft})^2 + (\tfrac{2}{12} \text{ ft})^2] = 6.48 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Entire Body. Adding the values obtained,

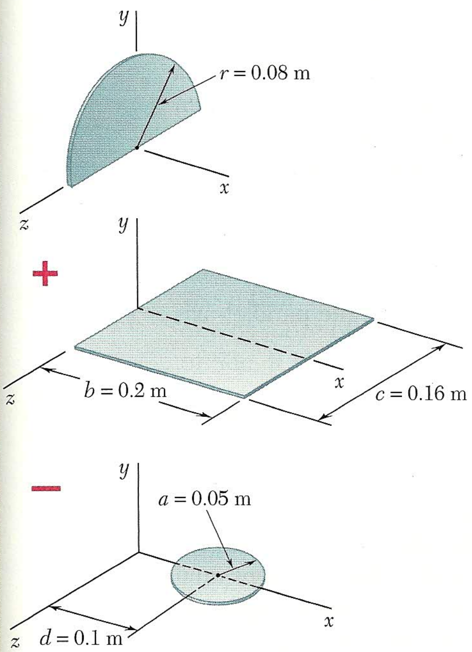
$$I_x = 4.88 \times 10^{-3} + 2(2.59 \times 10^{-3}) \qquad I_x = 10.06 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$
$$I_y = 0.977 \times 10^{-3} + 2(4.17 \times 10^{-3}) \qquad I_y = 9.32 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$
$$I_z = 4.88 \times 10^{-3} + 2(6.48 \times 10^{-3}) \qquad I_z = 17.84 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$



Dimensions in mm

SAMPLE PROBLEM 9.13

A thin steel plate which is 4 mm thick is cut and bent to form the machine part shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass moments of inertia of the machine part with respect to the coordinate axes.



SOLUTION

We observe that the machine part consists of a semicircular plate and a rectangular plate from which a circular plate has been removed.

Computation of Masses. ***Semicircular Plate***

$$V_1 = \tfrac{1}{2}\pi r^2 t = \tfrac{1}{2}\pi(0.08\text{ m})^2(0.004\text{ m}) = 40.21 \times 10^{-6}\text{ m}^3$$
$$m_1 = \rho V_1 = (7.85 \times 10^3\text{ kg/m}^3)(40.21 \times 10^{-6}\text{ m}^3) = 0.3156\text{ kg}$$

Rectangular Plate

$$V_2 = (0.200\text{ m})(0.160\text{ m})(0.004\text{ m}) = 128 \times 10^{-6}\text{ m}^3$$
$$m_2 = \rho V_2 = (7.85 \times 10^3\text{ kg/m}^3)(128 \times 10^{-6}\text{ m}^3) = 1.005\text{ kg}$$

Circular Plate

$$V_3 = \pi a^2 t = \pi(0.050\text{ m})^2(0.004\text{ m}) = 31.42 \times 10^{-6}\text{ m}^3$$
$$m_3 = \rho V_3 = (7.85 \times 10^3\text{ kg/m}^3)(31.42 \times 10^{-6}\text{ m}^3) = 0.2466\text{ kg}$$

Mass Moments of Inertia. Using the method presented in Sec. 9.13, we compute the mass moments of inertia of each component.

Semicircular Plate. From Fig. 9.28, we observe that for a circular plate of mass m and radius r

$$I_x = \tfrac{1}{2}mr^2 \qquad I_y = I_z = \tfrac{1}{4}mr^2$$

Because of symmetry, we note that for a semicircular plate

$$I_x = \tfrac{1}{2}(\tfrac{1}{2}mr^2) \qquad I_y = I_z = \tfrac{1}{2}(\tfrac{1}{4}mr^2)$$

Since the mass of the semicircular plate is $m_1 = \frac{1}{2}m$, we have

$$I_x = \tfrac{1}{2}m_1 r^2 = \tfrac{1}{2}(0.3156\text{ kg})(0.08\text{ m})^2 = 1.010 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$
$$I_y = I_z = \tfrac{1}{4}(\tfrac{1}{2}mr^2) = \tfrac{1}{4}m_1 r^2 = \tfrac{1}{4}(0.3156\text{ kg})(0.08\text{ m})^2 = 0.505 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Rectangular Plate

$$I_x = \tfrac{1}{12}m_2 c^2 = \tfrac{1}{12}(1.005\text{ kg})(0.16\text{ m})^2 = 2.144 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$
$$I_z = \tfrac{1}{3}m_2 b^2 = \tfrac{1}{3}(1.005\text{ kg})(0.2\text{ m})^2 = 13.400 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$
$$I_y = I_x + I_z = (2.144 + 13.400)(10^{-3}) = 15.544 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Circular Plate

$$I_x = \tfrac{1}{4}m_3 a^2 = \tfrac{1}{4}(0.2466\text{ kg})(0.05\text{ m})^2 = 0.154 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$
$$I_y = \tfrac{1}{2}m_3 a^2 + m_3 d^2$$
$$= \tfrac{1}{2}(0.2466\text{ kg})(0.05\text{ m})^2 + (0.2466\text{ kg})(0.1\text{ m})^2 = 2.774 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$
$$I_z = \tfrac{1}{4}m_3 a^2 + m_3 d^2 = \tfrac{1}{4}(0.2466\text{ kg})(0.05\text{ m})^2 + (0.2466\text{ kg})(0.1\text{ m})^2$$
$$= 2.620 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Entire Machine Part

$$I_x = (1.010 + 2.144 - 0.154)(10^{-3})\text{ kg}\cdot\text{m}^2 \qquad I_x = 3.00 \times 10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$
$$I_y = (0.505 + 15.544 - 2.774)(10^{-3})\text{ kg}\cdot\text{m}^2 \qquad I_y = 13.28 \times 10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$
$$I_z = (0.505 + 13.400 - 2.620)(10^{-3})\text{ kg}\cdot\text{m}^2 \qquad I_z = 11.29 \times 10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$