9-3.2 Square-Threaded Screws

Square-threaded screws are essentially wedges that have been wound around a cylindrical shaft. These simple devices can be found in nearly every facet of our lives. Screws are used as fasteners to hold machinery together. Screws are used in jacks to raise heavy loads and on the feet of heavy appliances such as refrigerators to level them. Screws are also used in vises and clamps to squeeze objects together. In each of these cases and many more like them, friction on the threads keeps the screws from turning and loosening.

For example, consider the simple C-clamp of Fig. 9-21. When a twisting moment M is applied to the screw, the clamp tightens and exerts an axial force W on whatever is held in the clamp. As the screw turns and tightens, however, a small segment of the screw's thread will travel around and up the groove in the frame (Fig. 9-22). The distance that the screw moves in the axial direction during one revolution (from point A to A') is called the lead of the screw. For a single-threaded screw, the lead is the same as the distance between the adjacent threads (Fig. 9-23). If a screw has two independent threads that wind around it, the lead would be twice the distance between the adjacent threads

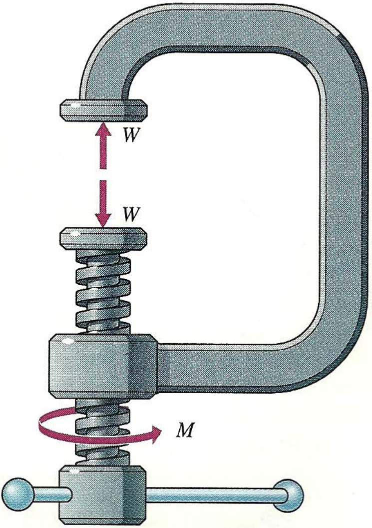

Figure 9-21 Twisting moment applied to the screw of a simple C-clamp.

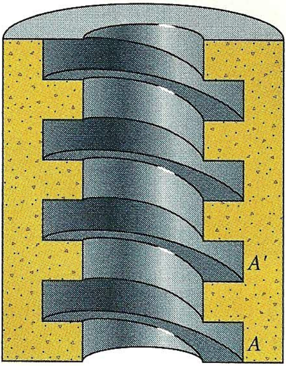

Figure 9-22 Lead for a single-threaded screw.

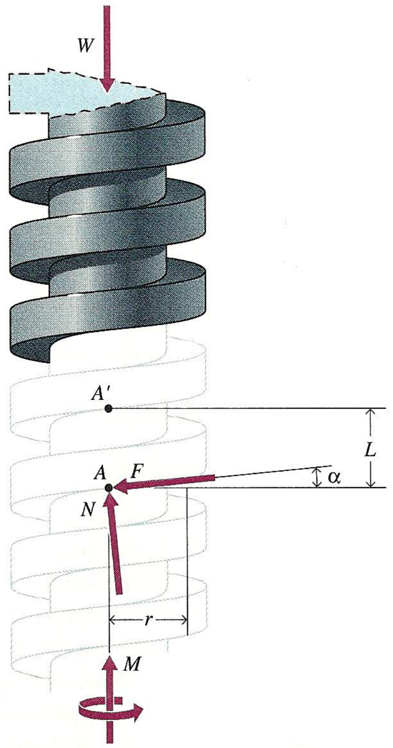

Figure 9-23 Normal and friction forces on a screw.

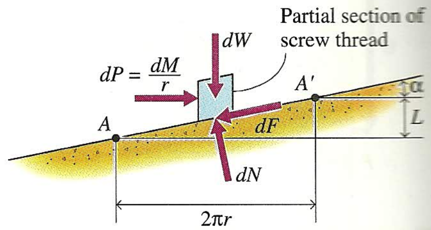

Figure 9-24 Free-body diagram for a partial section of a screw.

During each complete turn of the screw a small segment of the screw's thread will travel a distance $2\pi r$ around the shaft while advancing the distance L. It is as if the small segment of the screw thread is being pushed up a wedge or inclined plane of angle $\alpha = \tan^{-1}(L/2\pi r)$ (Fig. 9-24). In addition to the normal and friction forces on the thread, the free-body diagram for a typical segment of the screw includes a portion of the axial force dW and a force dP due to the twisting moment, $dM = r\ dP$. If the equilibrium equations for each little segment of the screw are added together, the resulting set of equations would be the same as the equilibrium equations for the free-body diagram shown in Fig. 9-25 in which $W = \int dW$ is the total axial force, $P = \int dP = (\int dM/r) = M/r$ is the total pushing force due to the moment M, and $F = \int dF$ and $N = \int dN$ are the total friction and normal forces, respectively.

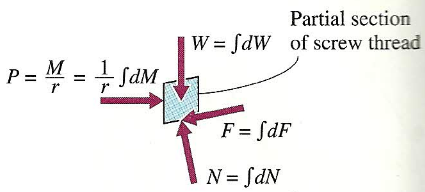

Figure 9-25 Free-body diagram for a screw being advanced.

If the twisting moment M is just sufficient to turn the screw, then R, the resultant of the friction and normal forces, will act at the angle of static friction ϕ_s to the normal and force equilibrium is neatly expressed by the force triangle (Fig. 9-26) from which

$$\tan(\alpha + \phi_s) = \frac{M/r}{W}$$

Therefore,

$$M = r\,W \tan(\alpha + \phi_s) \tag{9-7}$$

is the minimum torque M necessary to advance the screw against a load W.

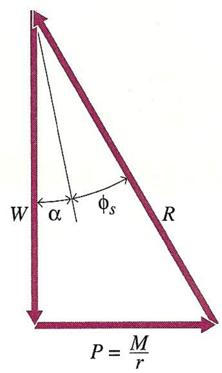

Figure 9-26 Force triangle when the minimum moment needed to advance the screw is being applied.

When the twisting moment M is removed or reduced to near zero, the screw will tend to unwind and the friction force will change direction (Fig. 9-27). If the lead angle α is greater than the angle of static friction ϕ_s, then the screw will not be in equilibrium when the twisting moment is removed, but will require a twisting moment of

$$M = rW \tan(\alpha - \phi_s) \tag{9-8a}$$

to maintain equilibrium (Fig. 9-28*a*). However, if the lead angle α is less than the angle of static friction ϕ_s, then the screw will be in equilibrium even when the twisting moment is removed. This condition is called self-locking and is a design criterion in most screw designs. In this case a reverse moment of

$$M = rW \tan(\phi_s - \alpha) \tag{9-8b}$$

is required to remove the screw (Fig. 9-28*b*).

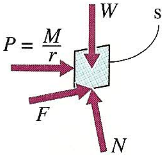

Figure 9-27 Free-body diagram for a screw as it unwinds.

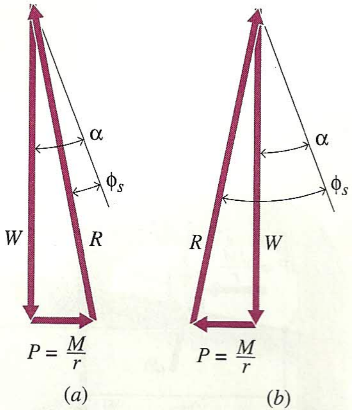

Figure 9-28 Free-body diagrams for a screw when the lead angle is greater than or less than the angle of static friction.

EXAMPLE PROBLEM 9-10

In the C-clamp of Fig. 9-21, the screw has a mean radius of 3 mm and a single thread with a pitch of 2 mm. If the coefficient of friction is 0.2, determine

a. The minimum twisting moment necessary to produce a clamping force of 600 N.
b. The minimum twisting moment necessary to release the clamp when the clamping force is 600 N.
c. The minimum coefficient of friction for which the clamp is self-locking.

SOLUTION

The screw has a single thread, so the lead is equal to the pitch

$$\alpha = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left(\frac{2}{6\pi}\right) = 6.06°$$

The distance a screw advances during one turn is called the lead L of the screw.

The pitch angle α for a screw can be determined by using the triangle formed when a single thread is visualized as being unwound for one turn of the screw.

The angle of static friction is

$$\phi_s = \tan^{-1} 0.2 = 11.31°$$

a. Since the twisting moment is just sufficient to tighten the screw, the free-body diagram and force triangle of Figs. 9-25 and 9-26 apply and

$$\begin{aligned} M &= rW \tan(\alpha + \phi_s) = 0.003(600) \tan 17.37° \\ &= 0.563 \text{ N} \cdot \text{m} \end{aligned} \qquad \text{Ans.}$$

b. Now the twisting moment is just sufficient to loosen the screw, so the free-body diagram and force triangle of Figs. 9-27 and 9-28*b* apply, and

$$\begin{aligned} M &= rW \tan(\phi_s - \alpha) = 0.003(600) \tan 5.25° \\ &= 0.1654 \text{ N} \cdot \text{m} \end{aligned} \qquad \text{Ans.}$$

c. When $M = 0$, $\phi_s \geq \phi = \alpha$. The minimum coefficient of friction corresponds to $\phi = \phi_s = 6.06°$, and hence

$$\mu_s = \tan 6.06° = 0.106 \qquad \text{Ans.}$$