FRAMES AND MACHINES

6.9. STRUCTURES CONTAINING MULTIFORCE MEMBERS

Under trusses, we have considered structures consisting entirely of pins and straight two-force members. The forces acting on the two-force members were known to be directed along the members themselves. We now consider structures in which at least one of the members is a *multiforce* member, that is, a member acted upon by three or more forces. These forces will generally not be directed along the members on which they act; their direction is unknown, and they should be represented therefore by two unknown components.

Frames and machines are structures containing multiforce members. *Frames* are designed to support loads and are usually stationary, fully constrained structures. *Machines* are designed to transmit and modify forces; they may or may not be stationary and will always contain moving parts.

6.12. MACHINES

Machines are structures designed to transmit and modify forces. Whether they are simple tools or include complicated mechanisms, their main purpose is to transform *input forces* into *output forces*. Consider, for example, a pair of cutting pliers used to cut a wire (Fig. 6.22*a*). If we apply two equal and opposite forces **P** and −**P** on their handles, they will exert two equal and opposite forces **Q** and −**Q** on the wire (Fig. 6.22*b*).

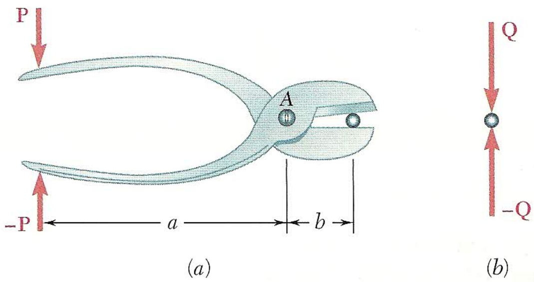


Fig. 6.22

Photo 6.5 The lamp shown can be placed in many positions. By considering various free bodies, the force in the springs and the internal forces at the joints can be determined.

To determine the magnitude Q of the output forces when the magnitude P of the input forces is known (or, conversely, to determine P when Q is known), we draw a free-body diagram of the pliers *alone*, showing the input forces **P** and −**P** and the *reactions* −**Q** and **Q** that the wire exerts on the pliers (Fig. 6.23). However, since

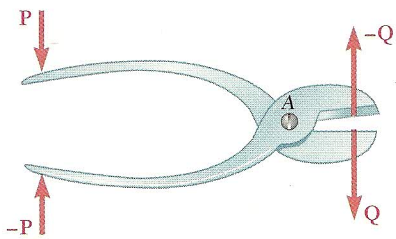


Fig. 6.23

a pair of pliers forms a nonrigid structure, we must use one of the component parts as a free body in order to determine the unknown forces. Considering Fig. 6.24*a*, for example, and taking moments about A, we obtain the relation $Pa = Qb$, which defines the magnitude Q in terms of P or P in terms of Q. The same free-body diagram can be used to determine the components of the internal force at A; we find $A_x = 0$ and $A_y = P + Q$.

In the case of more complicated machines, it generally will be necessary to use several free-body diagrams and, possibly, to solve simultaneous equations involving various internal forces. The free bodies should be chosen to include the input forces and the reactions to the output forces, and the total number of unknown force components involved should not exceed the number of available independent equations. It is advisable, before attempting to solve a problem, to determine whether the structure considered is determinate. There is no point, however, in discussing the rigidity of a machine, since a machine includes moving parts and thus *must* be nonrigid.

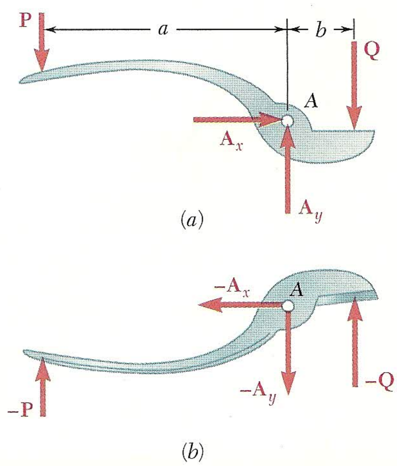
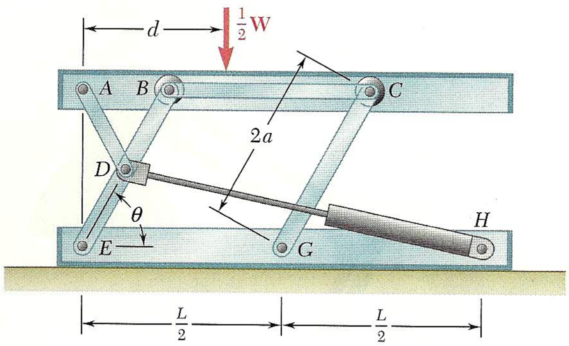
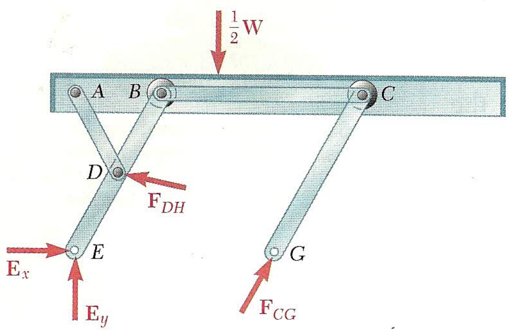


Fig. 6.24



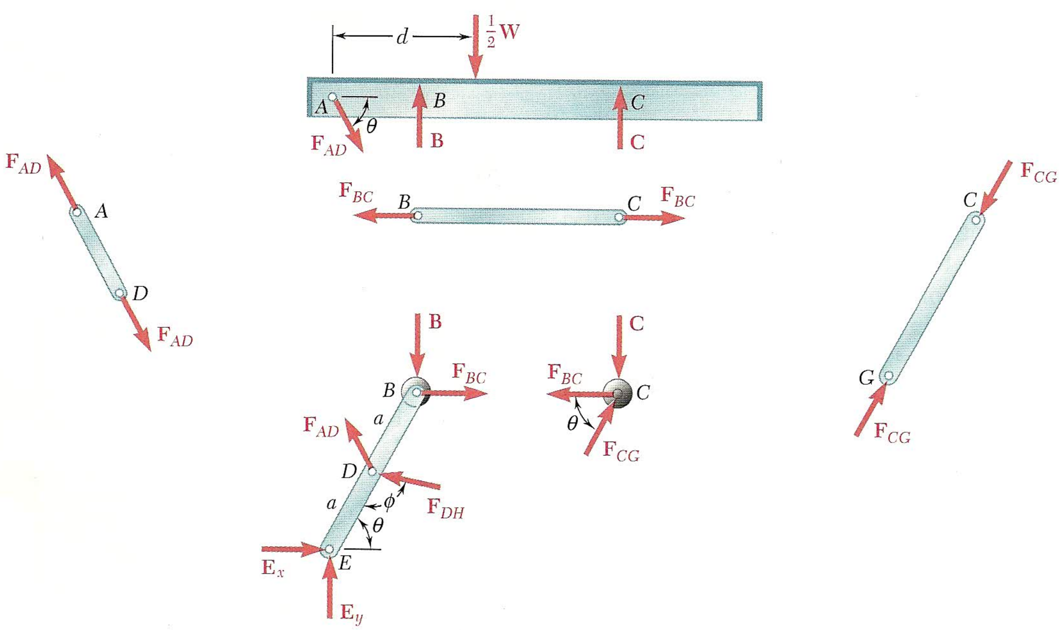
SAMPLE PROBLEM 6.7

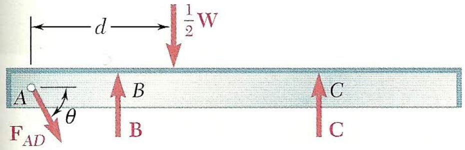
A hydraulic-lift table is used to raise a 1000-kg crate. It consists of a platform and two identical linkages on which hydraulic cylinders exert equal forces. (Only one linkage and one cylinder are shown.) Members *EDB* and *CG* are each of length $2a$, and member *AD* is pinned to the midpoint of *EDB*. If the crate is placed on the table, so that half of its weight is supported by the system shown, determine the force exerted by each cylinder in raising the crate for $\theta = 60°$, $a = 0.70$ m, and $L = 3.20$ m. Show that the result obtained is independent of the distance d.



SOLUTION

The machine considered consists of the platform and the linkage. Its free-body diagram includes a force $\mathbf{F}_{DH}$ exerted by the cylinder, the weight $\frac{1}{2}\mathbf{W}$, and reactions at *E* and *G* that we assume to be directed as shown. Since more than three unknowns are involved, this diagram will not be used. The mechanism is dismembered and a free-body diagram is drawn for each of its component parts. We note that *AD*, *BC*, and *CG* are two-force members. We already assumed member *CG* to be in compression. We now assume that *AD* and *BC* are in tension; the forces exerted on them are then directed as shown. Equal and opposite vectors will be used to represent the forces exerted by the two-force members on the platform, on member *BDE*, and on roller *C*.

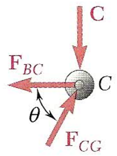


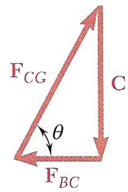


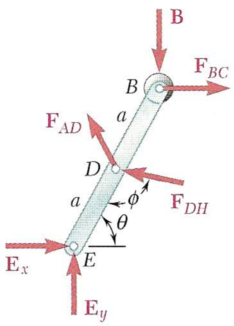
Free Body: Platform *ABC*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \qquad F_{AD}\cos\theta = 0 \qquad F_{AD} = 0$$
$$+\uparrow\Sigma F_y = 0: \qquad B + C - \tfrac{1}{2}W = 0 \qquad B + C = \tfrac{1}{2}W \tag{1}$$

Free Body: Roller *C*. We draw a force triangle and obtain $F_{BC} = C\cot\theta$.







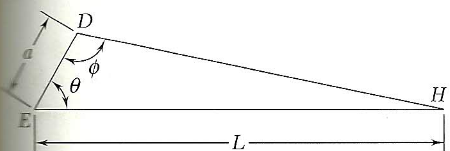
Free Body: Member *BDE*. Recalling that $F_{AD} = 0$,

$$+\circlearrowleft\Sigma M_E = 0: \qquad F_{DH}\cos(\phi - 90°)a - B(2a\cos\theta) - F_{BC}(2a\sin\theta) = 0$$
$$F_{DH}a\sin\phi - B(2a\cos\theta) - (C\cot\theta)(2a\sin\theta) = 0$$
$$F_{DH}\sin\phi - 2(B + C)\cos\theta = 0$$

Recalling Eq. (1), we have

$$F_{DH} = W\frac{\cos\theta}{\sin\phi} \tag{2}$$

and we observe that *the result obtained is independent of d.* ◀



Applying first the law of sines to triangle *EDH*, we write

$$\frac{\sin\phi}{EH} = \frac{\sin\theta}{DH} \qquad \sin\phi = \frac{EH}{DH}\sin\theta \tag{3}$$

Using now the law of cosines, we have

$$(DH)^2 = a^2 + L^2 - 2aL\cos\theta$$
$$= (0.70)^2 + (3.20)^2 - 2(0.70)(3.20)\cos 60°$$
$$(DH)^2 = 8.49 \qquad DH = 2.91\text{ m}$$

We also note that

$$W = mg = (1000\text{ kg})(9.81\text{ m/s}^2) = 9810\text{ N} = 9.81\text{ kN}$$

Substituting for $\sin\phi$ from (3) into (2) and using the numerical data, we write

$$F_{DH} = W\frac{DH}{EH}\cot\theta = (9.81\text{ kN})\frac{2.91\text{ m}}{3.20\text{ m}}\cot 60°$$

$F_{DH} = 5.15$ kN ◀