8-6.1 Cables Subjected to Concentrated Loads

A cable subjected to concentrated loads $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$ at discrete points along its length is depicted in Fig. 8-21*a*. The cable is anchored to rigid walls at ends *A* and *B* with pins. If the loads are much larger than the weight of the cable, the weight of the cable can be neglected in the analysis and the segments of the cable can be considered as straight two-force bars.

Assume for the following discussion that loads $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$ together with distances x_1, x_2, x_3, and span a are known. The distances y_1, y_2, and y_3 are unknowns to be determined. A free-body diagram of the cable is shown in Fig. 8-21*b*. Since the distances y_1, y_2, and y_3 are unknown, the slopes of the cable segments at ends *A* and *B* are not known; therefore, the reactions at *A* and *B* are represented by two components each. Since four unknowns are involved, the three equations of equilibrium for this free-body diagram (Fig. 8-21*b*) are not sufficient to determine the reactions at *A* and *B*. The information that can be determined is as follows:

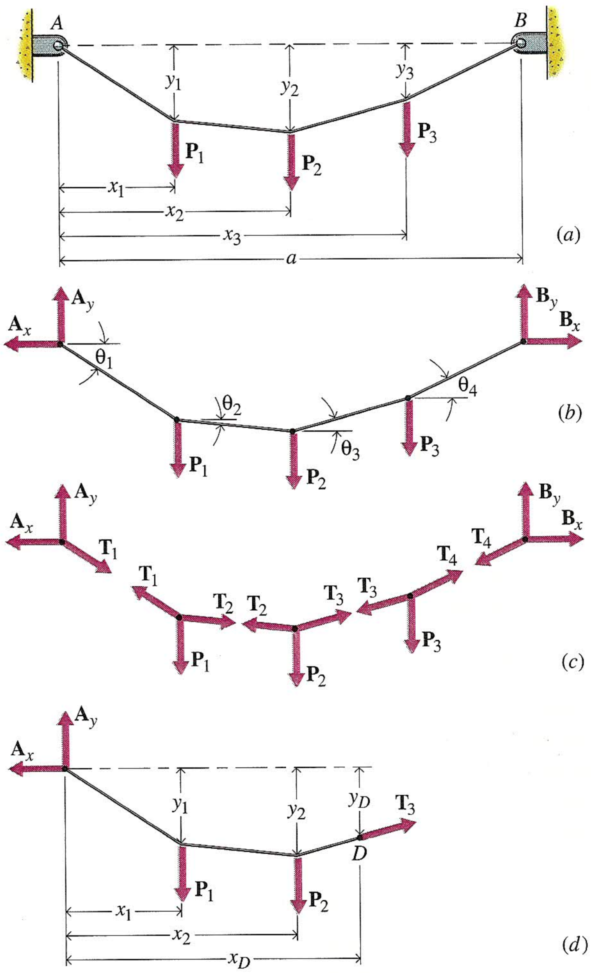

Figure 8-21 Flexible cable supporting a system of concentrated loads.

From the equilibrium equation $\Sigma M_A = 0$:

$$+\circlearrowleft B_y a - P_1 x_1 - P_2 x_2 - P_3 x_3 = 0$$

$$B_y = \frac{1}{a}(P_1 x_1 + P_2 x_2 + P_3 x_3)$$

From the equilibrium equation $\Sigma F_y = 0$:

$$+\uparrow A_y + B_y - P_1 - P_2 - P_3 = 0$$
$$A_y = P_1 + P_2 + P_3 - B_y$$
$$= P_1 + P_2 + P_3 - \frac{1}{a}(P_1 x_1 + P_2 x_2 + P_3 x_3)$$

From the equilibrium equation $\Sigma F_x = 0$:

$$+\!\rightarrow B_x - A_x = 0$$
$$B_x = A_x$$

Previously, in the discussion of frames and trusses, additional equations were obtained by considering the equilibrium of a portion of the structure. For a cable, if the weight of the cable is neglected, the internal forces in the different segments of the cable can be represented

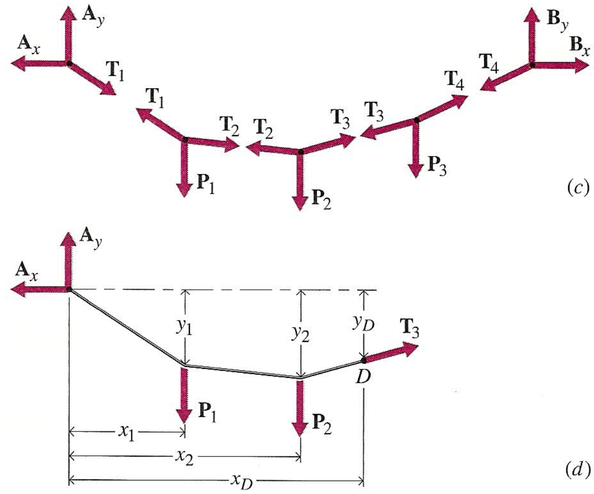

Fig. 8-21

as shown in Fig. 8-21*c*. From this series of free-body diagrams and the equilibrium equation $\Sigma F_x = 0$, it is observed that

$$A_x = T_1 \cos \theta_1 = T_2 \cos \theta_2 = T_3 \cos \theta_3 = T_4 \cos \theta_4 = B_x$$

This equation indicates that the horizontal component of the tensile force at any point in the cable is constant and equal to the horizontal component of the pin reaction at the supports. The maximum tension T will occur in the segment with the largest angle of inclination θ since $\cos \theta$ will be minimum in this segment. Such a segment must be adjacent to one of the two supports. The equilibrium equation $\Sigma F_x = 0$ and $\Sigma F_y = 0$ applied to the pins at supports A and B yield

$$\begin{aligned} A_x &= T_1 \cos \theta_1 & \qquad B_x &= T_4 \cos \theta_4 \\ A_y &= T_1 \sin \theta_1 & \qquad B_y &= T_4 \sin \theta_4 \end{aligned}$$

from which

$$\begin{aligned} T_1 &= \sqrt{A_x^2 + A_y^2} & \qquad T_4 &= \sqrt{B_x^2 + B_y^2} \\ \theta_1 &= \tan^{-1} \frac{A_y}{A_x} & \qquad \theta_4 &= \tan^{-1} \frac{B_y}{B_x} \end{aligned} \qquad (a)$$

If either the maximum tension or the maximum slope is specified for a given problem, Eqs. *a* can be used to determine the unknown horizontal components of the support reactions. Once either A_x or B_x is known, all the remaining unknowns, T_1, T_2, T_3, T_4, y_1, y_2, and y_3 can be determined by using the free-body diagrams shown in Fig. 8-21*c*. The procedure is illustrated in Example Problem 8-11.

The unknown horizontal components of the support reactions can also be determined if the vertical distance from a support to any point along the cable is known. For example, consider the free-body diagram shown in Fig. 8-21*d*, where it is assumed that the distance y_D is known. The equilibrium equation $\Sigma M_D = 0$ yields

$$A_y x_D - P_1(x_D - x_1) - P_2(x_D - x_2) - A_x y_D = 0$$

from which

$$A_x = \frac{1}{y_D}[A_y x_D - P_1(x_D - x_1) - P_2(x_D - x_2)]$$

Finally, problems of this type can be solved by specifying the length of the cable. In this case, the length of each segment of the cable is written in terms of the vertical distances y_1, y_2, and y_3 and the horizontal distances x_1, x_2, x_3, and span a. The required additional equation is then obtained by equating the sum of the individual segment lengths to the total length L:

$$L = \sqrt{x_1^2 + y_1^2} + \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} + \sqrt{(x_3 - x_2)^2 + (y_2 - y_3)^2} + \sqrt{(a - x_3)^2 + y_3^2}$$

Since the terms for the segment lengths in this equation involve square roots of the unknown vertical distances, any solution is extremely tedious and time-consuming if performed by hand calculation. Computer solution is recommended for this formulation of cable problems.

EXAMPLE PROBLEM 8-11

A cable supports concentrated loads of 500 lb and 200 lb as shown in Fig. 8-22*a*. If the maximum tension in the cable is 1000 lb, determine

a. The support reactions A_x, A_y, D_x, and D_y.
b. The tensions T_1, T_2, and T_3 in the three segments of the cable.
c. The vertical distances y_B and y_C from the level of support A.
d. The length L of the cable.

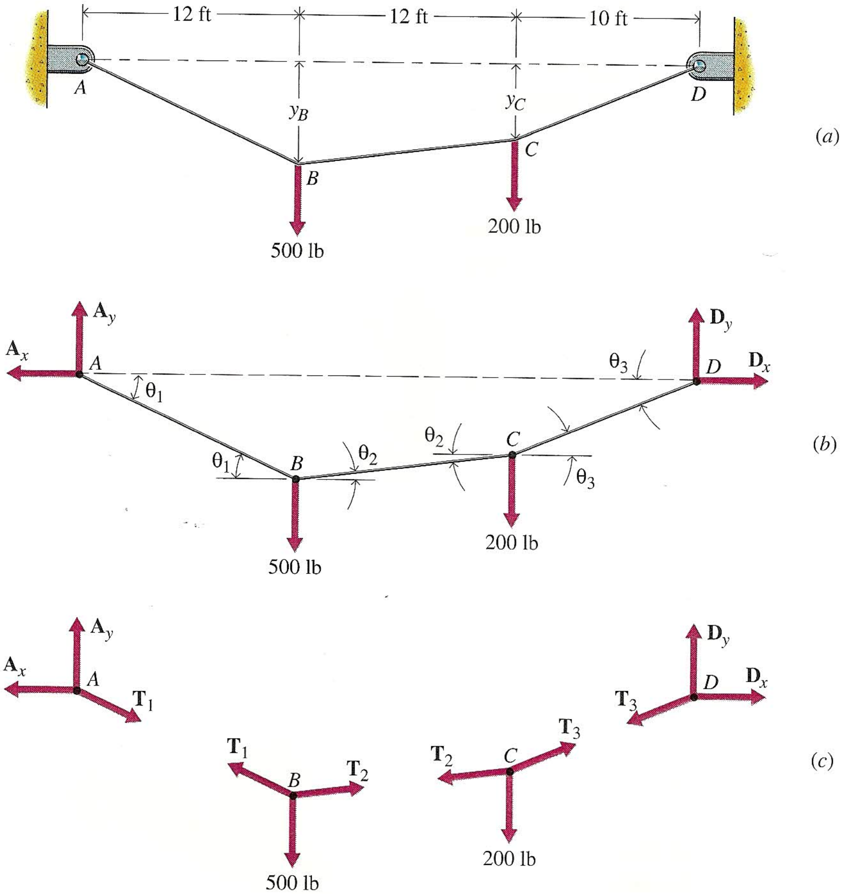

Fig. 8-22

SOLUTION

a. A free-body diagram for the cable is shown in Fig. 8-22*b*. From the equilibrium equation $\Sigma M_D = 0$,

$$A_y = \frac{1}{34}[500(22) + 200(10)] = 382.4 \cong 382 \text{ lb} \qquad \text{Ans.}$$

The maximum tension of 1000 lb will occur in interval AB of the cable. Thus, from the free-body diagram at point A of the cable (Fig. 8-22*c*),

$$A_x = \sqrt{T_1^2 - A_y^2} = \sqrt{1000^2 - 382.4^2} = 924.0 = 924 \text{ lb} \qquad \text{Ans.}$$

Since the horizontal component of the tensile force at any point in the cable is constant, the maximum tension in the cable occurs in the segment with the largest angle of inclination.

Also,

$$\theta_1 = \tan^{-1}\frac{A_y}{A_x} = \tan^{-1}\frac{382.4}{924.0} = 22.48°$$

Return now to the free-body diagram for the entire cable shown in Fig. 8-22*b*. From the equilibrium equation $\Sigma F_x = 0$,

$$D_x = A_x = 924.0 = 924 \text{ lb} \qquad \text{Ans.}$$

From the equilibrium equation $\Sigma F_y = 0$,

$$D_y = 500 + 200 - A_y = 700 - 382.4 = 317.6 \cong 318 \text{ lb} \qquad \text{Ans.}$$

b. Free-body diagrams at points B and C (Fig. 8-22*c*) can be used to determine the tensions T_2 and T_3. From the equilibrium equation $\Sigma F_x = 0$ and $\Sigma F_y = 0$,

At point B:

$$T_{2x} = T_2 \cos\theta_2 = T_1 \cos\theta_1 = 1000 \cos 22.48° = A_x = 924.0 \text{ lb}$$
$$T_{2y} = T_2 \sin\theta_2 = 500 - T_1 \sin\theta_1 = 500 - 1000 \sin 22.48° = 117.64 \text{ lb}$$
$$T_2 = \sqrt{T_{2x}^2 + T_{2y}^2} = \sqrt{924.0^2 + 117.64^2} = 931.4 \cong 931 \text{ lb} \qquad \text{Ans.}$$
$$\theta_2 = \tan^{-1}\frac{T_{2y}}{T_{2x}} = \tan^{-1}\frac{117.64}{924.0} = 7.256°$$

At point C:

$$T_{3x} = T_3 \cos\theta_3 = T_2 \cos\theta_2 = 931.4 \cos 7.256° = A_x = 924.0 \text{ lb}$$
$$T_{3y} = T_3 \sin\theta_3 = 200 + T_2 \sin\theta_2 = 200 + 931.4 \sin 7.256° = 317.6 \text{ lb}$$
$$T_3 = \sqrt{T_{3x}^2 + T_{3y}^2} = \sqrt{924.0^2 + 317.6^2} = 977.1 \cong 977 \text{ lb} \qquad \text{Ans.}$$
$$\theta_3 = \tan^{-1}\frac{T_{3y}}{T_{3x}} = \tan^{-1}\frac{317.6}{924.0} = 18.969°$$

As a check at point D:

$$T_3 = \sqrt{D_x^2 + D_y^2} = \sqrt{924.0^2 + 317.6^2} = 977.1 \cong 977 \text{ lb} \qquad \text{Ans.}$$

c. Once the angles are known, the vertical distances y_B and y_C are

$$y_B = 12 \tan\theta_1 = 12 \tan 22.48° = 4.966 \cong 4.97 \text{ ft} \qquad \text{Ans.}$$
$$y_C = 10 \tan\theta_3 = 10 \tan 18.969° = 3.439 \cong 3.44 \text{ ft} \qquad \text{Ans.}$$

d. The cable length L is obtained from the segment lengths as

$$L = \sqrt{12^2 + 4.966^2} + \sqrt{12^2 + (4.966 - 3.439)^2} + \sqrt{10^2 + 3.439^2}$$
$$= 35.66 \cong 35.7 \text{ ft} \qquad \text{Ans.}$$