9-3.5 Flat Belts and V-Belts

Many types of power machinery rely on belt drives to transfer power from one piece of equipment to another. Without friction, the belts would slip on their pulleys and no power transfer would be possible. Maximum torque is applied to the pulley when the belt is at the point of impending slip, and that is the case discussed here.

Although the analysis presented is for flat belts, it also applies to any shape belt as well as circular ropes, as long as the only contact between the belt and the pulley is on the bottom surface of the belt. This section ends with a brief discussion of V-belts, which indicates the kind of modifications required when friction acts on the sides of the belt instead of the bottom.

Figure 9-33*a* shows a flat belt passing over a circular drum. The tensions in the belt on either side of the drum are T_1 and T_2 and the bearing reaction is R. Friction in the bearing is neglected for this analysis, but a torque M is applied to the drum to keep it from rotating. If there is no friction between the belt and the drum, the two tensions must be equal, $T_1 = T_2$, and no torque is required for moment equilibrium, $M = 0$. If there is friction between the belt and the drum, however, then the two tensions need not be equal and a torque $M = r(T_2 - T_1)$ is needed to satisfy moment equilibrium. Assuming that $T_2 > T_1$, this means that friction must exert a counterclockwise moment on the drum (Fig. 9-33*b*) and the drum will exert an opposite frictional resistance on the belt (Fig. 9-33*c*). Because the friction force depends on the normal force and the normal force varies around the drum, care must be taken in adding up the total frictional resistance.

The free-body diagram (Fig. 9-34) of a small segment of the belt includes the friction force ΔF and the normal force ΔP. The tension in the belt increases from T on one side of the segment to $T + \Delta T$ on the other side. Equilibrium in the radial direction gives

$$\nwarrow \Sigma F_r = \Delta P - T \sin\left(\frac{\Delta\theta}{2}\right) - (T + \Delta T)\sin\left(\frac{\Delta\theta}{2}\right) = 0$$

or

$$\Delta P = 2T \sin\left(\frac{\Delta\theta}{2}\right) + \Delta T \sin\left(\frac{\Delta\theta}{2}\right) \qquad (a)$$

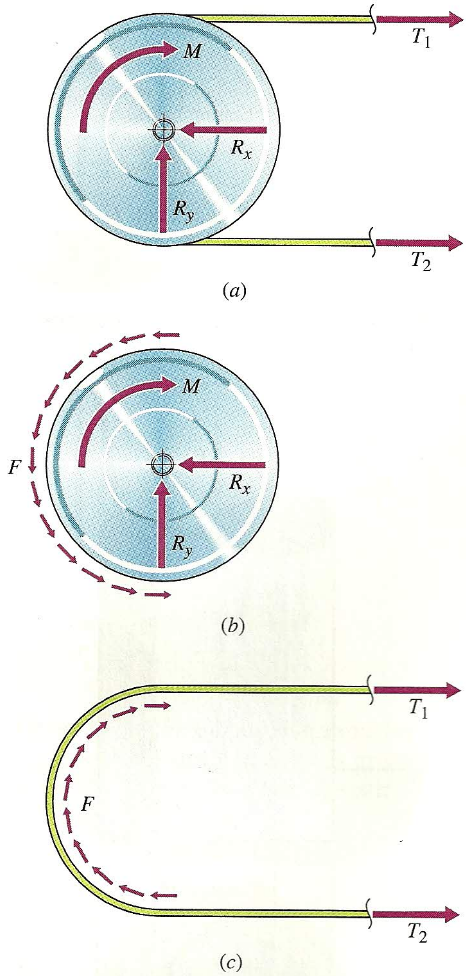


Figure 9-33 Free-body diagrams for a flat belt in contact with a circular drum.

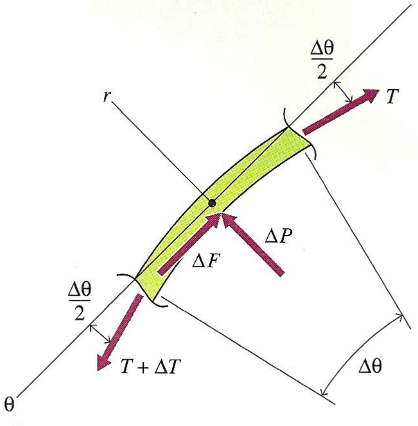


Figure 9-34 Free-body diagram for a small segment of a flat belt.

while equilibrium in the circumferential (θ-) direction gives

$$\nwarrow \Sigma F_\theta = (T + \Delta T)\cos\left(\frac{\Delta\theta}{2}\right) - T\cos\left(\frac{\Delta\theta}{2}\right) - \Delta F = 0$$

or

$$\Delta T \cos\left(\frac{\Delta\theta}{2}\right) = \Delta F$$

In the limit as $\Delta\theta \to 0$ the normal force ΔP on the small segment of the belt must vanish according to Eq. *a*. But when the normal force vanishes ($\Delta P \to 0$), there can be no friction on the belt either ($\Delta F \to 0$). Therefore, the change in tension across the small segment of the belt must also vanish ($\Delta T \to 0$) in the limit as $\Delta\theta \to 0$ according to Eq. *b*.

Assuming that slip is impending gives $\Delta F = \mu_s \Delta P$, and Eqs. *a* and *b* can be combined to give

$$\Delta T \cos(\Delta\theta/2) = \mu_s\, 2T \sin(\Delta\theta/2) + \mu_s\, \Delta T \sin(\Delta\theta/2)$$

which, after dividing through by $\Delta\theta$, is

$$\frac{\Delta T}{\Delta\theta}\cos(\Delta\theta/2) = \mu_s T \frac{\sin(\Delta\theta/2)}{\Delta\theta/2} + \frac{\mu_s \Delta T}{2}\,\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}$$

Finally, taking the limits as $\Delta\theta \to 0$ and recalling that

$$\lim_{\Delta\theta\to 0}\frac{\Delta T}{\Delta\theta} \to \frac{dT}{d\theta} \qquad \lim_{x\to 0}\cos x \to 1 \qquad \lim_{x\to 0}\frac{\sin x}{x} \to 1$$

gives

$$\frac{dT}{d\theta} = \mu_s T$$

Equation *e* can be rearranged in the form

$$dT/T = \mu_s\, d\theta \tag{9-15}$$

which, since the coefficient of friction is a constant, can be immediately integrated from θ_1, where the tension is T_1, to θ_2, where the tension is T_2, to get

$$\ln(T_2/T_1) = \mu_s(\theta_2 - \theta_1) = \mu_s\beta \tag{9-16a}$$

or

$$T_2 = T_1 e^{\mu_s\beta} \tag{9-16b}$$

where $\beta = \theta_2 - \theta_1$ is the central angle of the drum for which the belt is in contact with the drum. The angle of wrap β must be measured in radians and must obviously be positive. Angles greater than 2π radians are possible and simply mean that the belt is wrapped more than one complete revolution around the drum.

It must be emphasized that Eq. 9-16 assumes impending slip at all points along the belt surface and therefore gives the maximum change in tension that the belt can have. Since the exponential function of a positive value is always greater than 1, Eq. 9-16 gives that T_2 (the tension in the belt on the side toward which slip tends to occur) will always be greater than T_1 (the tension in the belt on the side away from which slip tends to occur). Of course, if slip is not known to be impending, then Eq. 9-16 does not apply and T_2 may be larger or smaller than T_1.

V-belts as shown in Fig. 9-35*a* are handled similarly to the above. A view of the belt cross section (Fig. 9-35*b*), however, shows that there are now two normal forces and there will also be two frictional forces (acting along the edges of the belt and pointing into the plane of the figure). Equilibrium in the circumferential (θ-) direction now gives

$$\Delta T \cos(\Delta\theta/2) = 2\Delta F$$

while equilibrium in the radial direction gives

$$2\Delta P \sin(\alpha/2) = 2T \sin(\Delta\theta/2) + \Delta T \sin(\Delta\theta/2)$$

Continuing as above results finally in

$$T_2 = T_1 e^{(\mu_s)_{enh}\beta} \tag{9-17}$$

in which $(\mu_s)_{\text{enh}} = [\mu_s/\sin(\alpha/2)] > \mu_s$ is an *enhanced coefficient of friction*. That is, V-belts always give a larger T_2 than flat belts for a given coefficient of friction μ_s and a given angle of wrap β.

Equations 9-16 and 9-17 can also be used when slipping is actually occurring by replacing the static coefficient of friction μ_s with the kinetic coefficient of friction μ_k.

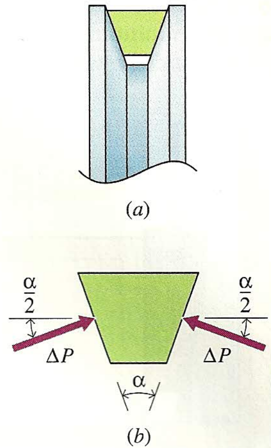
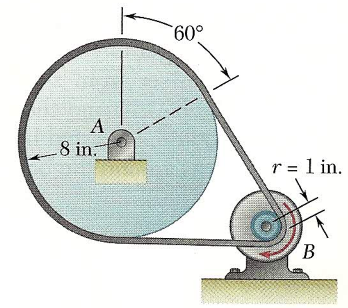


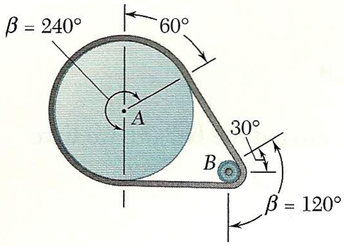
Figure 9-35 Cross section of a V-belt showing normal forces on the inclined surfaces.

SAMPLE PROBLEM 8.8

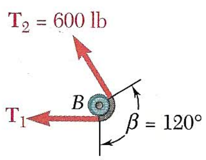


A flat belt connects pulley A, which drives a machine tool, to pulley B, which is attached to the shaft of an electric motor. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt. Knowing that the maximum allowable tension in the belt is 600 lb, determine the largest couple which can be exerted by the belt on pulley A.

SOLUTION



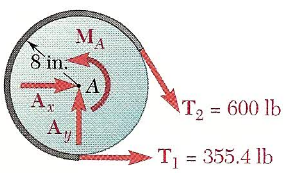
Since the resistance to slippage depends upon the angle of contact β between pulley and belt, as well as upon the coefficient of static friction μ_s, and since μ_s is the same for both pulleys, slippage will occur first on pulley B, for which β is smaller.



Pulley *B*. Using Eq. (8.14) with $T_2 = 600$ lb, $\mu_s = 0.25$, and $\beta = 120° = 2\pi/3$ rad, we write

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad \frac{600 \text{ lb}}{T_1} = e^{0.25(2\pi/3)} = 1.688$$

$$T_1 = \frac{600 \text{ lb}}{1.688} = 355.4 \text{ lb}$$



Pulley *A*. We draw the free-body diagram of pulley A. The couple $\mathbf{M}_A$ is applied to the pulley by the machine tool to which it is attached and is equal and opposite to the torque exerted by the belt. We write

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (600 \text{ lb})(8 \text{ in.}) + (355.4 \text{ lb})(8 \text{ in.}) = 0$$

$$M_A = 1957 \text{ lb} \cdot \text{in.} \qquad M_A = 163.1 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

Note. We may check that the belt does not slip on pulley A by computing the value of μ_s required to prevent slipping at A and verifying that it is smaller than the actual value of μ_s. From Eq. (8.13) we have

$$\mu_s \beta = \ln \frac{T_2}{T_1} = \ln \frac{600 \text{ lb}}{355.4 \text{ lb}} = 0.524$$

and, since $\beta = 240° = 4\pi/3$ rad,

$$\frac{4\pi}{3} \mu_s = 0.524 \qquad \mu_s = 0.125 < 0.25$$