5-7 FORCES ON SUBMERGED SURFACES

A fluid (either a liquid or a gas) at rest can, by definition, transmit compressive forces but not tensile or shear forces. Since a shear force acts tangent to a surface, a fluid at rest can exert only a compressive normal force (known as a pressure) on a submerged surface. The pressure, called a hydrostatic pressure (equal in all directions), is due to the weight of the fluid above any point on the submerged surface; therefore, fluid pressures vary linearly with depth in fluids with a constant specific weight. The absolute pressure p_A at a depth d is

$$p_A = p_0 + \gamma d = p_0 + \rho g d \tag{5-20}$$

where

p_0 = atmospheric pressure at the surface of the fluid
γ = specific weight of the fluid
ρ = density of the fluid
g = gravitational acceleration

In the U.S. Customary system of units, the specific weight γ of fresh water is 62.4 lb/ft^3. In the SI system of units, the density ρ of fresh water is 1000 kg/m^3. The gravitational acceleration g is 32.2 ft/s^2 in the U.S. Customary system and 9.81 m/s^2 in the SI system.

In general, pressure-measuring instruments record pressures above atmospheric pressure. Such pressures are known as gage pressures, and it is obvious from Eq. 5-20 that the gage pressure p_g is

$$p_g = p_A - p_0 = \gamma d = \rho g d \qquad (5\text{-}21)$$

For the analysis of many engineering problems involving fluid forces, it is necessary to determine the resultant force **R** due to the distribution of pressure on a submerged surface and the location of the intersection of the line of action of the resultant force with the submerged surface. The point P on the submerged surface where the line of action of the resultant force **R** intersects the submerged surface is known as the center of pressure.

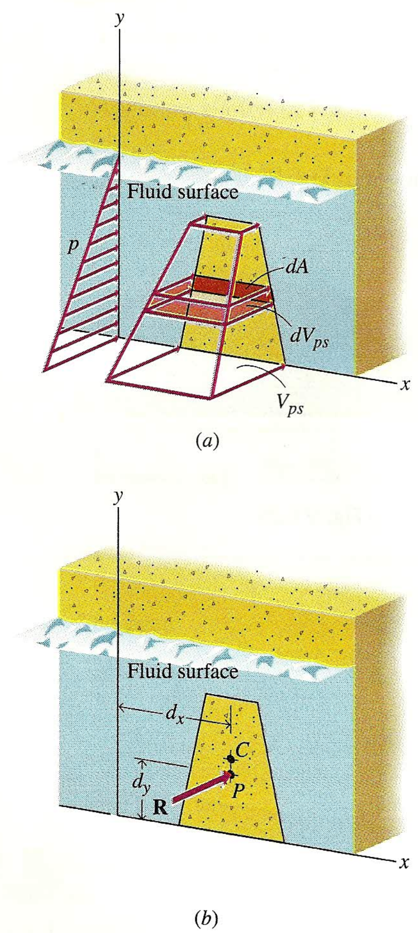


Figure 5-27 Replacing a distributed fluid pressure load on a submerged plane surface with a resultant force **R**.

5-7.1 Forces on Submerged Plane Surfaces

For the case of fluid pressures on submerged plane surfaces, the load diagram (area) introduced in Section 5-6 for a distributed load along a line becomes a pressure solid (volume), as shown in Fig. 5-27*a*, since the intensity of a distributed load (pressure) on the submerged surface varies over an area instead of a length. When the distributed pressure p is applied to an area in the xy-plane, the ordinate $p(x, y)$ along the z-axis represents the intensity of the force (force per unit area). The magnitude of the increment of force $d\mathbf{R}$ on an element of area dA is

$$dR = p\, dA = dV_{ps}$$

where dV_{ps} is an element of volume of the pressure solid, as shown in Fig. 5-27*a*. The magnitude of the resultant force **R** acting on the submerged surface is

$$R = \int_A p\, dA = \int_V dV_{ps} = V_{ps} \qquad (5\text{-}22)$$

where V_{ps} is the volume of the pressure solid.

The line of action of the resultant force **R** with respect to the x- and y-axes (called the center of pressure) can be located by using the principle of moments.

For moments about the y-axis:

$$Rd_x = \int x\, dR = \int_A x\, p\, dA = \int_V x\, dV_{ps} = x_{Cps}\, V_{ps} \qquad (5\text{-}23a)$$

For moments about the x-axis:

$$Rd_y = \int y\, dR = \int_A y\, p\, dA = \int_V y\, dV_{ps} = y_{Cps}\, V_{ps} \qquad (5\text{-}23b)$$

Figure 5-27 indicates that the line of action of the resultant force **R** passes through the centroid C_V of the volume of the pressure solid. If the pressure is uniformly distributed over the area, the center of pressure P will coincide with the centroid C_A of the area. If the pressure is not uniformly distributed over the area, the center of pressure P and the centroid C_A of the area will be at different points, as shown in Fig. 5-27b.

5-7.2 Forces on Submerged Curved Surfaces

Equations 5-22 and 5-23 apply only to submerged plane surfaces; however, many surfaces of interest in engineering applications—such as those associated with pipes, dams, and tanks—are curved. For such problems, the resultant force **R** and the intersection of its line of action with the curved surface can be determined by integration for each individual problem, but general formulas applicable to a broad class of problems cannot be developed. To overcome this difficulty, the procedure illustrated in Fig. 5-28 has been developed.

In Fig. 5-28*a*, a cylindrical gate with a radius a and a length L is being used to close an opening in the wall of a tank containing a fluid. The pressure distribution on the gate is shown in Fig. 5-28*b*. From such a distribution, horizontal and vertical components of the resultant force can be determined by integration and combined to yield the resultant force **R**. The pressure-solid approach can also be used to determine the resultant force **R** if horizontal and vertical planes are used to isolate the gate and a volume of fluid in contact with the gate, as shown in Fig. 5-28*c*. The force exerted on the horizontal fluid surface by the fluid pressure is

$$F_{1v} = p_1 A_h = \gamma(d - a)(aL)$$

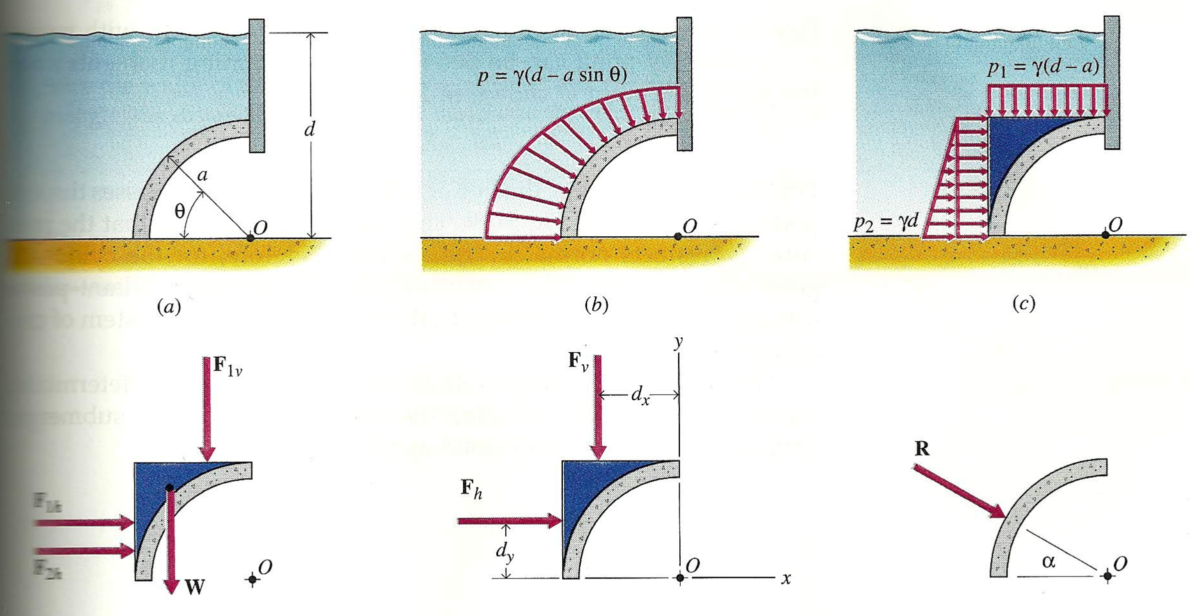


Figure 5-28 Replacing a distributed fluid pressure load on a submerged curved surface with a resultant force **R**.

Similarly on the vertical surface,

$$F_{1h} = p_1 A_v = \gamma(d - a)(aL)$$
$$F_{2h} = (p_2 - p_1)A_v = \gamma a(aL)$$

The volume of fluid V_f has a weight W, which is given by the expression

$$W = \gamma V_f = \gamma\left(a^2 - \frac{1}{4}\pi a^2\right)L$$

The four forces F_{1v}, F_{1h}, F_{2h}, and W together with their lines of action are shown in Fig. 5-28*d*. The two vertical forces and the two horizontal forces can be combined to give

$$F_v = F_{1v} + W$$
$$F_h = F_{1h} + F_{2h}$$

where F_v and F_h are the rectangular components of a resultant force **R**. That is, **R** is the resultant of F_{1v}, F_{1h}, F_{2h}, and W, which are the forces exerted by the adjoining water and the earth on the volume of water in contact with the gate. This force is the same as the force exerted by the water on the gate because the volume of water in contact with the gate is in equilibrium and the force exerted on the water by the gate is equal in magnitude and opposite in direction to the force exerted on the gate by the water. The magnitude of the resultant is

$$R = \sqrt{(F_h)^2 + (F_v)^2}$$

The slope of the line of action of the resultant is given by the expression

$$\alpha = \tan^{-1}\frac{F_v}{F_h}$$

Finally, the location of the line of action of the resultant with respect to an arbitrary point can be determined by summing moments about the point. For point O shown in Fig. 5-28*e*,

$$Rd = F_v d_x - F_h d_y$$

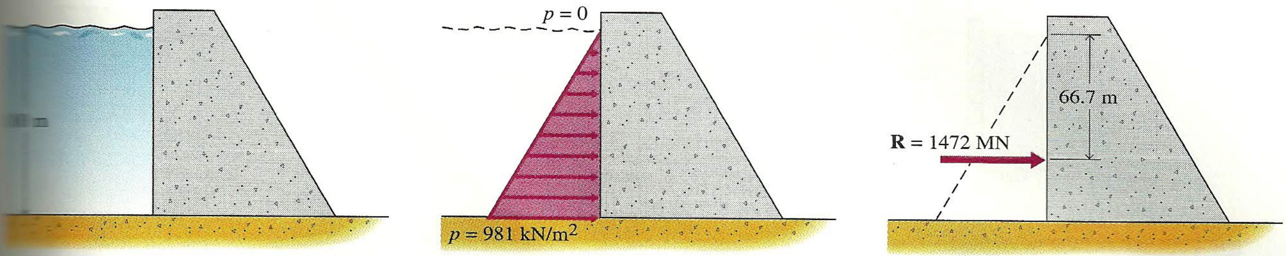
For the cylindrical gate, the line of action of the resultant passes through point O, as shown in Fig 5-28*f*. This results from the fact that the pressure always acts normal to the surface; therefore, for the cylindrical gate, the line of action of each increment $d\mathbf{R}$ of the resultant passes through point O. In other words, the increments form a system of concurrent forces.

The following examples illustrate the procedure for determining the resultant force and locating the center of pressure for submerged surfaces by using pressure-solid approaches.

EXAMPLE PROBLEM 5-16

The water behind a dam is 100 m deep as shown in Fig. 5-29*a*. Determine

a. The magnitude of the resultant force **R** exerted on a 30-m length of the dam by the water pressure.
b. The distance from the water surface to the center of pressure.



(*a*) (*b*) (*c*)

Fig. 5-29

SOLUTION

A cross section through the pressure solid is shown in Fig. 5-29*b*. At the base of the dam, the pressure is

$$p = \gamma d = \rho g d = 1000(9.81)(100)$$
$$= 981(10^3)\ \text{N/m}^2 = 981\ \text{kN/m}^2$$

a. Thus, for the 30-m length of dam, the volume of the pressure solid is

$$V_{ps} = \frac{1}{2}pA = \frac{1}{2}(981)(10^3)(100)(30) = 1471.5(10^6)\ \text{N} \cong 1472\ \text{MN}$$

$$R = V_{ps} \cong 1472\ \text{MN} \qquad \text{Ans.}$$

b. Since the width of the pressure solid is constant and the cross section is a triangle, the distance from the water surface to the centroid of the solid is

$$d_P = \frac{2}{3}d = \frac{2}{3}(100) = 66.67\ \text{m} \cong 66.7\ \text{m} \qquad \text{Ans.}$$

The results are shown in Fig. 5-29*c*.

When a fluid is at rest, it is capable of transmitting contact forces only in a direction perpendicular to the surface of contact. The resultant **R** is represented by the volume of the pressure solid and its line of action passes through the centroid of the volume.

EXAMPLE PROBLEM 5-17

The width of the rectangular gate shown in Fig. 5-30 is 8 ft. Determine the magnitude of the resultant force **R** exerted on the gate by the water pressure and the location of the center of pressure d_p with respect to the hinge at the top of the gate.

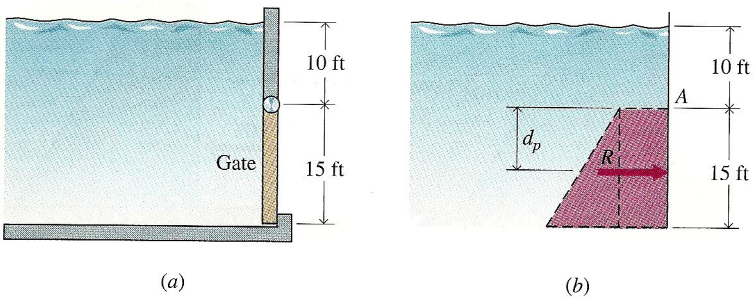


Fig. 5-30

SOLUTION

A cross section through the pressure solid is shown in Fig. 5-30*b*. At the top of the gate, the pressure is

$$p_T = \gamma d_T = 62.4(10) = 624 \text{ lb/ft}^2$$

At the bottom of the gate the pressure is

$$p_B = \gamma d_B = 62.4(25) = 1560 \text{ lb/ft}^2$$

The volume of the pressure solid can be separated into two parts.

For part 1:

$$V_{ps1} = p_T h w = 624(15)(8) = 74{,}880 \text{ lb}$$

For part 2:

$$V_{ps2} = \frac{1}{2}(p_B - p_T)hw = \frac{1}{2}(1560 - 624)(15)(8) = 56{,}160 \text{ lb}$$

Therefore,

$$\begin{aligned} R = R_1 + R_2 = V_{ps1} + V_{ps2} &= 74{,}880 + 56{,}160 \\ &= 131{,}040 \text{ lb} \cong 131.0 \text{ kip} \end{aligned} \qquad \text{Ans.}$$

Summing moments about point A yields

$$\begin{aligned} M_A &= R_1 y_{C1} + R_2 y_{C2} \\ &= 74{,}880(7.5) + 56{,}160(10) = 1{,}123{,}200 \text{ ft} \cdot \text{lb} \end{aligned}$$

The principle of moments states that

$$R d_P = R_1 y_{C1} + R_2 y_{C2} = M_A$$

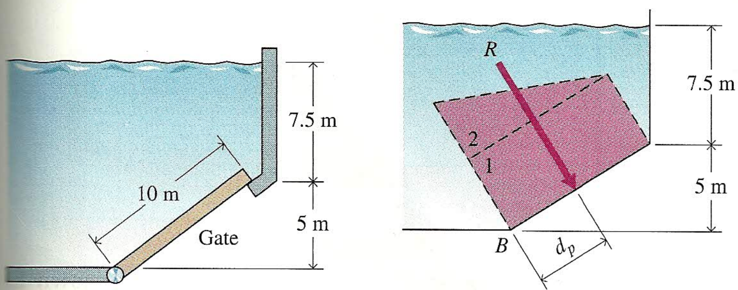
Therefore,

$$d_P = \frac{M_A}{R} = \frac{1{,}123{,}200}{131{,}040} = 8.571 \text{ ft} \cong 8.57 \text{ ft} \qquad \text{Ans.}$$

When a fluid is at rest, it is capable of transmitting contact forces only in a direction perpendicular to the surface of contact. Dividing the pressure distribution into two parts simplifies the calculation.

EXAMPLE PROBLEM 5-18

The width of the rectangular gate shown in Fig. 5-31*a* is 4 m. Determine the magnitude of the resultant force **R** exerted on the gate by the water pressure and the location of the center of pressure d_P with respect to the hinge at the bottom of the gate.



(*a*) (*b*)

Fig. 5-31

SOLUTION

A cross section through the pressure solid is shown in Fig. 5-31*b*. At the top of the gate the pressure is

$$p_T = \rho g d_T = 1000(9.81)(7.5) = 73{,}575 \text{ N/m}^2$$

At the bottom of the gate the pressure is

$$p_B = \rho g d_B = 1000(9.81)(12.5) = 122{,}625 \text{ N/m}^2$$

The volume of the pressure solid can be separated into two parts.

For part 1:

$$V_{ps1} = p_T L w = 73{,}575(10)(4) = 2.943(10^6) \text{ N}$$

For part 2:

$$\begin{aligned} V_{ps2} &= \frac{1}{2}(p_B - p_A)Lw \\ &= \frac{1}{2}(122{,}625 - 73{,}575)(10)(4) = 0.981(10^6) \text{ N} \end{aligned}$$

Therefore,

$$\begin{aligned} R = R_1 + R_2 = V_{ps1} + V_{ps2} &= 2.943(10^6) + 0.981(10^6) \\ &= 3.924(10^6) \text{ N} \cong 3.92 \text{ MN} \end{aligned} \qquad \text{Ans.}$$

Summing moments about point B yields

$$\begin{aligned} M_B = V_{ps1}d_{C1} + V_{ps2}d_{C2} &= 2.943(10^6)(5.00) + 0.981(10^6)(3.333) \\ &= 17.985(10^6) \text{ N} \cdot \text{m} \cong 17.99(10^6) \text{ N} \cdot \text{m} \end{aligned}$$

The principle of moments states that

$$Rd_P = R_1 d_{C1} + R_2 d_{C2} = M_B$$

Therefore,

$$d_P = \frac{M_B}{R} = \frac{17.985(10^6)}{3.924(10^6)} = 4.583 \text{ m} \cong 4.58 \text{ m} \qquad \text{Ans.}$$

When a fluid is at rest, it is capable of transmitting contact forces only in a direction perpendicular to the surface of contact. Dividing the pressure distribution into two parts simplifies the calculation.