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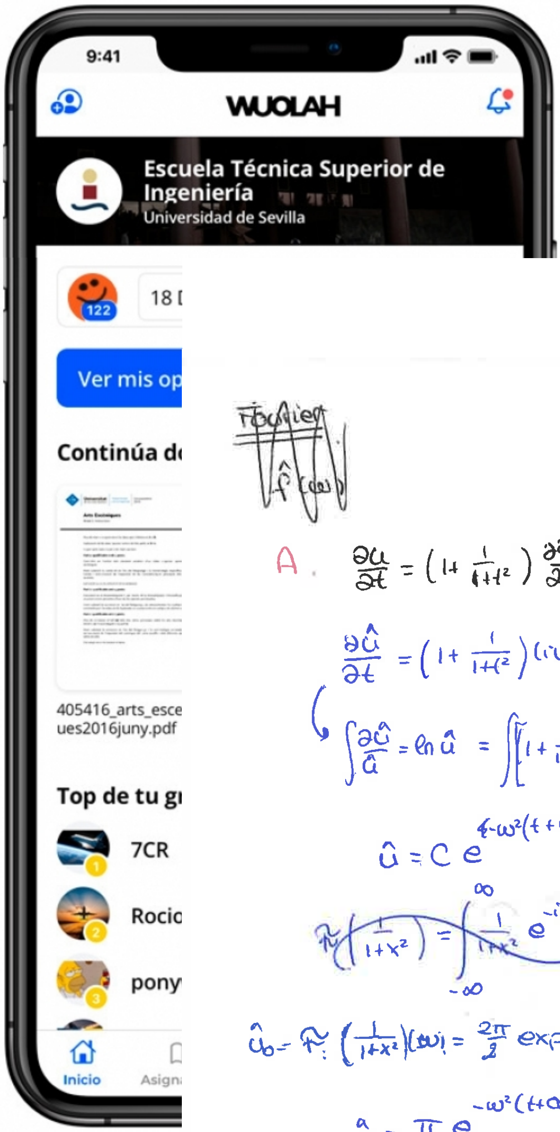
Exprum

Ampliación de Matemáticas

3º Grado en Ingeniería Aeroespacial

Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio
Universidad Politécnica de Madrid



~~Fourier~~ $\cancel{\hat{f}(\omega)}$

01-2018.

A. $\frac{\partial u}{\partial t} = \left(1+\frac{1}{1+t^2}\right)\frac{\partial^2 u}{\partial x^2} + u \qquad x\in\mathbb{R}.$

$u(\omega,t)$. $\quad \hat u(2,1/\sqrt3)$?

$u(t=0) = \frac{1}{1+x^2}$

$$\frac{\partial \hat u}{\partial t} = \left(1+\frac{1}{1+t^2}\right)(i\omega)^2\hat u + \hat u .$$

$$\int \frac{\partial \hat u}{\hat u} = \ln \hat u = \int\left[\left(1+\frac{1}{1+t^2}\right)(-\omega^2)+1\right]dt = (-\omega^2+1)t - \omega^2\,\mathrm{arctg}(t) + C .$$

$$\hat u = C\, e^{-\omega^2(t+\mathrm{arctg}\, t)+t} \qquad C = \hat u(t=0).$$

$$\cancel{\mathcal{F}\left(\frac{1}{1+x^2}\right) = \int_{-\infty}^{\infty}\frac{1}{1+x^2}e^{-i\omega x}dx = \frac{1}{1+x^2}\frac{1}{i\omega}e^{-i\omega x} - \int\frac{e^{-i\omega x}}{i\omega}\mathrm{arctg}\,x\,dx}$$

$$\hat u_0 = \mathcal{F}\left(\frac{1}{1+x^2}\right)(\omega) = \frac{2\pi}{2}\exp(-|\omega|)$$ (creo que es dato)

$$\hat u = \pi\, e^{-\omega^2(t+\mathrm{arctg}\, t)+t-|\omega|}$$

$\mathrm{tg}^{-1} = \pi/6$

$$\boxed{\hat u(2,1/\sqrt3) = \pi\exp\left(-4\left(\frac{\sqrt3}{3}+\mathrm{arctg}\left(\frac{\sqrt3}{3}\right)\right)+\frac{\sqrt3}{3}-|2|\right) = \pi\exp\left(-\sqrt3-\frac{2\pi}{3}-2\right)}$$

B. $\frac{\partial^2 w}{\partial t^2} + 4\frac{\partial w}{\partial t} + 8w = g(t) \qquad w(0)=0,\ w'(0)=1 \qquad ¿\mathcal{L}(w)(2)?$

$$g(t) = \begin{cases}(e^t-e)t & t\in(0,1)\\ 0 & t\notin(0,1)\end{cases}$$

$$s^2W - sw(0) - w'(0) + 4sW - 4w(0) + 8W = G$$ donde W y G son los $\mathcal{L}(w)$ y $\mathcal{L}(g)$.

$$(s^2+4s+8)W = G+1 \Rightarrow W = \frac{G+1}{s^2+4s+8}$$

$$g(t) = (e^t-e)[H(t)-H(t-1)] = (e^t-e)H(t) - (e^t-e)H(t-1) = (e^t-e)H(t) - (e\cdot e^{t-1}-e)H(t-1)$$

$\mathcal{L}(e^t)(s) = \frac{1}{s-1}$

$\mathcal{L}(e)(s) = \frac{e}{s}$

$\mathcal{L}(f(t-a)H(t-a)) = e^{-as}\mathcal{L}(f)(s)$

$$\to G = \mathcal{L}(g) = \frac{1}{s-1} - \frac{e}{s} - \left[e\cdot\frac{e^{-s}}{s-1} - e\cdot\frac{1}{s}e^{-s}\right]$$

$$G(2) = 1 - \frac{e}{2} - \left[\frac{e^{-2+1}}{1} - \frac{e^{-2+1}}{2}\right] = 1 - \frac{e}{2} - \frac{1}{2e} = \frac{2e-e^2-1}{2e}$$

$$W(2) = \frac{\frac{4e-e^2-1}{2e}}{4+8+8} = \frac{1}{20}\left(2-\frac{e^2+1}{2e}\right).$$

$$g(t) = H(t)(e^t-e)t - H(t-1)\cdot e(e^{t-1}-1)(t-1) - H(t-1)e(e^{t-1}-1), \Rightarrow G = \frac{1}{(s-1)^2} - \frac{e}{s^2} - e^{-s}\left(\frac{e}{(s-1)^2}-\frac{e}{s^2}\right) - e^{-s}\left(\frac{e}{s-1}-\frac{e}{s}\right)$$

$H(t-1)e(e^{t-1})(t-1+1)$

$$\Rightarrow W(2) = \frac{1}{20}\left(2-\frac{e^2+5}{4e}\right)$$

C. $\frac{\partial^2\omega}{\partial z^2} - (z^2 + i z^4)\omega = 0$

$\frac{\partial^2\omega}{\partial z^2} - (z^2 + i z^4)\omega = 0$

$\omega(0) = 0$
$\omega'(0) = 1$

¿ $Im(\omega(x)) = 0$, $x \in \mathbb{R}$?

$\omega = z + \sum_{k=2}^{\infty} C_k z^k = \sum_{k=0}^{\infty} C_k z^k$ $\quad$ ~~$c_0 = C_0 = 0$~~ $\quad C_1 = 1$.

$\frac{\partial\omega}{\partial z} = 1 + \sum_{k=2}^{\infty} k C_k z^{k-1}$

$\frac{\partial^2\omega}{\partial z^2} = \sum_{k=2}^{\infty} k(k-1) C_k z^{k-2}$

Ecuación: $\sum_{k=2}^{\infty} k(k-1) C_k z^{k-2} - \sum_{k=1} C_k z^{k+2} - \sum_{k=1}^{\infty} i C_k z^{k+4} = 0$

término z^0: $2 \cdot 1 C_2 = 0$

" z^1, z^2: $\rightarrow C_3 = C_4 = 0$

" z^3: $5 \cdot 4 \cdot C_5 - C_1 = 0 \rightarrow C_5 = \frac{C_1}{20} = \frac{1}{20}$

" z^4: $6 \cdot 5 C_6 - \cancel{C_2} = 0 \rightarrow C_6 = 0$

" z^5: $7 \cdot 6\, C_7 - \cancel{C_3} - i C_1 = 0 \rightarrow C_7 = \frac{i \cdot 1}{7 \cdot 6}$

...

" z^n: $(n+2)(n+1) C_{n+2} = C_{n-2} + i\, C_{n-4} \Rightarrow \boxed{C_{n+2} = \frac{C_{n-2} + i\, C_{n-4}}{(n+2)(n+1)}}$

Si $Im(\omega(x)) = 0 \rightarrow Im\left(\frac{\partial^k \omega}{\partial x}(x)\right) = 0 \ \forall k$.

↳ imposible porque $C_7 = \frac{i}{7 \cdot 6} = \frac{\omega^{(7}}{7!}(0) \Rightarrow$ FALSO

D. $\frac{d^2\omega}{dz^2}(z) = \frac{-1}{z}\frac{d\omega}{dz} + \frac{1}{z^2}\frac{(1+z)z}{16 \sin z}\omega$

¿ $\underbrace{\lim_{z\to 0} \frac{\omega_1(z) - \sqrt[4]{z}}{\sqrt[4]{z}}}_{L_1}$? ¿ $\underbrace{\lim_{z\to\infty} \frac{\omega(z)}{\sqrt{z}}}_{L_2}$?

↳ ¿ L_1 puede ser 3 ?

$b(z) = -1$
$a(z) = \frac{(1+z)z}{16 \sin z}$

$b(0) = -1$
$a(0) = \lim_{z\to 0} \frac{(1+z)\cancel{z}}{16 \sin z} = \frac{1+0}{16} = \frac{1}{16}$

$\sin z \simeq z + O(z^3)$

$\begin{pmatrix} 0 & 1 \\ 1/16 & 1+(-1) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1/16 & 0 \end{pmatrix}$

$\begin{vmatrix} -\lambda & 1 \\ 1/16 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 = 1/16 \rightarrow \lambda = \pm 1/4$

$\lambda_1 - \lambda_2 \notin \mathbb{N}$

$\Rightarrow \omega = C_1 \underbrace{z^{1/4} p_1(z)}_{\omega_1} + C_2 z^{-1/4} p_2(z)$

$p_1(0) = p_2(0) = 1$.

$p_1(z)$ y $p_2(z)$ funciones analíticas.

• $L_1 = \lim_{z\to 0} \frac{C_1 p_1(z) z^{1/4} - z^{1/4}}{z^{1/4}} = \lim_{z\to 0} \frac{C_1 - p_1(0)}{1} = C_1 - 1$

si $C_1 = 4 \rightarrow L_1 = 3$

• $L_2 = \lim_{z\to\infty} \frac{C_1 p_1(z) z^{1/4} + C_2 z^{-1/4} p_2(z)}{z^{1/2}} = \lim_{z\to\infty} C_1 p_1(\infty) \cdot z^{-1/4} + C_2 p_2(\infty) z^{-3/4} = \infty$

$p_1(z) = \sum_k a_k z^k \quad p_2(z) = \sum_k b_k z^k$.

E.

$$z\frac{d^2\omega}{dz^2} = \left(-4+\frac{\gamma}{z^3}\right)\frac{d\omega}{dz}$$

¿ω presenta singularidad esencial? ¿cuando?

$$\frac{d^2\omega}{dz^2} + \left(\frac{+4}{z} - \frac{\gamma}{z^4}\right)\frac{d\omega}{dz} = 0$$

$$z^4\frac{d^2\omega}{dz^2} + \left(+4z^3-\gamma\right)\frac{d\omega}{dz} = 0$$

$$\frac{d}{dz}\left(z^4\frac{d\omega}{dz} - \gamma\omega\right) = 0 \Rightarrow z^4\frac{d\omega}{dz} - \gamma\omega = C_1 \Rightarrow \frac{d\omega}{dz} = \frac{1}{z^4}(C_1+\gamma\omega)$$

$$\Rightarrow \int\frac{d\omega}{C_1+\gamma\omega} = \int\frac{1}{z^4}dz \Rightarrow \frac{\ln(C_1+\gamma\omega)}{\gamma} = \frac{z^{-3}}{-3} + C \Rightarrow \ln(C_1+\gamma\omega) = C - \frac{\gamma}{3z^3}$$

$$\Rightarrow C_1+\gamma\omega = C_2\cdot e^{-\gamma/3z^3} \Rightarrow \omega = k_1 + k_2 e^{-\gamma/3z^3} = k_1 + k_2\left(1+\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{-\gamma}{3}\right)^n\left(\frac{1}{z^3}\right)^n\right)$$

si $k_2 \neq 0 \rightarrow \omega$: singularidad esencial en $z=0$

acercándome a $z=0$ por:

- $y=0,\ x>0$: $e^{\frac{-\gamma}{3(0^+)^3}} = e^{-\infty} = 0 \Rightarrow \omega \Rightarrow k_1 + 0 = k_1$
- $x=0,\ y>0$: $e^{-\frac{\gamma}{3(i0^+)^3}} = e^{-\frac{\gamma}{3(-i)0^+}} = e^{\frac{\gamma}{30^+i}} \Rightarrow e^{\infty} \Rightarrow \infty \Rightarrow \omega = \infty$