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Exprum

Ampliación de Matemáticas

3º Grado en Ingeniería Aeroespacial

Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio
Universidad Politécnica de Madrid

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A. $\frac{\partial u}{\partial t} = (1+\cos t)\frac{\partial^2 u}{\partial x^2} + 2t\,u$ nota: $\mathcal{F}\left[\frac{1}{1+x^2}\right](\omega) = \pi\exp(-|\omega|)$

$u(x,0) = \frac{1}{1+x^2}$ ¿ $\hat{u}(2,\frac{\pi}{2})$?

$$\frac{\partial \hat{u}}{\partial t} = (1+\cos t)(i\omega)^2\,\hat{u} + 2t\,\hat{u} = (-\omega^2(1+\cos t)+2t)\,\hat{u}$$

$$\int\frac{\partial\hat{u}}{\hat{u}} = \ln\hat{u} = \int\left[-\omega^2(1+\cos t)+2t\right]dt = -\omega^2 t - \omega^2 \operatorname{sen} t + t^2 + C$$

$$\hat{u} = C\,e^{-\omega^2(t+\operatorname{sen} t)+t^2}$$

$$\hat{u}(t=0) = C = \mathcal{F}\left[\frac{1}{1+x^2}\right] = \pi\exp(-|\omega|)$$

$$\hookrightarrow \hat{u} = \pi\exp\left(-\omega^2(t+\operatorname{sen} t)+t^2-|\omega|\right)$$

$$\hat{u}(2,\pi/2) = \pi\exp\left(-4\left(\frac{\pi}{2}+1\right)+\frac{\pi^2}{4}-2\right) = \pi\exp\left(-6-2\pi+\frac{\pi^2}{4}\right)$$

B. $\frac{\partial^2 w}{\partial t^2} + 4\frac{\partial w}{\partial t} + 8w = g(t)$ $w(0)=0$, $w'(0)=1$ ¿ $\mathcal{L}(w)(3)$?

$$g(t) = \begin{cases} 1-t & t\in[0,1) \\ t-1 & t\in[1,\infty) \end{cases}$$

$$g(t) = (1-t)[H(t)-H(t-1)] + (t-1)H(t-1) = (1-t)H(t) + 2(t-1)H(t-1)$$

$$G(s) = \mathcal{L}(g) = \mathcal{L}\left((1-t)H(t) + 2\underbrace{(t-1)H(t-1)}_{f(t-1)}\right) = \frac{1}{s} - \frac{1}{s^2} + 2\mathcal{L}\left(f(t-1)H(t-1)\right) \overset{*}{=} \frac{1}{s} - \frac{1}{s^2} + 2e^{-s}\left[\frac{1}{s^2}\right]$$

$\boxed{\mathcal{L}(1) = \frac{1}{s} \quad \mathcal{L}(t^\nu) = \frac{\Gamma(\nu+1)}{s^{\nu+1}} = \frac{\nu!}{s^{\nu+1}} \quad (\nu\in\mathbb{N}) \quad \text{llamo } f(t)=t}$

$\boxed{*: \mathcal{L}(f(t-a)H(t-a)) = e^{-as}\cdot\mathcal{L}(f)(s)}$

$$G(s) = \frac{1}{s} + \frac{2e^{-s}-1}{s^2}$$

Ecuación: $s^2W - s\underbrace{w(0)}_{0} - \underbrace{w'(0)}_{1} + 4sW - 4\underbrace{w(0)}_{0} + 8W = G$

$$(s^2+4s+8)W = G+1$$

$$W(s) = \frac{G+1}{s^2+4s+8}$$

$$G(3) = \frac{1}{3} + \frac{1}{9}(2e^{-3}-1) = \frac{1}{9}(2e^{-3}+2)$$

$$\boxed{W(3) = \frac{1}{9+12+8}\cdot\left(\frac{1}{9}(2e^{-3}+2)+1\right) = \frac{1}{29}\cdot\frac{1}{9}(2e^{-3}+11)}$$

C. $\dfrac{\partial^2 \omega}{\partial z^2} - i \cdot \operatorname{sen} z \cdot \omega = 0$ $\quad \omega(0)=1 \quad \omega'(0)=1$ $\quad$ ¿ $C_0+C_1+C_2+C_3+C_4$?

$$\omega = 1+z+\sum_{k=2}^{\infty} C_k z^k = \sum_{k=0}^{\infty} C_k z^k \qquad C_0=1 \quad C_1=1.$$

$$\frac{d^2\omega}{dz^2} = \sum_{k=2}^{\infty} k\cdot(k-1) C_k z^{k-2}$$

$$\operatorname{sen} z = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^{2k-1}}{(2k-1)!}$$

Ecuación:

$$\sum_{k=2}^{\infty} k(k-1)C_k z^{k-2} - i\overbrace{\left(\sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^{2k-1}}{(2k-1)!}\right)}^{\cancel{z}+\frac{z^3}{3!}+\frac{z^5}{5!}+\cdots}\overbrace{\left(\sum_{k=0}^{\infty} C_k z^k\right)}^{1+z+C_2z^2+\cdots} = 0$$

término z^0: $\quad 2C_2 = 0$

z^1: $\quad 3\cdot 2C_3 - i = 0 \to C_3 = \frac{i}{6}$

z^2: $\quad 4\cdot 3C_4 - i = 0 \to C_4 = i/12$

$\Rightarrow C_0 + C_1 + C_2 + C_3 + C_4 = 1+1+\frac{i}{6}+\frac{i}{12} = 2+\frac{3i}{12}$

$= \boxed{2+\frac{i}{4}}$

D. $z\dfrac{d^2\omega}{dz^2} + \dfrac{d\omega}{dz} - \dfrac{(1+z)}{16\tanh(z)}\omega = 0$ $\quad$ ¿ $\exists\, \omega$ " $\lim\limits_{z\to 0} \dfrac{\omega_1(z)-\sqrt[4]{z}}{\sqrt[4]{z}} = 3$?

$$\frac{d^2\omega}{dz^2} = \frac{1}{z}\underbrace{(-1)}_{b(z)}\frac{d\omega}{dz} + \frac{1}{z^2}\underbrace{\frac{z(1+z)}{16\tanh(z)}}_{a(z)}\omega$$

$\tanh(z) = \dfrac{\operatorname{sh}(z)}{\operatorname{ch}(z)} \xrightarrow{z\to 0} \dfrac{z+O(z^2)}{1+O(z^2)} \approx z$

$b(0) = -1$

$a(0) = \lim\limits_{z\to 0} \dfrac{z(1+z)}{16\tanh(z)} = \lim\limits_{z\to 0} \dfrac{\cancel{z}(1+z)}{16\cancel{z}} = \dfrac{1}{16}$

$$\begin{pmatrix} 0 & 1 \\ 1/16 & 0 \end{pmatrix} \qquad \begin{vmatrix} -\lambda & 1 \\ 1/16 & -\lambda \end{vmatrix} = 0 \to \lambda^2 - 1/16 = 0 \to \lambda = \pm 1/4.$$

$1-1$

$\omega_1 = C_1 z^{1/4} p_1(z) \qquad \omega_2 = C_2 z^{-1/4} p_2(z) \qquad p_1(z), p_2(z)$ analíticos " $p_1(0)=p_2(0)=1.$

$$\lim_{z\to 0} \frac{\omega_1 - z^{1/4}}{z^{1/4}} = \lim_{z\to 0} \frac{\cancel{z^{1/4}}(C_1 p_1(z) - 1)}{\cancel{z^{1/4}}} = C_1\cdot 1 - 1 = 3 \Leftrightarrow \underline{C_1 = 4}$$