WUOLAH

diciembre 2017.pdf

Exprum

Ampliación de Matemáticas

3º Grado en Ingeniería Aeroespacial

Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio
Universidad Politécnica de Madrid

06-17.

E.

$$\lim_{x\to 0}\frac{\sin(J_1(x)) - J_1(x)}{J_3(x)}$$

$J_1(x) = \frac{x}{2} + o(x^2)$.

$J_3(x) = \frac{1}{\Gamma(4)}\frac{x^3}{2^3} + o(x^4) = \frac{x^3}{2^4\cdot 3} + o(x^4)$

$\sin(J_1(x)) = J_1(x) - \frac{J_1^3}{3!} + \dots$

$$\lim_{x\to 0}\frac{J_1 + \frac{J_1^3}{3!} - J_1}{J_3} = \frac{-\frac{1}{6}\cdot\frac{x^3}{2^3}}{\frac{x^3}{2^4\cdot 3}} = -\frac{1}{3}\ (-1)\ \to \cancel{15}\quad 19.$$

12-17.

A. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + (1+2t)u$

$\mathcal{F}(\exp(-ax^2)) = \sqrt{\frac{\pi}{a}}\cdot\exp\left(-\frac{\omega^2}{4a}\right)$ ¿$\hat{u}(2,2)$?

$u(t=0) = x\exp(-x^2)$.

$\frac{\partial\hat{u}}{\partial t} = -\omega^2\hat{u} + (1+2t)\hat{u}$

$\int\frac{\partial\hat{u}}{\hat{u}} = \int(2t+1-\omega^2)dt \to \hat{u} = Ce^{(t^2-\omega^2 t+t)}$

$\hat{u}(t=0) = \mathcal{F}(x\exp(-x^2)) = i\cdot\frac{d\mathcal{F}(\exp(-x^2))}{d\omega} = i\,\frac{d\sqrt{\pi}\exp(-\omega^2/4)}{d\omega} = -\frac{i\omega}{2}\sqrt{\pi}\,e^{-\omega^2/4}$

$\hat{u} = -\frac{i\omega}{2}\sqrt{\pi}\,e^{(t^2-\omega^2 t+t-\omega^2/4)}$

$\hat{u}(2,2) = -i\sqrt{\pi}\,e^{(4-8+2-1)} = -i\sqrt{\pi}\,e^{-3}$

B. $g(t) = \begin{cases} t(t-1) & t\in(0,1) \\ 0 & t\notin \end{cases}$. $\omega(0) = 0 \quad \omega'(0) = 1$. $\left|\mathcal{L}(\omega)(s=2)?\right|$

$W = \mathcal{L}(\omega) \quad G = \mathcal{L}(g)$

$\frac{\partial^2\omega}{\partial t^2} + 4\frac{\partial\omega}{\partial t} + 8\omega = g(t)$.

$s^2W - s\omega(0) - \omega'(0) + 4(sW - \omega(0)) + 8W = G$

$(s^2+4s+8)W = G+1$

$W(s) = \frac{G+1}{s^2+4s+8}$.

¿$G = \mathcal{L}(g)$?

$(t-1+1)(t-1) = (t-1)^2 + (t-1)$

$g(t) = t(t-1)[H(t) - H(t-1)] = \overbrace{t(t-1)}^{f(t)}H(t) - t(t-1)H(t-1) = f(t)H(t) - \overbrace{[(t-1)^2+(t-1)]}^{h(t-1)}H(t-1)$

$f(t) = t(t-1)\quad t\in\mathbb{R}^+$

$h(t) = t(t+1)$

$F = \mathcal{L}(f) = \mathcal{L}(t^2-t) \overset{*_1}{=} \frac{2}{s^3} - \frac{1}{s^2}$

$H = \mathcal{L}(h) = \frac{2}{s^3} + \frac{1}{s^2}$

$*_1\ \mathcal{L}(t^\nu) = \frac{\Gamma(\nu+1)}{s^{\nu+1}} = \frac{\nu!}{s^{\nu+1}}$ $(\nu+1)\in\mathbb{N}$

$G = \frac{2}{s^3} - \frac{1}{s^2} - e^{-s}\left(\frac{2}{s^3} + \frac{1}{s^2}\right)$

$G(2) = 0 - e^{-2}\left(\frac{1}{4}+\frac{1}{4}\right) = -\frac{e^{-2}}{2}$

$W(2) = \frac{-e^{-2}\frac{1}{2}+1}{4+8+8} = \frac{1-e^{-2}/2}{20} = \frac{2-e^{-2}}{40}$

$*_2: \mathcal{L}(f(t-a)H(t-a)) = e^{-as}\mathcal{L}(f)(s)$

C. $\frac{\partial^2 \omega}{\partial z^2} - (iz+z^4)\omega = 0$ $\omega(0)=0$ $\frac{d\omega}{dz}(0) = i$ ¿ $Re(\omega(x))=0$?

$\omega(z) = iz + \sum_{k=2}^{\infty} C_k z^k = \sum_{k=1}^{\infty} C_k z^k$ $\underline{C_1 = i}$

$\frac{\partial \omega}{\partial z} = \sum k C_k z^{k-1}$; $\frac{\partial^2 \omega}{\partial z^2} = \sum_{k=1}^{\infty} k(k-1) C_k z^{k-2}$

Ecuación: $\overbrace{\sum_{k=1}^{\infty} k(k-1)C_k z^{k-2}}^{\partial^2\omega/\partial z^2} - \overbrace{\sum_{k=1}^{\infty} i C_k z^{k+2}}^{iz^2\omega} - \overbrace{\sum_{k=1}^{\infty} C_k z^{k+4}}^{z^4\omega} = 0$

término z^0: $2\cdot 1\cdot C_2 = 0 \to \boxed{C_2 = 0}$

" z^1: $3\cdot 2 C_3 = 0 \to \boxed{C_3 = 0}$

" z^2: $4\cdot 3 C_4 = 0 \to \boxed{C_4 = 0}$ ~~$\to 12C_4 - i^2 = 0 \to C_4 = 1/12$~~

" z^3: $5\cdot 4 C_5 - i\underbrace{C_1}_{i^2} = 0 \to 20 C_5 + 1 = 0 \to \boxed{C_5 = -1/20}$

...

" z^n: $(n+2)(n+1)C_{n+2} - i C_{n-2} - C_{n-4} = 0 \Rightarrow C_{n+2} = \frac{iC_{n-2} + C_{n-4}}{(n+2)(n+1)}$

¿ $Re(\omega(x)) = 0$? $\to$ verdad $\Leftrightarrow Re(\omega^{(k}(x)) = 0$ $\forall x$ ~~y~~

$\hookrightarrow$ Falso porque $\frac{\omega^{(5}(0)}{5!} = -1/20$

D. $\frac{d^2\omega}{dz^2} = -\frac{1}{z}\frac{d\omega}{dz} + \frac{(1+z)^2}{2z \operatorname{sen} z}\omega$ ¿ cómo son $\underbrace{\lim_{z\to 0} \frac{\omega_1(z) - z^{\sqrt{2}/2}}{z^{\sqrt{2}/2}}}_{L_1}$ y $\underbrace{\lim_{z\to 0} \frac{\omega(z)}{z^{\sqrt{2}/2}}}_{L_2}$?

$\frac{d^2\omega}{dz^2} = \frac{1}{z}\overbrace{(-1)}^{b(z)}\frac{d\omega}{dz} + \frac{1}{z^2}\overbrace{\frac{z(1+z)^2}{2\operatorname{sen} z}}^{a(z)}\omega$

$b(0) = -1$

$a(0) = \lim_{z\to 0} \frac{z(1+z)^2}{2 \underbrace{\operatorname{sen} z}_{z+O(z^3)}} = \lim_{z\to 0} \frac{\cancel{z}(1+z)^2}{2\cancel{z}} = 1/2$

$\begin{pmatrix} 0 & 1 \\ 1/2 & 1-1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1/2 & 0 \end{pmatrix} \Rightarrow \begin{vmatrix} -\lambda & 1 \\ 1/2 & -\lambda \end{vmatrix} = 0 \to \lambda^2 - 1/2 = 0 \Leftrightarrow \lambda = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\lambda_1 - \lambda_2 \notin \mathbb{N}$:

$\omega_1(z) = C_1(z^{\sqrt{2}/2} p_1(z))$ $\omega_2(z) = C_2(z^{-\sqrt{2}/2} p_2(z))$

suponiendo $C_1 = 1$: $\to$ ¿ L_1 es 0 en caso: $C_1 = 1$

$\hookrightarrow \boxed{L_1 = \lim_{z\to 0} \frac{\cancel{z^{\sqrt{2}/2}}(p_1(z)-1)}{\cancel{z^{\sqrt{2}/2}}} = 1-1 = 0}$

$L_2 = \lim_{z\to 0} C_1 p_1(z) \frac{\cancel{z^{\sqrt{2}/2}}}{\cancel{z^{\sqrt{2}/2}}} + C_2\, p_2(z)\cdot z^{-\sqrt{2}} =$ $\to$ si $C_2 = 0 \Rightarrow L_2 = C_1 p_1(0) = \underline{C_1}$; $\to$ si $C_2 \neq 0 \Rightarrow L_2 = C_1 + \frac{1}{0} = \underline{\infty}$