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Aeropro

Ampliación de Matemáticas

3º Grado en Ingeniería Aeroespacial

Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio
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Examen 25-01-2017 (2º parcial)

$\boxed{1}$ $f(x)=1 \quad -1 \leq x \leq 1$ $\qquad \mathcal{F}(f \cdot f) = \cancel{\mathcal{F}(f^2)} = \hat{f}(w)\hat{f}(w) = [\hat{f}(w)]^2$

$f(x)=0 \quad x<-1 \quad 1 \leq x$

¿$\mathcal{F}(f \cdot f)$? ¿$F(0)$?

$$\hat{f}(w) = \int_{-1}^{1} \underbrace{1}_{f(x)} e^{-iwx}\,dx = \frac{1}{-iw} e^{-iwx}\Big|_{-1}^{1} = \frac{1}{-iw}\left(e^{-iw} - e^{iw}\right) = \frac{e^{iw}-e^{-iw}}{iw}$$

$$\hat{f}(w) = \frac{e^{iw}-e^{-iw}}{2iw} = \frac{1}{w}\cdot 2\operatorname{sen} w$$

$$\mathcal{F}(f \cdot f)(w) = \hat{f}\cdot\hat{f}(w) = \left(\frac{2\operatorname{sen} w}{w}\right)^2 = \frac{4\operatorname{sen}^2 w}{w^2}$$

$$F(0) = \lim_{w\to 0} \frac{4\operatorname{sen}^2 w}{w^2} \approx \frac{4w^2}{w^2} = 4$$

$\boxed{2}$ $\dfrac{d^2w}{dt^2}(t) + 2\dfrac{dw}{dt}(t) + 2w(t) = g(t)$ $\quad$ con $g(t) = t(t-1)$ con $t \in [\ ,\ ]$

$w(0)=0 \qquad \dfrac{dw}{dt}(0) = 1$

$$(z^2+2z+2)\,\mathcal{L}[w(t)](z) = 1 + \mathcal{L}[g(t)](z)$$

$$g(t) = t(t-1)\big(H(t)-H(t-1)\big) = H(t)(\underbrace{t^2 - t}_{(t-1)t}) - H(t-1)\big((t+1)-1\big)(t-1)$$

↳ Pasar de variable $g(t) \longrightarrow H(t)$

$\mathcal{L}[t^v] = \dfrac{\Gamma(v+1)}{z^{v+1}}$ (fórmula)

$a=1$

$\mathcal{L}(H(t-a)f(t-a)) = \exp(-az)\cdot\mathcal{L}[f(t)](z)$

$(\Gamma(n+1) = \Gamma(n))$

$$\mathcal{L}(g(t))(z) = \frac{\overbrace{\Gamma(2+1)}^{v\ =2}}{z^{2+1}} - \frac{\Gamma(1+1)^{=1}}{z^{1+1}} - \underbrace{\exp(-z)}_{a=1}\cdot\Big(\underbrace{\mathcal{L}\big(t(1+t)\big)}_{\frac{\Gamma(3)}{z^3}+\frac{\Gamma(2)}{z^2}}\Big)$$

z^{2+1} → grado de t

$2 = \Gamma(3)$ $\qquad \Gamma(2) = 1$

$$\mathcal{L}(g(t))(z) = \frac{2}{z^3} - \frac{1}{z^2} - \exp(-z)\left(\frac{2}{z^3}+\frac{1}{z^2}\right)$$

$g \to w$

$\mathcal{L}[w(t)](z)\cdot\bigcirc = 1 + \mathcal{L}[g(t)](z)$

$$\mathcal{L}[\cancel{g}\,w(t)](z) = \frac{1}{z^2+2z+2}\left(1 + \frac{1}{z^2}\left(\frac{2}{z} - 1 - \exp(-z)\left(\frac{2}{z}-1\right)\right)\right)$$

$$\mathcal{L}[w(t)](z) = \frac{1}{10}\left(1 + \frac{1}{4}\left(1 - 1 - e^{-2}(1+1)\right)\right) = \frac{1}{10}\left(1 - \frac{1}{2}e^{-2}\right)$$

$$\boxed{\mathcal{L}[w(t)](z) = \frac{1}{20}(2 - e^{-2})}$$

3 $\dfrac{d^2w}{dz^2} - (z+z^4)w = 0$

$w(0)=0 \quad \dfrac{dw}{dz}(0)=1$

$w(z) = \sum_{k=0}^{\infty} C_k z^k$

$w(z) = 1\cdot w(0) + z\cdot\dfrac{dw}{dz}(0) + \sum_{k=2}^{\infty} C_k z^k = z + \sum_{k=2}^{\infty} C_k z^k$

Sustituyendo en la ecuación:

$\sum_{k=2}^{\infty} k(k-1)C_k z^{k-2} - (z+z^4)\left(z + \sum_{k=2}^{\infty} C_k z^k\right) = 0$

$\dfrac{d^2w}{dz^2} = \dfrac{d}{dz}\left(1 + \sum_{k=2}^{\infty} C_k\cdot k z^{k-1}\right) = \sum_{k=2}^{\infty} C_k (k-1) k z^{k-2}$

$\sum_{k=2}^{\infty} k(k-1)C_k z^{k-2} - \boxed{(z^2+z^5)} - \sum_{k=2}^{\infty} C_k z^{k+1} - \sum_{k=2}^{\infty} C_k z^{k+4} = 0$

Buscamos valores de k tales que lo podamos unir con las series ("que las z tengan el mismo orden")

$k=1 \longrightarrow -(z^2+z^5) - C_1 z^2 - C_1 z^5 = 0 \longrightarrow C_1 = 1$

Ya lo sabemos C.I. $dw/dz(0) = 1$ Por tanto, $C_1 = dw/dz$

$k=4 \longrightarrow 4\cdot 3 C_4 z^2 - (z^2+z^5) - C_4 z^5 - C_4 z^8 = 0$

4 Test...

Dado que todos $C_k \geq 0$, no puede presentar extremos relativos (es extrictamente creciente). Además, $w(z)>0\ \forall z$ de donde $\dfrac{d^2w}{dz^2}(z)>0\ \forall z$ y por tanto, no presenta puntos de inflexión la gráfica de w.

5 $z\exp(z)\dfrac{d^2w}{dz^2} - \ln(1+z)\dfrac{dw}{dz} + \dfrac{(1+z)^2}{4\operatorname{sen} z}w = 0 \quad ¿\lim_{z\to 0}\ ?$

Reescribimos la ecuación.

$\dfrac{d^2w}{dz^2} = \dfrac{\ln(1+z)}{z\exp(z)}\dfrac{dw}{dz} - \dfrac{(1+z)^2}{4z\operatorname{sen}(z)\exp(z)}w$

Se puede observar que $z=0$ es un punto singular

$\dfrac{d^2w}{dz^2}(z) = \dfrac{b(z)}{z}\cdot\dfrac{dw}{dz}(z) + \dfrac{a(z)}{z^2}w(z)$ ¡FALTA!

$C = \begin{bmatrix} 0 & 1 \\ a(0) & b(0)+1 \end{bmatrix}$

$\dfrac{d^2w}{dz^2} = \dfrac{\ln(1+z)}{z\cdot\exp(z)}\dfrac{dw}{dz} + \dfrac{(1+z)^2}{4z\operatorname{sen} z\exp z}w\cdot\dfrac{z}{z}$

$b(z) = \exp(-z)\ln(1+z) \longrightarrow b(0) = 0$

$a(z) = -\exp(-z)\cdot\dfrac{z(1+z)^2}{4\operatorname{sen} z} \longrightarrow a(0) = \lim_{z\to 0} -\dfrac{z(1+z)^2}{4\operatorname{sen} z\exp z} \simeq \lim_{z\to 0}\dfrac{z(1+z)^2}{4z\operatorname{sen} z} = \dfrac{1}{4}$

$C = \begin{bmatrix} 0 & 1 \\ 1/4 & 1 \end{bmatrix}$

Autovalores:

$$-\lambda(1-\lambda) - \frac{1}{4} = 0 \longrightarrow \lambda^2 - \lambda + \frac{1}{4} = 0 \longrightarrow \left(\lambda - \frac{1}{2}\right)^2 = 0 \longrightarrow \lambda = \frac{1}{2} \text{ doble}$$

Dado que $\lambda_1 = \lambda_2 = 1/2$

$P_1(0) = 1$

P_2 puede ser nula

$u_1 = z^\lambda P_1 C_1$

$u_2 = C_2 (z P_1 \ln z + z^\lambda P_2(z))$

Por tanto, $w = C_1\sqrt{z}\,P_1(z) + C_2\sqrt{z}\,(P_1 \ln z + z^\lambda P_2(z))$

$$\lim_{z\to 0} \frac{w_2}{\sqrt{z}\ln z} = \lim_{z\to 0}\left(\frac{\cancel{\sqrt{z}}\,\overbrace{P_1(z)}^{1}\cancel{\ln(z)}}{\cancel{\sqrt{z}}\,\cancel{\ln z}} + \frac{\cancel{\sqrt{z}}\,\cancel{P_2(z)}^{0}}{\cancel{\sqrt{z}}\cdot \ln z}\right) = 1$$

$\hookrightarrow 1/z = z$

$w_2 = \sqrt{z}\,P_1(z)\ln z + \sqrt{z}\,P_2(z)$ ya que $C_1 = 0$ y $C_2 = 1$

Tendríamos que hacer el límite con ambos términos pero el segundo no converge:

$$\lim_{z\to 0}\frac{w_2}{\sqrt{z}} = \lim_{z\to 0}\left(\frac{\cancel{\sqrt{z}}\,\overbrace{P_1(z)}^{1}\underbrace{\ln(z)}_{-\infty}}{\cancel{\sqrt{z}}} + \frac{\sqrt{z}\,\cancel{P_2(z)}^{0}}{\sqrt{z}}\right) =$$

$\boxed{6}$ $J_1(x) = \frac{x}{\cdot 2} + \sigma(x^2)$ No se si es el enunciado, creo que no.

$\Gamma(2)$

No entra en el parcial

$J_2(x) = \frac{x^2}{2\cdot 4} + \sigma(x^3)$

$\Gamma(3)$

$J_3(x) = \frac{x^3}{3\cdot 2\cdot 8} + \sigma(x^4)$

$\Gamma(4)$

$$\lim_{x\to 0} \frac{J_3(x) + 4J_2(x) - xJ_1(x)}{\int_0^x J_2(t)\,dt} = \lim \frac{\frac{1}{6}\left(\frac{x}{2}\right)^3 + \frac{1}{2}\left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)}{\int_0^x \frac{1}{2}\left(\frac{x}{2}\right)^2 dx \cdot \frac{1}{24}x^3}$$

$$\lim \frac{\frac{1}{48}\cdot x^3}{\frac{1}{24}x^3} = \frac{1}{2}$$