Physics Internal Assessment

To what extent does the measured radius of curvature of a beta particle in a uniform magnetic field deviate from relativistic predictions?

Jacob Bruner

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1 Introduction

1.1 Motivation

As a mathematical-physics enthusiast, I have always been fascinated by the intricacies of particle behavior close to the speed of light. I distintly recall my fascination at a young age with Carl Segan's episode "Einstein's Special Relativity" in the Cosmos series. In particular, I've always been amazed by how our reality can defy our intuition. The prospect of testing high-velocity relativistic theory in action has always been a long-standing intrigue of mine. This lab presents an opportunity to delve deeper into this concept and understand the nuances of this phenomenon, fueling my excitement to uncover the results and see how they align with my expectations. I am eager to see how my understanding of the relativistic predictions will compare to the experimental observations and gain a deeper appreciation for the complexities of particle behavior in magnetic fields.

For my Internal Assessment, I want to explore the behavior of a beta particle in a uniform magnetic field. Since beta particles emitted from a radioactive source have a velocity that is a significant fraction of the speed of light c, the cosmological speed-limit, relativistic techniques must be employed to attempt at accurately modeling the particle's motion. Because of this, I want to explore **to what extent does the measured radius of curvature of a beta particle in a uniform magnetic field deviate from relativistic predictions?**

1.2 Background Theory

Prior to the 20th-century, the prevailing theory in Physics was that a omnipresent *'luminiferous aether'* was responsible for propagating forces and electromagnetic waves.[1] Yet, as experimental methods grew more sophisticated, a number of theoretical inconsistencies came to the fore. Most notable of which was the incompatibility of Newtonian mechanics with Maxwell's electromagnetic field equations.[2] It turned out to be intractable to mathematically define this 'aether,' prompting German-physicist Albert Einstein to simplify his assumptions and debut his *Theory of Special Relativity* in 1905.

Einstein's theory is founded upon considering two physical 'laws' as being the most important and fundamental to all physical phenomena:

1. The speed of light in vacuum is the same for all observers.
2. The laws of physics are invariant in all inertial frames of reference (zero acceleration).

From this point, Einstein explored the implications of these postulates, including unintuitive phenomena like time dilation, length contraction, and mass-energy equivalance—all of which preserve this 'cosmological speed limit.' His theory modifies a number of formulas from classical mechanics including coordinate transformations from one frame of reference to another, force equations, and energy equations. These will be important momentarily.

[1] Albert Einstein. *The Genesis of General Relativity. Volume 1-4.* 1st ed. Boston Studies in the Philosophy and History of Science. Springer, 2007. ISBN: 9781402039997, p. 24.

[2] Ibid., p. 40.

2 Predictions and Special Relativity

In an electromagnetic field, a charged particle in 3-space will experience, in full generality, a force $\vec{\mathbf{F}}$ given by $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$,[3] where q denotes the particle's charge, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ denote the magnetic and electric field vectors at the point of the particle, and $\vec{\mathbf{v}}$ denotes the velocity of the particle. If this particle is moving through a uniform magnetic field generated by two parallel magnets orthogonal to the direction of the magnetic field, this formula simplifies to $F_m = q|v||B|$, directed orthogonal to the magnetic field vector and the velocity by the right-hand rule, since $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = |v||B| \sin\theta$ with $\theta = \frac{\pi}{2}$.

This magnetic force supplies the centripetal force for the particle to move in a circle, satisfying the criterion for circular motion with the force (acceleration) directed orthogonal to the velocity. Using the equation for the centripetal force, $F_c = mv^2/r$, it is possible to solve the resulting force-balance equation.

Yet, when the speed v approaches the speed of light (as it does in a nuclear decay), the effects of Einstein's relativity become more apparent and a factor must be included in the equation. The 'Lorentz factor' describes the relationships between time and space for an object moving at constant speed. Formally, it is the ratio of the time coordinates of an event in the moving reference frame and the same event in a stationary observer's reference frame. Denoted by the Greek letter gamma γ, it looks as follows:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

2.1 Predicted Results

From the description of the force on a charged particle in a magnetic field, it's clear a beta particle (β) will experience a force directed orthogonal to both the its velocity vector and the magnetic field vector given by $q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. This orthogonality condition is sufficient to supply the centripetal force for circular motion (tangent to the radius). This can be seen in Equation 6, where the radius of curvature r can be calculated relativistically from the mass m, velocity $\vec{\mathbf{v}}$, charge q, and magnetic field strength $\vec{\mathbf{B}}$.

For this reason, it is predicted that the beta-minus particle will experience a rightward centripetal force (by the right-hand rule with a negative charge) supplied by the uniform cylindrical magnetic field, until the boundary of the cylinder, where the magnetic field becomes non-uniform and is assumed to be negligible. Hence, the beta-minus particle will travel tangent to its circular path until interacting with matter.

2.1.1 Calculating Speed

First, the speed of the β^- particle (an electron) must be ascertained using the published mean β^- kinetic energy released of $187.1\,\text{keV} \pm 10.\,\text{keV}$.[4] Or, in terms of percent uncertainty, $187.1\,\text{keV} \pm 5.3\%$ First consider the formula for kinetic energy E_k in terms of the Lorentz factor γ,

$$E_k = (\gamma - 1)m_0c^2, \tag{2}$$

[3] Richard Feynman. *Mainly Electromagnetism and Matter*. Vol. 2. The Feynman Lectures. California Institute of Technology, 1965. ISBN: 0465023827, p. 10.

[4] National Nuclear Data Center. URL: `https://www.nndc.bnl.gov/nudat3/`.

with

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{3}$$

where v is the speed of the particle, m_0 is the rest mass of the particle, and c is the speed of light. Rearranging (2), we can calculate the Lorentz factor γ directly:

$$\gamma = \frac{E_k}{m_0c^2} + 1 \tag{4}$$

Now using the rest mass of an electron, published to four significant figures, of $511.5\,\mathrm{keV}/c^2$ with negligibly small uncertainty,[5] and the total decay energy released to calculate gamma:

$$\begin{aligned}\gamma &= \frac{187.1\,\mathrm{keV} \pm 5.3\%}{(511.5\,\mathrm{keV}c^{-2})c^2} + 1\\ \gamma &= 1.366 \pm 5.3\%.\end{aligned}$$

Using (3), it is now possible to ascertain the relativistic speed v. Rearranging,

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\\ \frac{1}{\gamma^2} &= 1-\frac{v^2}{c^2}\\ 1-\frac{1}{\gamma^2} &= \frac{v^2}{c^2}\\ v^2 &= c^2\left(1-\frac{1}{\gamma^2}\right)\\ \Rightarrow \quad v &= c\sqrt{1-\frac{1}{\gamma^2}} \qquad (5)\end{aligned}$$

Plugging in the given values and simplifying the equation, one gets:

$$\begin{aligned}v &= (2.998\times 10^9)\sqrt{1-\frac{1}{(1.366\pm 5.3\%)^2}}\\ v &= .681c \pm 5.3\%\\ v &= 2.042\times 10^8\ \mathrm{m\,s^{-1}} \pm 5.3\% \qquad (6)\end{aligned}$$

So the speed of the beta minus particle is approximately .681 times the speed of light, with a percent uncertainty of $\pm 5.3\%$ since $\frac{\Delta v}{v} = \left|\frac{1}{2}\right||-2|\frac{\Delta\gamma}{\gamma}$.

2.1.2 Calculating Predicted Curvature

Solving for the curvature experienced by the beta particle requires considering the forces involved. In particular, the particle will experience a Lorentz force directed orthogonal to both its velocity and the magnetic—which acts as the centripital force to propegate

[5] *The NIST Reference on Constants, Units, and Uncertainty*. 2018. URL: `https://physics.nist.gov/cgi-bin/cuu/Value?mec2mev`.

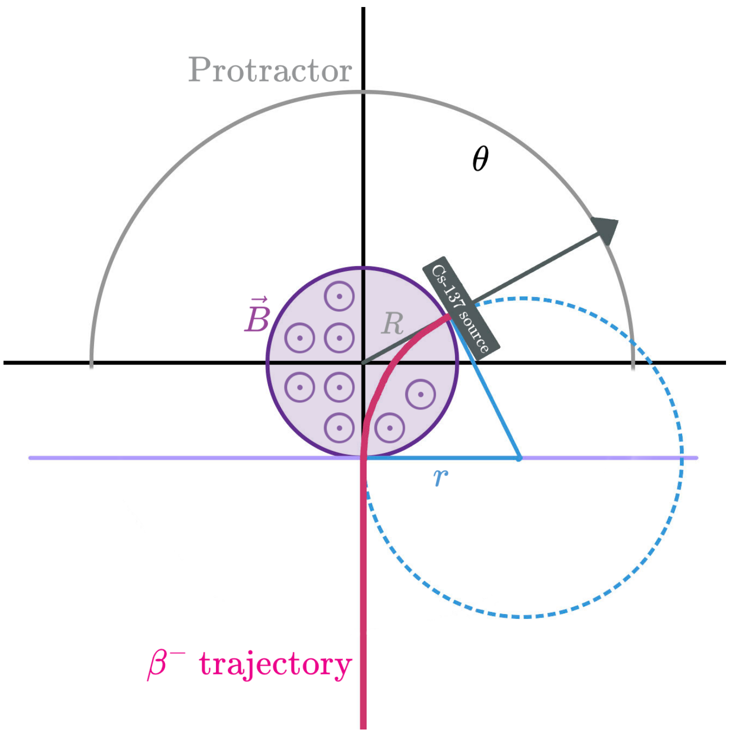


Figure 1: A diagram depicting the geometry of the experimental setup with the beta-minus particle's trajectory in magenta and the uniform magnetic field pointing into the plane of the diagram.

the particle in circlar motion when it enters the uniform cyindrical field. Firstly, the relativistic centripetal force F_c can be calculated by,

$$F_c = \frac{\gamma m |\vec{\mathbf{v}}|^2}{r}$$

Substituting the Lorentz factor:

$$F_c = \frac{m|\vec{\mathbf{v}}|^2}{r} \cdot \frac{1}{\sqrt{1 - \frac{|\vec{\mathbf{v}}|^2}{c^2}}}$$

In the uniform magnetic field, the centripetal force is provided by the Lorentz force on the particle, and, since $\vec{\mathbf{v}}$ and $\vec{\mathbf{b}}$ always lie in orthogonal planes, this can be written:

$$F_L = q\,|\vec{\mathbf{v}}|\,|\vec{\mathbf{B}}|.$$

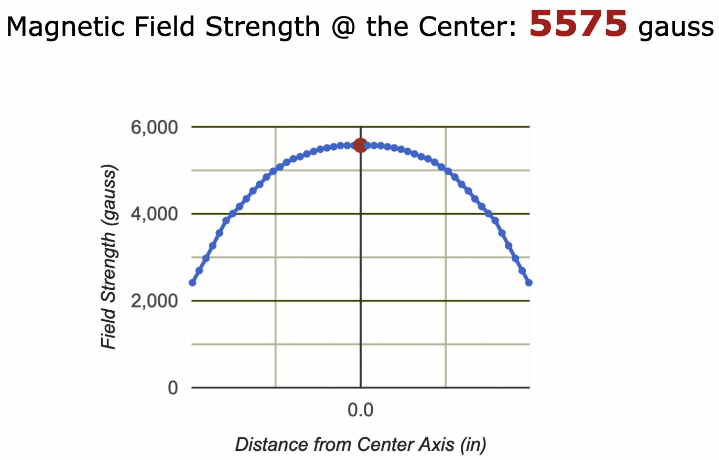
Then, solving the force-balance equation,

$$F_c = F_L$$

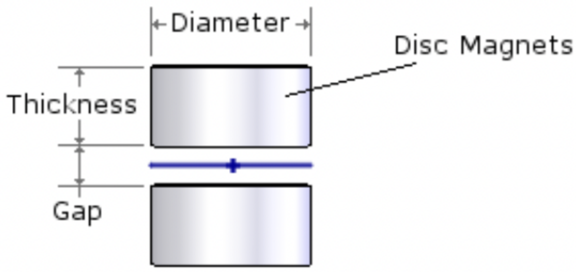
$$\frac{mv^2}{r} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = q\,|\vec{\mathbf{v}}|\,|\vec{\mathbf{B}}|$$

$$\Rightarrow \quad r = \frac{m|\vec{\mathbf{v}}|}{q\,|\vec{\mathbf{B}}|} \cdot \sqrt{1 - \frac{|\vec{\mathbf{v}}|^2}{c^2}} \tag{6}$$

For the value of $|\vec{\mathbf{B}}|$ in the experiment, the manufacturer of the magnets' finite element method simulation was used to determine the field strength. This experiment was conducted with two N35-grade neodyium disk magnets of with diameter 10mm and thickness 6mm which were separated by a 4mm gap. The published residual induction (magnetization) of an N35 magnet is 11700 Gauss.[6] The results of the simulation are in Figure 2.



(a) Field strength (Gauss) versus distance from center axis of magnets.



(b) Diagram of setup.

Figure 2: Simulation of magnetic field strength with varying distance from the center of the magnetic field.[7]

For this reason, an average magnetic field strength $|\vec{\mathbf{B}}|$ of 4979 Gauss or 0.4979 T was noted. Due to the 'uniform' assumption of this magnetic field, it is difficult to estimate an uncertainty for the field strengh, since this value resulted from computer simulation. This value will be represented without uncertainty for now, but will be discussed later.

This value for magnetic field strength can be substituted into equation (6) so that the radius of curvature can be computed:

$$r = \frac{(9.109 \times 10^{-31}\,\mathrm{kg})(2.042 \times 10^{8}\,\mathrm{m\,s^{-1}})}{(1.602 \times 10^{-19}\ \mathrm{C})(0.4979\ \mathrm{T})} \cdot \sqrt{1 - \frac{(2.042 \times 10^{8}\,\mathrm{m\,s^{-1}})^2}{(2.998 \times 10^{8}\,\mathrm{m\,s^{-1}})^2}}$$

$$r = 1.707 \times 10^{-3}\,\mathrm{m} \pm 11\% = 1.707\,\mathrm{mm} \pm 11\%.$$

[6] *Neodymium Magnets Fact Sheet*. 2016. URL: https://www.albmagnets.com/content/9-grade-of-neodymium-magnets.

Now, leveraging the geometry of the problem, as shown in Figure 3, the relationship between the radius of curvature r and the protracter angle θ is, given the radius of the magnets R, the following:

$$r = R\cot\theta,$$

and, likewise,

$$\theta = \arctan\left(\frac{R}{r}\right).$$

Substituting the above derived values, one can predict the angle θ. Although, first, the absolute error of $\pm 0.190\,\text{mm}$ is calculated.[8]

$$\theta = \arctan\left(\frac{5\,\text{mm}}{1.707\,\text{mm} \pm 0.190\,\text{mm}}\right)$$
$$\theta = 1.241 \pm 0.020\ \text{rad}$$
$$\theta = 1.241 \pm 1.6\%\ \text{rad} = 71.15^\circ \pm 1.6\%$$

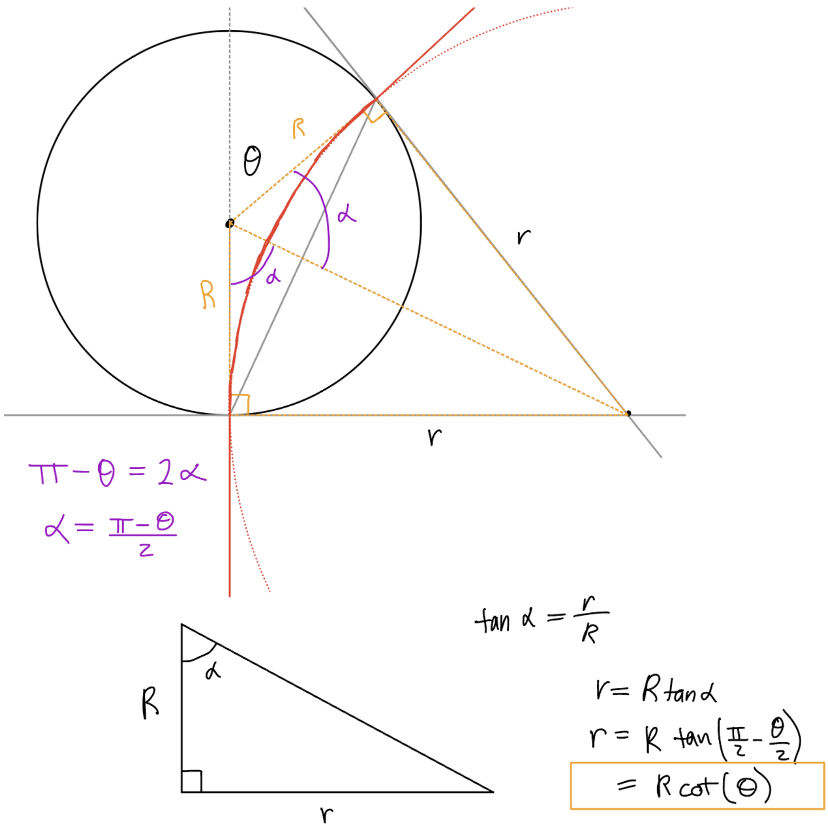


Figure 3: Diagram depicting the geometry of the experiment that shows the relationship between radius of curvature and measured protractor angle.

[8]Since, for $f(x) = \arctan(x)$, $\frac{df}{dx} = \frac{1}{1+x^2}$, the absolute propegated uncertainty is $\Delta_f \approx \frac{\Delta_x}{1+x^2}$.

3 Experimental Design

3.1 Variables

Independent Variable:
Variable: Protractor Angle
The protractor angle, measured in degrees, is the only independent variable in this experimental design and it determines the angle of the radioactive source to the uniform magnetic field and the Geiger counter. It will begin at the top-most mark of the semicircular protractor, noted 0°. This angle will then be incremented by intervals of 5° until 90°. This value is measured by visual inspection of the protractor marks and the rotating apparatus.

Dependent Variable:
Variable: Counts of radiation events over 10 minutes
The only dependent variable measured in this experiment is the counts of radiation events occuring over each 10 minute, or 600 second, trial. This quantity depends on the incident angle of the radioactive source because the negatively-charged electrons released in beta-minus decay should undergo a lorentz force changing their trajectory in the magnetic field. This value is measured through the use of a Geiger counter with a 'count' mode and a stopwatch. Additionally, the units or magnitude of the measurement shouldn't be highly relavant to the conclusion as long as a clear maximum counts can be determined at a specific protractor angle.

Control Variables	**Implications**	**Solutions**
Background radiation	Not controlling for background radiation can influence the quality of the data by causing inconsistent results, especially if background radiation is a significant fraction of the measured radiation from the source. Here, background radiation accounted for about $180 - 280$ counts over 10 minutes.	Measure background radiation with the Geiger counter at semi-regular intervals or before, if needed, restarting trials or moving the experiment. This measured value can be subtracted from each data point, or, as was done here, the experiment can be conducted under more sheilding to limit background influence.

3.2 Materials

The experiment requires a Geiger counter, a sample of Cs-137 (or similar beta-minus emitter), a protractor, a stopwatch (if not in the Geiger counter), two identical cylindrical permanent magnets, and fastening materials to create and secure the apparatus.

The crucial component is the mechanism by which the permanent magnets are secured opposite each other. Using magnets that exert a considerable attractive force on one another, a consideration is to ensure the apparatus can withstand the stress forces in the first place. To solve this, a 3d apparatus had been printed already to secure the magnets a distance apart.

Another important component is the rotating mechanism to move the sample to a specific angle. In this case, this was 3d printed as well, however make-shift devices can be employed. For example, this experiment was attempted with a popsicle stick fixed on an axle at the center of the protractor with the radioactive source fixed atop the stick's surface to rotate. But, more success was had with a custom solution that allowed the radioactive source to rotate more freely.

The protractor's origin must be centered at the center of the bottom of the cylindrical magnets so the readings are accurate. The end window of the Geiger counter must be secured to face normal to the (semi-circular) protractor at the level of the uniform magnetic field. Securing this Geiger counter properly is paramount, as slight movements can dramatically affect the data quality. In this experiment, a secure fastening was accomplished with liberal application of gaffer tape on multiple ends of the Geiger counter.

3.3 Method

With the radioactive source very distant from the Geiger counter, first measure the background radiation in the lab over a time span of at least 10 minutes. Then ascertain the likely direction of the force the particle will experience either mathematically with the orientation of $q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ or through inspection with a Geiger counter.

Then with the apparatus firmly secured, in this case through gaffer tape, begin the first trial by moving the protractor to the center mark (typically $90°$) and set the Geiger counter in `count` mode to count the total number of decay events over the ten minute period. Start the stopwatch at the same time as turning on the Geiger counter (or, start timing if the stopwatch is internal). At 10 min, or 600 sec, stop the count of the decay events and record the measured value in a data table beginning with $0°$. Repeat this process while incrementing the angle of the radioactive source to the normal by $5°$ each trial *in the previously ascertained direction the particle will travel*, stopping at $90°$ when the radioactive source is flush with the protractor edge. Then, complete this process three times for a total of three trials per angle.

Note that the duration of each trial can be modified at the expense of the quality of data. Shorter trials are more subject to the stochastic nature of radiation and will experience huge inconsistencies if not averaged over a long period of time.

3.4 Safety

When dealing with a radioactive source, care must be taken to ensure the utmost safety of the experimenter and, importantly, those in their surroundings. Foremost, one should not conduct this experiment with any source that is imminently dangerous to human exposure, since the experimental hypothesis should not vary with increasing radioactivity. This can be measured in the Curies of the source, with the Cs-137 used here being only 5μCi. Care was taken to store this source in a labeled metal box behind layers of wall to prevent undue exposure when not in use. Additionally, steps must be taken to ensure the radioactive source is sealed to prevent consumption. The source used here came in the form of a sealed coin. Gloves may be used proactively if necessary to ensure that there is as little risk of contact as possible.

More steps can be taken to limit experimental exposure, including the use of shielding materials during the experiment and in storage. For beta radiation, materials like metals (layers of aluminum foil), water (wet towels), or simply plenty of air can be effective in dramatically reducing exposure. In this experiment, metal and wet towels were used as sheilding.[9]

[9] Although, in certain cases, like with the Cs-137 used here, metal shielding can re-emit beta particles

Lastly, the steps taken to ensure safety depend on the specific radioactive isotope used. Sources that emit alpha or gamma in addition to beta must be handled appropriately, for instance by employing more dense lead shielding in the case of the latter. The source used in this experiment, Cs-317, is a notable emitter of gamma, so additional precaution was taken to ensure it was shielded to a safe level at all times.

4 Raw Data

Angle θ/degrees $\pm\Delta\theta = \pm 0.5°$	Total Count of β^- Decays over 10 min N/decays $\pm\Delta N = \pm 400$			
	Trial 1	Trial 2	Trial 3	Average
0.0	5,549	5,682	5,642	5,624
5.0	5,571	5,740	5,760	5,690
10.0	5,785	5,824	5,815	5,808
15.0	5,840	5,991	5,930	5,920
20.0	5,915	6,170	6,072	6,052
25.0	6,184	6,259	6,210	6,218
30.0	6,387	6,522	6,552	6,487
35.0	6,570	6,616	6,499	6,562
40.0	6,865	7,208	6,828	6,967
45.0	7,458	7,847	7,349	7,551
50.0	8,697	9,095	8,661	8,818
55.0	12,020	12,370	11,830	12,070
60.0	16,830	16,540	17,310	16,890
65.0	19,630	18,670	19,290	19,200
70.0	20,620	20,110	21,090	20,610
75.0	20,210	19,450	20,070	19,910
80.0	18,330	18,230	18,710	18,420
85.0	16,180	15,940	16,220	16,110
90.0	15,240	15,150	15,480	15,290

Table 1: The effect of changing the incident angle of a radiation source on the count of Cs-137 β^- decay events over a 10 minute period in a uniform magnetic field.

The divisions on the protractor were marked at 1° increments. Hence, an absolute uncertainty of ±0.5° is noted.

The LND 712 sensor of the Geiger counter, a Vernier *Digital Radiation Monitor*, has a stated typical uncertainty of ±10% in the user manual. During data collection, however, it was observed that the real-world percent uncertainties were far lower than the stated 10%. For this reason, standard deviations were calculated on each trial, then normalized by the average result to yield an average percent uncertainty of about 2%. Using the largest average value, an absolute uncertainty of ±400. was determined, which is denoted in Table 1.

as gamma frequency electromagnetic radiation through Bremsstrahlung radiation.

5 Data Analysis

The average counts over 10 minutes from Table 1 were plotted on Figure 4 with error bars corresponding to the ±400. absolute uncertainty.

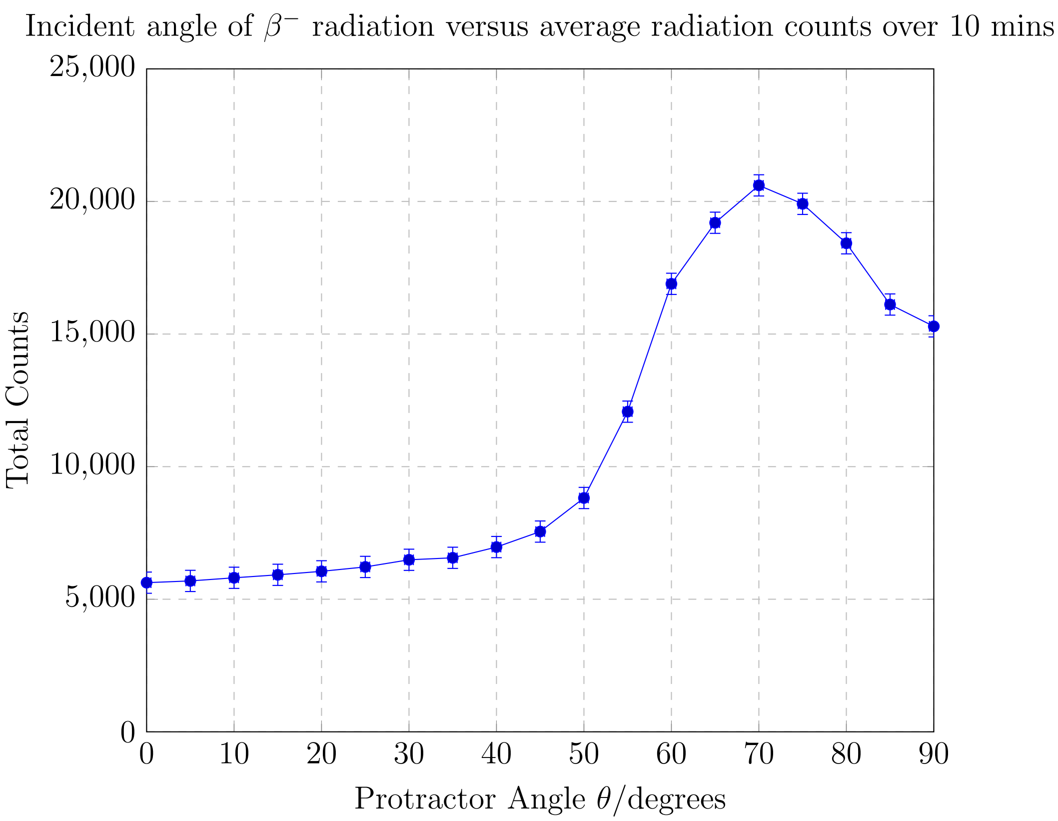


Figure 4: Graph depicting the effect of incident angle of beta-minus radiation on average counts of radiation measured over a 10 minute period in three trials. Note, horizontal error-bars are omitted for being too small.

By visual inspection of Figure 4, it is clear there is a maximum counts-per-ten-minutes at a protractor angle of 70°. This indicates that at this source angle, the number of beta particles reaching the Geiger counter is greatest, demonstrating to a high degree of confidence that this is the expected angle of greatest radiation strength given by the Lorentz force on the beta particle. And, indeed, this is in accordance with the predicted value of $\theta = 71.15° \pm 1.6\%$.

In particular, the percent error of the measured value from the predicted value is computed:

$$err = \frac{\Delta\theta}{\theta}$$
$$err = \frac{1.15}{70}$$
$$err = 1.6\%$$

The calculated percent error is exactly within the calculated uncertainty value to two significant figures, indicating strong support for the experimental hypothesis.

6 Conclusion

In evaluating the research question, "to what extent does the measured radius of curvature of a beta particle in a uniform magnetic field deviate from relativistic predictions?" a thorough experimental design was devised and carried out to reach this conclusion. As a result of the findings of this experiment, it is clear that the behaviour of a beta particle in a magnetic field does not deviate from the relativistic predictions. This conclusion is supported by the data presented in Figure 4 since the observed maximum counts per ten minutes occurred at an angle of 70°, which falls within the ±1.6% percent uncertainty of the predicted angle of 71.15°. This result is one of many in a body of literature supporting Einstein's Theory of Relativity broadly speaking—although this isn't too big a surprise. More importantly, this specific result about the behavior of Cs-137 in a magnetic field aligns with other experiments regarding its beta decay pattern.[10] What can be meaningfully concluded from this study is that the formulas for relativistic circular motion and mass-energy equivalence hold up in this high-velocity circumstance with a beta particle and a magnetic field.

6.1 Evalutation

Although the experimental results yielded data which strongly supported the experimental hypothesis and predicted values, errors were present that affected the skew of the data. Namely, the assumption that the magnetic field emanating from the cylindrical magnets was uniform is clearly a computational trick. For a more precise analysis of the not-necessarily circular motion of the beta particle, more advanced techniques could be applied to reduce this error, including computer simulations. This was attempted to be mitigated by averaging the magnetic field inside the cylindrical space, but this average was only an approximation. Additionally, the uncertainties calculated and used from the instruments should also be commented on. In a few cases, like with the Geiger counter, the published uncertainty values were excessively high compared to the deviations found in the recorded data. In other cases, the noted uncertainties of the measurement tools were too small compared to the effect they had on the experiment. With respect to the control of variables and design of the investigation, it is clear that the choices made resulted in a clear, reproducible experiment. From the data collected, it is not evident that the results had been significantly skewed by other factors or unaccounted physical phenomena, demonstrating that this experimental setup succeeded in isolating the relevant relationships. One limitation of the interpretation of the data is the lack of a clear explanation as to the considerable spread of the counts from the maximum point. The most convincing explanation involves the stochastic nature of beta-negative-decay, where the speed of the emitted electron could vary significantly based on the kinetic energy it is released with. Although the mean kinetic energy in the Cs-137 decay event of 187.1 keV ± 10. keV was used, the maximum energy can reach 511 keV, providing a speed much closer to the speed of light. This shorter duration (or, vice-versa, longer duration) in the magnetic field corresponds to the spread seen in the data. However, this interpretation cannot be justified without more careful consideration of the standard deviations involved in the decay processes.

[10] L. M. Langer and R. J. D. Moffat. "The Twice-Forbidden Transition of Cs^{137} and the Law of Beta-Decay". In: *Phys. Rev.* 82 (5 June 1951), pp. 635–638. DOI: `10.1103/PhysRev.82.635`. URL: `https://link.aps.org/doi/10.1103/PhysRev.82.635`.

6.2 Improvements

Among the possible improvements that could be made to the experimental design of this exploration, some of the most realistic and relevant changes include exploring different relationships between relativistic variables. For instance, it could have been more insightful to test the effect of changing magnetic field strength on the radius of curvature of the particle instead of only testing with one magnetic field strength. Another possibility would be to automate the process of taking data to permit the experimenter to reduce the experimental error of their process and to allow them to take more data points more safely. Unfortunately, the data collection process for this experiment was repetitive and time-consuming, which prevented the scope of the research from being expanded beyond just testing if relativity held experimentally.

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