Probabilités et Statistique II Chapitre 5. Lois discrètes de probabilités

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Discrete Uniform

In[ ]:=
```
PDF[DiscreteUniformDistribution[{a, b}], x]
```

Out[ ]=

$$\begin{cases} \frac{1}{1-a+b} & a \le x \le b \\ 0 & \text{True} \end{cases}$$

```
In[◦]:= d = DiscreteUniformDistribution[{1, 10}]

Out[◦]=
DiscreteUniformDistribution[{1, 10}]

In[◦]:= PDF[d, x]

Out[◦]=
```

$$\begin{cases} \frac{1}{10} & 1 \le x \le 10 \\ 0 & \text{True} \end{cases}$$

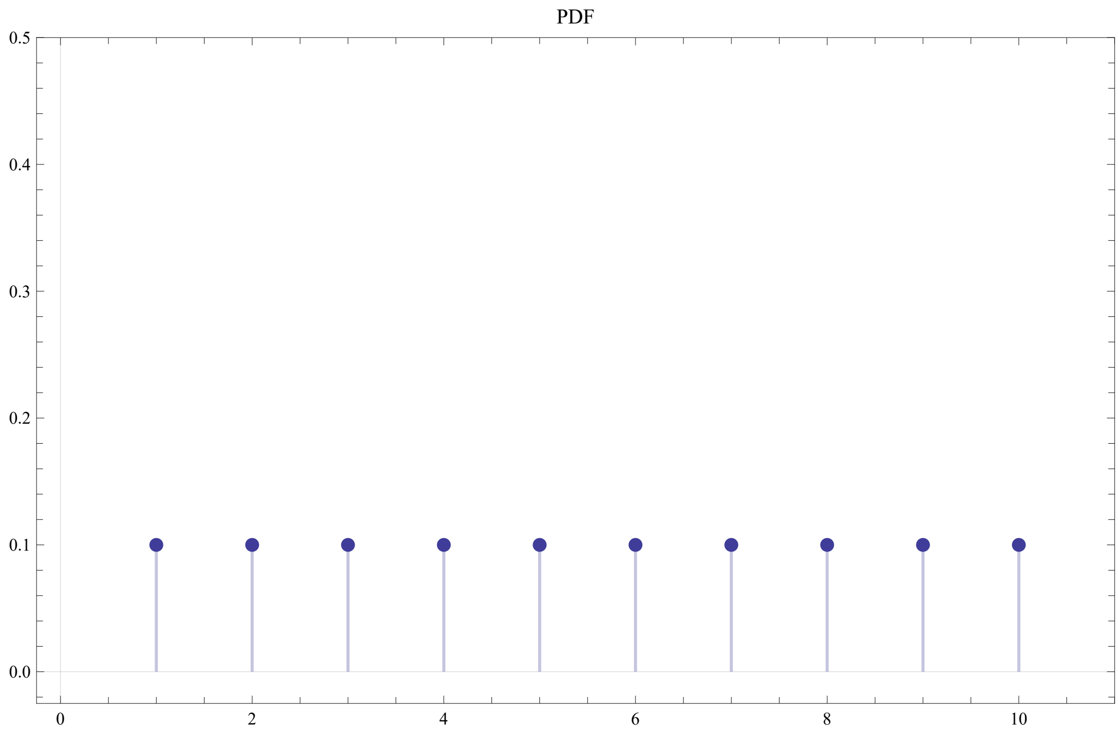
In[ ]:=
```
PiecewiseExpand[Piecewise[{{1/10, 1 ≤ x ≤ 10}}, 0]]
```

Out[ ]=

$$\begin{cases} \frac{1}{10} & 1 \le x \le 10 \\ 0 & \text{True} \end{cases}$$

In[ ]:=
```
pdf = DiscretePlot[PiecewiseExpand[Piecewise[{{1/10, 1 ≤ x ≤ 10}}, 0]],
  {x, 1, 10}, ExtentSize → 0, PlotLabel → "PDF",
  PlotStyle → ColorData[1, 1], PlotTheme → "Scientific",
  ImageSize → Large, PlotRange → {{-0.25, 11}, {-0.025, 0.5}}]
```

Out[ ]=



In[]:= CDF[d, x]

Out[]=

$$\begin{cases} \frac{\text{Floor}[x]}{10} & 1 \le x < 10 \\ 1 & x \ge 10 \\ 0 & \text{True} \end{cases}$$

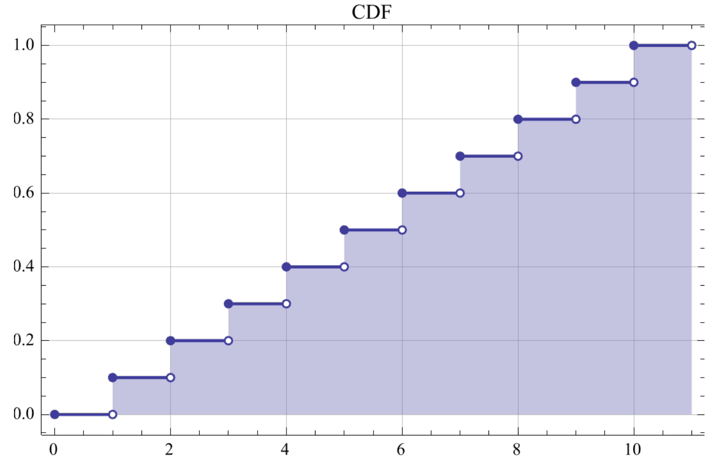
In[]:= PiecewiseExpand$\left[\text{Piecewise}\left[\left\{\left\{\frac{\text{Floor}[x]}{10}, 1 \le x < 10\right\}, \{1, x \ge 10\}\right\}, 0\right]\right]$

Out[]=

$$\begin{cases} \frac{1}{10} & 1 \le x < 2 \\ \frac{1}{5} & 2 \le x < 3 \\ \frac{3}{10} & 3 \le x < 4 \\ \frac{2}{5} & 4 \le x < 5 \\ \frac{1}{2} & 5 \le x < 6 \\ \frac{3}{5} & 6 \le x < 7 \\ \frac{7}{10} & 7 \le x < 8 \\ \frac{4}{5} & 8 \le x < 9 \\ \frac{9}{10} & 9 \le x < 10 \\ 1 & x \ge 10 \\ 0 & \text{True} \end{cases}$$

In[∘]:=
```
cdf = DiscretePlot[PiecewiseExpand[
    Piecewise[{{Floor[x]/10, 1 ≤ x < 10}, {1, x ≥ 10}}, 0]], {x, 0, 10},
   ExtentSize → Right, ExtentMarkers → {"Filled", "Empty"},
   PlotLabel → "CDF", PlotStyle → ColorData[1, 1],
   GridLines → Automatic,
   PlotTheme → "Scientific", ImageSize → Medium]
```

Out[∘]=



Mean Discrete Uniform Distribution

The aim is to replicate the demonstration than can be found here :
https://en.wikibooks.org/wiki/Statistics/Distributions/Discrete_Uniform

The discrete uniform distribution (not to be confused with the continuous uniform distribution) is where the probability of equally spaced possible values is equal. Mathematically this means that the probability density function is identical for a finite set of evenly spaced points. An example of would be rolling a fair 6-sided die. In this case there are six, equally like probabilities.

One common normalization is to restrict the possible values to be integers and the spacing between possibilities to be 1. In this setup, the only two parameters of the function are the minimum value (a), the maximum value (b). (Some even normalize it more, setting a=1.) Let n=b-a+1 be the number of possibilities.

The probability density function is then $\sum_{i=0}^{n} f(x_i) \times (x_i)$:

Let $S = \{a, a+1, \ldots, b-1, b\}$. The mean (noted as $E[X]$) can then be derived as follows:

$$E[X] = \sum_{x \in S} x\, f(x) = \sum_{i=0}^{n-1} \left(\frac{1}{n} (a+i) \right)$$

In[ ]:= **EXU =** $\sum_{i=0}^{n-1} \left(\frac{1}{n} (a+i) \right)$

Out[ ]=

$$\frac{1}{2} (-1 + 2 a + n)$$

$$E[X] = \frac{1}{n}\left(\sum_{i=0}^{n-1} a + \sum_{i=0}^{n-1} i\right)$$

In[•]:=
```
Simplify[1/n (Sum[a, {i, 0, n-1}] + Sum[i, {i, 0, n-1}])]
```

$$\text{Simplify}\left[\frac{1}{n}\left(\sum_{i=0}^{n-1} a + \sum_{i=0}^{n-1} i\right)\right]$$

Out[•]=

$$\frac{1}{2}\,(-1 + 2\,a + n)$$

Use the Closed Form for Triangular Numbers, with m (m=n-1)

In[ ]:= $\sum_{i=0}^{m} i$

Out[ ]=

$\frac{1}{2} m (1 + m)$

In[ ]:= **Expand[(m (1 + m)) / 2 == (m + m^2) / 2]**

Out[ ]=

True

$$E[X] = \frac{1}{n}\left(n\,a + \frac{(n-1)^2 + (n-1)}{2}\right)$$

In[ ]:=
```
Simplify[1/n (n × a + ((n - 1)^2 + (n - 1))/2)]
```

Out[ ]=

$$\frac{1}{2}\,(-1 + 2\,a + n)$$

$$E[X] = \frac{2\,n\,a + n^2 - 2\,n + 1 + n - 1}{2\,n}$$

In[ ]:=
```
Simplify[(2 n × a + n^2 - 2 n + 1 + n - 1)/(2 n)]
```

Out[ ]=

$$\frac{1}{2}\,(-1 + 2\,a + n)$$

$$E[X] = \frac{2a + n - 1}{2}$$

In[ ]:= **Simplify$\left[\frac{2a + n - 1}{2}\right]$**

Out[ ]=

$\frac{1}{2}(-1 + 2a + n)$

```
(* n=b-a+1 *)
```

In[ ]:= **n = b - a + 1**

Out[ ]=

1 - a + b

In[∘]:= Simplify$\left[\frac{2\,a\ + n - 1}{2}\right]$

Out[∘]=

$$\frac{a+b}{2}$$

In[∘]:=
```
Mean[DiscreteUniformDistribution[{a, b}]]
```

Out[∘]=

$$\frac{a+b}{2}$$

In[∘]:= $\frac{a+b}{2} == \frac{a+b}{3}$

Out[∘]=

$$\frac{a+b}{2} == \frac{a+b}{3}$$

In[∘]:=
```
Expand[%]
```

Out[∘]=

$$\frac{a}{2}+\frac{b}{2} == \frac{a}{3}+\frac{b}{3}$$

In[∘]:= $\frac{a+b}{2} == \frac{a+b}{(1+1)}$

Out[∘]=

True

In[∘]:=
```
Expand[%]
```

Out[∘]=

True

With variable, the Expand function indicates that the expression are not equivalent by letting the expression intact instead of ‘True’.

In[ ]:=
```
Remove[n]
```

In[ ]:=
```
Simplify[(2 a + n - 1)/2]
```

Out[ ]=
$$\frac{1}{2}\,(-1 + 2\,a + n)$$

Variance Discrete Uniform Distribution

The aim is to replicate the demonstration than can be found here :
https://en.wikibooks.org/wiki/Statistics/Distributions/Discrete_Uniform

In[ ]:= **EXU = $\sum_{i=0}^{n-1} \left(\frac{1}{n} (a + i)\right)$**

Out[ ]=

$\frac{1}{2} (-1 + 2 a + n)$

$$Var(X) = E\left[(X - E[X])^2\right] = \sum_{x \in S} f(x)\,(x - E[X])^2 = \sum_{i=0}^{n-1}\left(\frac{1}{n}\left((a+i) - \frac{a+b}{2}\right)^2\right)$$

In[∘]:= $\text{VXU} = \sum_{i=0}^{n-1}\left(\frac{1}{n}\,((a+i) - \text{EXU})^2\right)$

Out[∘]=

$$\frac{1}{12}\left(-1 + n^2\right)$$

$$Var(X) = \frac{1}{n}\sum_{i=0}^{n-1}\left(\frac{a + 2\,i - b}{2}\right)^2$$

In[∘]:= $\frac{1}{n}\sum_{i=0}^{n-1}\left(\frac{a + 2\,i - b}{2}\right)^2$

Out[∘]=

$$\frac{1}{12}\left(2 - 6\,a + 3\,a^2 + 6\,b - 6\,a\,b + 3\,b^2 - 6\,n + 6\,a\,n - 6\,b\,n + 4\,n^2\right)$$

$$Var(X) = \frac{1}{4n}\sum_{i=0}^{n-1}\left(a^2 + 4\,a\,i - 2\,a\,b + 4\,i^2 - 4\,i\,b + b^2\right)$$

In[∘]:= $\frac{1}{4\,n}\sum_{i=0}^{n-1}\left(a^2 + 4\,a\times i - 2\,a\times b + 4\,i^2 - 4\,i\times b + b^2\right)$

Out[∘]=

$$\frac{1}{12}\left(2 - 6\,a + 3\,a^2 + 6\,b - 6\,a\,b + 3\,b^2 - 6\,n + 6\,a\,n - 6\,b\,n + 4\,n^2\right)$$

$$Var(X) = \frac{1}{4n}\left[\sum_{i=0}^{n-1}\left(a^2 - 2ab + b^2\right) + \sum_{i=0}^{n-1}(4ai - 4ib) + \sum_{i=0}^{n-1} 4i^2\right]$$

In[ ]:= **Simplify$\left[\frac{1}{4n}\left(\sum_{i=0}^{n-1}\left(a^2 - 2a\times b + b^2\right) + \sum_{i=0}^{n-1}(4a\times i - 4i\times b) + \sum_{i=0}^{n-1}\left(4i^2\right)\right)\right]$**

Out[ ]=

$$\frac{1}{12}\left(2 + 3a^2 + 3b^2 - 6a(1 + b - n) - 6b(-1 + n) - 6n + 4n^2\right)$$

$$Var(X) = \frac{1}{4n}\left[n\left(a^2 - ab + b^2\right) + 4(a - b)\sum_{i=0}^{n-1} i + 4\sum_{i=0}^{n-1} i^2\right]$$

In[ ]:= **Simplify$\left[\frac{1}{4n}\left(n\left(a^2 - 2a\times b + b^2\right) + 4(a - b)\sum_{i=0}^{n-1}(i) + 4\sum_{i=0}^{n-1}\left(i^2\right)\right)\right]$**

Out[ ]=

$$\frac{1}{12}\left(2 + 3a^2 + 3b^2 - 6a(1 + b - n) - 6b(-1 + n) - 6n + 4n^2\right)$$

In[ ]:=
```
(1 / 12) (2 + 3 a^2 + 3 b^2 - 6 a (1 + b - n) - 6 b (-1 + n) - 6 n + 4 n^2) ==
  (1 / 12) (2 - 6 a + 3 a^2 + 6 b - 6 a b + 3 b^2 - 6 n + 6 a n - 6 b n + 4 n^2)
```

Out[ ]=

$$\frac{1}{12}\left(2+3a^2+3b^2-6a(1+b-n)-6b(-1+n)-6n+4n^2\right) == \frac{1}{12}\left(2-6a+3a^2+6b-6ab+3b^2-6n+6an-6bn+4n^2\right)$$

In[ ]:=
```
Expand[%]
```

Out[ ]=

True

Remember that $\sum_{i=0}^{m}\left(i^2\right) = [m\,(m+1)\,(2m+1)]\,/\,6$:

In[∘]:= $\sum_{i=0}^{m}\left(i^2\right)$

Out[∘]=

$$\frac{1}{6}\, m\,(1+m)\,(1+2m)$$

$$Var(X) = \frac{1}{4n}\left[n\,(b-a)^2 + 4\,(a-b)\,[(n-1)\,n/2] + 4\,[(n-1)\,n\,(2n-1)/6]\right]$$

In[∘]:= $\text{Simplify}\left[\frac{1}{4n}\left(n\,(b-a)^2 + 4\,(a-b)\left(\frac{(n-1)\,n}{2}\right) + 4\left(\frac{(n-1)\,n\,(2n-1)}{6}\right)\right)\right]$

Out[∘]=

$$\frac{1}{12}\left(2 + 3\,(a-b)^2 + 6\,(a-b)\,(-1+n) - 6n + 4n^2\right)$$

In[ ]:= **% == $\frac{1}{12}\left(2 - 6 a + 3 a^2 + 6 b - 6 a b + 3 b^2 - 6 n + 6 a n - 6 b n + 4 n^2\right)$**

Out[ ]=

$$\frac{1}{12}\left(2 + 3 (a - b)^2 + 6 (a - b) (-1 + n) - 6 n + 4 n^2\right) == \frac{1}{12}\left(2 - 6 a + 3 a^2 + 6 b - 6 a b + 3 b^2 - 6 n + 6 a n - 6 b n + 4 n^2\right)$$

In[ ]:= **Expand[%]**

Out[ ]=

True

In[∘]:=
```
Simplify[1/(4 n) (n (n - 1)^2 - 2 (n - 1) (n - 1) n + (2 (n - 1) n (2 n - 1))/3)]
```

$$\text{Simplify}\left[\frac{1}{4\,n}\left(n\,(n-1)^2-2\,(n-1)\,(n-1)\,n+\left(\frac{2\,(n-1)\,n\,(2\,n-1)}{3}\right)\right)\right]$$

Out[∘]=

$$\frac{1}{12}\left(-1+n^2\right)$$

```
(* n=b-a+1 *)
```

In[∘]:=
```
n
```

Out[∘]=

n

In[∘]:=
```
Remove[n]
```

$$Var(X) = \frac{1}{4}\left[-(n-1)^2 + 2\,(n-1)\,(2\,n-1)/3\right]$$

In[ ]:=
```
Simplify[1/4 (-(-1 + n)^2 + 2/3 (-1 + n) (-1 + 2 n))]
```

Out[ ]=

$$\frac{1}{12}\left(-1 + n^2\right)$$

$$Var(X) = \frac{1}{12}\left[-3(n-1)^2 + 2(n-1)(2n-1)\right]$$

In[ ]:= **Simplify**$\left[\frac{1}{12}\left(-3(n-1)^2 + 2(n-1)(2n-1)\right)\right]$

Out[ ]=

$$\frac{1}{12}\left(-1 + n^2\right)$$

$$Var(X) = \frac{1}{12}\left[-3\left(n^2 - 2n + 1\right) + 2\left(2n^2 - 3n + 1\right)\right]$$

In[ ]:=
```
Simplify[1/12 (-3 (n^2 - 2 n + 1) + 2 (2 n^2 - 3 n + 1))]
```

Out[ ]=

$$\frac{1}{12}\left(-1 + n^2\right)$$

$$Var(X) = \frac{n^2 - 1}{12}$$

In[ ]:= $\frac{1}{12}\left(-1 + n^2\right) == \frac{n^2 - 1}{12}$

Out[ ]=
```
True
```

In[ ]:= **Expand[%]**

Out[ ]=
```
True
```

In[ ]:= **n = b - a + 1**

Out[ ]=

$1 - a + b$

In[ ]:= **1 - a + b**

Out[ ]=

$1 - a + b$

In[ ]:= **Variance[DiscreteUniformDistribution[{a, b}]]**

Out[ ]=

$$\frac{1}{12}\left(-1+(1-a+b)^2\right)$$

In[ ]:= $\boldsymbol{\frac{1}{12}\left(-1+(n)^2\right)}$

Out[ ]=

$$\frac{1}{12}\left(-1+(1-a+b)^2\right)$$

In[ ]:= **Remove[n]**

In[ ]:= $$\frac{1}{12}\left(-1+n^2\right) == \frac{n^2-2}{12}$$

Out[ ]=

$$\frac{1}{12}\left(-1+n^2\right) == \frac{1}{12}\left(-2+n^2\right)$$

In[ ]:= **Expand[%]**

Out[ ]=

$$-\frac{1}{12}+\frac{n^2}{12} == -\frac{1}{6}+\frac{n^2}{12}$$

With variable, the Expand function indicates that the expression are not equivalent.

Discrete Bernoulli

In[ ]:=
```
PDF[BernoulliDistribution[p], x]
```

Out[ ]=

$$\begin{cases} 1-p & x == 0 \\ p & x == 1 \\ 0 & \text{True} \end{cases}$$

In[ ]:= **d = BernoulliDistribution[1 / 2]**

Out[ ]=

$$\text{BernoulliDistribution}\left[\frac{1}{2}\right]$$

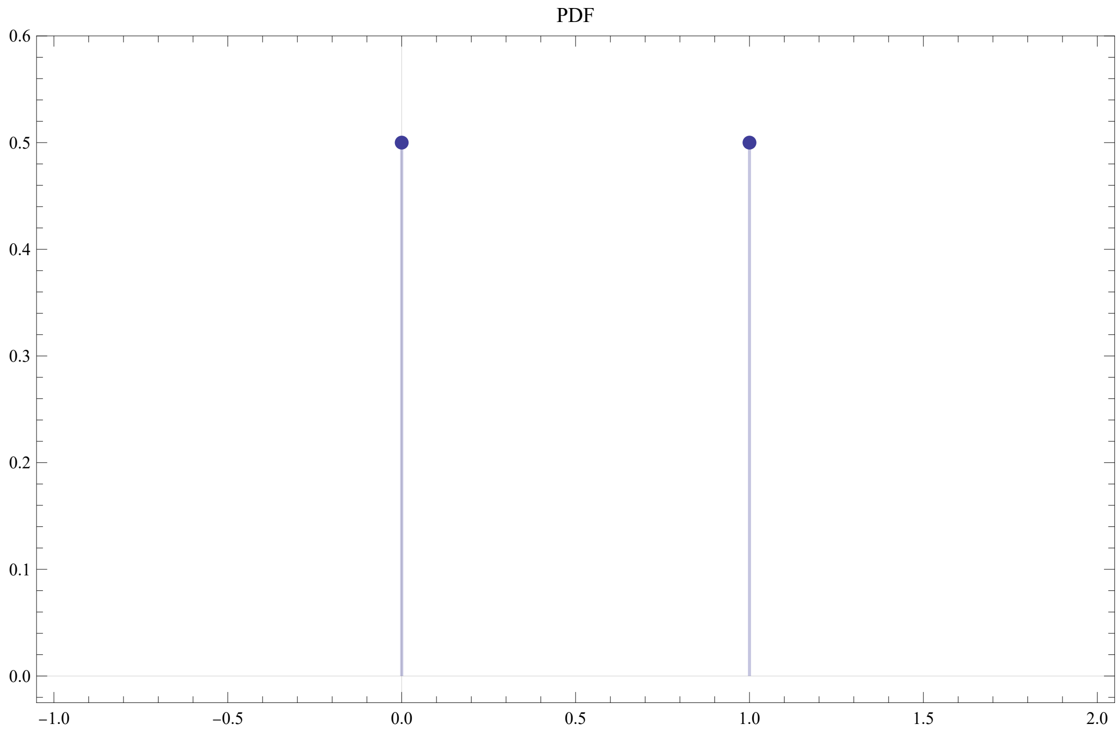
In[ ]:= **PDF[d, x]**

Out[ ]=

$$\begin{cases} \frac{1}{2} & x == 0 \,||\, x == 1 \\ 0 & \text{True} \end{cases}$$

In[∘]:=
```
pdf = DiscretePlot[Piecewise[{{1/2, x == 0 || x == 1}}, 0],
  {x, 0, 1}, ExtentSize → 0, PlotLabel → "PDF",
  PlotStyle → ColorData[1, 1], PlotTheme → "Scientific",
  ImageSize → Large, PlotRange → {{-1.05, 2.05}, {-0.025, 0.6}}]
```

Out[∘]=



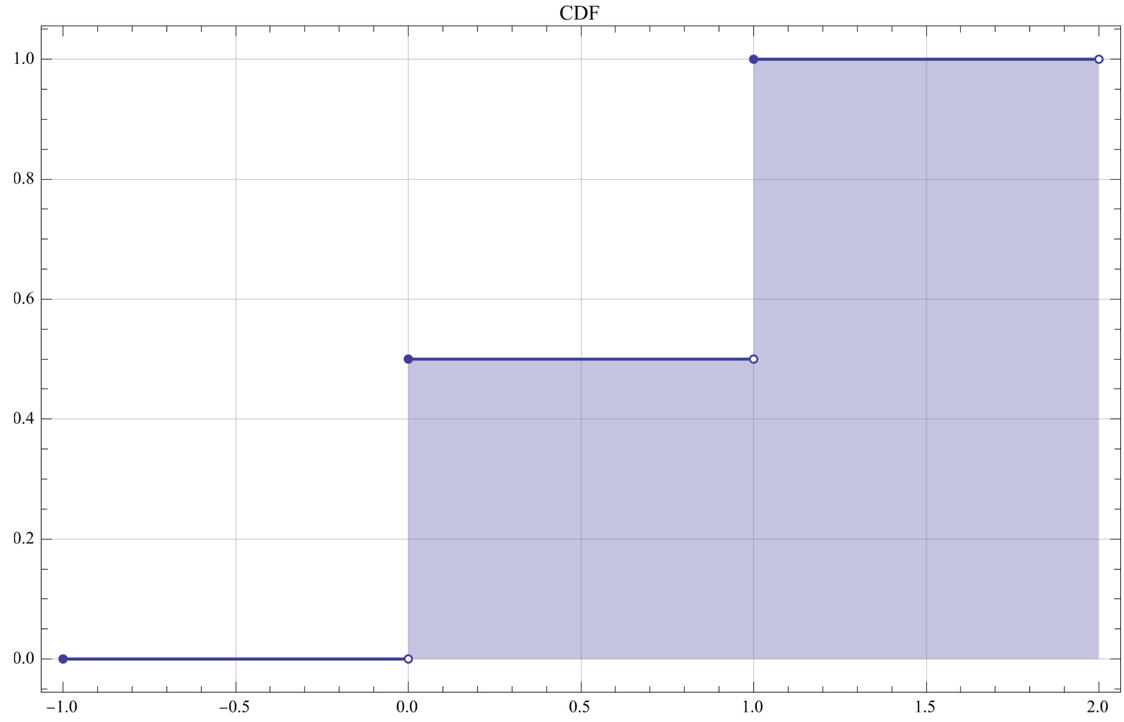
In[∘]:= **PiecewiseExpand[Piecewise[{{0, x < 0}, {$\frac{1}{2}$, 0 ≤ x < 1}}, 1]]**

Out[∘]=

$$\begin{cases} \frac{1}{2} & 0 \le x < 1 \\ 1 & x \ge 1 \\ 0 & \text{True} \end{cases}$$

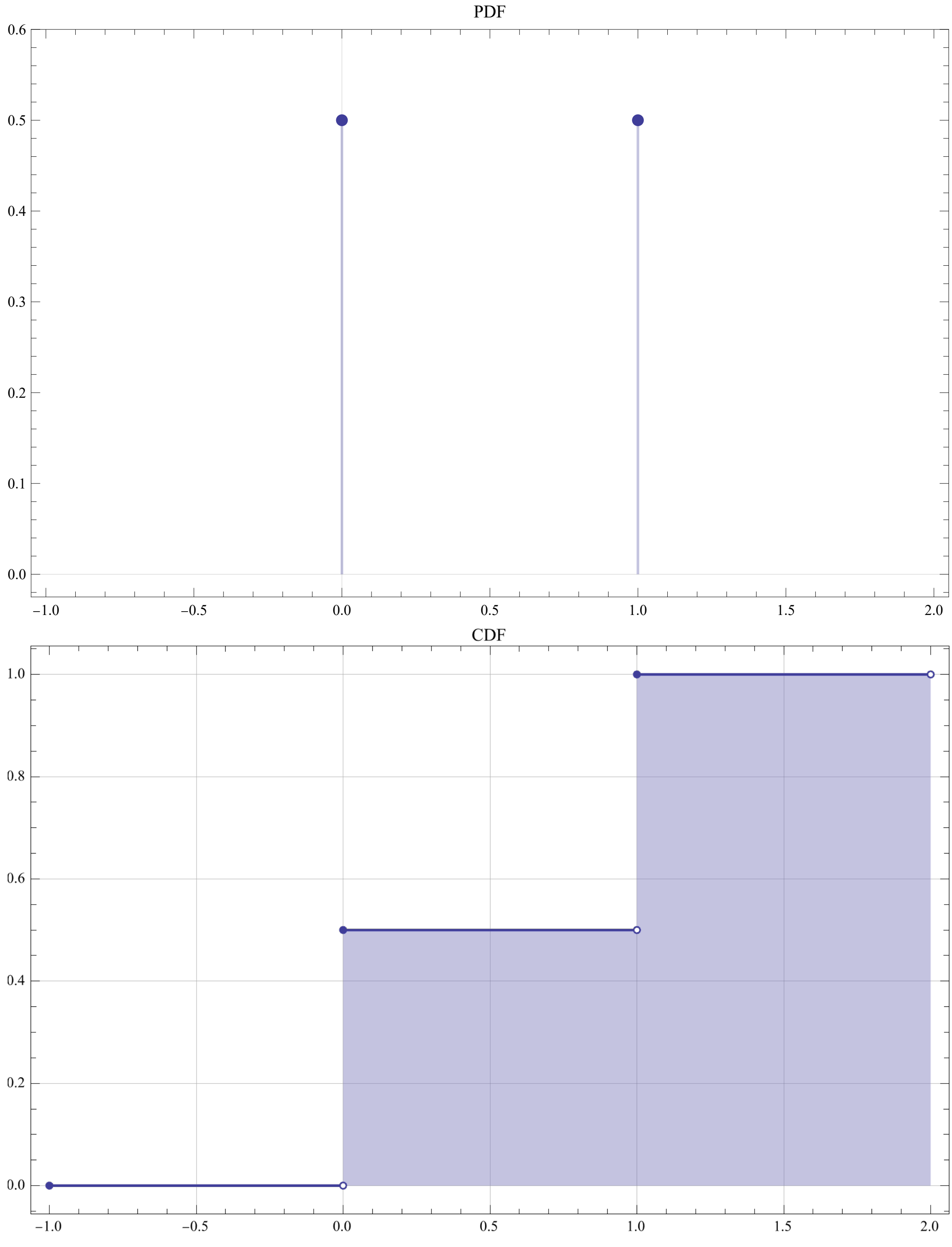
In[∘]:=
```
cdf = DiscretePlot[PiecewiseExpand[
    Piecewise[{{0, x < 0}, {1/2, 0 ≤ x < 1}}, 1]], {x, -1, 1},
   ExtentSize → Right, ExtentMarkers → {"Filled", "Empty"},
   PlotStyle → ColorData[1, 1], GridLines → Automatic,
   PlotLabel → "CDF", PlotTheme → "Scientific", ImageSize → Large]
```

Out[∘]=



In[◦]:= `Grid[{{pdf}, {cdf}}]`

Out[◦]=



Demonstration (Mean)

https://en.wikibooks.org/wiki/Statistics/Distributions/Bernoulli

There is no more basic random event than the flipping of a coin. Heads or tails. It's as simple as you can get! The "Bernoulli Trial" refers to a single event which can have one of two possible outcomes with a fixed probability of each occurring. You can describe these events as "yes or no" questions. For example:

Will the coin land heads?
Will the newborn child be a girl?
Are a random person's eyes green?
Will a mosquito die after the area was sprayed with insecticide?
Will a potential customer decide to buy my product?
Will a citizen vote for a specific candidate?
Is an employee going to vote pro-union?
Will this person be abducted by aliens in their lifetime?

The Bernoulli Distribution has one controlling parameter: the probability of success. A "fair coin" or an experiment where success and failure are equally likely will have a probability of 0.5 (50%). Typically the variable p is used to represent this parameter.

If a random variable X is distributed with a Bernoulli Distribution with a parameter p we write its probability mass function as:

$$f(x) = \begin{cases} p, & \text{if } x = 1 \\ 1 - p, & \text{if } x = 0 \end{cases} \quad 0 \leq p \leq 1$$

This distribution may seem trivial, but it is still a very important building block in probability. The Binomial distribution extends the Bernoulli distribution to encompass multiple "yes" or "no" cases with a fixed probability. Take a close look at the examples cited above. Some similar questions will be presented in the next section which might give an understanding of how these distributions are related.

Mean [edit | edit source]

The mean (E[X]) can be derived:

$$\mathrm{E}[X] = \sum_i f(x_i) \cdot x_i$$

$$\mathrm{E}[X] = p \cdot 1 + (1-p) \cdot 0$$

$$\mathrm{E}[X] = p$$

In[ ]:=
```
p * 1 + (1 - p) * 0
```

Out[ ]=

p

In[ ]:=
```
Mean[BernoulliDistribution[p]]
```

Out[ ]=

p

Demonstration (Variance)

https://en.wikibooks.org/wiki/Statistics/Distributions/Bernoulli

Variance [edit | edit source]

$$\mathrm{Var}(X) = \mathrm{E}[(X - \mathrm{E}[X])^2] = \sum_i f(x_i) \cdot (x_i - \mathrm{E}[X])^2$$

$$\mathrm{Var}(X) = p \cdot (1-p)^2 + (1-p) \cdot (0-p)^2$$

$$\mathrm{Var}(X) = [p(1-p) + p^2](1-p)$$

$$\mathrm{Var}(X) = p(1-p)$$

In[ ]:= `p * (1 - p)^2 + (1 - p) * (0 - p)^2`

Out[ ]=

$(1-p)^2 p + (1-p) p^2$

In[ ]:= `Factor[(1 - p)^2 p + (1 - p) p^2]`

Out[ ]=

$-((-1+p)\, p)$

In[ ]:= Simplify$\left[(1-p)^2 p + (1-p)\, p^2\right]$

Out[ ]=
$-((-1+p)\, p)$

In[ ]:= Equal[- ((-1 + p) p) == (1 - p) p]

Out[ ]=
True

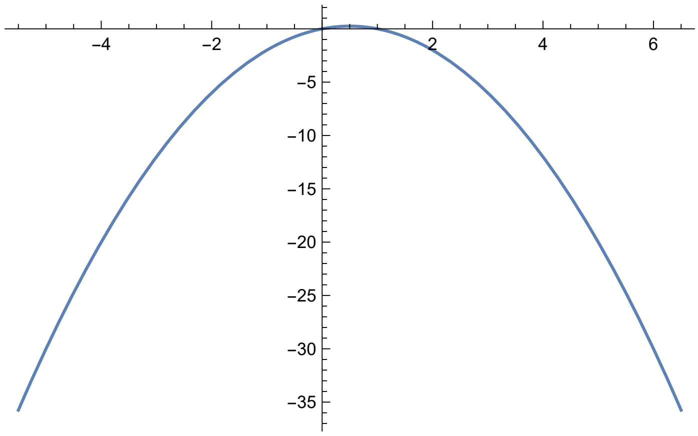
In[ ]:= **Variance[BernoulliDistribution[p]]**

Out[ ]=

(1 - p) p

In[ ]:= **Plot[(1 - p) p, {p, -5.5, 6.5}]**

Out[ ]=



Binomial Distribution

In[•]:=
```
PDF[BinomialDistribution[n, p], x]
```

Out[•]=

$$\begin{cases} (1-p)^{n-x}\, p^x\, \text{Binomial}[n, x] & 0 \le x \le n \\ 0 & \text{True} \end{cases}$$

In[•]:=
```
d = BinomialDistribution[3, 0.95]
```

Out[•]=
```
BinomialDistribution[3, 0.95]
```

```
In[•]:= PDF[d, x]
Out[•]=
```

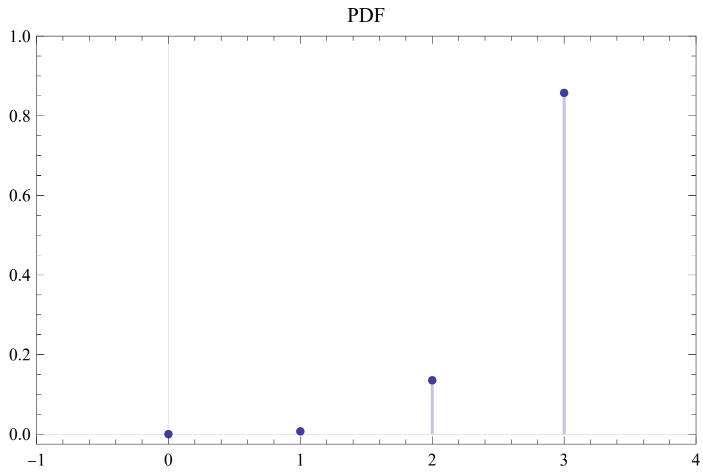
$$\begin{cases} 0.05^{3-x} \times 0.95^{x}\ \text{Binomial}[3, x] & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

```
In[•]:= PiecewiseExpand[Piecewise[{{0.05^(3-x) × 0.95^x Binomial[3, x], 0 ≤ x ≤ 3}}, 0]]
Out[•]=
```

$$\begin{cases} 0.000125 \times 19.^{x}\ \text{Binomial}[3, x] & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

In[ ]:=
```
pdf = DiscretePlot[Piecewise[{{0.05^(3-x) × 0.95^x Binomial[3, x], 0 ≤ x ≤ 3}}, 0],
  {x, 0, 3}, ExtentSize → 0, PlotLabel → "PDF", PlotStyle → ColorData[1, 1],
  PlotTheme → "Scientific", ImageSize → Medium, PlotRange → {{-1, 4}, {-0.025, 1}}]
```

Out[ ]=



In[•]:= **CDF[d, x]**

Out[•]=

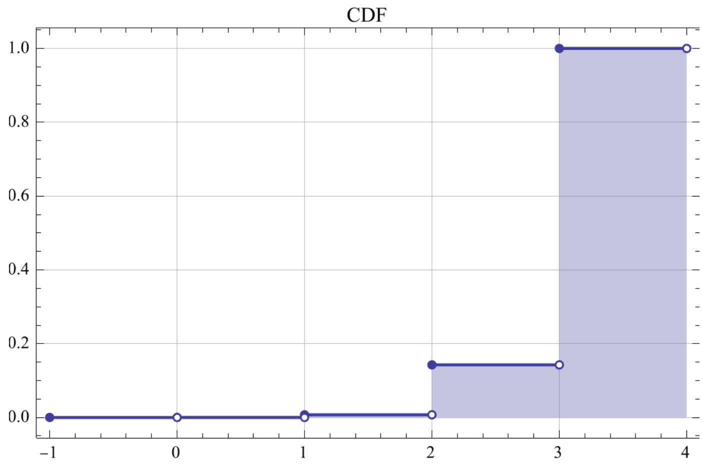
$$\begin{cases} \text{BetaRegularized}[0.05, 3 - \text{Floor}[x], 1 + \text{Floor}[x]] & 0 \le x < 3 \\ 1 & x \ge 3 \\ 0 & \text{True} \end{cases}$$

In[•]:= **cudf = PiecewiseExpand[%]**

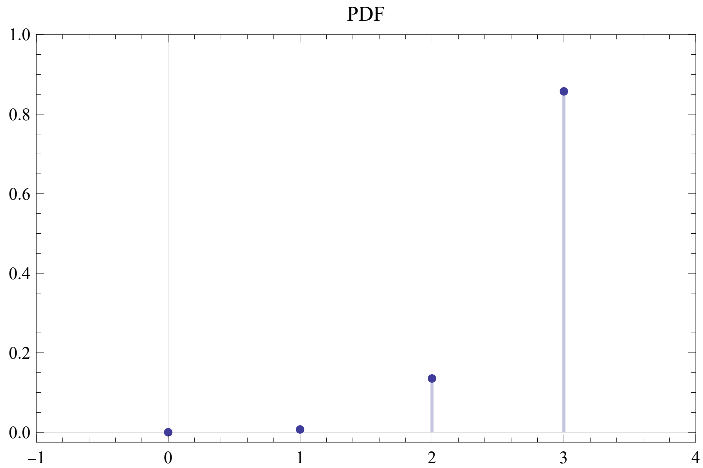
Out[•]=

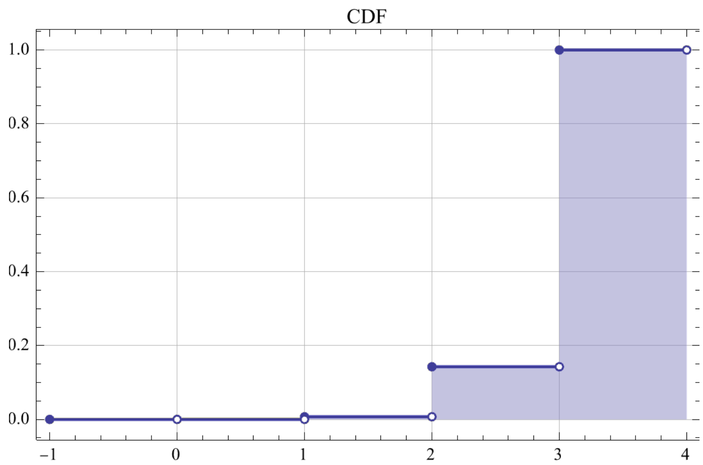
$$\begin{cases} 0.000125 & 0 \le x < 1 \\ 0.00725 & 1 \le x < 2 \\ 0.142625 & 2 \le x < 3 \\ 1 & x \ge 3 \\ 0 & \text{True} \end{cases}$$

```
In[]:= cdf = DiscretePlot[cudf, {x, -0.5, 4}, ExtentSize → 1,
        ExtentMarkers → {"Filled", "Empty"}, PlotStyle → ColorData[1, 1], PlotLabel → "CDF",
        GridLines → Automatic, PlotTheme → "Scientific", ImageSize → Medium]
Out[]=
```




```
In[]:= Grid[{{pdf}, {cdf}}]
Out[]=
```





Demonstration (Mean)

https://en.wikibooks.org/wiki/Statistics/Distributions/Binomial

```
(* (1) Write the expression for the Mean:  ∑_{i=0}^n f(x_i) × (x_i) *)
```

In[ ]:= $EX = \sum_{x=0}^{n} \text{Binomial}[n, x]\, p^x\, (1-p)^{n-x}\, x;$

In[ ]:= $EX == \sum_{x=0}^{n} \frac{n!}{x!\,(n-x)!}\, p^x\, (1-p)^{n-x}\, x$

Out[ ]=

```
True
```

```
(* (2) The sum start from x=1 *)
```

In[ ]:= $EX == \frac{n!}{0!\,(n-0)!}\, p^{0}\,(1-p)^{n-0}\, 0 + \sum_{x=0}^{n} \frac{n!}{x!\,(n-x)!}\, p^{x}\,(1-p)^{n-x}\, x$

Out[ ]=

True

(* (3) Replace n! by n(n-1)!, x! by x(x-1)! and simplify x *)

In[∘]:= $EX == \sum_{x=1}^{n} \frac{n\,(n-1)\,!}{x\,(x-1)\,!\,(n-x)\,!}\,p^{x}\,(1-p)^{n-x}\,x$

Out[∘]=

True

In[∘]:= $\text{Equal}\left[EX == np\sum_{x=1}^{n} \frac{(n-1)\,!}{x\,(x-1)\,!\,(n-x)\,!}\,p^{x-1}\,(1-p)^{n-x}\,x\right]$

Out[∘]=

True

In[∘]:= $\text{Equal}\left[EX == np\sum_{x=1}^{n} \frac{(n-1)\,!}{(x-1)\,!\,(n-x)\,!}\,p^{x-1}\,(1-p)^{n-x}\right]$

Out[∘]=

True

(* (4) w=x-1 and m=n-1 *)

In[∘]:= $w = x - 1$

Out[∘]=

$-1 + x$

In[∘]:= $m = n - 1$

Out[∘]=

$-1 + n$

In[∘]:= $\text{Equal}\left[EX == np\sum_{x=1}^{n} \frac{(m)\,!}{w\,!\,(m-w)\,!}\,p^{w}\,(1-p)^{m-w}\right]$

Out[∘]=

True

```
(* (5) Check that the sum is equal to 1, definition of a PDF *)
```

In[ ]:=
$$\sum_{x=1}^{n} \frac{(m)!}{w!\,(m-w)!}\, p^{w}\, (1-p)^{m-w}$$

Out[ ]=

1

```
(* (6) The demonstration is finished *)
```

In[∘]:= **EX**

Out[∘]=

n p

In[∘]:= $\sum_{x=0}^{n} \text{Binomial}[n, x]\, p^{x}\, (1-p)^{n-x}\, x$

Out[∘]=

n p

In[∘]:= **Mean[BinomialDistribution[n, p]]**

Out[∘]=

n p

Demonstration (Variance)

https://en.wikibooks.org/wiki/Statistics/Distributions/Binomial

(* (1) Write the expression for the "Squared Mean": $\sum_{i=0}^{n} f(x_i) \times (x_i)^2$ *)

In[ ]:= $EX2 = \sum_{x=0}^{n} \text{Binomial}[n, x]\, p^x\, (1-p)^{n-x}\, x^2;$

In[ ]:= $\text{Equal}\left[EX2 == \sum_{x=0}^{n} \frac{n!}{x!\,(n-x)!}\, p^x\, (1-p)^{n-x}\, x^2\right]$

Out[ ]=

True

```
(* (2) The sum start from x=1 *)
```

In[◦]:= $\text{Equal}\left[EX2 == \frac{n!}{0!\,(n-0)!}\,p^{0}\,(1-p)^{n-0}\,0 + \sum_{x=1}^{n} \frac{n!}{x!\,(n-x)!}\,p^{x}\,(1-p)^{n-x}\,x^{2}\right]$

Out[◦]=

True

```
(* (3) Replace n! by n(n-1)!, x! by x(x-1)! and simplify x *)
```

In[◦]:= $EX2 == \sum_{x=1}^{n} \frac{n\,(n-1)!}{x\,(x-1)!\,(n-x)!}\,p^{x}\,(1-p)^{n-x}\,x^{2}$

Out[◦]=

True

In[◦]:= $\text{Equal}\left[EX2 == np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!}\,p^{x-1}\,(1-p)^{n-x}\,x\right]$

Out[◦]=

True

```
(* (4) Remind that w=x-1 and m=n-1 *)
```

In[ ]:= $\text{Equal}\left[\text{EX2} == \text{np}\sum_{x=1}^{n}\frac{(m)\,!}{w\,!\,(m-w)\,!}\,p^{w}\,(1-p)^{m-w}\,(w+1)\right]$

Out[ ]= True

In[ ]:= $\text{Equal}\left[\text{EX2} == \text{np}\sum_{x=1}^{n}\frac{(m)\,!}{w\,!\,(m-w)\,!}\,p^{w}\,(1-p)^{m-w}\,(w) + \text{np}\sum_{x=1}^{n}\frac{(m)\,!}{w\,!\,(m-w)\,!}\,p^{w}\,(1-p)^{m-w}\right]$

Out[ ]= True

```
(* (5) Check that the sums *)
```

In[ ]:= $np \sum_{x=1}^{n} \frac{(m)\,!}{w\,!\,(m-w)\,!}\, p^{w}\, (1-p)^{m-w}\, (w)$

Out[ ]= $(-1+n)\; np\; p$

In[ ]:= `Collect[(-1 + n) np p, n]`

Out[ ]= $-np\; p + n\; np\; p$

In[ ]:= $np \sum_{x=1}^{n} \frac{(m)\,!}{w\,!\,(m-w)\,!}\, p^{w}\, (1-p)^{m-w}$

Out[ ]= np

In[ ]:= **EX2**

In[ ]:= $p\left(n - n\,p + n^2\,p\right)$

Out[ ]=

$p\left(n - n\,p + n^2\,p\right)$

In[ ]:= **Equal**$\left[p\left(n - n\,p + n^2\,p\right) == np\,(np - p + 1)\right]$

Out[ ]=

```
True
```

In[ ]:= **FullSimplify**[$np\,(np - p + 1)$]

Out[ ]=

$np\,(1 + np - p)$

```
(* (6) The demonstration is finished *)
```

In[∘]:= `VX = EX2 - (EX)^2`

Out[∘]=

$-n^2 p^2 + p \left(n - n p + n^2 p\right)$

In[∘]:= `Simplify[VX]`

Out[∘]=

$-n (-1 + p) p$

In[∘]:= `-n (-1 + p) p`

Out[∘]=

$-n (-1 + p) p$

In[∘]:= `Equal[-n (-1 + p) p == np (1 - p)]`

Out[∘]=

True

In[∘]:= `Variance[BinomialDistribution[n, p]]`

Out[∘]=

$n (1 - p) p$

Example 1

In[∘]:=
```
BinomialDistribution[3, 0.95]
```
Out[∘]=
```
BinomialDistribution[3, 0.95]
```

In[∘]:=
```
PDF[BinomialDistribution[3, 0.95], x]
```
Out[∘]=

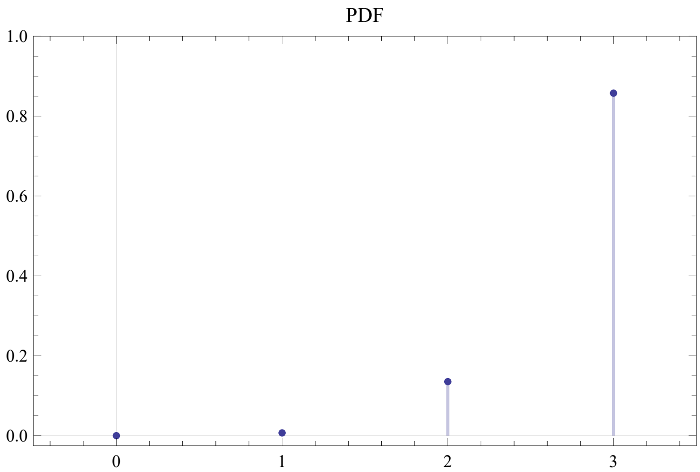
$$\begin{cases} 0.05^{3-x} \times 0.95^{x} \text{ Binomial}[3, x] & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

In[∘]:=
```
Table[Piecewise[{{0.05^(3-x) × 0.95^x Binomial[3, x], 0 ≤ x ≤ 3}}, 0], {x, 1, 20}]
```
Out[∘]=
```
{0.007125, 0.135375, 0.857375, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

In[∘]:=
```
PiecewiseExpand[Piecewise[{{0.05^(3-x) × 0.95^x Binomial[3, x], 0 ≤ x ≤ 3}}, 0]]
```
Out[∘]=

$$\begin{cases} 0.000125 \times 19.^{x} \text{ Binomial}[3, x] & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

In[∘]:=
```
DiscretePlot[Piecewise[{{0.000125 × 19.^x Binomial[3, x], 0 ≤ x ≤ 3}}, 0],
 {x, 0, 20}, ExtentSize → 0, PlotLabel → "PDF", PlotStyle → ColorData[1, 1],
 PlotTheme → "Scientific", ImageSize → Medium, PlotRange → {{-.5, 3.5}, {-0.025, 1}}]
```
Out[∘]=



Example 2

In[∘]:=
```
PDF[BinomialDistribution[5, 0.90], x]
```

Out[∘]=

$$\begin{cases} 0.1^{5-x} \times 0.9^{x}\,\mathrm{Binomial}[5, x] & 0 \le x \le 5 \\ 0 & \mathrm{True} \end{cases}$$

In[∘]:=
```
Table[Piecewise[{{0.1^(5-x) × 0.9^x Binomial[5, x], 0 ≤ x ≤ 5}}, 0], {x, 0, 5}]
```

Out[∘]=
```
{0.00001, 0.00045, 0.0081, 0.0729, 0.32805, 0.59049}
```

In[∘]:=
```
mean = (0.00001 * 0) + (0.00045 * 1) +
  (0.0081 * 2) + (0.0729 * 3) + (0.32805 * 4) + (0.59049 * 5)
```

Out[∘]=
```
4.5
```

In[∘]:=
```
variance = (0.00001 * 0^2) + (0.00045 * 1^2) + (0.0081 * 2^2) +
  (0.0729 * 3^2) + (0.32805 * 4^2) + (0.59049 * 5^2) - mean^2
```

Out[∘]=
```
0.45
```

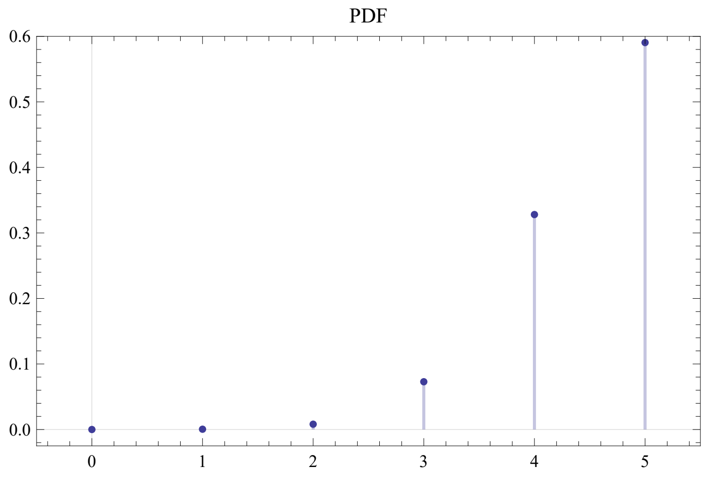
In[∘]:=
```
PiecewiseExpand[Piecewise[{{0.1^(5-x) × 0.9^x Binomial[5, x], 0 ≤ x ≤ 5}}, 0]]
```

Out[∘]=

$$\begin{cases} 0.00001 \times 9.^{x}\,\mathrm{Binomial}[5, x] & 0 \le x \le 5 \\ 0 & \mathrm{True} \end{cases}$$

In[∘]:=
```
DiscretePlot[Piecewise[{{0.00001 × 9.^x Binomial[5, x], 0 ≤ x ≤ 5}}, 0],
 {x, 0, 20}, ExtentSize → 0, PlotLabel → "PDF", PlotStyle → ColorData[1, 1],
 PlotTheme → "Scientific", ImageSize → Medium, PlotRange → {{-.5, 5.5}, {-0.025, 0.6}}]
```

Out[∘]=



Poisson Distribution

In[∘]:=
```
PDF[PoissonDistribution[λ], x]
```

Out[∘]=

$$\begin{cases} \frac{e^{-\lambda}\lambda^{x}}{x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:=
```
d = PoissonDistribution[1.5]
```

Out[∘]=
```
PoissonDistribution[1.5]
```

In[∘]:=
```
PDF[d, x]
```

Out[∘]=

$$\begin{cases} \frac{0.22313 \times 1.5^{x}}{x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

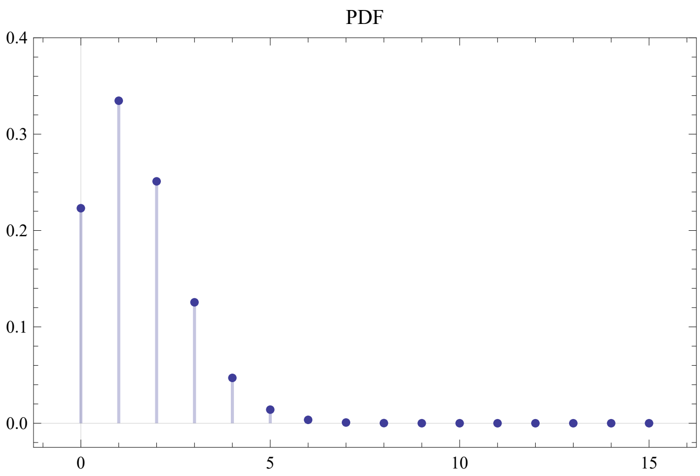
In[ ]:=
```
PiecewiseExpand[Piecewise[{{0.22313 × 1.5^x / x!, x ≥ 0}}, 0]]
```

Out[ ]=

$$\begin{cases} \frac{0.22313 \times 1.5^x}{x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[ ]:=
```
pdf = DiscretePlot[Piecewise[{{0.22313 × 1.5^x / x!, x ≥ 0}}, 0], {x, 0, 15}, ExtentSize → 0,
  PlotLabel → "PDF", PlotStyle → ColorData[1, 1], PlotTheme → "Scientific",
  ImageSize → Medium, PlotRange → {{-1.25, 16.25}, {-0.025, 0.4}}]
```

Out[ ]=



In[∘]:=
```
CDF[d, x]
```

Out[∘]=

$$\begin{cases} \text{GammaRegularized}[1+\text{Floor}[x], 1.5] & x \geq 0 \\ 0 & \text{True} \end{cases}$$

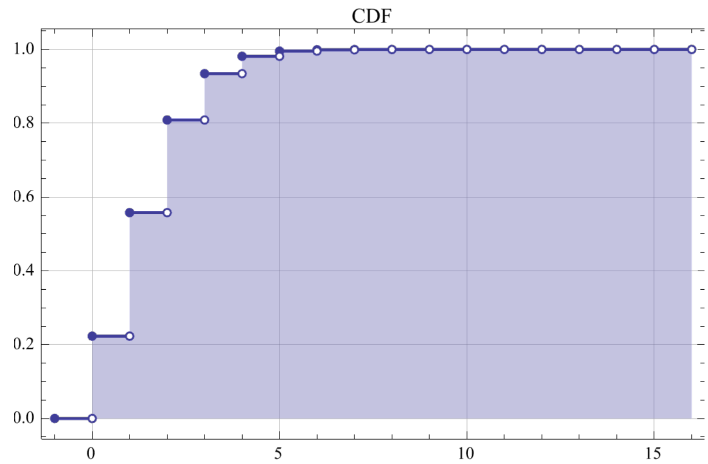
In[∘]:=
```
PiecewiseExpand[Piecewise[{{GammaRegularized[1 + Floor[x], 1.5], x ≥ 0}}, 0]]
```

Out[∘]=

$$\begin{cases} \text{GammaRegularized}[1+\text{Floor}[x], 1.5] & x \geq 0 \\ 0 & \text{True} \end{cases}$$

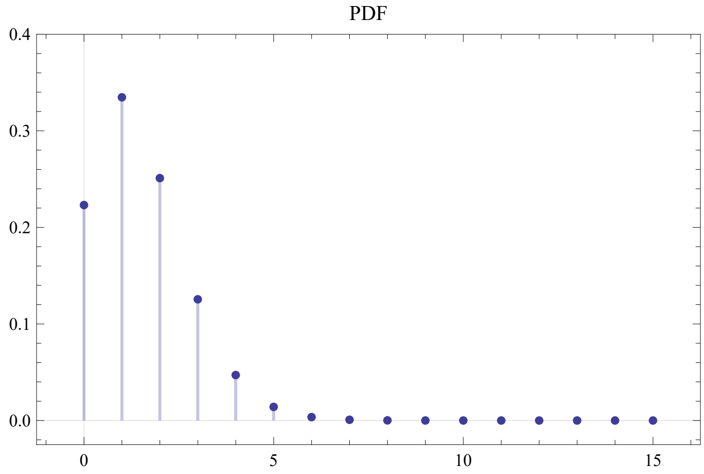
In[∘]:=
```
cdf = DiscretePlot[
  PiecewiseExpand[Piecewise[{{GammaRegularized[1 + Floor[x], 1.5], x ≥ 0}}, 0]],
  {x, -1, 15}, ExtentSize → Right, ExtentMarkers → {"Filled", "Empty"},
  PlotLabel → "CDF", GridLines → Automatic, PlotStyle → ColorData[1, 1],
  PlotTheme → "Scientific", ImageSize → Medium]
```

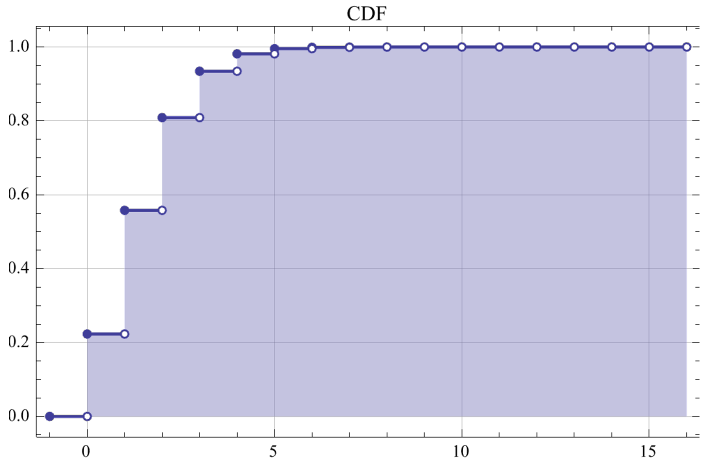
Out[∘]=



In[•]:=
```
Grid[{{pdf}, {cdf}}]
```

Out[•]=





Demonstration (Mean)

https://en.wikibooks.org/wiki/Statistics/Distributions/Poisson

We calculate the mean as follows:

$$\mathrm{E}[X] = \sum_i f(x_i) \cdot x_i = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} x$$

$$\mathrm{E}[X] = \frac{e^{-\lambda}\lambda^0}{0!} \cdot 0 + \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} x$$

$$\mathrm{E}[X] = 0 + e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda\lambda^{x-1}}{(x-1)!}$$

$$\mathrm{E}[X] = \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$\mathrm{E}[X] = \lambda e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

Remember that $e^{\lambda} = \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$

$$\mathrm{E}[X] = \lambda e^{-\lambda} e^{\lambda} = \lambda$$

In[∘]:= $\sum_{x=0}^{\infty} \frac{\left(e^{-\lambda}\lambda^x\right)}{x!} x$

Out[∘]=

λ

In[∘]:= $\mathrm{Sum}\left[\frac{\left(e^{-\lambda}\lambda^x\right)}{x!} x, \{x, 0, \infty\}\right]$

Out[∘]=

λ

Demonstration (Variance)

https://en.wikibooks.org/wiki/Statistics/Distributions/Poisson ; https://proofwiki.org/wiki/Variance_of_Poisson_Distribution

Proof 1

From the definition of Variance as Expectation of Square minus Square of Expectation:

$$\operatorname{var}(X) = \mathsf{E}(X^2) - (\mathsf{E}(X))^2$$

From Expectation of Function of Discrete Random Variable:

$$\mathsf{E}(X^2) = \sum_{x \in \Omega_X} x^2 \Pr(X = x)$$

So:

$$\begin{aligned}
\mathsf{E}(X^2) &= \sum_{k \ge 0} k^2 \frac{1}{k!} \lambda^k e^{-\lambda} && \text{Definition of Poisson Distribution} \\
&= \lambda e^{-\lambda} \sum_{k \ge 1} k \frac{1}{(k-1)!} \lambda^{k-1} && \text{Note change of limit: term is zero when } k = 0 \\
&= \lambda e^{-\lambda} \left(\sum_{k \ge 1} (k-1) \frac{1}{(k-1)!} \lambda^{k-1} + \sum_{k \ge 1} \frac{1}{(k-1)!} \lambda^{k-1} \right) && \text{straightforward algebra} \\
&= \lambda e^{-\lambda} \left(\lambda \sum_{k \ge 2} \frac{1}{(k-2)!} \lambda^{k-2} + \sum_{k \ge 1} \frac{1}{(k-1)!} \lambda^{k-1} \right) && \text{Again, note change of limit: term is zero when } k - 1 = 0 \\
&= \lambda e^{-\lambda} \left(\lambda \sum_{i \ge 0} \frac{1}{i!} \lambda^i + \sum_{j \ge 0} \frac{1}{j!} \lambda^j \right) && \text{putting } i = k - 2, j = k - 1 \\
&= \lambda e^{-\lambda} (\lambda e^{\lambda} + e^{\lambda}) && \text{Taylor Series Expansion for Exponential Function} \\
&= \lambda (\lambda + 1) \\
&= \lambda^2 + \lambda
\end{aligned}$$

Then:

$$\begin{aligned}
\operatorname{var}(X) &= \mathsf{E}(X^2) - (\mathsf{E}(X))^2 \\
&= \lambda^2 + \lambda - \lambda^2 && \text{Expectation of Poisson Distribution: } \mathsf{E}(X) = \lambda \\
&= \lambda
\end{aligned}$$

■

In[ ]:= $\sum_{x=0}^{\infty} \frac{\left(e^{-\lambda} \lambda^{x}\right)}{x!} x^2$

Out[ ]=

$\lambda + \lambda^2$

In[ ]:= `Variance[PoissonDistribution[λ]]`

Out[ ]=

λ

Example 1

In[∘]:=
```
PoissonDistribution[2]
```

In[∘]:=
```
PoissonDistribution[2]
```

Out[∘]=
```
PoissonDistribution[2]
```

In[∘]:=
```
PDF[PoissonDistribution[2], x]
```

Out[∘]=

$$\begin{cases} \frac{2^x}{e^2 x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:=

$$\text{PiecewiseExpand}\left[\text{Piecewise}\left[\left\{\left\{\frac{2^x}{e^2 x!}, x \geq 0\right\}\right\}, 0\right]\right]$$

Out[∘]=

$$\begin{cases} \frac{2^x}{e^2 x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:=

$$a = \text{Table}\left[\text{Piecewise}\left[\left\{\left\{\frac{2^x}{e^2 x!}, x \geq 0\right\}\right\}, 0\right], \{x, 0, 10\}\right]$$

```
b = Table[Piecewise[{{x, x ≥ 0}}, 0], {x, 0, 10}]
c = N[a, 4]
```

Out[∘]=

$$\left\{\frac{1}{e^2}, \frac{2}{e^2}, \frac{2}{e^2}, \frac{4}{3 e^2}, \frac{2}{3 e^2}, \frac{4}{15 e^2}, \frac{4}{45 e^2}, \frac{8}{315 e^2}, \frac{2}{315 e^2}, \frac{4}{2835 e^2}, \frac{4}{14175 e^2}\right\}$$

Out[∘]=
```
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Out[∘]=
```
{0.1353, 0.2707, 0.2707, 0.1804, 0.09022, 0.03609,
 0.01203, 0.003437, 0.0008593, 0.0001909, 0.00003819}
```

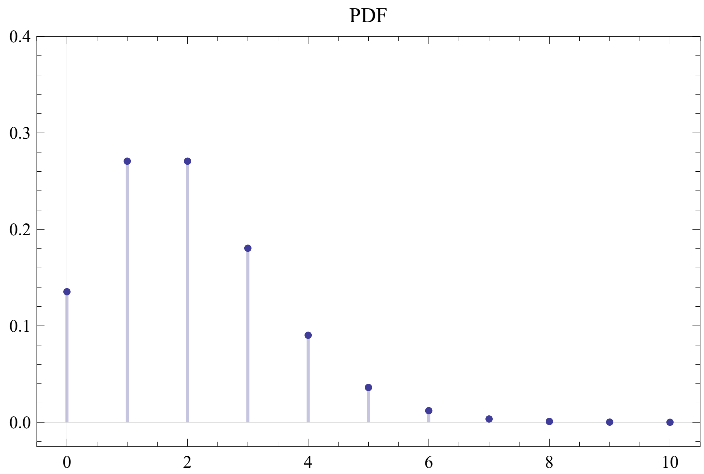
In[∘]:=
```
acol = Column[a];
bcol = Column[b];
Grid[{b, c}]
```

Out[∘]=

0	1	2	3	4	5	6	7	8	9	10
0.1353	0.2707	0.2707	0.1804	0.09022	0.03609	0.01203	0.003437	0.00085 93	0.00019 09	0.00003 819

In[◦]:=
```
DiscretePlot[Piecewise[{{2^x/(ⅇ^2 x!), x ≥ 0}}, 0], {x, 0, 20}, ExtentSize → 0,
  PlotLabel → "PDF", PlotStyle → ColorData[1, 1], PlotTheme → "Scientific",
  ImageSize → Medium, PlotRange → {{-.5, 10.5}, {-0.025, 0.4}}]
```

Out[◦]=



Example 2

In[ ]:= PoissonDistribution[2]

Out[ ]= PoissonDistribution[2]

In[ ]:= dist = TransformedDistribution[x + y, {x ≈ PoissonDistribution[2], y ≈ PoissonDistribution[1]}];

In[ ]:= Mean[dist]

Out[ ]= 3

In[ ]:= PDF[PoissonDistribution[2], x]

Out[ ]=

$$\begin{cases} \frac{2^x}{e^2 x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[ ]:= PDF[PoissonDistribution[2], 2]

Out[ ]=

$$\frac{2}{e^2}$$

In[ ]:= $N\left[\frac{2}{e^2}\right]$

Out[ ]= 0.270671

In[ ]:= Mean[PoissonDistribution[2]]

Out[ ]= 2

In[ ]:= PoissonDistribution[1]

Out[ ]= PoissonDistribution[1]

In[ ]:= PDF[PoissonDistribution[1], x]

Out[ ]=

$$\begin{cases} \frac{1}{e\, x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[ ]:= PDF[PoissonDistribution[1], 2]

Out[ ]=

$$\frac{1}{2\, e}$$

In[ ]:= $N\left[\frac{1}{2\, e}\right]$

Out[ ]= 0.18394

In[ ]:= **PoissonDistribution[3]**

Out[ ]=

PoissonDistribution[3]

In[ ]:= **PDF[PoissonDistribution[3], x]**

Out[ ]=

$$\begin{cases} \frac{3^x}{e^3\, x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[ ]:= **PDF[PoissonDistribution[3], 2]**

Out[ ]=

$$\frac{9}{2\, e^3}$$

In[ ]:= $$\frac{3^2}{e^3\, 2\,!}$$

Out[ ]=

$$\frac{9}{2\, e^3}$$

In[ ]:= $$N\left[\frac{9}{2\, e^3}\right]$$

Out[ ]=

0.224042

In[ ]:= **Mean[PoissonDistribution[3]]**

Out[ ]=

3

Proxy Binomial and Poisson

In[∘]:=
```
BinomialDistribution[100, 0.02]
```
Out[∘]=
```
BinomialDistribution[100, 0.02]
```

In[∘]:=
```
PDF[BinomialDistribution[100, 0.02], x]
```
Out[∘]=

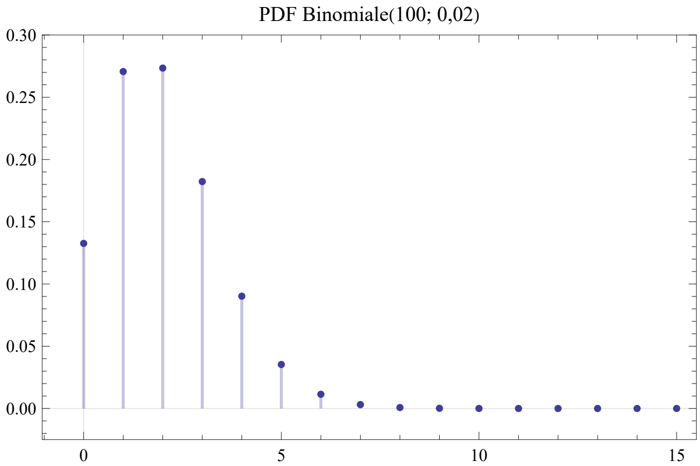
$$\begin{cases} 0.02^x \times 0.98^{100-x}\,\text{Binomial}[100, x] & 0 \le x \le 100 \\ 0 & \text{True} \end{cases}$$

In[∘]:=
```
PiecewiseExpand[Piecewise[{{0.02^x × 0.98^(100-x) Binomial[100, x], 0 ≤ x ≤ 100}}, 0]]
```
Out[∘]=

$$\begin{cases} 0.13262 \times 0.0204082^x\,\text{Binomial}[100, x] & 0 \le x \le 100 \\ 0 & \text{True} \end{cases}$$

In[∘]:=
```
Table[Piecewise[{{0.13262 × 0.0204082^x Binomial[100, x], 0 ≤ x ≤ 100}}, 0], {x, 0, 6}]
```
Out[∘]=
```
{0.13262, 0.270652, 0.273414, 0.182276, 0.090208, 0.0353468, 0.0114216}
```

In[∘]:=
```
binomiale =
 DiscretePlot[Piecewise[{{0.13262 × 0.0204082^x Binomial[100, x], 0 ≤ x ≤ 100}}, 0],
  {x, 0, 20}, ExtentSize → 0, PlotLabel → "PDF Binomiale(100; 0,02)",
  PlotStyle → ColorData[1, 1], PlotTheme → "Scientific",
  ImageSize → Medium, PlotRange → {{-1.05, 15.5}, {-0.025, 0.3}}]
```
Out[∘]=



In[∘]:=
```
PoissonDistribution[2]
```
Out[∘]=
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PoissonDistribution[2]
```

In[∘]:=
```
PDF[PoissonDistribution[2], x]
```
Out[∘]=

$$\begin{cases} \frac{2^x}{e^2\, x!} & x \ge 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:= $\texttt{PiecewiseExpand}\left[\texttt{Piecewise}\left[\left\{\left\{\frac{2^{x}}{e^{2}\,x\,!},\ x \geq 0\right\}\right\},\ 0\right]\right]$

Out[∘]=

$$\begin{cases} \frac{2^{x}}{e^{2}\,x\,!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:= $\texttt{N}\left[\texttt{Table}\left[\texttt{Piecewise}\left[\left\{\left\{\frac{2^{x}}{e^{2}\,x\,!},\ x \geq 0\right\}\right\},\ 0\right],\ \{x,\ 0,\ 6\}\right]\right]$

Out[∘]=

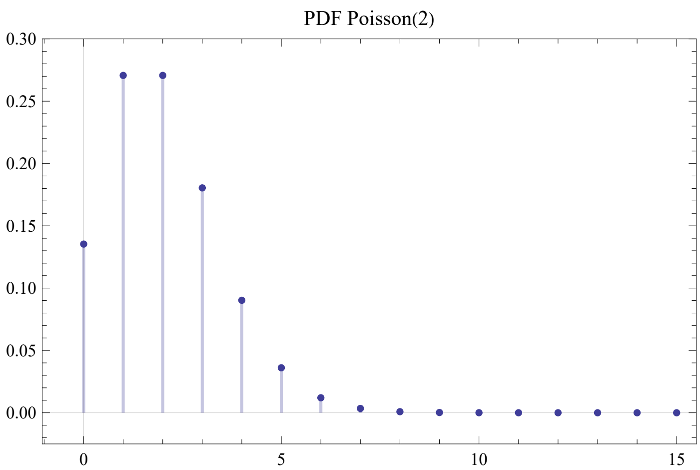
{0.135335, 0.270671, 0.270671, 0.180447, 0.0902235, 0.0360894, 0.0120298}

In[∘]:= $\texttt{poisson} = \texttt{DiscretePlot}\left[\texttt{Piecewise}\left[\left\{\left\{\frac{2^{x}}{e^{2}\,x\,!},\ x \geq 0\right\}\right\},\ 0\right],\ \{x,\ 0,\ 20\},\ \texttt{ExtentSize} \to 0,\right.$

PlotLabel → "PDF Poisson(2)", PlotStyle → ColorData[1, 1], PlotTheme → "Scientific",

ImageSize → Medium, PlotRange → {{-1.05, 15.5}, {-0.025, 0.3}}]

Out[∘]=



In[∘]:= TableForm[Table[{poisson, binomiale}, 1]]

Out[∘]//TableForm=