Probabilités et Statistique II Chapitre 4. Variables aléatoires continues

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In[ ]:=
```
g[x_] := Piecewise[{{y = 0, x < 0}, {y = 1 - e^(-0.1 x), x ≥ 0}}]
```

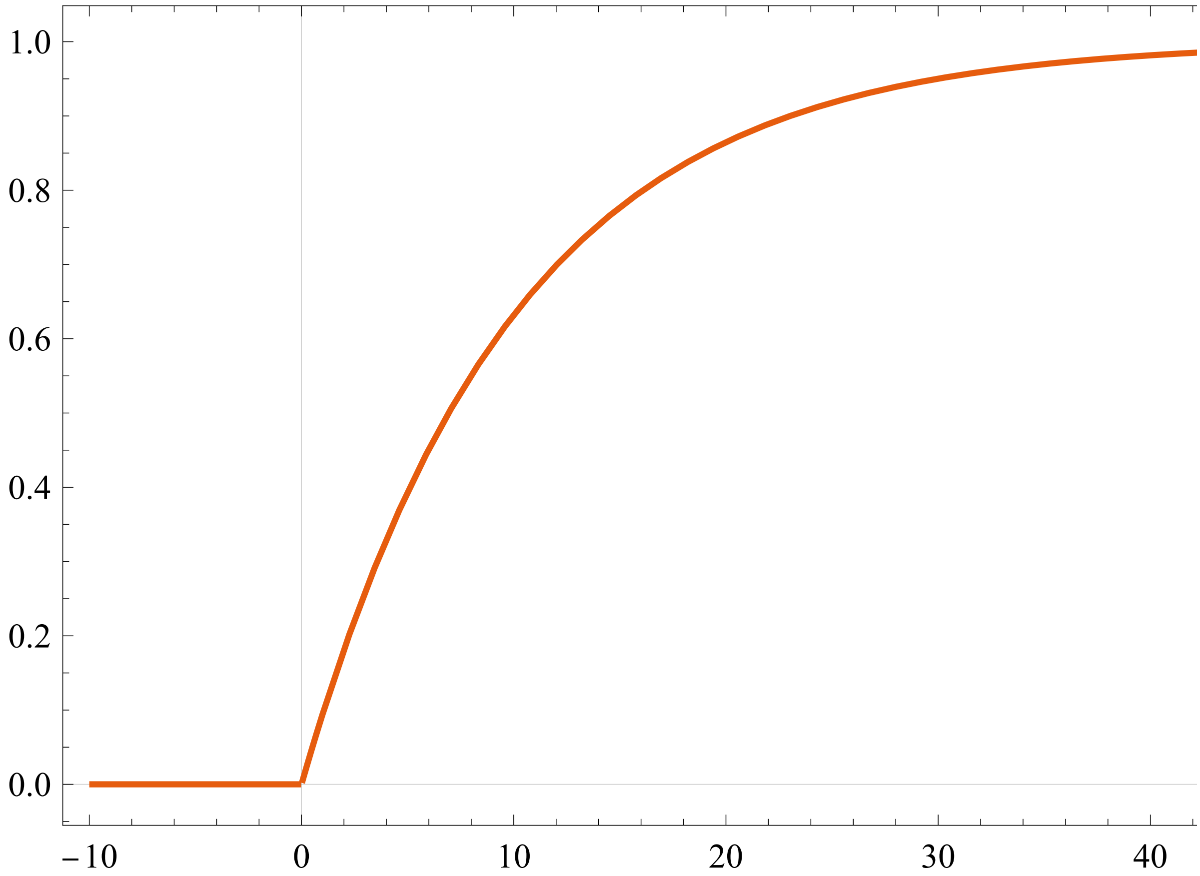
In[ ]:=
```
g[x]
```

Out[ ]=

$$\begin{cases} 0 & x < 0 \\ 1 - e^{-0.1\,x} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[ ]:=
```
Magnify[
 Plot[Evaluate[g[x]], {x, -10, 50}, PlotTheme → "Scientific", ImageSize → Medium], 2]
```

Out[ ]=



- Derivate of the CDF is the PDF

In[ ]:=
```
f[x_] := y = 1 - e^(-0.1 x)
```

In[ ]:=
```
f[x]
```

Out[ ]=

$1 - e^{-0.1\,x}$

In[ ]:=
```
f'[x]
```

Out[ ]=

$0.1\,e^{-0.1\,x}$

In[ ]:= derivative of 1- E^(-0.1*x)

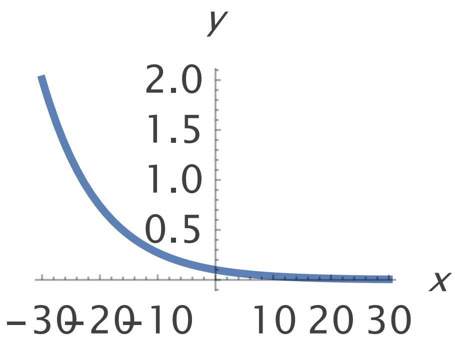
Derivative:

Approximate form

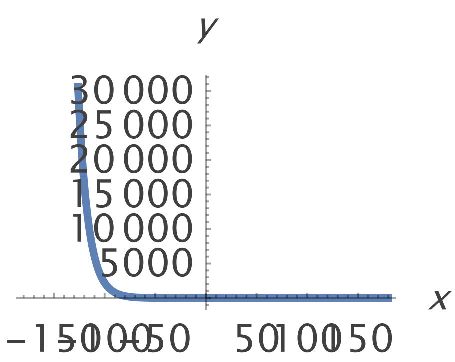
Step-by-step solution

$$\frac{d}{dx}\left(1 - e^{-0.1\,x}\right) = 0.1\,e^{-0.1\,x}$$

Plots:



min max



min max

Alternate form assuming x is real:

$$0.1\, e^{-0.1\, x} + 0$$

Roots: Step–by–step solution

(no roots exist)

Properties as a real function:

Domain:

$\mathbb{R}$ (all real numbers)

Range:

$\{ y \in \mathbb{R} : y > 0 \}$ (all positive real numbers)

Injectivity:

injective (one-to-one)

R is the set of real numbers »

Periodicity: Approximate form

periodic in x with period $20\, i\, \pi$

Series expansion at x = 0:

$$0.1 - 0.01\, x + 0.0005\, x^2 - 0.0000166667\, x^3 + 4.16667 \times 10^{-7}\, x^4 + O(x^5)$$

(Taylor series)

Big-O notation »

Indefinite integral: Approximate form

Step-by-step solution

$$\int 0.1\, e^{-0.1\, x}\, dx = -e^{-0.1\, x} + \text{constant}$$

Limit:

$$\lim_{x \to \infty} 0.1\, e^{-0.1\, x} = 0 \approx 0$$

WolframAlpha

In[∘]:= `h[x_] := Piecewise[{{y = 0, x < 0}, {y = 0.1 e^(-0.1 x), x ≥ 0}}]`

$$h[x\_] := \text{Piecewise}\left[\left\{\{y = 0,\ x < 0\},\ \left\{y = 0.1\, e^{-0.1\, x},\ x \geq 0\right\}\right\}\right]$$

In[∘]:= `h[x]`

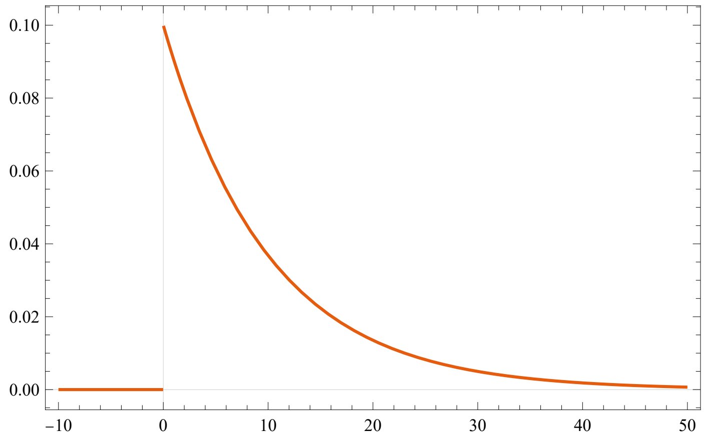
Out[∘]=

$$\begin{cases} 0 & x < 0 \\ 0.1\, e^{-0.1\, x} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:=

```
graph1 = Legended[Plot[Evaluate[h[x]], {x, -10, 50},
   PlotTheme → "Scientific", ImageSize → Medium], h[x]]
```

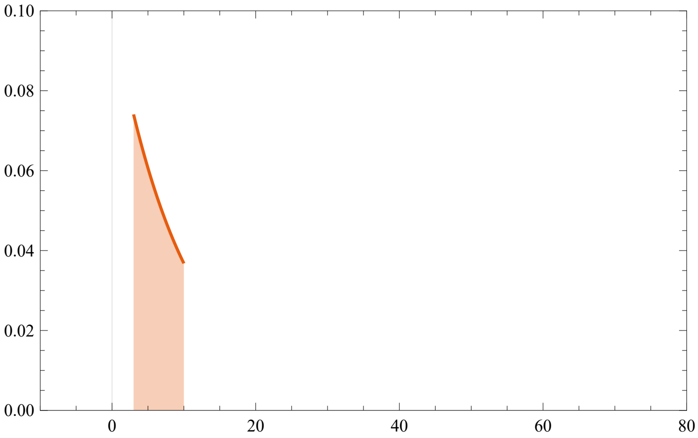
Out[∘]=



$$\begin{cases} 0 & x < 0 \\ 0.1\, e^{-0.1\, x} & x \geq 0 \end{cases}$$

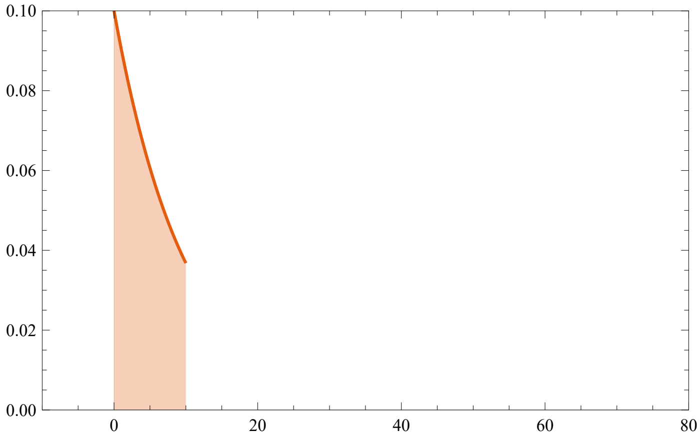
In[∘]:=
```
graph2 = Plot[{Evaluate[h[x]]}, {x, 3, 10},
  PlotTheme → "Scientific" , ImageSize → Medium, Filling → Axis,
  AxesOrigin → {0, 0}, PlotRange → {{-10, 80}, {0, 0.10}}]
```

Out[∘]=



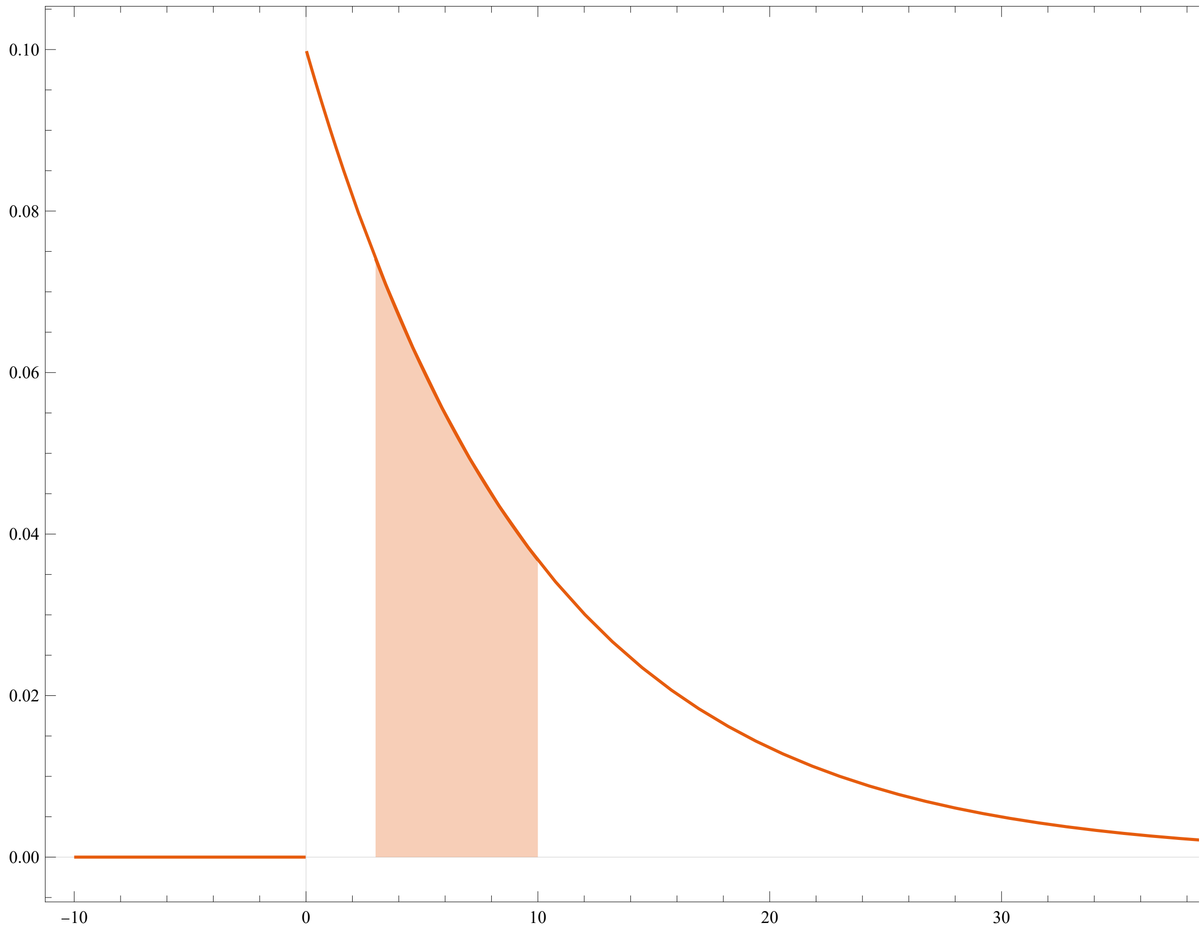
In[ ]:=
```
graph3 = Plot[{Evaluate[h[x]]}, {x, 0, 10},
  PlotTheme → "Scientific" , ImageSize → Medium, Filling → Axis,
  AxesOrigin → {0, 0}, PlotRange → {{-10, 80}, {0, 0.10}}]
```

Out[ ]=



In[ ]:= `Show[graph1, graph2, PlotRange → All]`

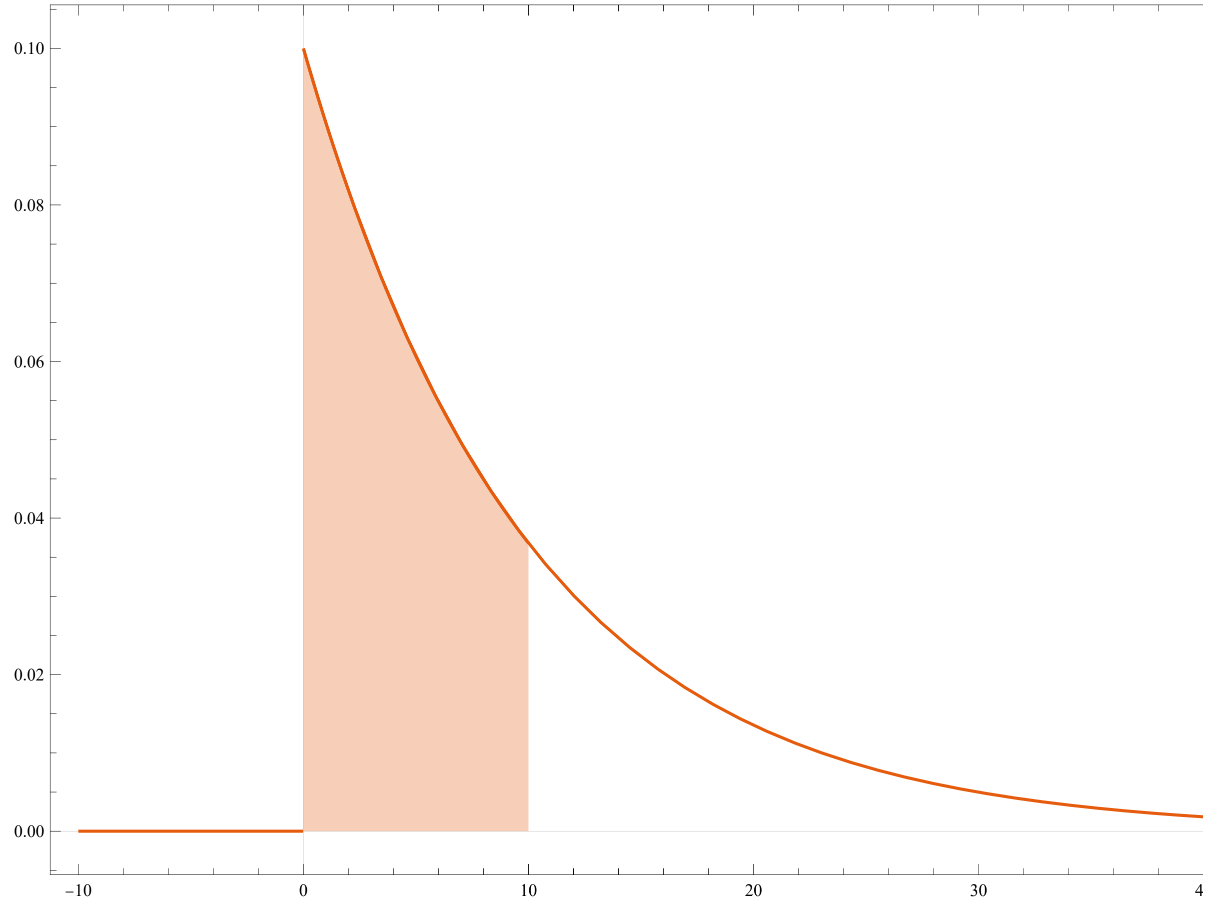
Out[ ]=



In[ ]:=

```
Show[graph1, graph3, PlotRange → All]
```

Out[ ]=



In[ ]:=
```
Integrate[0.1 ⅇ^(-0.1 x), {x, 0, +∞}]
```

Out[ ]=
```
1.
```

In[ ]:= Integrate[0.1 E^(-0.1 x), {x, 0, +∞}]

Definite integral:

Step-by-step solution

$$\int_0^\infty 0.1\, e^{-0.1\, x}\, dx = 1$$

Indefinite integral:

Approximate form

Step-by-step solution

$$\int 0.1\, e^{-0.1\, x}\, dx = -e^{-x/10} + \text{constant}$$

WolframAlpha

In[ ]:=
```
g[x_] := Piecewise[{{y = 0, x < 10},
    {y = ((10 - x) (x - 70) / 36000), 10 ⩽ x ⩽ 70}, {y = 0, x > 70}}]
```

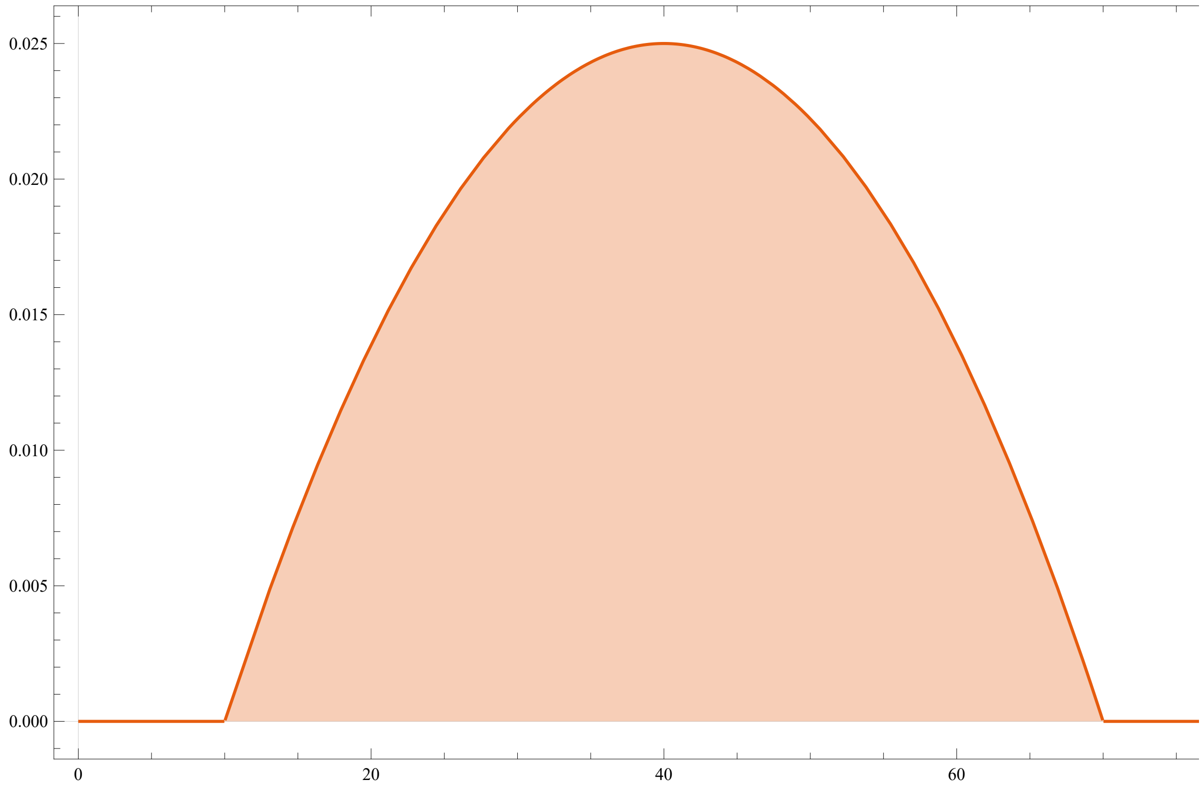
In[ ]:=
```
g[x]
```

Out[ ]=

$$\begin{cases} 0 & x < 10 \\ \frac{(10-x)\ (-70+x)}{36\,000} & 10 \le x \le 70 \\ 0 & \text{True} \end{cases}$$

In[•]:=
```
Legended[Plot[Evaluate[g[x]], {x, 0, 80},
  PlotTheme → "Scientific", Filling → Axis], g[x]]
```

Out[•]=



```
In[ ]:= Integrate[((10 - x) (x - 70) / 36000), {x, 10, 70}]

Out[ ]=
1
```

In[ ]:=
```
Integrate[((10-x)(x-70)/36000), {x, 10, 70}]
```

In[ ]:=
```
Integrate[((10 - x) (x - 70) / 36000), x]
```

Out[ ]=

$$\frac{-700\,x+40\,x^2-\frac{x^3}{3}}{36000}$$

```
In[ ]:= Integrate[((10 - x) (x - 70)/36000)]
```

In[ ]:=
```
pdf = Piecewise[{{y = 0, x < 10},
  {y = ((10 - x) (x - 70) / 36 000), 10 ≤ x ≤ 70}, {y = 0, 10 > x > 70}}]
```

Out[ ]=

$$\begin{cases} 0 & x < 10 \\ \frac{(10-x)\,(-70+x)}{36\,000} & 10 \le x \le 70 \\ 0 & \text{True} \end{cases}$$

In[ ]:=
```
cdf = Integrate[pdf, x]
```

Out[ ]=

$$\begin{cases} 0 & x \le 10 \\ \frac{5}{54} + \frac{-700\,x+40\,x^2-\frac{x^3}{3}}{36\,000} & 10 < x \le 70 \\ 1 & \text{True} \end{cases}$$

In[ ]:=
```
PiecewiseExpand[pdf]
```

Out[ ]=

$$\begin{cases} -\frac{(-70+x)\,(-10+x)}{36\,000} & 10 \le x \le 70 \\ 0 & \text{True} \end{cases}$$

In[ ]:=
```
PiecewiseExpand[cdf]
```

Out[ ]=

$$\begin{cases} 1 & x > 70 \\ \frac{10\,000-2100\,x+120\,x^2-x^3}{108\,000} & 10 < x \le 70 \\ 0 & \text{True} \end{cases}$$

In[∘]:= **(-700 x + 40 x^2 - x^3/3)/36000 when x=10**

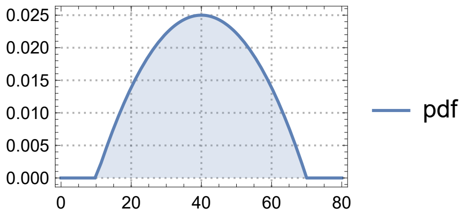
```
In[∘]:= ReplaceAll[(-700 x + 40 x^2 - x^3 / 3) / 36 000, {x → 10}]
```

Out[∘]=

$$-\frac{5}{54}$$

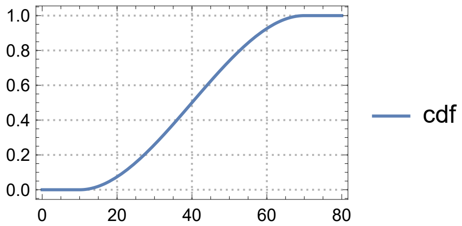
```
In[ ]:= pdf1 = Plot[pdf, {x, 0, 80},
          PlotTheme → "Detailed", Filling → Axis, ImageSize → Small]
```

Out[ ]=



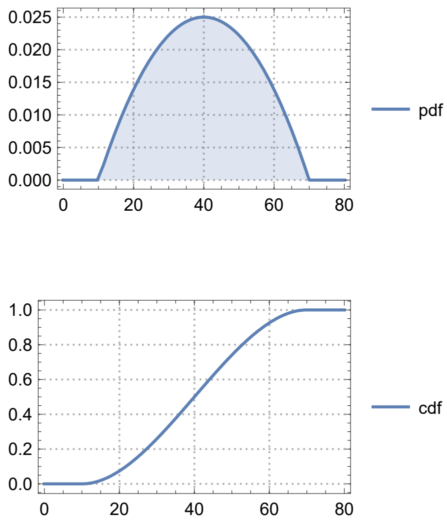

```
In[ ]:= cdf1 = Plot[cdf, {x, 0, 80}, PlotTheme → "Detailed", ImageSize → Small]
```

Out[ ]=



In[•]:= `GraphicsColumn[{pdf1, cdf1}, Spacings → 1]`

Out[•]=



In[•]:= `Integrate[x, {x, 0, 1}]`

Out[•]=

$\frac{1}{2}$

In[•]:= **Integrate x from 0 to 1**

Assuming "from 0 to 1" is referring to variable ranges

| Use "from" as a word instead

Definite integrals:

Hide steps

$$\int_0^1 x\, dx = \frac{1}{2}$$

Possible intermediate steps:

Compute the definite integral:

$$\int_0^1 x\, dx$$

Apply the fundamental theorem of calculus.

The antiderivative of x is $\frac{x^2}{2}$:

$$= \left.\frac{x^2}{2}\right|_0^1$$

Evaluate the antiderivative at the limits and :

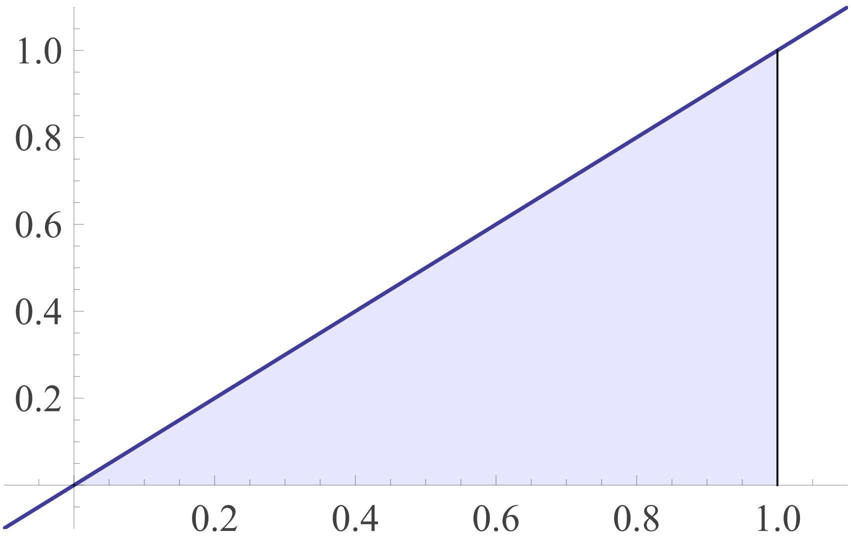
Evaluate the antiderivative at the limits and subtract.

$$\left.\frac{x^2}{2}\right|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

Answer:

$$= \frac{1}{2}$$

Visual representation of the integral:

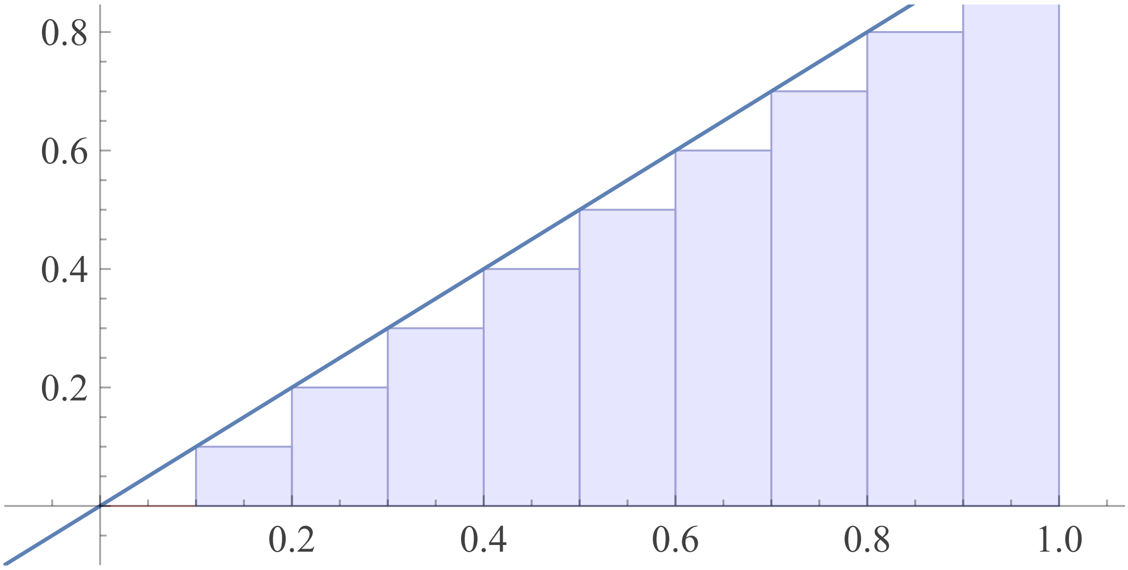


Riemann sums:

More cases

left sum	$\frac{n-1}{2\,n} = \frac{1}{2} - \frac{1}{2\,n} + O\left(\left(\frac{1}{n}\right)^2\right)$

(assuming subintervals of equal length)



integral: $\frac{1}{2} \approx 0.5$

Riemann sum: 0.45

error: 0.05

number of subintervals

summation method | left endpoint | midpoint | right endpoint

Indefinite integral:

Step-by-step solution

$$\int x \, dx = \frac{x^2}{2} + \text{constant}$$

WolframAlpha

In[∘]:= `Integrate[(x - 1/2)^2, {x, 0, 1}]`

$$\texttt{Integrate}\left[\left(x-\frac{1}{2}\right)^2, \{x, 0, 1\}\right]$$

Out[∘]=

$$\frac{1}{12}$$

In[∘]:= Integrate (x-(1/2))^2 from 0 to 1

Definite integrals:

More digits | Hide steps

$$\int_0^1 \left(x-\frac{1}{2}\right)^2 dx = \frac{1}{12} \approx 0.083333$$

Possible intermediate steps:

Compute the definite integral:

$$\int_0^1 \left(x-\frac{1}{2}\right)^2 dx$$

For the integrand $\left(x-\frac{1}{2}\right)^2$, substitute

$u = x - \frac{1}{2}$ and $du = dx$. This gives a new lower

bound $u = 0 - \frac{1}{2} = -\frac{1}{2}$ and upper bound

$u = 1 - \frac{1}{2} = \frac{1}{2}$:

$$= \int_{-1/2}^{1/2} u^2 \, du$$

Apply the fundamental theorem of calculus.

The antiderivative of u^2 is $\frac{u^3}{3}$:

$$= \left.\frac{u^3}{3}\right|_{-1/2}^{1/2}$$

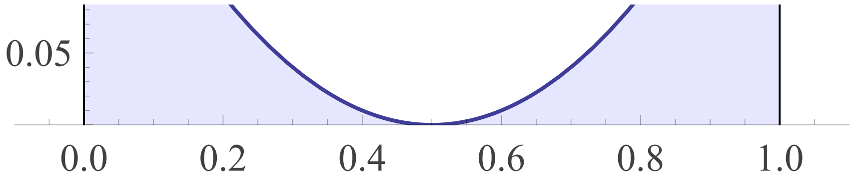
Evaluate the antiderivative at the limits and :
subtract.

$$\left.\frac{u^3}{3}\right|_{-1/2}^{1/2} = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \left(\frac{1}{3}\left(-\frac{1}{2}\right)^3\right) = \frac{1}{12}$$

Answer:

$$= \frac{1}{12}$$

Visual representation of the integral:

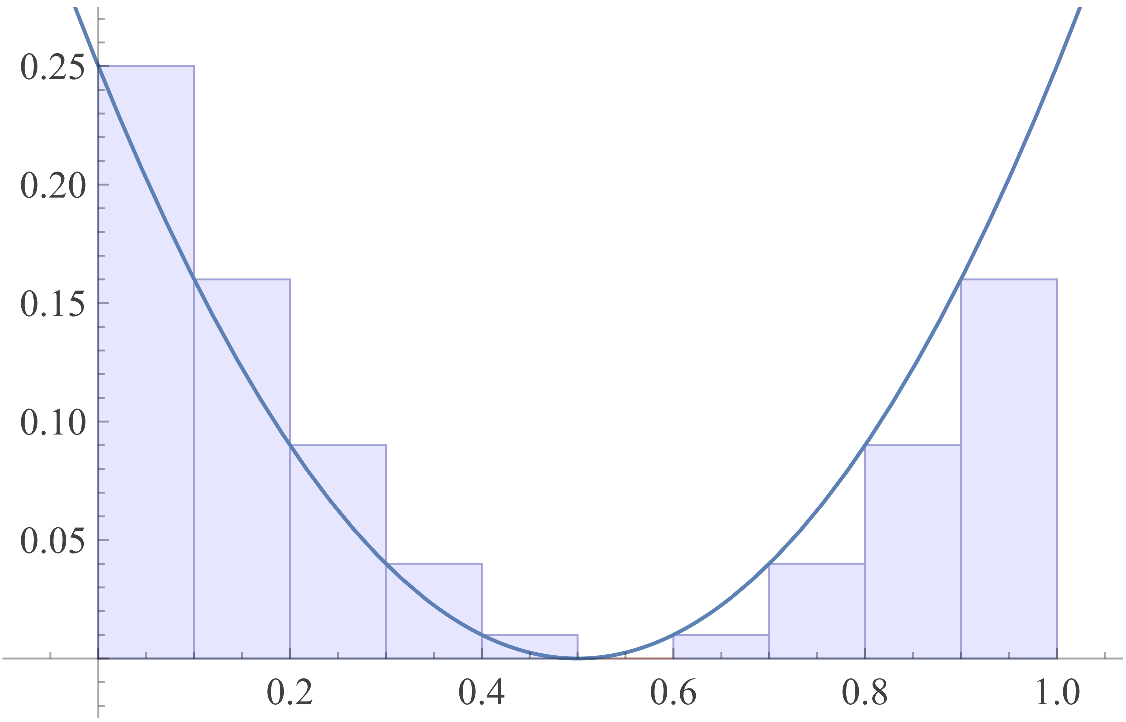


Riemann sums:

More cases

left sum	$\frac{1}{6n^2} + \frac{1}{12}$

(assuming subintervals of equal length)



integral: $\frac{1}{12} \approx 0.0833333$

Riemann sum: 0.085

error: 0.00166667

number of subintervals

summation method | left endpoint | midpoint | right endpoint

Indefinite integral:

Step–by–step solution

$$\int\left(x-\frac{1}{2}\right)^2 dx = \frac{x^3}{3} - \frac{x^2}{2} + \frac{x}{4} + \text{constant}$$

WolframAlpha