

Abstract

In this project, we generalize ANNs to infinite dimensional Banach spaces by developing a practical analog to the feedforward propagation algorithm. Using this new class of algorithms, GANN, we prove a new universal approximation theorem for bounded linear operators and show that representation of weights by samples from a multivariate weight polynomial can drastically reduce the dimensionality of a learning problem. Lastly, we give a practical implementation of the error back-propagation algorithm in this space for the classification of continuous data.

Introduction

Although neural networks have proven an extremely effective mechanism of machine learning [1], theoretically they remain a black-box model. In answer to this problem [2] examined the notion of infinite hidden nodes with a network proving that such a construction becomes a Gaussian kernel. Then, [2] described a model for affine neural networks with continuous hidden layers in alignment with [1]. These authors showed effectively the viability of a "continuous" neural network, but left many similar constructions unexplored. [1].

Artificial Neural Networks

Definition 1. We say $\mathcal{N} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a feedforward neural network if for an input vector \mathbf{x} ,

$$\begin{aligned} \mathcal{N} : \sigma_j^{(l+1)} &= g \left(\sum_{i \in Z^{(l)}} w_{ij}^{(l)} \sigma_i^{(l)} + \beta^{(l)} \right) \\ \sigma_j^{(1)} &= g \left(\sum_{i \in Z^{(0)}} w_{ij}^{(0)} x_i + \beta^{(0)} \right), \end{aligned} \quad (1)$$

where $1 \leq l \leq L - 1$. Furthermore we say $\{\mathcal{N}\}$ is the set of all neural networks.

Functional Neural Networks

Suppose that we wish to map one functional space to another with a neural network. Consider the standard model of an ANN as the number of neural nodes for every layer becomes uncountable. The index for each node then becomes real-valued, along with the weight and input vectors.

Definition 2. We call $\mathcal{F} : L^1(X) \rightarrow L^1(Y)$ a functional neural network if,

$$\begin{aligned} \mathcal{F} : \sigma^{(l+1)}(j) &= g \left(\int_{R^{(l)}} \sigma^{(l)}(i) w^{(l)}(i, j) di \right) \\ \sigma^{(0)}(j) &= \xi(j). \end{aligned} \quad (2)$$

Furthermore let $\{\mathcal{F}\}$ denote the set of all functional neural networks.

Important Result

Theorem. Given a functional neural network \mathcal{F} then some layer $l \in \mathcal{F}$, the let $K : C(R^{(l)}) \rightarrow C(R^{(l)})$ be a bounded linear operator. If we denote the operation of layer l on layer $l - 1$ as $\sigma^{(l+1)} = g(\Sigma_{l+1} \sigma^{(l)})$, then for every $\epsilon > 0$, there exists a weight polynomial $w^{(l)}(i, j)$ such that the supremum norm over $R^{(l)}$

$$\|K\sigma^{(l)} - \Sigma_{l+1}\sigma^{(l)}\|_{\infty} < \epsilon \quad (3)$$

Mathematical Section

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$$E = mc^2 \quad (4)$$

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$$\cos^3 \theta = \frac{1}{4} \cos \theta + \frac{3}{4} \cos 3\theta \quad (5)$$

Methods

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Conclusion

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Additional Information

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- Duis porta consequat lorem

References

- [1] J. M. Smith and A. B. Jones. *Book Title*. Publisher, 7th edition, 2012.
- [2] A. B. Jones and J. M. Smith. Article Title. *Journal title*, 13(52):123–456, March 2013.

Acknowledgements

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Placeholder
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Figure 1: Figure caption

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