

# Principles of modeling

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# Agenda: What today teaches or repeats

## Conceptual modeling steps

- Do I really need a process model?
- (Cognitive) measurement vs. computational vs. process models

## Testability/Precision/Complexity

- What's a response surface?
- Tension between flexibility and testability

- Computational complexity

## Inferential Fallacies

- What0s reverse inference

## Important Other Criteria

- Inclusiveness
- Referencing
- The toothbrush problem

# Cognitive Process Modeling Steps

# An Informal Cooking Recipe

## Cognitive Process Model Development Steps

- a. Decide what to model
- b. Specify in what stages the model transforms data
- c. Ensure model makes precise, testable predictions
- d. Check for fallacies, e.g. reverse inference
- e. Check of model's processes not implausible?

(Jarecki, Tan, Jenny, [2020](#))

**a. What to model**

# What's the Goal and Content of my Model?

## Modeling goals

- Implement processes (at a given level of abstraction)? (> *cognitive process model*)
- Describe responses (in a particular domain)? (> *cognitive formal model*)
- Measurement of cognitive constructs? (> *cognitive measurement model*)
- Statistical test? (> *statistical model*)

## Explain and/or predict?

- Explanation of phenomena?
- Prediction of new phenomena?

**Important, note:** there is no hierarchy as long as you are clear on what you do.

# What's the Goal of Modeling?

## Implement Cognitive Processes

- Theory contains statements about how the mind transforms information
- *Computation Assumption*: We assume that the mind does something *like* computing something (it's a metaphor)
- *Multi-realizability*: A theoretical statement can be realized in many different ways

b. Specify stages



# Specify stages

Write down what your model does to the input – this is what this school is mostly about

c. Ensure precise, testable predictions

# Response Scope of a Model

## Response Scope as data patterns

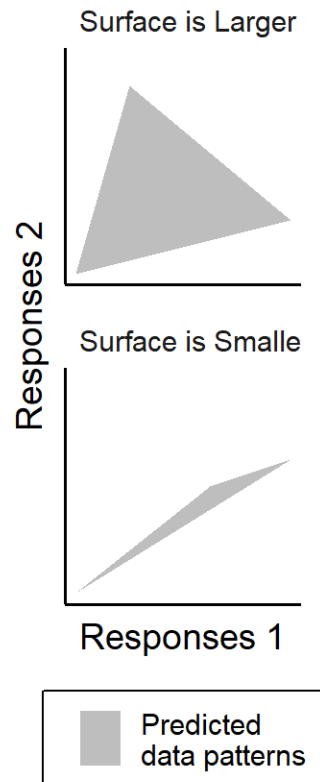
- **How much a model can handle**  
(Farrell & Lewandowsky, 2018)
- How many different data patterns a model can predict

## (Other meanings of scope)

- Purpose of the model and guides interpreting model variables  
(Hodges, [2013](#))
- Which model variables represent which properties of the cognitive system and sets the level of abstraction (Jarecki, Tan, Jenny, [2020](#))

# Response Surface of a Model

A model's **response surface**, meaning the amount of the total data space occupied by the model



Response Surface tends to be larger in models

- with many free parameters which can adapt the model to different patterns in the data
- with a more complex functional form, e.g. an cosine curve compared to a linear curve

# Response Surface - Example

## Three models

that predict a response  $\backslash(y\backslash)$  given a variable  $\backslash(t\backslash)$  with 1 free model parameter  $\backslash(\theta\backslash)$ :

- Linear model  $\backslash(y=1-\theta t\backslash)$
- Power model  $\backslash(y=t^\theta\backslash)$
- Blackhole model  $\backslash(y=(1.102^{-\theta}) \sin(5\theta + \pi t/12) + 1)/2\backslash)$

*How large is their response surface, meaning how many response patterns can the models – with the values of the free parameter – produce?*

Say, we have two responses for inputs  $\backslash(t_1, t_2\backslash)$ . Get the models' response surfaces by computing response predictions  $\backslash(y_1, y_2\backslash)$  given inputs  $\backslash(t_1, t_2\backslash)$  for each value that parameter  $\backslash(\theta\backslash)$  is allowed to take. If we then plot the possible response combinations  $\backslash(y_1\backslash), \backslash(y_2\backslash)$  against each other, we see the model's response patterns.

(Pitt, Myung, Zhang, 2002)

# Response Surface - Example

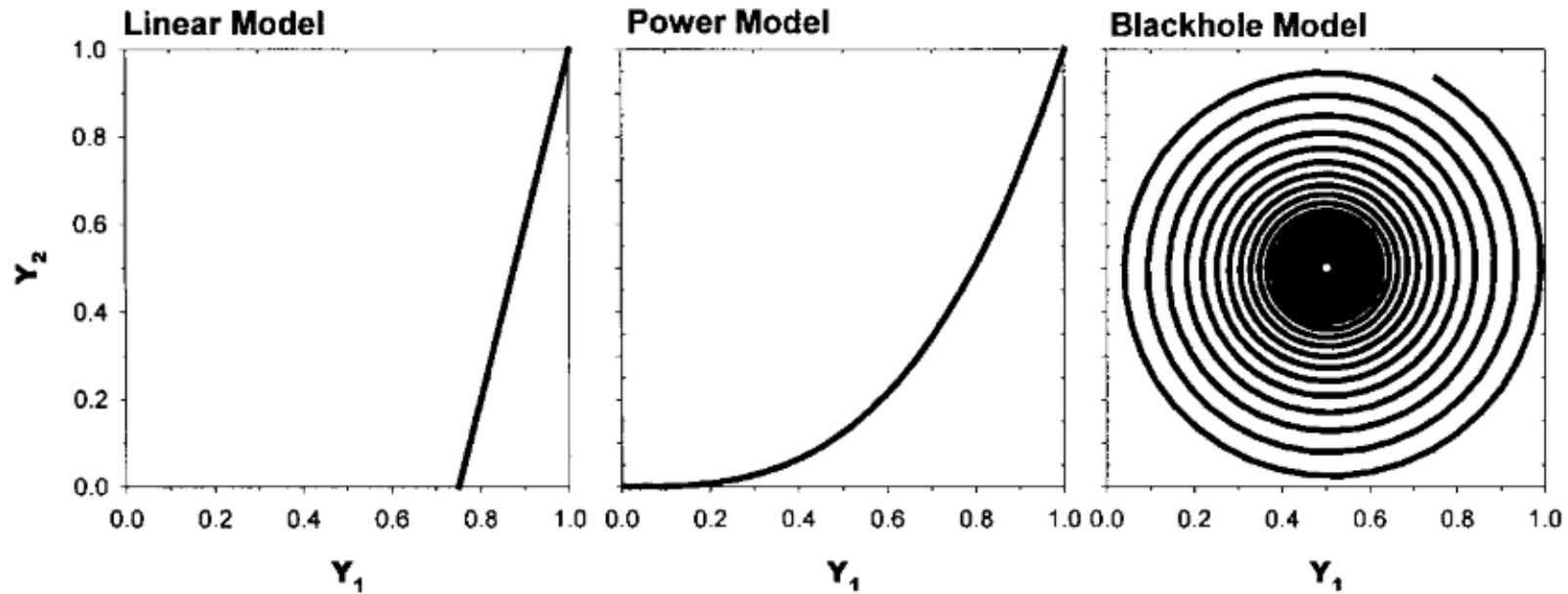


Figure 4. Response curves of three one-parameter models that have the same number of parameters but differ in functional form, each obtained for  $t_1 = 2$  and  $t_2 = 8$ .

This is called response surface analysis (RSA).

(Pitt, Myung, Zhang, 2002)

# Response Surface

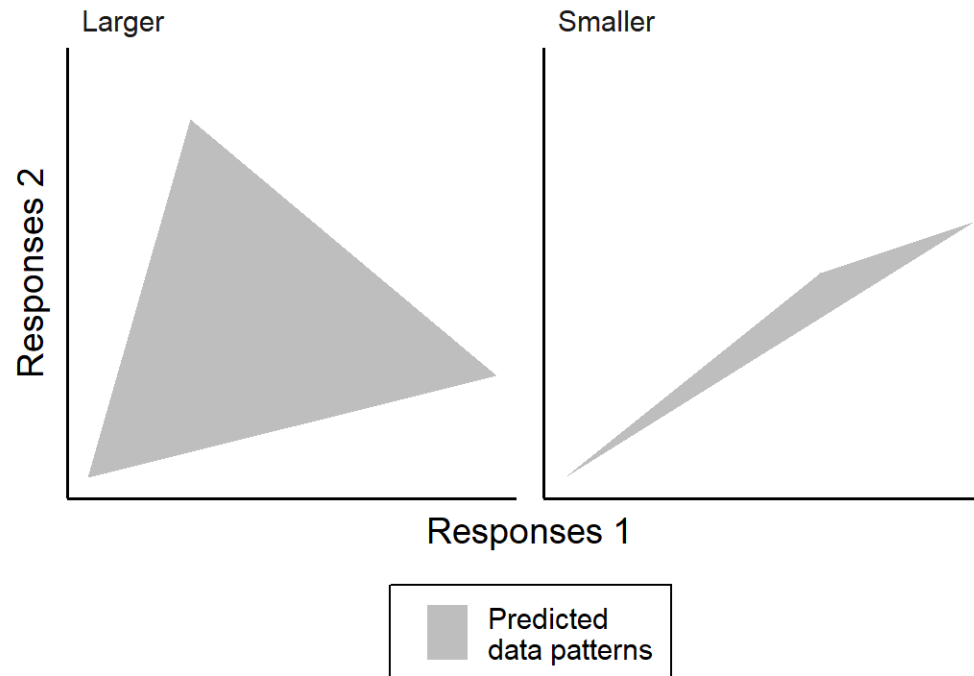
The response surface is a measure for how many data patterns our model can handle.

If you want to know more, check out

- The Bible :)
- Pitt, M. A., Myung, I. J., & Zhang, S. (2002). Toward a method of selecting among computational models of cognition. *Psychological Review*, 109(3), 472–491. <https://doi.org/10.1037/0033-295X.109.3.472>
- Roberts, S., & Pashler, H. (2000). How persuasive is a good fit? A comment on theory testing. *Psychological Review*, 107(2), 358–367. <https://doi.org/10.1037/0033-295X.107.2.358>

# Response Surface - But Wait?

How large do we want the response surface of our models to be?



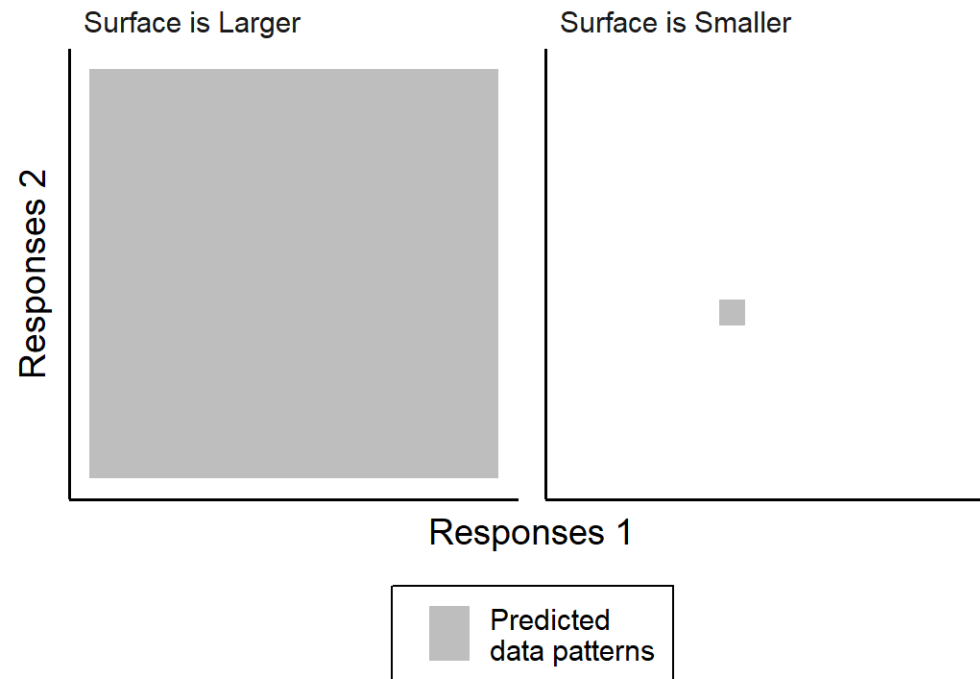


**Falsifiability**

# Falsifiability

- Is there anything a model can't handle?
- The potential of samples of data to speak against a model's prediction

# vs. Flexibility



# Response Surface - Three Important Aspects

When designing and testing models, consider

- Model's flexibility:** how large is the response surface compared to the possible data space?
- (Expected) **data variability:** how firmly can the observed data rule out what the model cannot fit?
- Other outcomes:** Might the model have been able to produce any plausible result?

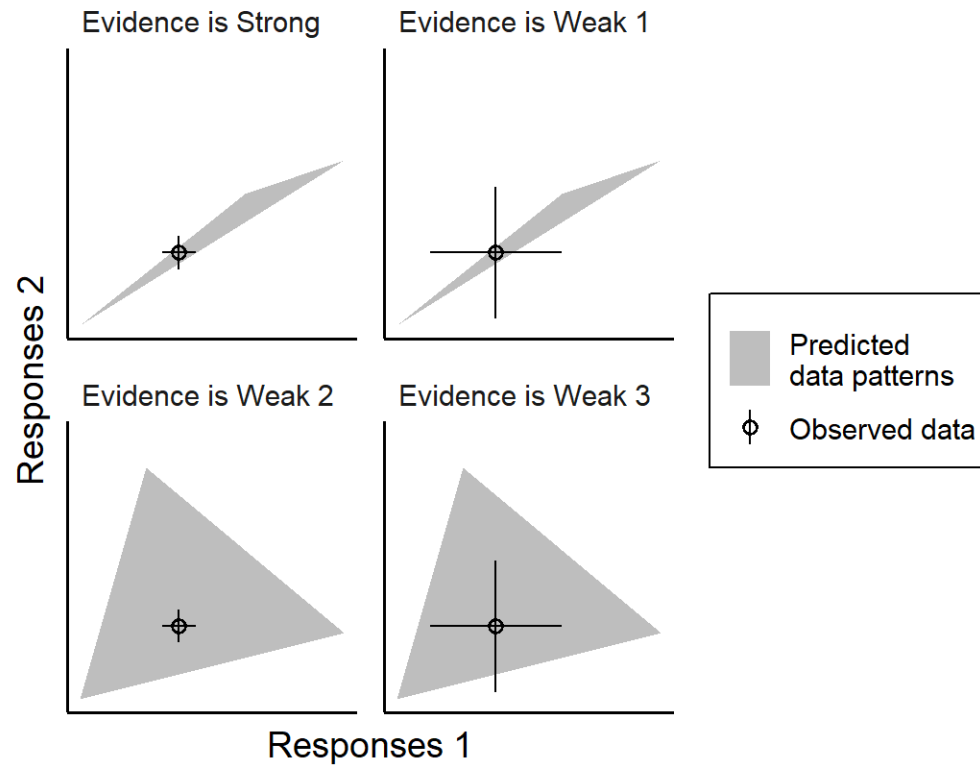
We want falsifiable, precise models

Data / Model	Model very flexible	Model precise
Data very variable	any result goes	maybe unrealistic
Data stable	not informative	<b>falsifiable</b>

(Roberts & Pashler, 2000)

The nice thing is that modeling actually allows us to quantify model flexibility (theoretical cognitive psychology).

# Response Surface - Potential Evidence



(Farrell & Lewandowsky, 2018; Roberts & Pashler, 2002)

**Fit, Generalizability**

# Fit versus Generalizability

## Model Fit

- How good a model handles old things
- AKA: *goodness of fit*, *GOF*
- Precision with which a model fits a particular sample of observed data (Myung & Pitt, [2005](#))
- Quantifiable by e.g., mean distance of predictions vs. observations; or probability of data sample given model predictions

## Model Generalizability

- How good a model handles new things
- AKA: *predictive GOF*, *predictive fit*
- model's ability to fit all data samples generated by the same cognitive process, not just the currently observed sample (i.e. the model's expected GOF with respect to new data samples) (Myung & Pitt, [2005](#))
- Quantifiable by e.g. GOF of model predictions for new experimental data; or GOF for old data but leaving out some data

## d. A Modeling Fallacy: Reverse Inference



# Reverse Inference

If a model's predictions ...

- **match** a data sample well
- **fail to match** a data sample well

... can we infer support for the process implemented within the model?

Computational Cognitive Models, Cognitive Process Models, Cognitive  
Measurement Models

# Reverse Inference

## Cognitive Process Models

- Intermediate Stages/Computations have psychological interpretations and predictive power
- Estimated parameters are either fixed (e.g. population distribution for salience) a function of inputs (e.g., attention depends on salience)
- E.g., can be fixed and tested

## Cognitive Measurement Models

- Estimated parameters have psychological interpretations
- Inter-individual differences
- E.g., prospect theory model's parameters can be interpreted as risk aversion/risk seekingness measures

e. Not too Implausible?

# Worthwhile to check if the model's processes don't assume things that you (the scientific community) know to be false

- E.g., Cognitive constraints
- E.g., Limits on human cognitive capacities
- E.g., Machine-learning neuronal networks often use infinite learning and perfect retrieval

Van Rooij, I. (2008). The tractable cognition thesis. *Cognitive Science*, 32(6), 939–984. <https://doi.org/10.1080/03640210801897856>

# Limits on Human Cognitive Capacities

## Computational Complexity

- how much processing power the model assumes
- demands on computational resources as a function of the size of the input (Van Rooij, [2008](#); introduction to the topic: Papadimitriou & Steiglitz; 1988)
  - time: *how long does it take to compute the input/output mapping*
  - space: *how much computational memory does it take to compute the input/output mapping*
- Computational measures
- Data-based measures - talk to senior researchers

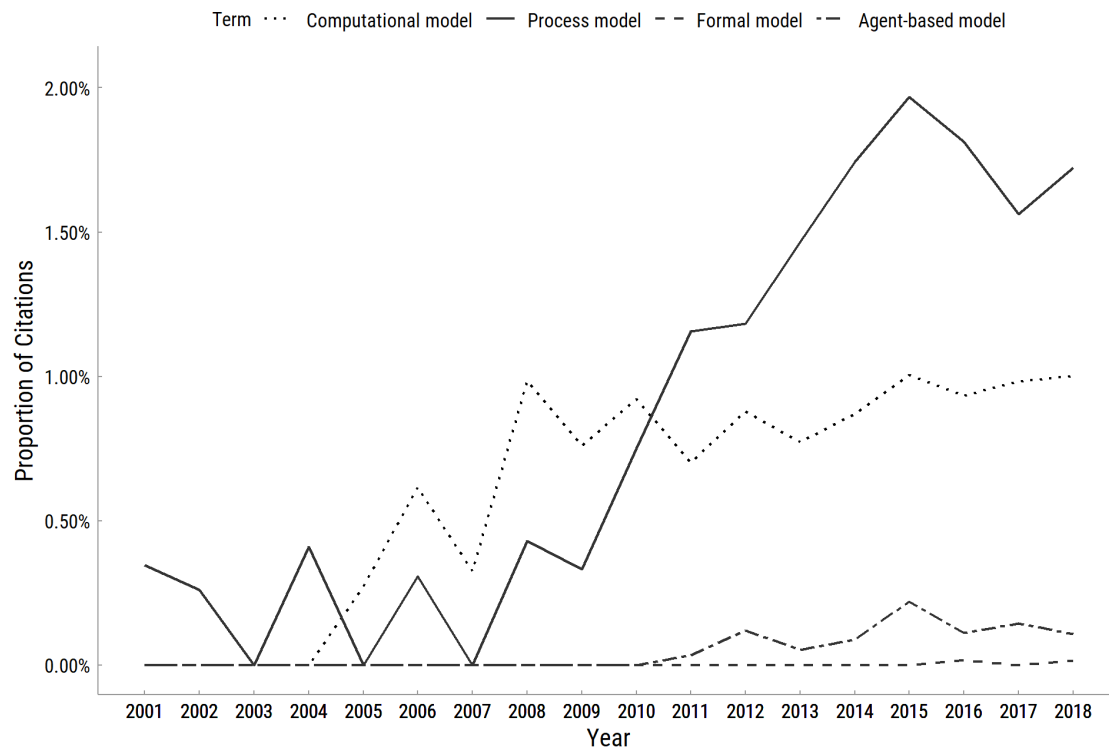
# Important Other Criteria

- Inclusiveness
- Theory interoperability
- Referencing other's work

# The Toothbrush Problem

## Relative Citation Frequency

Citations of models in CogSci and JDM relative to citations in CogSci and JDM



# The Toothbrush Problem

## Proliferation of Models

- In 2016: **100+ cognitive models** only in the field of judgment and decision making
- There is not a lot of consensus as to which models are process models
- *Although almost all (51 of 62) agreed that process models are important, they disagreed about which models constituted process models with an inter-rater agreement of Fleiss-Cuzick's  $\kappa = .27$ , far below the .60 (Jarecki, Tan, Jenny, 2020)*

## Toothbrush problem

Models are like toothbrushes: Everybody has their own toothbrush and I won't use yours



# Referencing Other's Work as Criterion

For instance:

- GCM Model that Steve introduced was developed in the 1980ies
- Some theorists in other fields pick the mechanism up in 99% similar in the mid-1990ies without referencing the work

*I am error-tolerant*, a real overlooking can happen, but if ever anybody alerts me (you) to that the ideas in my model existed prior, *cite and embrace* rather than trying to justify that my model is actually new