Simple Measurement Models for Complex Working Memory Tasks

KO,
Summer School 2022

Measurement Models vs. Explanatory Models

- Explanatory Models:
  – Goal: explain experimental effects
  – Fit all conditions with common parameters
  – Examples: GCM, SIMPLE, TBRS*
- Measurement Models:
  – Goal: measure interpretable latent variables
  – Fit each condition separately
  – Examples: SDT, diffusion model

Goal

- Measurement model for WM tasks:
  - Estimate theoretically interpretable parameters
  - Correlate them with other variables
  - Study experimental effects on them
- Hierarchical Bayesian Framework
  - Simple, closed-form likelihood

The Experiment: Complex Span

- Encode 5 red words for serial recall
- Distractors (black words) to be processed
  - Decide for all words: Larger / small than a soccer ball?

RING HORSE FAN PHONE FLY FOOT ...

t

2 Conditions: Free Time after Distractors (0.2 vs. 1.5 s)

Recall:
Select from Candidate Set

?

PIN	FAN	FLY	COW	TYRE
RING	BENCH	FOOT	LEMON	HOUSE
DOG	PHONE	CAR	HORSE	RIVER

Structure of Candidate Set

PIN	FAN	FLY	COW	TYRE
RING	BENCH	FOOT	LEMON	HOUSE
DOG	PHONE	CAR	HORSE	RIVER

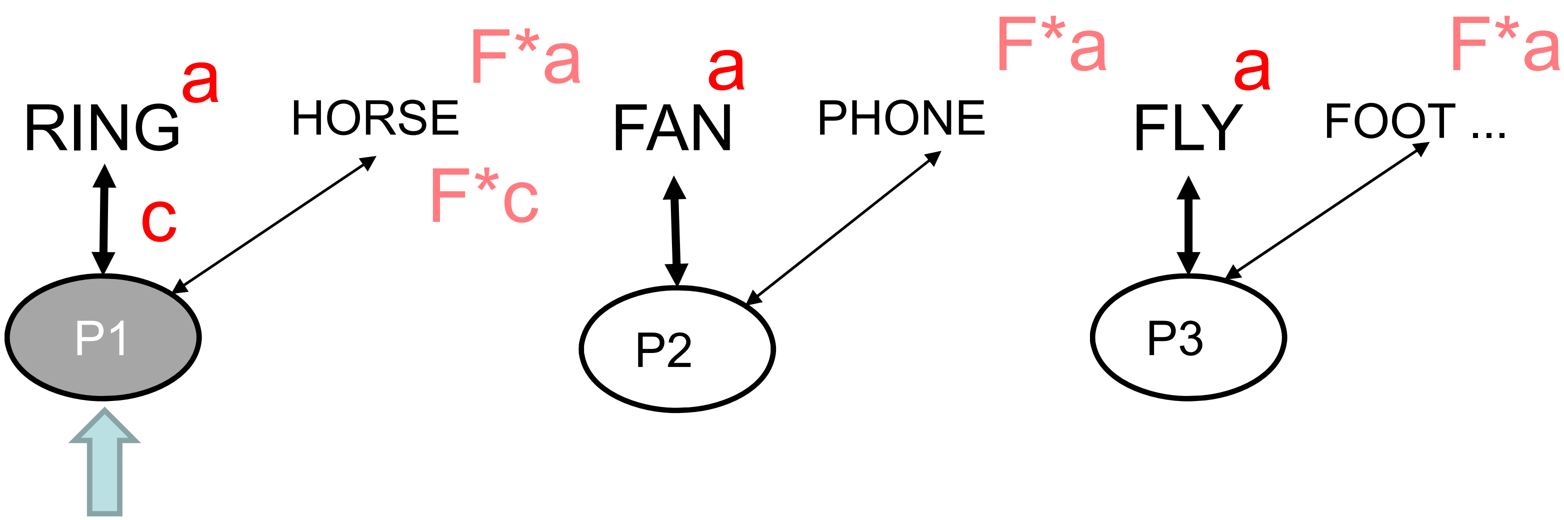
Multinomial Data Structure

Frequencies of 5 response categories

- correct item
- other list item
- distractor in probed position
- distractor in other position
- not-presented lure

Model Assumptions

- Recall = selection from a candidate set
- P(selection of *i*) = *f*(activation of *i*)
- Two sources of activation
  - persistent activation: a
  - re-activation through cue-based retrieval: c



Basic Model Equations

- A(correct) = b + a + c
- A(other item) = b + a
- A(distractor in position) = b + F*(a + c)
- A(other distractor) = b + F*a
- A(NPL) = b

c = Cueing: Strength of item-position binding

a = Activation: Strength of individual stimuli

b = Baseline (scaling parameter fixed to 0.1)

F = Filtering of distractors

From Activation to Selection Probability

- Luce‘s choice rule: $$p(i) = \frac{A(i)}{\sum_{j=1}^{n} A(j)}$$

Note of caution:
Activation is distributed over all 15 response candidates
→ A(i) for each category = A(i) for candidates * number of candidates in the category

$$p(i) = \frac{A(i)n(i)}{\sum_{j=1}^{ncat} A(j)n(j)}$$

Multinomial Likelihood

$$Binomial: P(k \mid p, n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{(n-k)}$$

$$Multinomial: P(\mathbf{k} \mid \mathbf{p}, n) = \frac{n!}{k_1! k_2! k_3! \ldots k_j!} p_1^{k_1} p_2^{k_2} p_3^{k_3} \ldots p_j^{k_j}$$

$$\mathbf{k} = \left[k_1, k_2, k_3, \ldots k_j\right]$$

$$\mathbf{p} = \left[p_1, p_2, p_3, \ldots p_j\right]$$

k_j = frequency of responses in category j
p_j = probability of a response in category j
n = number of trials

Generative Model Equations

$$\forall_j \in (1,\ldots J), \forall_c \in (1,2):$$

$$\mathbf{k}_{j,c} \sim Multinomial(\mathbf{p}_{j,c}, N_{j,c})$$

$$\forall_k \in (1,\ldots K): p_{j,c}(k) = \frac{A_{j,c}(k)n(k)}{\sum_{i=1}^{K} A_{j,c}(k)n(k)}$$

$$A_{j,c}(1) = 0.1 + a_{j,c} + c_{j,c}$$

$$A_{j,c}(2) = 0.1 + a_{j,c}$$

$$A_{j,c}(3) = 0.1 + F_{j,c}(a_{j,c} + c_{j,c})$$

$$A_{j,c}(4) = 0.1 + F_{j,c}a_{j,c}$$

$$A_{j,c}(5) = 0.1$$

Generative Model, continued

Individual-level parameters
~ group distribution

$$a_{j,c} \sim N(\mu_a, \sigma_a)$$

$$c_{j,c} \sim N(\mu_c, \sigma_c)$$

$$F_{j,c} \sim Beta(a_f, b_f)$$

(Hyper-) Priors for group-level
parameters

$$\mu_a \sim \Gamma(0.25, 0.05)$$

$$\mu_c \sim \Gamma(4, 0.2)$$

$$\sigma_a \sim \Gamma(1, 0.01)$$

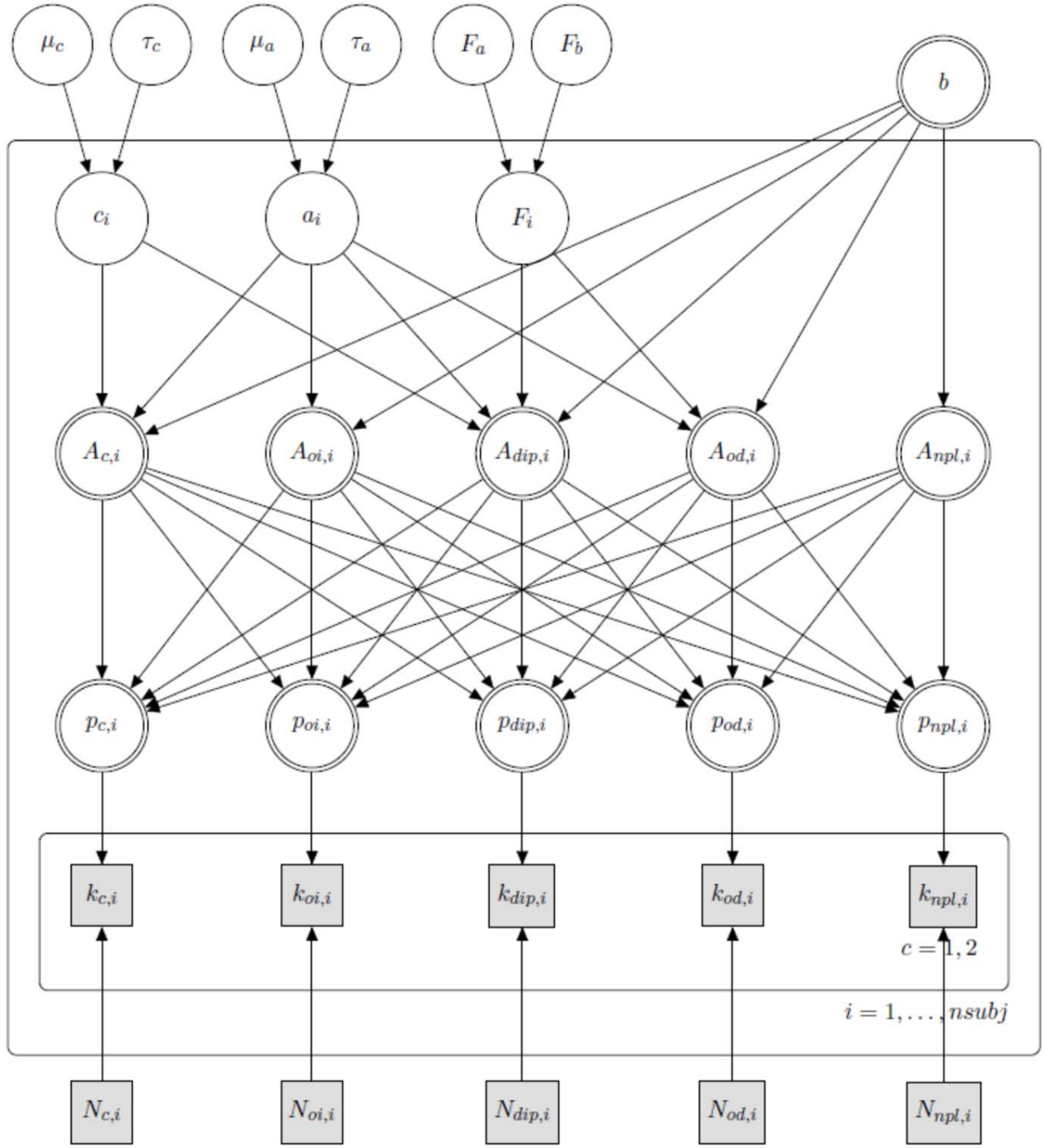
$$\sigma_c \sim \Gamma(1, 0.01)$$

$$a_f \sim \Gamma(1, 0.1)$$

$$b_f \sim \Gamma(1, 0.1)$$

Gamma(Shape, Rate)

Graphical Model



Exercise

- CspanMMM.R
- Build the JAGS model: CspanR.txt
- Some help:
  - Multinomial in JAGS: *dmulti*(**p**, N)
  - Truncated Normal: *dnorm*(1,0.1) T(0,)